

THE THEORY OF
SPECULATION UNDER ALTERNATIVE
REGIMES OF MARKETS

by

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Speculation is ordinarily understood to mean the purchase of a good for later re-sale rather than for use, or the temporary sale of a good with the intention of later re-purchase -- in the hope of profiting from an intervening price change.

The best-known theory of speculation, associated most prominently with J. M. Keynes and J. R. Hicks,^{1/} may be called the risk-transfer hypothesis. On this view, speculators are relatively risk-tolerant individuals who are rewarded for accepting price risks from more risk-averse "hedgers." A risk-averse trader who is or anticipates being long on the commodity (e.g., a farmer with a crop of wheat approaching harvest) may hedge by selling in a forward or "futures" market for future delivery at a known price. An individual who is or anticipates being short the commodity (e.g., a miller of wheat) may hedge by buying now for future delivery at a known price. Speculators in the forward or futures market may be on the long or the short side of any single such transaction, but in aggregate their commitments must offset any net imbalance of the long and short hedgers' positions. In the Keynes-Hicks view what the hedgers are doing is divesting themselves of price risks; it is these price risks that are, correspondingly, being accepted by speculators.

An alternative explanation of speculation, due to Holbrook Working, denies any such fundamental difference between the motivations of what are conventionally called speculators and hedgers. According to this alternative knowledgeable-forecasting hypothesis, what may look like risk-transfer behavior is only the interaction of traders with more and less optimistic beliefs about approaching developments that will affect prices. An individual

who expects prices to rise will make speculative purchases; one who expects them to fall will sell. On this view, futures markets do not serve mainly to facilitate the transfer of risk. Rather, they provide an instrumentality whereby a consensus of beliefs about future supply-demand influences is brought to bear (by the establishment of a current price for later deliveries) upon current production-consumption decisions.^{2/}

The first question to be addressed here is the logic of these hypotheses as to the fundamental nature of speculation. A second question is the closely related issue of the equilibrium pattern of price movements over time. Is the normal relation between current prices of futures contracts and the later spot prices such as to reward speculators for bearing price risks? Or is the current futures price merely the mathematical expectation of the spot price ultimately realized,^{3/} so that there is no reward (on average) for bearing price risk?^{4/} And, apart from the possible role of risk-transfer, how does the pattern of price movements reflect the relation between current consensus opinion and later revealed actuality emphasized by the knowledgeable-forecasting theory?

A predecessor paper to this one^{5/} developed a general-equilibrium model of the speculative process that led to the following key conclusions:

- (A) Contra the Keynes-Hicks theory, differences in risk-tolerances alone do not motivate speculative trading. If all individuals shared the same beliefs there would be no speculation -- even if traders had different degrees of risk-aversion. However, given differences of belief so that speculation occurs, the extent of an individual's speculative commitments will vary with his risk-tolerance.

(B) If "concordant beliefs" exist (in a sense to be explained below),
apart from time-discount the current price will be the mathematical
expectation of the stochastically varying spot price to be realized.

I.e., in terms of these probabilities prices will be a "martingale."

These propositions are too strong in that, like almost all the useful propositions of economic theory, they derive from a model that is a conscious oversimplification of reality. The fundamental nature of the conceptualization of speculation employed here, and of the "idealizing assumptions" leading to the above propositions, will be set forth in the Section following. The main thrust of the present paper will be to examine the robustness of Propositions A and B above with respect to modifications of the regime of markets postulated for trading under uncertainty.

I. FUNDAMENTALS OF SPECULATION, AND IDEALIZING ASSUMPTIONS

Several not-yet-generally comprehended fundamentals of speculation, that underly the analysis of this paper, will be asserted here with very brief commentary.^{6/}

1. Speculation occurs only in "informative situations"

Earlier theorists have not, generally speaking, appreciated the crucial role of the conditions of information emergence for speculative behavior. Speculation is premised upon anticipations of price changes. In a world of uncertainty, for each set of individuals' beliefs (over contingencies affecting the to-be-realized later "spot" prices) there is a corresponding equilibrium for current "futures" prices; these current prices can change only if beliefs change.^{7/} In short, speculation occurs in "informative situations" -- where some or all traders anticipate that additional public information^{8/} as to factors influencing supply or demand, and thereby spot prices, will emerge

before the close of trading. In such a situation some traders may transact away from their endowment positions (portfolios), not to their final productive-consumptive choices but instead to trading positions. Their intention is thus to revise portfolios, to attain their final positions on more advantageous terms after the anticipated price shift has occurred. These traders are the speculators -- the term, as used here, taken as inclusive of those ordinarily called hedgers. (It will be seen in Part V below that portfolio revision is not an essential or infallible sign of speculative activity. For the present, however, we can use this as a working definition.)

2. In informative situations individuals must adjust both to "price risk" and "quantity risk."

Given an informative situation, the prospect of stochastic price change generates the "price risk" that the traditional speculation literature emphasizes. But price is an endogenous variable: price uncertainty is necessarily the resultant of an underlying uncertainty about the exogenous elements determining demand and supply influences upon price. In the simplest case price uncertainty reflects merely a corresponding stochastic variability of the aggregate of individuals' commodity endowments. For example, the wheat price will be high if the crop (and, therefore, the average wheat endowment per individual) turns out to be small; price will be low if the crop (average wheat endowment) turns out big.^{2/} It is therefore not price risk alone that governs individuals' speculative/hedging behavior, but the interaction of price risk with quantity risk. The traditional speculation literature has overlooked the fundamental influence of quantity risk upon traders' decisions.

3. In an informative situation there are two inter-related market equilibria.

The prospect of information emergence implies that there will be two distinct, though of course inter-related, market equilibria. The first is associated with the "prior-round" trading that occurs before the anticipated public information emerges, the second with the "posterior-round" trading that takes place afterward. In the prior round, traders' commitments (transactional movements from endowment positions to trading positions) take place in the face of uncertainty as to quantity endowments and posterior prices -- though, of course, the prior-round price vector is itself deterministic. In posterior trading the underlying quantity uncertainty has been at least partially resolved. If, as will mainly be assumed below, the emergent information is conclusive as to the state of the world, the uncertainty will be fully resolved before posterior trading begins. Then the prior-round/posterior-round dichotomy corresponds to the distinction between futures and later spot markets. Section IV contains comments on the consequences of successive partial (less than conclusive) information injections -- leading to a multiple sequence of prior and posterior trading rounds before uncertainty is ultimately resolved.

4. Speculative behavior is conditioned upon the scope of markets.

The traditional speculation literature allows trading only in simple certainty claims to commodities. This will be called here a regime of "Unconditional Markets" (UM).^{10/} In contrast, the predecessor paper cited above studied a regime called "Semi-Complete Markets" (SCM).^{11/} Consider a world of two goods N and Z , and two states of the world \underline{a} and \underline{b} , where N is riskless (all individuals' N -endowments are uniform over states) but Z is risky (individuals' $Z_{\underline{a}}$ and $Z_{\underline{b}}$ endowments generally differ). Then under

Semi-Complete Markets (SCM) the marketable claims are the trio N , Z_a , and Z_b . (The riskless commodity N would naturally serve as numeraire.) Under Unconditional Markets UM only certainty claims N and ζ (the latter being a 1:1 package of Z_a and Z_b entitlements) can be traded. Another regime of interest is "Fully Complete Markets" (FCM), with trading permitted in all four definable claims N_a , N_b , Z_a , and Z_b . While Fully Complete Markets would seem a more natural polar case than SCM to oppose to the regime of Unconditional Markets, there can in fact be no speculation under FCM! In the prior round each trader would be able to buy a portfolio covering his desired consumption baskets in the light of the alternative possible information-events as well as over the different state-contingencies. Then, even if a posterior market were available, no-one would need to use it. With only one round of trading, there can be no speculation.^{12/}

Other interesting structures of markets can be defined. One, a kind of inverse of Semi-Complete Markets, would be a situation where conditional claims to the riskless commodity (N_a and N_b) could be traded but only unconditional claims to the risky commodity Z . This might be called a regime of "Numeraire Contingency Markets" (NCM). As it is intuitively evident that NCM trading can and will always match the results of SCM trading, NCM will not be considered further here.^{13/} Still another structure can be called "Equity Markets" (EM). Under EM, against unconditional claims to numeraire N could be traded unit shares ω defined as entitlements to a proportionate interest in the aggregate social endowment of Z , whatever that might turn out to be. It would be as if all the production of the risky good Z were carried out by a single firm, in the presence of a securities market where stock in that firm could be purchased and sold against the numeraire commodity N .^{14/}

The EM regime, like the UM regime, has only two classes of tradable claims -- but EM, unlike UM, does permit some trading in risky contingencies.

