

Informational Equilibrium*

by

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I. INTRODUCTION

Among recent advances in the economics of information, one of the more controversial has been the development of the notion of an informational equilibrium. By this is meant an equilibrium, in a world of incomplete markets, in which either the observed actions of better-informed agents, or the resulting equilibrium prices, yield valuable information to worse-informed agents.

For example, in his path-breaking dissertation, Spence (1973) suggested that difficulties in observing human traits correlated with labor productivity, and in monitoring productivity, would result in an equilibrium where wage offers were based on the educational credentials of the job seeker. That is, firms would use education as a screening device to sift out workers of lower productivity. As Spence emphasized, a crucial precondition for such an equilibrium is that those with greater productivity are also the faster learners in school and hence those with lower opportunity costs. Given this assumption, the higher productivity individuals, facing wage offers contingent upon educational performance, find it in their interest to accumulate higher credentials, and thereby provide a signal to potential employers.

Working independently, Rothschild and Stiglitz (1976), and also Wilson (1976) have proposed a similar model of information transmission in insurance markets. Instead of offering a single price per unit of coverage, it is shown that firms have an incentive to charge higher prices for greater coverage. In this way, higher risk individuals, for whom additional coverage yields greater marginal benefits, are separated from lower risk individuals. As we shall see, both the labor and insurance market examples belong to the same generic class of information transmission models.

There are several aspects of these models which have aroused considerable debate. First, there are those (e.g., Barzel (1976)) who have criticized Spence's conclusion that in general there will be "overinvestment" in the signal. The core of this criticism is that in the absence of alternative means of communicating information, there may be no feasible Pareto-improving system of taxes and lump sum transfers. Then unless one imposes a specific welfare criterion, Spence's normative inference is re-

placed by the positive (and trivial) proposition that investment in the signal will be greater than in a world of costless information.

Without disputing the above, it should be pointed out that at least in some simple models, there is a system of taxes and transfers that, if adopted, would lead the economy onto the full information Pareto frontier. This provides some justification for describing signalling equilibria as "inefficient".

Second, especially in reference to Spence's labor market model, it has been argued (Layard and Psacharopoulos (1974)) that firms are able to monitor an employee's productivity at relatively low cost. If so, signalling in labor markets is, at most, a phenomenon of minor importance. Hopefully, ongoing empirical research will help to resolve this issue.

Third, and most fundamental, have been the doubts raised about the viability of informational equilibria. Analysing similar models in which there are a finite number of classes of agents, Rothschild/Stiglitz and Wilson have demonstrated the possibility of there being no Nash equilibrium.

To establish such a result is, of course, to establish an inconsistency in the assumptions made about the decision rules of the individual agents. One possible way out of this dilemma, proposed by Rothschild/Stiglitz, is to assume that firms only consider alternatives which are in some sense "close" to an initial set of offers. However, it will be shown below that for a continuum of classes there is no "local" Nash informational equilibrium. This suggests that any definition of "closeness" which ensures existence in finite class models is sensitive to the number of these classes; that is, essentially ad hoc.

Given such a conclusion, it is natural to ask whether there exists a strategic equilibrium in which each agent takes the reactions of other agents at least partially into account. Rather than attempt to specify a truly dynamic interaction model, the approach taken below is to search for a plausible "imperfectly competitive" equilibrium concept.

The paper is organized as follows. In Section II the notion of an informationally consistent price function is developed, and a class of imperfect information models is described. Spence's labor market model and the Rothschild/Stiglitz insurance model are shown to be members of this class.

In Section III a gap in the literature, in regard to the existence of informationally consistent price functions, is closed. Given the assumptions of the model, it is shown that there exists a differentiable family of such functions.¹ In addition, it is shown that all informationally consistent price functions must belong to this family.

Then, in Section IV, a result is established which implies that no members of the family are Nash equilibria. It is demonstrated that for any informationally consistent price function, there is always an alternative offer at the lowest quality level (e.g., lowest skill level) which, if introduced by a single agent, yields the latter increased profits.² Conditions are also established under which there are new profitable offers at higher quality levels.