To complete the preliminaries, several of the "idealizing assumptions" will be listed here: (i) The standard theoretical postulates (costless trading, price-taking behavior, instantaneous market-clearing) necessary to assure that all transactions take place at equilibrium prices. (ii) Just two goods, N riskless and Z risky -- and two states of the world a and b reflected in the size of Z-endowments. (This assumptions is reconsidered in Section IV.) (iii) All individuals have state-independent, additive utility functions in N and Z, i.e., there is zero complementarity in preference.^{15/} (iv) There is no "real time" intervening between prior and posterior trading. This assumption isolates the effect of informational emergence from time-involved processes like growth, depreciation, storage, utility time-discount, etc. Mere passage of time can create patterns of price movement that have nothing to do with speculation, so long as uncertainty is not involved. (v) The most radical of the idealizing assumptions to be employed here is that prices will reflect "concordant" beliefs in the market. This means that essentially all the market weight in price determination will be contributed by traders who share identical prior probability beliefs about which state of the world will obtain. This does not mean that belief-deviant traders are rare or unimportant, but only that they cancel one another out so far as effects on prices are concerned. In practice one would want to interpret "concordant" beliefs as average beliefs, though this translation is not strictly warranted.^{16/}

Because of the oversimplifications involved in these and certain other assumptions (in particular, one called "correct conditional forecasting" to be discussed below), the statements summarized in Propositions A and B are

obviously too sweeping. Even if the model were valid in the sense of providing a usable representation of speculation phenomena, the propositions should be interpreted in a rather more modest sense such as: (A') Speculators are primarily individuals whose probability beliefs (rather than whose risk-tolerances) deviate from those more typical of individuals in the market. (B') The price-revision relation between prior and posterior rounds of trading (between futures and later spot prices) will approximate a martingale when calculated in terms of "concordant" probability beliefs (in practice, in terms of a suitable average of traders' beliefs).

II. THE NON-INFORMATIVE SITUATION IN ALTERNATIVE MARKET REGIMES

As a base point let us consider first the solutions obtained under alternative market regimes for a non-informative situation. Here since traders do not anticipate the emergence of information leading to any price change before the close of trading, there is only one trading round. So no portfolio revision (and hence no speculation) takes place. Each individual moves at once to his optimum consumptive position (which is, in general, a gamble over possible states of the world).

A trader's endowment, also in general a gamble, can be expressed in "prospect notation" as:

$$(1) \quad E \equiv [(n_a^e, z_a^e), (n_b^e, z_b^e); p, 1-p]$$

That is, with subjective probability belief p the individual anticipates the particular endowment vector n_a^e, z_a^e associated with the advent of state-a; he assigns, of course, the complementary probability $1-p$ to the endowment vector n_b^e, z_b^e associated with state-b. But since commodity N is riskless, the N -endowment is invariant over states -- i.e., $n_a^e = n_b^e = n^e$. Using this feature, and suppressing the probability parameter p , it will sometimes be convenient

to employ the more compact notation:

$$(1') \quad E \equiv (n^e; z_a^e, z_b^e)$$

The individual's problem is to select among "simple consumptive gambles" C , expressed (in the two alternative notations) as:

$$(2) \quad C \equiv [(n, z_a), (n, z_b); p, 1-p] \equiv (n; z_a, z_b)$$

The individual maximizes his expected state-independent utility:

$$(3) \quad U(C) \equiv U(n; z_a, z_b) \equiv pu(n, z_a) + (1-p)u(n, z_b)$$

The difference between market regimes is manifested to the individual in the form of the trading opportunities available. Under Semi-Complete Markets SCM his budget constraint is the wealth-value of the endowment combination:

$$(4) \quad n + P_{Za} z_a + P_{Zb} z_b = n^e + P_{Za} z_a^e + P_{Zb} z_b^e \equiv W^e \quad \begin{array}{l} \text{Budget equation} \\ \text{(SCM)} \end{array}$$

That is, he may buy or sell any desired numbers of units of the contingent claims Z_a or Z_b , each having its own price in terms of the numeraire commodity N . Maximizing expected utility subject to this constraint leads to:

$$(5) \quad p \frac{\partial u / \partial z_a}{\partial u / \partial n} = P_{Za} \quad \text{and} \quad (1-p) \frac{\partial u / \partial z_b}{\partial u / \partial n} = P_{Zb} \quad \begin{array}{l} \text{Optimality} \\ \text{conditions} \\ \text{(SCM)} \end{array}$$

Given the additivity assumption, the denominators in the two equations are equal.

Finally, the market-clearing conditions serve to determine the equilibrium prices P_{Za} and P_{Zb} :

$$(6) \quad \Sigma n = \Sigma n^e, \quad \Sigma z_a = \Sigma z_a^e, \quad \Sigma z_b = \Sigma z_b^e \quad \begin{array}{l} \text{Equilibrium} \\ \text{conditions} \\ \text{(SCM)} \end{array}$$

Of course, one of these conditions is implied by the other two.

Under the more restricted regime of Unconditional Markets UM, however, the trading constraint is quite different. Already, under SCM, trading in conditional claims to the riskless commodity N was ruled out. Under UM there

is no trading in conditional claims even to the risky commodity Z; only unconditional rights to either N or Z may be exchanged. A unit unconditional right to Z, symbolized as ζ , can be regarded as a sandwich consisting of unit conditional Z-claims to both possible states. P_ζ , the unit price of the sandwich, can evidently be interpreted as the price of Z in this market regime ($P_\zeta \equiv P_Z$). In general, however, we would not expect P_ζ under UM to equal the sum $P_{Za} + P_{Zb}$ of the conditional prices under SCM. It follows that, in general, individuals would not attain the same optimal consumptive gambles C^* as in SCM. In particular, under UM individuals with inconvenient endowment compositions will generally find it impossible to move to their SCM-preferred gambles when Z_a, Z_b claims can be bought and sold only in a 1:1 ratio.

This difficulty expresses itself in the form of the budget constraint. Endowed wealth, interpreted as the market value of endowment, is no longer the effective bound on attainable combinations under UM. The restrictions on trading can make endowment compositions partially (or even wholly) unmarketable. The individual must therefore account separately for all the commodities:

$$(7) \quad \begin{cases} n + P_\zeta \zeta = n^e \\ z_a - \zeta = z_a^e \\ z_b - \zeta = z_b^e \end{cases} \quad \begin{array}{l} \text{Budget} \\ \text{constraints} \\ \text{(UM)} \end{array}$$

And in consequence, the optimality conditions reduce to the single equation:

$$(8) \quad p \frac{\partial u / \partial z_a}{\partial u / \partial n} + (1-p) \frac{\partial u / \partial z_b}{\partial u / \partial n} = P_\zeta \quad \begin{array}{l} \text{Optimality} \\ \text{condition} \\ \text{(UM)} \end{array}$$

The market-clearing conditions (either implying the other) can here be expressed as:

$$(9) \quad \sum n = \sum n^e \quad \text{and} \quad \sum \zeta = 0 \quad \begin{array}{l} \text{Equilibrium} \\ \text{condition (UM)} \end{array}$$

NUMERICAL EXAMPLE 1

Assume that the economy consists of two equally numerous classes of individuals, differing only in endowment composition. Specifically, suppose as shown in Part 1 of Table 1 that we can think of a typical pair as a microcosm -- consisting of a "representative supplier" of the risky commodity Z with endowment $E = (n^e; z_a^e, z_b^e) = (0; 400, 160)$ and a "representative demander" of Z with endowment $E = (200; 0, 0)$. Suppose in addition that everyone has the identical utility function $u(n, z) = \log_e nz$, and identical (and therefore concordant) beliefs assigning the probability parameter $p = \pi = .6$ to state-a and $1-p = 1-\pi = .4$ to state-b. Then, under Semi-Complete Markets SCM, the equilibrium contingent-claim prices (in terms of N as numeraire) can be shown to be $P_{Za} = .3$ and $P_{Zb} = .5$. At these solution prices everyone has equal endowed wealth: $W^e = 200$. Consequently, all end up with the same optimal consumptive gamble $C^* = (n^*; z_a^*, z_b^*) = (100; 200, 80)$. These results, and the trading implied, are shown in the upper panel of Part 1 of the Table.

In the regime of Unconditional Markets UM, shown in the lower panel of Part 1, the result is quite different. The market-clearing price for the package ζ (i.e., for an unconditional claim to Z) works out as $P_\zeta = .9439$. This price is higher than the sum of the contingent-claim prices under SCM obtained above ($P_\zeta = .9439 > P_{Za} + P_{Zb} = .3 + .5 = .8$), as may be explained as follows. The supplier of Z wishes to sell off some Z-claims for the N-claims he lacks. But his endowed state-contingent holdings of Z ($z_a^e = 400$, $z_b^e = 160$) are highly unbalanced, which is an undesirable feature from the point of view of risk-aversion. He would have preferred to sell more of Z_a than of Z_b claims, as was possible under SCM. Selling units of ζ (certainty claims to Z) under UM does provide him with the N he desires, but only by

Table 1

ENDOWMENTS, TRANSACTIONS, AND SIMPLE CONSUMPTIVE GAMBLER
IN NON-INFORMATIVE SITUATION UNDER TWO MARKET REGIMES

	Endowment (E)	Trading	Consumptive optimum (C*)
	$n^e; z_a^e$	$\Delta n; \Delta z_a, \Delta z_b$	$n^*; z_a^* z_b^*$
Part 1: <u>Representative Supplier-Demander Pair With Concordant Beliefs ($\pi=.6$)</u>			
SEMI-COMPLETE MARKETS (SCM) [†]			
Supplier	0;400,160	+100;-200, -80	100; 200, 80
Demander	<u>200; 0, 0</u>	<u>-100;+200, +80</u>	<u>100; 200, 80</u>
TOTALS	200;400,160		200; 400, 160
UNCONDITIONAL MARKETS (UM) ^{††}			
Supplier	0;400,160	+100;-105.9,-105.9	100;294.1, 54.1
Demander	<u>200; 0, 0</u>	<u>-100;+105.9,+105.9</u>	<u>100;105.9, 105.9</u>
TOTALS	200;400,160		200; 400, 160
Part 2: <u>Traders with Representative Endowments and Deviant Beliefs</u>			
SEMI-COMPLETE MARKETS (SCM) [†]			
Optimist (p=.7)	100;200, 80	0; +33.3, -20	100;233.3, 60
Pessimist (p=.5)	100;200, 80	0; -33.3, +20	100;166.7, 100
UNCONDITIONAL MARKETS (UM) ^{††}			
Optimist (p=.7)	100;200, 80	+13.3; -14.1, -14.1	113.3;185.9, 65.9
Pessimist (p=.5)	100;200, 80	+3.6; -3.8, -3.8	103.6;196.2, 76.2
† SCM equilibrium prices: $P_{Za} = .3, P_{Zb} = .5$			
†† UM equilibrium price: $P_z = .9439$.			

further increasing the relative disproportion of his retained Z-holdings. Since the supplier is therefore somewhat reluctant to sell Z-claims in the 1:1 ratio dictated by UM, whereas the demander has no such reluctance in buying (since his endowment situation is already balanced at $z_a^e = z_b^e = 0$), the consequence is a relatively high price P_ζ .

At the equilibrium $P_\zeta \doteq .9439$ the supplier sells about 105.9 ζ -units (whereas, under SCM he would have sold 200 units of Z_a at $P_{Za} = .3$ and 80 units of Z_b at $P_{Zb} = .5$). The demander in either case pays just 100 units of N. The two classes of traders do not reach the same consumptive gambles under UM. The supplier of Z attains $C^* = (100; 294.1, 54.1)$, while for the demander of Z the consumptive optimum is $(100; 105.9, 105.9)$. In utility terms the Z-supplier ends up somewhat better off, the Z-demander somewhat worse off. It can be shown that the loss of the latter is greater than the gain to the former -- if the alternative of Semi-Complete Markets were available, those doing better under SCM would be able to compensate those who stand to gain under Unconditional Markets.

(END OF NUMERICAL EXAMPLE)

The Numerical Example above indicated a loss in efficiency under the regime of Unconditional Markets. This loss, due to the restriction imposed upon the consumption baskets attainable by individuals, is quite a general consequence of impaired trading opportunities. As compared with UM, a costless shift to a regime of Semi-Complete Markets SCM will (with appropriate compensation) generally be Pareto-preferred. Of course, providing the additional markets will in general be costly. Before coming to absolute conclusions as to efficiency the added expense would have to be weighed against the inefficiency of impaired trading.

It is of interest to compare UM with the Equity Markets EM regime. Under EM, as in UM, the Z_a, Z_b claims are tradable only jointly. But the ratio of the elements of the $Z_a:Z_b$ of the sandwich is not 1:1 but is, instead, equal to the ratio of the social totals $\bar{Z}_a:\bar{Z}_b$. Define the unit EM claim ω as the right to receive Z_a and Z_b in the respective amounts $\bar{Z}_a/100$ and $\bar{Z}_b/100$. Thus, an ω -entitlement is a claim to 1% of society's Z-output in either state.

Then, by analogy with the preceding:

$$(10) \quad \begin{cases} n + P_\omega \omega = n^e \\ z_a - \bar{Z}_a \omega = z_a^e \\ z_b - \bar{Z}_b \omega = z_b^e \end{cases} \quad \begin{array}{l} \text{Budget} \\ \text{Constraints} \\ \text{(EM)} \end{array}$$

$$(11) \quad p \bar{Z}_a \omega \frac{\partial u / \partial z_a}{\partial u / \partial n} + (1-p) \bar{Z}_b \omega \frac{\partial u / \partial z_b}{\partial u / \partial n} \quad \begin{array}{l} \text{Optimality} \\ \text{condition} \\ \text{(EM)} \end{array}$$

$$(12) \quad \Sigma n = \Sigma n^e \quad \text{and} \quad \Sigma \omega = 0 \quad \begin{array}{l} \text{Equilibrium} \\ \text{condition} \\ \text{(EM)} \end{array}$$

Considering Table 1 once again, let us now ask what would happen under Equity Markets EM? The unit claim would be a sandwich in the amounts $Z_a, Z_b = 4, 1.6$. In the example here, the SCM optimum could be attained under EM -- there is no efficiency loss! A trade between the parties of 50 ω -units, at the price $P_\omega = 4P_{Za} + 1.6P_{Zb} = 2$, would do the trick. But this is clearly an artifact, a result of the special construction of the example -- in particular, of the conditions that the utility functions are homothetic and identical, together with the fact that all the individual Z-endowments are in the same proportions as are the social totals (and therefore as the elements of ω). More generally, the optimum achievable under SCM could no more be attained under EM than under UM, and for the same reason -- the fixity of the proportions of the trading unit or sandwich. It seems a plausible conjecture that EM would commonly

do better than UM, however, i.e., would get closer to the SCM solution. If the society can at least roughly be dichotomized into "suppliers" (who want to trade off both Z_a and Z_b), and "demanders" (who want to acquire some of both), then UM will tend to do better -- since the trading unit ω will at least roughly reflect the relative $Z_a:Z_b$ proportions that the one party wants to sell and the other wants to buy. It is possible, however, to construct cases -- in particular, where some individuals are specialized in state-a endowments and others in state-b -- where UM does better than EM. In any case, it can be seen that the Equity Markets regime is qualitatively similar to Unconditional Markets in its working. For this reason, and in the interests of space saving, henceforth the main analysis will be limited to the traditional UM regime in comparison with the SCM regime needed to attain efficiency.

Let us briefly consider the impact of divergences in endowment position, in probability beliefs, and in risk-aversion upon the non-informative individual solutions obtained in these two market regimes.

(i) As to endowment position, the budget equation (4) shows that under SCM only the individual's endowed wealth W^e -- and not the detailed composition thereof -- will affect his achieved consumptive gamble. Under UM, in contrast, the significance of the specific commodity-state composition of endowments is revealed by the necessity of accounting separately as in (7) for all the goods entering into the utility function.

(ii) As to probability beliefs, the form of (5) shows that under SCM a belief-deviant individual assigning relatively high belief p_s to any state-s will accept a proportionately lower $\partial u / \partial z_s$ for that state -- implying a correspondingly larger purchase of contingent Z_s -claims to that state. Inability to trade separately in Z_s -claims means that under UM traders can no longer achieve an exact inverse proportionality between p_s and $\partial u / \partial z_s$.

In Part 1 of Table 1, the individuals constituting the supplier-demander pair had divergent endowments but concordant beliefs. In Part 2 are shown individuals with representative endowments (that are a cross-section of the social commodity totals) but deviant beliefs. In particular, we have an "optimist" who assigns probability $p=.7$ to the advent of state-a, and a "pessimist" who assigns $p=.5$ -- in contrast with the $\pi=.6$ belief of the concordant individuals. The upper panel of Part 2 shows how the consumptive solutions for the optimist and pessimist diverge, in the expected directions, from the C^* optimum positions of the concordant individuals. The lower panel of Part 2 shows the behavior of these deviant individuals under the more constrained regime UM. Here the high market price of Z-claims, as derived from the behavior of the concordant individuals shown in Part 1 of the Table ($P_Z = .9439$), induces both the optimist and pessimist to sell a number of units of Z for more N-claims. Apart from this effect, it will be seen that the 1:1 linkage of the Z_a and Z_b claims impairs -- indeed, practically eliminates -- the ability of the individuals to move from their endowment positions $E = (100; 200, 80)$ to anything like their preferred consumptive gambles C^* achievable under SCM.