Finally, in Section V, alternative equilibrium concepts are examined. After demonstrating that there is no informationally consistent local Nash equilibrium price function, two strategic equilibrium concepts are considered. While strategies in the two approaches are not dissimilar, it will be seen that the implications for equilibrium can be very different. Concluding remarks appear in Section VI.

II. A MODEL OF INFORMATION TRANSFER

An agent of type n , by selecting a freely observable element y from his choice set Y , is thereby able to offer for sale a commodity (or service) of value $S(n,y)$. In the absence of any direct means of observing either n or S prior to payment, potential buyers use the value of y as a signal of product "quality". They make bids w , contingent upon y , that is

$$w = w(y).$$

Type n 's welfare is dependent upon his choice of y and the price received for his product, that is

$$U = U(n,y,w(y)).$$

Assumptions:

- A1. $U(n,y,w)$ and $S(n,y)$ are twice differentiable for all $n > 0$, $y, w \geq 0$.
- A2. $S_1(n,y) > 0$, $S_2(n,y) \geq 0$, $U_3(n,y,w) > 0$.
- A3. For each n , $U(n,y,S(n,y))$ is quasi-concave, attaining a maximum at some unique finite value of y .
- A4. $\frac{\partial}{\partial n} \left(\frac{-U_2}{U_3} \right) < 0$.
- A5. n is distributed continuously on $[\underline{n}, \bar{n}]$.

Assumption A2 imposes natural restrictions on S and U . For a given y , higher n is associated with a higher quality product. Note that we include the special case in which the level of the signal has no impact on the value of the product offered ($S_2 = 0$). Assumption A4 is crucial. $\left(\frac{-U_2}{U_3} \right)$ is the opportunity cost in dollars of increasing the level of the signal y . It is necessary that this be everywhere lower for those capable of producing higher quality products. The remaining assumptions, together with A4 ensure the differentiability of $w(y)$.

Having described the model, we must now ask under what conditions the predictions by buyer of product quality are self-fulfilling. Following the terminology of Rothschild and Stiglitz, we shall say that the price function $w(y)$ is informationally consistent (INC) if

$$(I) w(y(n)) = S(n, y(n))$$

where

(II) $\forall n$, $y(n)$ yields the solution of

$$U^*(n) = \max_y U(n, y, w(y)).$$

Condition (II) simply states that each agent, upon observing the price function $w(y)$, chooses his maximizing value of y . Condition (I) is then the requirement that at each level of y , the bid price is equal to the value of the purchased product.

Before turning to a discussion of the existence of INC price functions, we note that the widely discussed labor market, and insurance market models, are both special cases.

In Spence (1974), n is underlying "ability", y is education, $S(n, y)$ is marginal value product and $w(y)$ is the market wage. The welfare of type n is the difference between the wage and his cost of obtaining an education y , that is

$$U(n, y, w(y)) = w(y) - c(n, y)$$

The alternative welfare function,

$$U(n, y, w(y)) = w(y)[\bar{t} - t(n, y)]$$

allows an interpretation more explicitly in terms of the time costs of education. $w(y)$ becomes the wage per year, \bar{t} is the life span and $t(n, y)$ is the time type n needs to obtain an education y .³

In the insurance model, insurance companies are assumed unable to observe the odds n , that an individual with income I will not incur a loss L . However, they obtain indirect information from the type of policy an individual purchases. Writing y as the amount by which a premium is discounted from maximum premium p , and $w(y)$ as the indemnity-

premium ratio, expected utility of type n is given by:

$$U(n,y,w(y)) = \frac{n}{1+n} V[I-(p-y)] + \frac{1}{1+n} V[I-L+w(y) (p-y)]$$

Assuming insurance is "fair", the indemnity-premium ratio must equal the odds of no loss. Then condition (I) reduces simply to

$$w(y) = n$$

It is easy to check that for each of the above interpretations, restrictions which ensure A1 - A4 are natural.