(iii) Finally, as to risk-aversion, a greater degree of risk-tolerance means a smaller change in $\partial u / \partial z$ for a given quantitative difference in the amount of Z held in the different states. Then (5) implies that, under SCM, a more risk-tolerant individual would be willing to hold relatively more of the more plentiful Z_a -claims and relatively less of the less plentiful Z_b -claims -- in short, the perfectly reasonable result that a more risk-tolerant trader would accept a wider risk than the typical individual. Under Unconditional Markets the result is somewhat different. The UM regime does not permit

a relatively risk-tolerant trader to widen (or a relatively risk-averse trader to narrow) his absolute Z-risk. Whatever the individual's endowed discrepancy between z_a^e and z_b^e may be, this discrepancy is preserved when Z-claims can only be traded on 1:1 basis. Equation (8) shows that the individual will equate a weighted average of his marginal utilities to the market price. Suppose a risk-averse individual tried to decrease his risk exposure by moving more heavily into N, thus holding less of the risky commodity Z. But in reducing the absolute scale of his Z-holdings, he is increasing their relative disproportion. And, in fact, it can be shown that under UM more risk-averse individuals will purchase relatively more of the risky commodity as a kind of insurance against the bad state-b contingency.^{17/}

III. THE INFORMATIVE SITUATION IN ALTERNATIVE MARKET REGIMES: EMERGENCE OF SPECULATION

In an informative situation, the anticipated emergence of new information affecting prices divides trading into a prior round and a posterior round. Then some or all individuals may be induced to speculate, i.e., to adopt trading positions -- portfolio holdings in the prior round that do not correspond to consumptive desires but rather to hopes for potential profit consequent upon anticipated price revisions. Of course, speculators will ultimately (in the posterior round) make trades permitting them to end up with desired consumptive gambles at the enhanced or diminished levels of wealth stemming from their degree of speculative success.

Continue to assume, for simplicity, that the emergent information will be conclusive as to which state is going to obtain. Then, under Semi-Complete Markets (SCM), a trader's optimizing problem can be formulated as:

$$(13a) \quad \text{Max } U = pu(n', z'_a) + (1-p)u(n'', z''_b) \text{ subject to}$$

$$(13b) \quad \begin{cases} n' + P'_{Za} z'_a = n^t + P'_{Za} z^t_a \equiv W' \\ n'' + P''_{Zb} z''_b = n^t + P''_{Zb} z^t_b \equiv W'' \\ n^t + P^o_{Za} z^t_a + P^o_{Zb} z^t_b = n^e + P^o_{Za} z^e_a + P^o_{Zb} z^e_b \equiv W^e \end{cases}$$

The optimal trading position $T^* = (n^t; z^t_a, z^t_b)$ is arrived at by the individual subject to the prior-round market prices denoted P^o_{Za} and P^o_{Zb} . The effective constraint in this round, represented by the third equation of (13b), is the level of endowed wealth W^e . As for the posterior round, one or the other of the constraints in the first two equations of (13b) will be effective. If state-a obtains the single-primed symbols of the first equation show the effective wealth W' (the posterior market value of the trading position), the choice variables n' and z'_a , and the posterior price P'_{Za} (the price of numeraire N remains unity, and Z_b -claims have become valueless). Similarly, the double-primed symbols of the second equation represent the variables conditional upon the advent of state-b, with only the price P''_{Zb} appearing.

Taking the two rounds of trading together, in an informative situation the individual can be regarded as selecting a "compound consumptive gamble" that may be denoted in prospect form as $D = [(n', z'_a), (n'', z''_b); p, 1-p]$. In the compound gamble permitted by the two rounds of trading, n' and n'' may differ -- something that could not be achieved in a single round of trading under Semi-Complete Markets.^{18/}

The posterior-round optimality conditions under SCM have the simple form:

$$(14) \quad \frac{\partial u / \partial z'_a}{\partial u / \partial n'} = P'_{Za} \quad \text{and} \quad \frac{\partial u / \partial z''_b}{\partial u / \partial n''} = P''_{Zb} \quad \begin{array}{l} \text{Posterior-round} \\ \text{optimality} \\ \text{conditions} \\ \text{(SCM)} \end{array}$$

Since there is posterior certainty, the probability belief parameter no longer plays any role.

For the prior-round optimality conditions, on the other hand, we must face the awkward problem that the choice of optimal trading position T^* depends not only upon the prior-round prices P_{Za}^0 and P_{Zb}^0 but also upon the posterior prices P_{Za}^1 and P_{Zb}^2 . But these latter have not yet been determined; indeed, one of them will never actually be realized, since only one state or the other is going to be observed! What information will traders have as to posterior prices to guide them in their prior-round decisions? In general, conditional posterior prices are not actually computable from public prior data.^{19/} But there is one set of anticipations -- called here "correct conditional forecasting" -- which, if universally held, will be self-fulfilling and thus consistent with equilibrium.^{20/} Specifically, suppose that traders anticipate that prices will move in proportion to changes in concordant beliefs:^{21/}

$$(15) \quad \frac{P_{Za}^1}{P_{Za}^0} = \frac{1}{\pi} \quad \text{and} \quad \frac{P_{Zb}^2}{P_{Zb}^0} = \frac{1}{1-\pi}$$

(The numerator of unity on the right-hand-side of each equation corresponds to posterior certainty, i.e., the information is to be conclusive.) Note that these anticipations concerning price revisions immediately imply the martingale property, calculated in terms of concordant beliefs. (The same price revisions would not be a martingale calculated in terms of any other set of beliefs.)

If (15) holds true the third constraint of (13b) can be reformulated as a relation among the endowed and conditional wealths:

$$(16) \quad \pi W' + (1-\pi)W'' = W^e$$

This leads to a simple condition for optimal prior trading:

$$(17) \quad \frac{p}{1-p} \frac{du'/dW'}{du''/dW''} = \frac{\pi}{1-\pi}$$

Prior-round
optimality
condition (SCM)

Thus, optimal prior trading involves determining the conditional posterior wealths W', W'' . With W' and W'' the trader can enter the first two equations

of (13b) to find the elements n^t, z_a^t, z_b^t of his optimal trading position T^* . But, as a point that will take on considerable significance below, note that the two wealths are insufficient to uniquely determine the three elements of T^* . In prior-round trading under SCM there is a degree of freedom; any one element of T^* can be arbitrarily selected.

It remains to be shown just what the equilibrium prior-round prices P_{Za}^O and P_{Zb}^O will be. Under the assumption of concordant beliefs the result is very neat:

$$(18) \quad P_{Za}^O = P_{Za} \quad \text{and} \quad P_{Zb}^O = P_{Zb}$$

That is, the prior-round prices in an informative situation are simply equal to the state-claim prices that would have ruled had the situation been non-informative!

This may be explained as follows. For individuals holding concordant beliefs, who account for essentially all the social weight in determining prices, equations (5) for the non-informative SCM optimum take the form

$$(5') \quad \pi \frac{\partial u / \partial z_a}{\partial u / \partial n} = P_{Za} \quad \text{and} \quad (1-\pi) \frac{\partial u / \partial z_b}{\partial u / \partial n} = P_{Zb}$$

But substituting from (15) into (14) yields an exactly analogous result for the prior-round informative SCM optimum:

$$(19) \quad \pi \frac{\partial u / \partial z_a'}{\partial u / \partial n'} = P_{Za}^O \quad \text{and} \quad (1-\pi) \frac{\partial u / \partial z_b''}{\partial u / \partial n''} = P_{Zb}^O$$

Then if $z_a = z_a'$, $z_b = z_b''$, and $n' = n'' = n$, equation (18) follows.

The prior and posterior markets will be in equilibrium if prices are such that individuals of concordant beliefs (who account collectively for essentially all the social weight determining prices) find it optimal to achieve the same optimal consumptive baskets D^* under an informative situation as the C^* baskets they would have chosen had the situation been non-informative.

And they will find it optimal to do so given "correct conditional forecasting." Put another way, prices will be a martingale if concordant-belief traders forecast that they will be!

Given the ruling prior-round prices P_{Za}^0 and P_{Zb}^0 , an individual of concordant beliefs has a range of options as to T^* , any of which could achieve the desired conditional wealth-pair W', W'' . In particular, he could stand pat with his endowment gamble in the prior round, using the posterior prices $P_{Za}' = P_{Za}^0/\pi$ or $P_{Zb}'' = P_{Zb}^0/(1-\pi)$ -- depending upon which state obtains -- to achieve $D^* = C^*$. Or, a once-and-for-all move from E would suffice to achieve optimality, with no need for portfolio revision.^{22/} On the other hand, individuals of deviant beliefs would all find it advantageous to trade in both rounds under SCM, i.e., they will speculate.

So much for the regime of Semi-Complete Markets. How different are results for the regime of Unconditional Markets, where only certainty claims may be traded?

Maintaining the same assumptions as before except for the change in market regime, the decision problem of an individual with belief parameter p can be expressed by equations analogous to (13a) and (13b):

$$\begin{aligned}
 (19a) \quad & \text{Max } U = pu(n', z'_a) + (1-p)u(n'', z''_b) && \text{subject to} \\
 (19b) \quad & \begin{cases} n' + P_{\zeta}' z'_a = n^t + P_{\zeta}' z_a^t \equiv W' \\ n'' + P_{\zeta}'' z''_b = n^t + P_{\zeta}'' z_b^t \equiv W'' \end{cases}
 \end{aligned}$$

Of course, P'_ζ is the price of the same contingent claim as P'_{Za} under SCM, and P''_ζ the same as P''_{Zb} .