III. EXISTENCE AND DIFFERENTIABILITY OF INFORMATIONALLY CONSISTENT PRICE FUNCTIONS

We begin by establishing that a particular family of twice differentiable functions $w = w(y, k)$ are INC price functions. It is then established that every INC price function must belong to this family.

Consider the first order differential equation

$$(1) \quad \frac{dn}{dy} + \frac{U_2 + U_3 S_2}{U_3 S_1} = 0 \quad \text{where } U = U(n, y, S(n, y)).$$

Since the term in parentheses is differentiable for all feasible n and y , (1) defines a one-parameter family of twice differentiable functions.

$$(2) \quad n = n(y, k).$$

From Assumption A2, $U_3 S_1$ is strictly positive. Moreover, from A3, $U_2 + U_3 S_2$ is a strictly decreasing function of y for all values of n , and is negative for large y . Then the curves $n = n(y, k)$ must have at most a single turning point $\langle y_0(k), n_0(k) \rangle$ with positive slope for all larger y . Three such curves are depicted in Figure 1.

Introduce $w = S(n, y) = S(n(y, k), y)$ from (2)
 $= w(y, k).$

Since n is twice differentiable, w is twice differentiable.

Moreover, rearranging (1) we have

$$U_2 + U_3 \left(S_1 \frac{dn}{dy} + S_2 \right) = 0.$$

Since the term in parentheses is simply $\frac{dw}{dy}$, this can be rewritten as

$$(3) \quad U_2 + U_3 \frac{dw}{dy} = \frac{\partial}{\partial n} \{U(n, y, w(y, k))\} = 0.$$

Since this is satisfied for all y , we can differentiate totally to obtain:

$$(4) \quad \frac{\partial}{\partial y} \left\{ U_2 + U_3 \frac{dw}{dy} \right\} + \frac{\partial}{\partial n} \left\{ U_2 + U_3 \frac{dw}{dy} \right\} \frac{dn}{dy} = 0.$$