We do not yet have the expression for the prior-round constraint, the analog of the third equation of (13b). It is more convenient to find an analog to the relation (16) among the wealths. In fact, using (15), (19b), and the budget accounting identities (10), we obtain:^{23/}

$$(19c) \quad \pi W' + (1-\pi)W'' = W^e + (P^o_{Za} + P^o_{Zb} - P^o_\zeta)\zeta$$

The symbol W^e here does not represent the actual market value of the endowment under Unconditional Markets UM. Rather, it is the hypothetical value that the same endowment would have under Semi-Complete Markets SCM. Similarly, P^o_{Zb} and P^o_{Za} are the corresponding hypothetical prior-round state-claim prices under SCM. The point of this development is that if the prior-round price of the unconditional Z-claim, P^o_ζ , satisfies $P^o_\zeta = P^o_{Za} + P^o_{Zb}$, then conditions (19) become literally identical with the conditions (13) governing choice of trading position T^* under SCM.

The upshot is that markets must clear if the unconditional prior-round price of the risky commodity Z is simply:

$$(20) \quad P^o_\zeta = P^o_{Za} + P^o_{Zb}$$

This price must be consistent with equilibrium, as it leads back to the very same equilibrium achieved under Semi-Complete Markets!^{24/} If (20) holds, each individual will want and be able in the initial round to attain the same posterior conditional wealth-pair W', W'' that he could achieve in the prior round under SCM. With the desired balance between W' and W'' achieved in selecting a trading position T^* , he will of course later engage in posterior-round transactions so as to reach the same compound consumptive gamble D^* that was optimal under SCM.

Martingale-type anticipations are involved once again in this equilibrium. For, it follows immediately from (15), (18), and (20) that:

$$(21) \quad P_{\zeta}^O = \pi P'_{\zeta} + (1-\pi) P''_{\zeta} = \pi P'_{Za} + (1-\pi) P''_{Zb}$$

That is, if all traders make "correct conditional forecasts," and believe that the prior-round price of the unconditional claim to Z is the mathematical expectation (calculated in terms of concordant probabilities) of the posterior-round price, this belief will be self-fulfilling in market equilibrium!

NUMERICAL EXAMPLE 2

Table 2 follows the pattern of Table 1, but provides for the two rounds of trading under an informative situation. Only the E, T*, and D* positions are shown; the implied transactions can be inferred.

Part 1 covers the interaction of the same representative supplier-demander pair as in Example 1. These individuals, with opposed endowments but concordant beliefs ($p=\pi=.6$), are supposed to represent a microcosm of the entire market. The upper panel of Part 1 shows that, under SCM, each of the pair attains a compound consumptive gamble D* that is identical with the C* achieved with a single trading round under SCM in Example 1. Specifically, with equilibrium prior prices $P_{Za}^O = P_{Za} = .3$ and $P_{Zb}^O = P_{Zb} = .5$ as given by equation (18), and posterior prices $P'_{Za} = .5$ and $P''_{Zb} = 1.25$ as given by equation (15), the optimal conditional wealths determined by equation (17) are $W' = W'' = 200$. Both traders can achieve these by moving immediately, in the prior round, to the trading position $T^* = (100; 200, 80)$ -- identical with the optimal simple consumptive gamble C* achieved under a non-informative situation. Then neither would trade in the posterior round with the result that $T^* = D^* = C^*$. Thus the efficient portfolios achieved by concordant traders in a non-informative situation under SCM are also attained in two trading rounds in an informative situation.

In the lower panel of Part 1, the informative-situation results under UM are shown. The equilibrium conditional posterior-round prices here are

$P'_{\zeta} (=P'_{Za}) = .5$ and $P''_{\zeta} (=P''_{Zb}) = 1.25$. The prior-round price implied by equation (21), the martingale proposition, is then $P^0_{\zeta} = .8$ (in contrast with the $P_{\zeta} = .9439$ obtained in Example 1 for the non-informative situation). In this case it so happens that the optimal wealth-pair of equation (17) -- $W', W'' = 200, 200$ -- can be achieved only by choosing $T^* = E$, i.e., there will be no prior-round trading. (This is, however, an accidental consequence of the particulars of this Example. More generally, both prior and posterior trading will be called for on the part of concordant individuals under UM.) This trading position makes possible the achievement of the optimal compound consumptive gamble $D^* = C^* = (100; 200, 80)$ even under the constrained UM market regime -- in contrast with the inefficient result under UM in Example 1, where only a single trading round was permitted.

Part 2 of Table 2 shows the results in these market regimes for individuals of representative endowments but deviant beliefs. In the upper panel of Part 2, compare the trading positions T^* attained under SCM with the consumptive optimum positions C^* shown for a single trading round in Table 1. (Here again, there is a degree of freedom that has been used to fix the first element $n^t = 100$ of T^*). The key point to notice is that the two rounds of trading permit a heavier commitment to deviant beliefs. For the optimist assigning a relatively higher probability to state-a, the trading position backs his belief more heavily than the simple consumptive gamble under SCM, $C^* = (100; 233.3, 60)$. For the pessimist assigning a relatively higher probability to state-b, $T^* = (100; 133.3, 120)$ compares similarly with $C^* = (100; 166.7, 100)$. Note also that if the speculation is successful, the belief-deviant trader can attain relatively high consumptions of both N and Z -- since with two rounds of trading the n' and n'' elements of the optimal D^* gamble can differ along with z'_a and z''_b .

Table 2

ENDOWMENTS, TRADING POSITIONS, AND COMPOUND CONSUMPTIVE GAMBLER
IN INFORMATIVE SITUATION UNDER TWO MARKET REGIMES

	Endowment (E)	Desired Posterior Wealths	Trading position (T*)	Compound consumptive gamble (D*)
	$n^e; z_a^e, z_b^e$	W', W''	$n^t; z_a^t, z_b^t$	$\begin{cases} n', z_a' \\ n'', z_b'' \end{cases}$
<u>Part 1: Supplier-Demander Pair with Concordant Beliefs ($\pi=.6$)</u>				
SEMI-COMPLETE MARKETS (SCM)†				
Supplier	0;400,160	200,200	100; 200, 80	$\begin{cases} 100, 200 \\ 100, 80 \end{cases}$
Demander	<u>200; 0, 0</u>	200,200	<u>100; 200, 80</u>	$\begin{cases} 100, 200 \\ 100, 80 \end{cases}$
TOTALS	200;400,160		200; 400,160	
UNCONDITIONAL MARKETS (UM)††				
Supplier	0;400,160	200,200	0; 400,160	$\begin{cases} 100, 200 \\ 100, 80 \end{cases}$
Demander	<u>200; 0, 0</u>	200,200	<u>200; 0, 0</u>	$\begin{cases} 100, 200 \\ 100, 80 \end{cases}$
TOTALS	100;400,160		200; 400,160	
<u>Part 2: Traders with Representative Endowments and Deviant Beliefs</u>				
SEMI-COMPLETE MARKETS (SCM)†				
Optimist ($p=.7$)	100;200, 80	233.3,150	100;266.7, 40	$\begin{cases} 116.7, 233.3 \\ 75, 60 \end{cases}$
Pessimist ($p=.5$)	<u>100;200, 80</u>	166.7,250	<u>100;133.3,120</u>	$\begin{cases} 83.3, 166.7 \\ 125, 100 \end{cases}$

Table 2 (continued)

	Endowment (E)	Desired Posterior Wealths	Trading position (T*)	Compound consumptive gamble (D*)
UNCONDITIONAL MARKETS (UM)††				
Optimist (p=.7)	100;200, 80	233.3,150	100;266.7, 40	$\left\{ \begin{array}{l} 116.7, 233.3 \\ 75, 60 \end{array} \right.$
Pessimist (p=.5)	<u>100;200, 80</u>	166.7,250	<u>100;133.3,120</u>	$\left\{ \begin{array}{l} 83.3, 166.7 \\ 125, 100 \end{array} \right.$

† SCM prior-round equilibrium prices: $P_{Za}^0 = .3$, $P_{Zb}^0 = .5$. Posterior-round equilibrium prices: $P'_{Za} = .5$, $P''_{Zb} = 1.25$.

†† UM prior-round equilibrium price: $P_{\zeta}^0 = .8$. Posterior-round equilibrium prices: $P'_{\zeta} = .5$, $P''_{\zeta} = 1.25$.

The lower panel of Part 2 shows, once again, that two rounds of trading under UM permit the achievement of the same wealth-pairs and so the same efficient consumptive gambles D^* as the C^* gambles under SCM.

(END OF NUMERICAL EXAMPLE)

To sum up, the development in this Section indicates that the opportunity to engage in both prior-round and posterior-round trading, afforded by the anticipated emergence of information, remedies the deficiency of the regime of Unconditional Markets that was observed for a non-informative situation. Specifically, in our model above every trader -- whether belief-deviant or not -- was able to achieve in two trading rounds under UM the exact same consumptive gamble that was optimal under the more ample regime of Semi-Complete Markets SCM.

The next Section is intended to evaluate the generality of this conclusion when the idealizing assumption of just two goods and two states is relaxed.

IV. ADEQUACY OF MARKETS AS RELATED TO NUMBERS OF STATES AND GOODS

The key result derived for the model of the previous Section was that the two rounds of trading in an informative situation allow every transactor, even under the incomplete Unconditional Markets regime, to achieve the same optimal consumptive gamble as was attainable under Semi-Complete Markets. But the preceding analysis specified a world of just two alternative states (a and b) and just two commodities (N and Z). This is too artificial to be acceptable without further consideration.

More generally, let S denote the number of states and G the number of goods. Then, in a non-informative situation -- or in prior trading in an informative situation -- under Semi-Complete Markets there will be dealing in $S(G-1)+1$ distinct claims or "contracts." (Contingent claims to $G-1$

different risky commodities will be traded, but only the single unconditional claim to the riskless numeraire commodity N.) In the posterior round, after the emergence of conclusive information, there is no more uncertainty. At that point trading takes place once again, but only in G contracts -- one for each good. Under Unconditional Markets, in contrast, there will be trading in only the G unconditional contracts in the prior round, followed by a second posterior round involving the same G contracts.

The key feature of the development in Section III is that, for each individual in an informative situation to achieve his optimal gamble, all that he need determine in the prior trading round are his S conditional posterior wealths -- one for each possible state of the world. If $G = S = 2$, as specified in Section III, trading under Semi-Complete Markets in $S(G-1) + 1 = 3$ contracts in the prior round provides an extra degree of freedom in choice of T^* . Since the restriction under Unconditional Markets UM to certainty trading in the risky commodity Z (the fact that Z_a and Z_b claims must be traded only in a 1:1 ratio) used up just this one degree of freedom, the UM regime could achieve the same result as the SCM regime.

More generally, under SCM the excess number of contracts (the degrees of freedom d.f. in choice of T^*) will be:

$$(22) \quad \text{d.f.} = S(G-1) + 1 - S = S(G-2) + 1$$

Under SCM there will always be such an excess for $G \geq 2$.^{25,26/} Under UM, the excess number of contracts is simply:

$$(23) \quad \text{d.f.} = G - S$$

Since the number of potentially distinguishable states of the world S is infinite, it might be thought that under the more realistic UM regime there would generally be a deficiency of markets. To this there are several

possible replies: (1) Given our limited mental capacities, the number of actually distinguished states entering into people's subjective calculations is likely to be a rather small number. (2) While Unconditional Markets may be a closer approximation of real world actuality than Semi-Complete Markets, there is in fact some trading in conditional claims (e.g., in insurance markets). Such markets are likely to be provided just where it is most important to do so in terms of meeting perceived gaps in market adequacy. (3) Most important of all is the consideration that information will not ordinarily be arriving in one single injection. If successive informational inputs are anticipated, all but the last being less than conclusive, repeated rounds of trading become available for rebalancing of portfolios. Multiple trading rounds thus compensate for inadequacy of markets in any given round.²⁷ Since the arrival of information is often an essentially continuous process over time, the number of degrees of freedom available tends to rise without limit, even if there are only Unconditional Markets at any point in time. (An explicit analysis of the complex problem of sequential information inputs will not be provided here, however.)

The conclusion, therefore, is that the applicability of the key theoretical results obtained does not depend in any essential way upon the illustrative assumptions of $S = 2$ and $G = 2$ goods.

V. BEHAVIORAL IMPLICATIONS -- SPECULATORS VS. HEDGERS

There is one significant difference in implications for trader behavior as between Semi-Complete Markets and Unconditional Markets. Under SCM anyone with concordant beliefs can, in the prior round of trading, choose a trading position T^* identical with his optimal consumptive gamble $D^* = C^*$. That is,

he could move directly to his consumptive optimum as soon as markets open, foregoing any opportunity for posterior trading at the changed prices to rule after emergence of the anticipated new information. And anyone with deviant beliefs would plan to deal in both the prior and the posterior markets; his prior-round choice of T^* would necessarily diverge from his ultimate consumptive choices. This distinction provided a simple identification of speculative behavior under the SCM regime: a speculator is one whose prior-round trading position is linked to prospective portfolio revision in the posterior round, whereas a nonspeculator is one who need not plan for portfolio revision.

This implication is no longer valid under the more constrained UM market regime. Here the limitations upon marketing are such that posterior trading is in general necessary for everyone, whether of concordant beliefs or of deviant beliefs. Since under UM, typically, everyone will be revising portfolios, the working definition of speculation employed above for the SCM regime (planning for posterior portfolio revision) is not ultimately satisfactory. A more general definition, applicable to both SCM and UM market regimes, is as follows: A speculator is a trader who, in an informative situation, plans to deal in the prior and posterior rounds in such a way as to achieve a compound consumptive gamble D^* that differs from the simple consumptive gamble C^* he would have chosen in a non-informative situation. Or, putting it less technically, a speculator is one who plans to profit from emergent information; for a non-speculator, on the other hand, planned contingent consumption is identical over information-events (but not, in general, over states of the world).²⁸

What of the conventional view in the speculation literature that distinguishes between (a) risk-averse individuals who use the prior round of trading to reduce their exposure to price risk ("hedgers") versus (b) risk-tolerant individuals who

trade so as to increase their exposure to price risk ("speculators")? Our analysis shows that this distinction is invalid. While relatively risk-tolerant individuals do (other things equal) seek wider gambles, if they hold representative beliefs their ultimate D^* gambles will not differ from the C^* gambles they would have chosen in a non-informative situation. They will therefore not be planning to profit from incoming information. Perhaps even more convincing a refutation, to increase one's exposure to price risk -- in a world where quantity risk also exists (and is, indeed, as we have seen, an ultimate determinant of price risk) -- is not necessarily to widen consumptive risk. Putting this the other way, a highly risk-averse trader will not, in general, accept an opportunity to "transfer" price risks even at fair odds! (Whereas a risk-averse individual would, by definition, always be willing to convert a quantity risk into a certainty at fair odds.) This is shown most clearly by a numerical illustration.

NUMERICAL EXAMPLE 3

In Table 2, the lower panel of Part 1 illustrated an informative situation under a regime of Unconditional Markets UM. At the equilibrium prior price $P_{\zeta}^0 = .8$ there was no actual trading in the prior round ($T^* = E$ for both traders). Each individual does as well as possible by engaging in posterior trading only.

But this behavior would be regarded, in the conventional view, as failure to use the prior market ("futures" market) to divest price risk! The martingale property, with "correct conditional forecasting," shows that such divestment could be attained at fair odds -- both traders agree that the prior-round price is the mathematical expectation of the unknown posterior price. This means that the representative demander and representative supplier here could, in the conventional view, mutually "hedge" without having to pay any premium at all to

speculators. Why do they then not do so? Because the risk-aversion argument is plausibly yet incorrectly applied to the divesting of price risk. It is quantity risk, not price risk, that enters into the utility function; the divesting of price risk may actually increase the riskiness (and therefore reduce the desirability) of the overall consumptive gamble attainable. And, in particular, in this case any trading by an individual in the prior round -- whether conventionally regarded as "hedging" or "speculating" -- will preclude his ever attaining a consumptive gamble as desirable as the $D^* = (100; 200, 80)$ achievable by not trading in the prior round.

For traders with representative beliefs, the prior-round optimality condition (17) reduces to the simple form $du'/dW' = du''/dW''$ -- implying, with (16), that $W' = W'' = 200$. But it is easy to see that, in this market regime, each individual can assure himself a conditional posterior wealth of 200 only by remaining at his endowment position. Suppose a demander was somehow induced to make a "hedging" purchase of ζ in the initial round, at the price $P_{\zeta}^0 = .8$, or a supplier was similarly induced to "hedge" by sale of ζ in the prior round. It can be verified that this must lead to an inferior outcome for each, as compared with the "no-hedging" behavior that leads to the optimal conditional wealths $W' = W'' = 200$.

(END OF NUMERICAL EXAMPLE)

Does the hedger/speculator distinction then retain any meaning or applicability whatsoever? There is one natural reinterpretation that follows from the general definition of speculation arrived at here. In the spirit of this underlying concept, we can define hedgers as that sub-class of speculators who trade with a view toward attaining D^* gambles that are less risky than their C^* gambles under SCM. The "speculators," where the word is used in contrast with "hedgers," are then those seeking more risky D^* gambles.

As previously emphasized, the crucial role in speculative behavior is played by belief-deviance -- by the degree of optimism $p-\pi$. We would anticipate that an optimistic individual, one for whom p exceeds the representative belief parameter π , would tend to adopt wider gambles -- stake more of his resource upon state-a obtaining. That this is true, up to a point, is illustrated by the final numerical example below.