Then rearranging and utilizing (3) we have

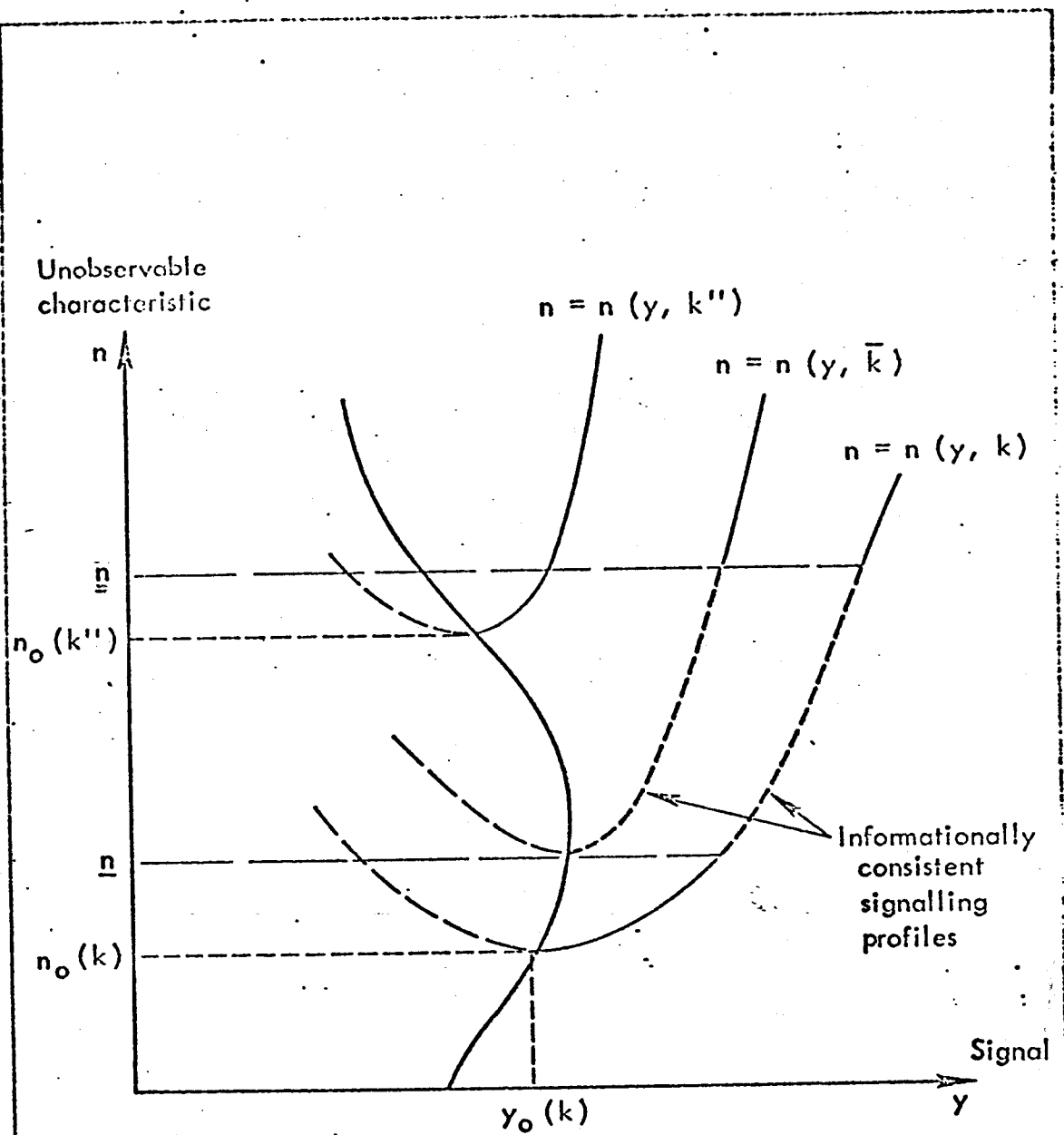


Fig. 1—Informational consistency

$$(5) \quad \frac{\partial^2}{\partial y^2} \{U(n, y, w(y, k))\} = \frac{\partial}{\partial y} \left\{ U_2 + U_3 \frac{dw}{dy} \right\} = -U_3 \frac{\partial}{\partial n} \left(\frac{U_2}{U_3} \right) \frac{dn}{dy}.$$

Given Assumption A4, the right hand side of (5) takes on the sign of $-\frac{dn}{dy}$. Therefore, as long as $n(y, k)$ is increasing in y , (3) and (5) form a set of sufficient conditions for the maximization of $U(n, y, w(y))$.

Then the second condition for informational consistency is satisfied. But by construction $w(y, k)$ satisfies condition (I). Therefore, from Figure 1, those members of the family of price functions $w(y, k)$, such that $n_0(k) \leq \underline{n}$, are indeed informationally consistent.

Finally, without loss of generality we may assume that $n(y, k)$ and hence $w(y, k)$ are strictly increasing in k . We have therefore proved:

Theorem 1: Given assumptions A1 - A5, there exists a family of twice differentiable price functions $w = w(y, k)$, $k \leq \bar{k}$ and $w_y, w_k > 0$, which are informationally consistent.

We now turn to the deeper issue of whether there might be other INC price functions. To prove that this is not the case, we begin by establishing a series of lemmas.

Lemma 1: Individuals with a smaller value of n choose a lower level of the signal.

Proof:

Consider the indifference curves for types n and n' intersecting, as in Figure 2, at the offer $\langle y', w' \rangle$, that is

$$\{\langle y, w \rangle \mid \langle y, w \rangle \sim_n \langle y', w' \rangle\} \text{ and } \{\langle y, w \rangle \sim_{n'} \langle y', w' \rangle\}$$

From A4, if $n < n'$, the former intersects the latter from below at $\langle y', w' \rangle$ only.

Then $\forall y > y', \langle y, w \rangle \succsim_n \langle y', w' \rangle \rightarrow \langle y, w \rangle \succ_{n'} \langle y', w' \rangle$

It follows immediately that if $\langle y', w' \rangle$ is maximal for type n' , any offer in the set $\langle y, w \mid y > y' \rangle$ cannot be maximal for type n , since such an offer would be strictly preferred by type n' .