NUMERICAL EXAMPLE 4

Table 3 shows the bearing of the belief parameter p upon the consumptive risks accepted by individuals. An informative situation under a regime of Unconditional Markets is assumed (where, following the analysis above, all traders achieve the same solutions as could be attained under Semi-Complete Markets). The numerical data are the same as in the preceding Examples:

- (1) The representative belief parameter is $\pi = .6$; (2) The prior-round equilibrium price is $P_{\zeta}^0 = .8$, and the posterior-round conditional prices are $P_{\zeta}^I = .5$ and $P_{\zeta}^{II} = 1.25$; (3) All traders have utility function $u = \log_e nz$;
- (4) But five different endowment compositions are shown, all with wealth-values $W^e = 200$ (calculated in terms of the prices that would have ruled under Semi-Complete Markets, not in terms of the actual prices under UM).

The main results can be interpreted as follows. Looking at the simple consumptive gambles C^* achievable under SCM (recall that these will not in general be achievable in a single trading round under UM), we see that the "optimist" with $p = .7$ widens his gamble in the direction of the more prosperous state-a -- in comparison with the individual of representative beliefs for whom $p = \pi = .6$. The "moderate pessimist" ($p = .5$) narrows his gamble, as one would expect. But note that the "extreme pessimist" ($p = .2$) is so confident about "bad news" (that state-b will obtain) as to widen his gamble in the direction of the latter state.

Consider now the compound consumptive gambles D^* achievable in two rounds of trading. Note, first, that for any given value of the belief parameter p the differently-endowed traders all end up with the same D^* . But to achieve this identity of result the prior-round trading ζ (and also the trading positions T^* not shown in the Table) must in general differ.

The initial-round purchases ζ do not vary, for any given p , as among the first three endowments tabulated (types 1,2, and 3). The reason is that these individuals are already holding representative endowed proportions of the risky state-claims Z_a and Z_b . The other two cases, individuals with asymmetrical compositions of Z -claims over states (types 4 and 5), must arrange initial-round exchanges that balance Z_a and Z_b in the course of achieving the desired contingent wealth-pair W' , W'' . Subject to this proviso, the optimists ($p = .7$) tend to be sellers of Z in the prior round. That is, they are optimistic about the social quantity of Z to be available for consumption, and so they expect on average a low price of Z in the posterior round.²⁹ Individuals of concordant beliefs ($p = \pi = .6$) all end up with the optimal gamble $D^* = C^* (100;200,80)$; here only endowment types 4 and 5 actually trade in the prior round (for the purpose of balancing their asymmetrical state-endowments). The moderate pessimists ($p = .5$), and even more so the extreme pessimists ($p = .2$), tend to be buyers of Z in the initial round; they attach greater belief to a Z -scarcity and hence to a higher posterior price.

(END OF NUMERICAL EXAMPLE)

VI. SUMMARY

The widely-accepted Keynes-Hicks theory explains hedging and speculation as the avoidance in the one case, and the acceptance in the other case, of price risk. We have found this explanation unacceptable, in view of the fact that it

TABLE 3: RISKS ACCEPTED, BY DEGREE OF OPTIMISM/PESSIMISM

BELIEF PARAMETER	p =	.7 (Optimist)	.6 (Represent- ative)	.5 (Moderate Pessimist)	.2 (Extreme Pessimist)
<hr/>					
SIMPLE CONSUMPTIVE GAMBLES (SCM)					
$C^* = (n^*; z_a^*, z_b^*) =$		$(100; 233\frac{1}{3}, 60)$	$(100; 200, 80)$	$(100; 166\frac{2}{3}, 100)$	$(100; 66\frac{2}{3}, 160)$
DESIRED POSTERIOR WEALTHS					
W', W''	=	$233\frac{1}{3}, 150$	$200, 200$	$166\frac{2}{3}, 250$	$66\frac{2}{3}, 400$
PRIOR-ROUND TRADING (UM)					
<u>Endowments</u>					
$E = (n^e; z_a^e, z_b^e)$					
1. $(100; 200, 80) \cdot \zeta =$		$-111\frac{1}{9}$	0	$111\frac{1}{9}$	$444\frac{4}{9}$
2. $(200; 0, 0) \cdot \zeta =$		$-111\frac{1}{9}$	0	$111\frac{1}{9}$	$444\frac{4}{9}$
3. $(0; 400, 160) \cdot \zeta =$		$-111\frac{1}{9}$	0	$111\frac{1}{9}$	$444\frac{4}{9}$
4. $(80; 400, 0) \cdot \zeta =$		$155\frac{5}{9}$	$266\frac{6}{9}$	$377\frac{7}{9}$	$711\frac{1}{9}$
5. $(120; 0, 160) \cdot \zeta =$		$-377\frac{7}{9}$	$-266\frac{6}{9}$	$-155\frac{5}{9}$	$177\frac{7}{9}$
ELEMENTS OF COMPOUND CONSUMPTIVE GAMBLES					
$D^* = [C', C''; p, 1-p]$					
$C' = (n', z_a')$	=	$(116\frac{2}{3}, 233\frac{1}{3})$	$(100, 200)$	$(83\frac{1}{3}, 166\frac{2}{3})$	$(33\frac{1}{3}, 66\frac{2}{3})$
$C'' = (n'', z_b'')$	=	$(75, 60)$	$(100, 80)$	$(125, 100)$	$(200, 160)$

is the interaction between price risk and quantity risk that governs the overall hazard accepted or avoided by individuals. Failure to appreciate the significance of the quantitative uncertainty that underlies and is the main cause of stochastic variation of prices is the key failing of the traditional speculation literature. According to the definition proposed here, speculation and hedging consist of trading in the prior and posterior markets (in an informative situation, since only with anticipated emergence of new information can differences between posterior and prior prices be anticipated) in such a way as to achieve compound consumptive gambles D^* that differ from the simple consumptive gambles C^* that would have been adopted in a non-informative situation. The hedgers are those for whom the net effect of the prior trading activity is to make D^* less risky than C^* -- with the reverse holding for speculators, if the latter term is understood as opposed to hedging. Alternatively, it is sometimes terminologically more convenient to think of speculation as a wider category of activity (including all prior-market trading leading to divergences between C^* and D^*), in which case hedging becomes a special risk-reducing subclass of speculative behavior.

Among the factors studied here as possibly involved in the speculative decision are: (1) The individual's beliefs about the emergence and content of new information -- which must logically bear a definite relation to his own prior estimates of the likelihood of alternative states of the world. (2) His utility function, involving both preferences as among different commodities and his degree of risk-tolerance (willingness to hold prospects yielding differential outcomes over the various states of the world). (3) The scale and composition of his endowment, as distributed over commodities and states of the world. (4) The extent of the markets available, and in particular whether or not conditional state-claims to commodities can be bought and sold separately.

The key results derivable from the models considered here can be summarized:

(A) Speculative trading is undertaken only by individuals whose opinions, as to the likelihood of future states of the world, diverge from representative beliefs in the market.

(B) Mutual equilibration of prior-round and posterior-round prices requires that the price-revision relation be a martingale, calculated in terms of concordant beliefs.

These results are applicable not only for "unrealistic" market regimes permitting trading in contingent claims to risky commodities (Semi-Complete Markets) but also, subject to certain limitations, to regimes in which only certainty claims can be traded (Unconditional Markets) as assumed in the traditional speculation literature.

As is always the case for theoretical models, these propositions depend upon a number of idealizing assumptions and hence could not be expected to be exactly applicable to actual behavior. But the main thrust of the results is to support the observation of Holbrook Working that speculative/hedging behavior is governed primarily by differences of belief, rather than by difference of risk-tolerance as postulated by the Keynes-Hicks risk-transfer theory.

FOOTNOTES

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1. Keynes (1930), v. 2, Ch. 29; Hicks (1946), pp. 137-39. Also Houthakker (1947, 1968); Cootner (1968).
2. Working (1953, 1962); see also Rockwell (1967).
3. Fama (1970), p. 384; Samuelson (1965).
4. If hedgers are mostly long the physical good (if they are predominantly suppliers or warehouseurs of the commodity) they must be mostly short in the futures market. Then the speculators must be net long in futures. This premise had led to the inference of "normal backwardation" -- that prices of contracts tend to rise as delivery date approaches, thus rewarding the speculators for making early purchase commitments and thereby bearing the price risk. The evidence does not conclusively support normal backwardation, though the issue remains in debate (compare Houthakker [1968], Rockwell [1967], Telser [1967]). Two main explanations for the supposed failure of normal backwardation have been proposed that are consistent with an underlying risk-transfer theory: (1) If hedgers were predominantly demanders rather than suppliers of the commodity, normal compensation for speculators would dictate a falling rather than a rising price trend over the life of the contract (Cootner [1968], p. 119); or, (2) If speculators are not risk-averse on balance, no net compensation is required (Friedman [1960]). An alternative explanation, of course, would be that the risk-transfer theory is simply incorrect.
5. Hirshleifer (1975). An earlier version was Hirshleifer (1972).

6. Fuller exposition appears in Hirshleifer (1975).
7. We are here making use of two of the idealizing assumptions to be mentioned below: that all trading takes place at equilibrium prices, and that decisions associated merely with the passage of time (e.g., storage activities) can be separated from speculation proper.
8. If traders are atomistic, merely private information will only negligibly affect prices. See Hirshleifer (1971), p. 564.
9. There are other exogenous influences whose probabilistic variation might induce a corresponding stochastic distribution of prices. Among them are possible changes in tastes, technology, social institutions, etc. These factors will not be considered here.
10. Called "Certainty Markets" in Hirshleifer (1975), p. 525.
11. Ibid.
12. See Feiger (1976). A comparable result for a one-commodity model appears in Hirshleifer and Rubinstein (1976).
13. NCM can be considered a more realistic equivalent of SCM. Under SCM there is trading of claims to "wheat given a good crop" and "wheat given a poor crop" against unconditional numeraire units (money, say). Under NCM there is trading only in unconditional claims to wheat, but traders can in effect make side-bets in numeraire (money) units as to whether the feast or famine state of the world is going to obtain.

There is one version of the (non-speculative) market structure described in Arrow (1964) that ought not be confused with the NCM regime as defined here. In that version Arrow provides for a prior round in which only state-claims to numeraire (N_a or N_b , in the notation here) can be traded, to be followed by a posterior round in which only unconditional commodity-claims are exchanged (after it is revealed which state is to obtain). In contrast, the NCM regime provides for prior trading in N_a , N_b , and Z-claims. NCM (like SCM, UM, and the other market regimes

here considered) allows for repeated trading of some types of claims, a necessary condition for the price revisions making speculation possible.

14. A somewhat parallel conceptualization appears in Diamond (1967). Diamond's market regime provides for an arbitrary number of firms and corresponding tradable equity claims, and is in that respect more general than EM here. But his one-period one-commodity model does not permit speculative behavior, though he suggests that the interaction of "technological uncertainty" (quantity risk) and "price uncertainty" (price risk) does have to be taken into account in more general models.
15. For discussion of the additivity assumption, see the comment by Salant (1976) and reply by Hirshleifer (1976). The martingale theorem (Proposition B) was derived, under the assumptions of a riskless commodity with state-independent additive utility, in Dreze (1970-71). Dreze did not consider the problem of speculation, however.
16. Rubinstein (1975) defines a concept called "consensus beliefs" -- a single probability distribution which, if unanimously held, would determine the same market prices as the actual heterogeneous beliefs of traders. In general, it is not possible to find a "consensus" or "representative" probability distribution as an average of individual beliefs.
17. Personal communication from John C. Riley.
18. Such a difference can be achieved, in a single trading round, only under Fully Complete Markets (FCM).
19. See Radner (1968).
20. A certain resemblance to the concept called "rational expectations" in Muth (1961) may be noted. See also Arrow (1975).
21. It is only necessary that belief-concordant traders make "correct conditional forecasts" to achieve the price effects of (15), as these traders account for essentially all the social weight.

22. Because of the degree of freedom in choice of elements of T^* , the individual might move in the prior round to a trading position having the same associated posterior wealths W' and W'' as D^* , yet requiring posterior trading to actually achieve D^* . But there is no advantage in not moving from E to D^* in one round of trading as described in the text. And any transaction costs, however minute, would make this direct movement strictly preferred.
23. Multiplying the first equation of (19b) by π and the second by $1-\pi$ and summing leads to: $\pi W' + (1-\pi)W'' = \pi(n^t + P_{Za}^t z_a^t) + (1-\pi)(n^t + P_{Zb}^t z_b^t)$.
- $$= n^t + P_{Za}^o z_a^t + P_{Zb}^o z_b^t$$
- $$= (n^e - P_Z^o \zeta) + P_{Za}^o (z_a^e + \zeta) + P_{Zb}^o (z_b^e + \zeta)$$
- Equation (19c) then follows.
24. As before, we will not attempt to show uniqueness.
25. This result assumes linear independence among the G goods with regard to posterior price distributions. If, for example, two goods had exactly proportional posterior prices over all possible states of the world, more contracts would be needed in the prior round. This qualification will henceforth be ignored.
26. If $G = 1$, the number of contracts exactly suffices if $S = 1$ and is deficient for larger S . This is not of practical significance, of course, but it suggests that single-commodity models of speculation (for which $G = 1$) should be viewed with reserve.
27. On this general point see Drèze (1970-71), pp. 144-145; Rubinstein (1975), p. 813, fn. 3.
28. Compare Hirshleifer and Rubinstein (1975), p. 799.
29. A low price of Z is, of course, "bad news" for suppliers, other things equal. But other things are not equal, since the price is low precisely when the quantity available is great. Socially speaking, naturally, a large quantity available is "good news."

REFERENCES

- Arrow, K.J., "The Role of Securities in the Optimal Allocation of Risk-Bearing,"
REV. EC. STUD., v. 31 (1964).
- _____, "On a Theorem of Arrow: Comment," REV. EC. STUD., v. 42 (July, 1975).
- Cootner, P.H., "Speculation, Hedging, and Arbitrage," INTERNATIONAL ENCYCLOPEDIA
OF THE SOCIAL SCIENCES (1968), v. 15.
- Diamond, P.A., "The Role of a Stock Market in a General Equilibrium Model with
Technological Uncertainty," AM. EC. REV., v. 57 (September, 1967).
- Drèze, J.H., "Market Allocation Under Uncertainty," EUROPEAN EC. REV., v. 2
(Winter 1970-71).
- Fama, E.F., "Efficient Capital Markets: A Review of Theory and Empirical Work,"
J. OF FIN., v. 25 (May, 1970).
- Feiger, George, "Speculation and Equilibrium: Comment," QUARTERLY JOURNAL OF
ECONOMICS v. 90 (1976, forthcoming).
- Friedman, M., "In Defense of Destabilizing Speculation," in R.W. Pfouts, ed.,
ESSAYS IN ECONOMICS AND ECONOMETRICS (Chapel Hill, N.C.: U. of North Carolina
Press, 1960). Reprinted in M. Friedman, THE OPTIMUM QUANTITY OF MONEY AND
OTHER ESSAYS (Chicago: Aldine, 1969).
- Hicks, J.R., VALUE AND CAPITAL, 2nd ed. (London: Oxford U.P., 1946).
- Hirshleifer, J., "The Private and Social Value of Information and the Reward
to Investment Activity," AM. EC. REV., v. 61 (September, 1971).
- _____, "Foundations of the Theory of Speculation: Information, Risk,
and Markets," Western Management Science Institute, UCLA, Working Paper No.
189 (Revised, June 1972).
- _____, "Speculation and Equilibrium: Information, Risk, and Markets,"
QUARTERLY JOURNAL OF ECONOMICS, v. 89 (November 1975).
- _____, "Speculation and Equilibrium: Response," QUARTERLY JOURNAL OF
ECONOMICS, v. 90 (1976, forthcoming).

Hirshleifer, J. & Rubinstein, M.E., "Speculation and Information in Securities Markets," PROCEEDINGS -- XX INTERNATIONAL MEETING, THE INSTITUTE OF MANAGEMENT SCIENCES, v. 2 (Jerusalem Academic Press, 1976).

Houthakker, H., "Can Speculators Forecast Prices?" REV. EC. & STAT., v. 39 (May, 1957).

_____, "Normal Backwardation," in J.N. Wolfe, ed., VALUE, CAPITAL, AND GROWTH (Edinburgh University Press, 1968).

Keynes, J.M., A TREATISE ON MONEY (New York, 1930).

Muth, J.F., "Rational Expectations and the Theory of Price Movements," ECONOMETRICA, v. 29 (July, 1961).

Radner, R., "Competitive Equilibrium Under Uncertainty," ECONOMETRICA, v. 36 (January, 1968).

Rockwell, C.S., "Normal Backwardation, Forecasting, and the Returns to Commodity Futures Traders," FOOD RESEARCH INSTITUTE STUDIES, v. 7 (1967), Supplement.

Rubinstein, Mark E., "Securities Market Efficiency in an Arrow-Debreu Economy," AM. EC. REV., v. 65 (December, 1975).

Salant, Stephen W., "Speculation and Equilibrium: Comment," QUARTERLY JOURNAL OF ECONOMICS v. 90 (1976, forthcoming).

Samuelson, P.A., "Proof that Properly Anticipated Prices Fluctuate Randomly," INDUST. MANAG. REV., v. 6 (Spring, 1965).

Telser, L.G., "The Supply of Speculative Services in Wheat, Corn, and Soybeans," FOOD RESEARCH INSTITUTE STUDIES, v. 7 (1967), Supplement.

Working, H., "Futures Trading and Hedging," AM. EC. REV., v. 43 (June, 1953).

_____, "New Concepts Concerning Futures Markets and Prices," AM. EC. REV., v. 52 (June, 1962).