Then $\forall n < n', y(n) \leq y'$. Moreover, if $w(y)$ is INC, we must have $w = w(y(n)) = S(n, y(n)) \leq S(n, y') < S(n', y') = w' = w(y')$.

Since w' is the market price for signal level y' , it follows immediately that $y(n) \neq y'$. Q.E.D.

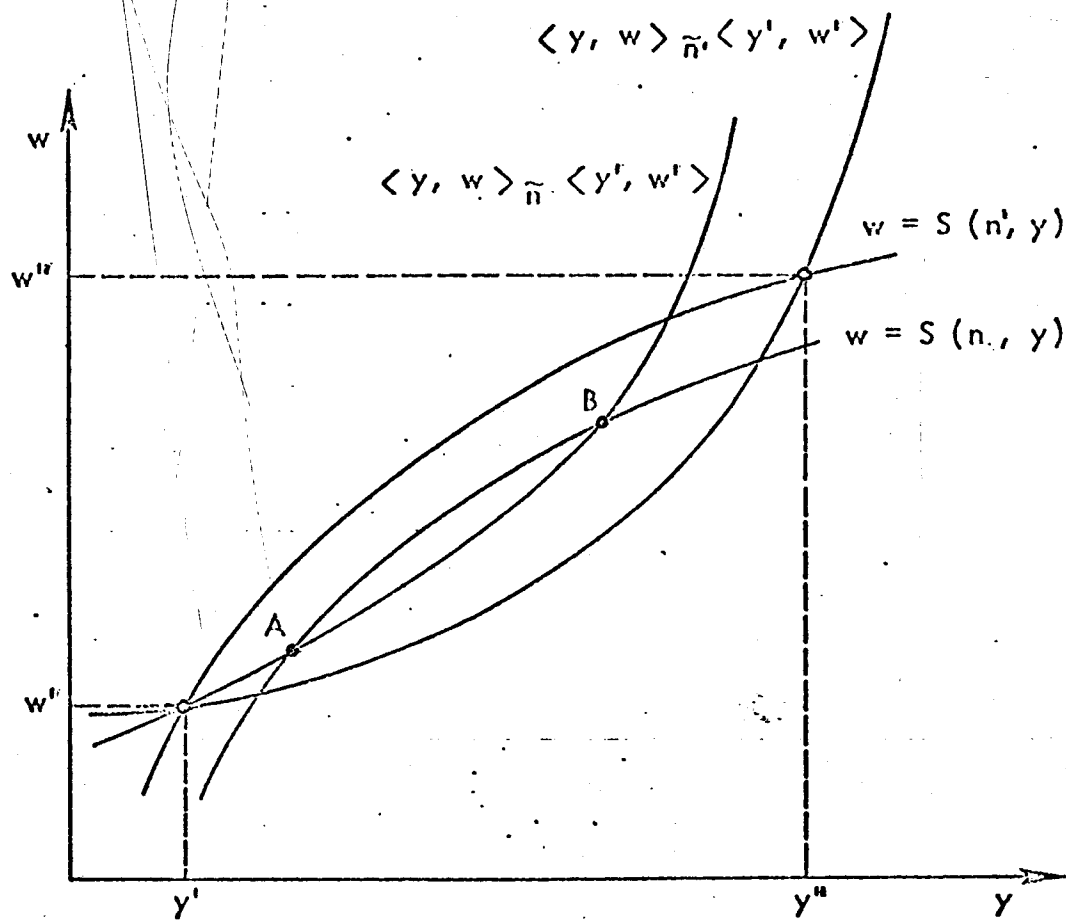


Fig. 2—Type n^1 indifferent between two levels of the signal

Lemma 2: An indifference curve for type n intersects $w = S(n, y)$ at most twice and lies above the latter for sufficiently large y (as depicted in Figure 2).

Proof:

Given A1 we can differentiate $U(n, y, S(n, y))$ with respect to y to obtain

$$\begin{aligned} \frac{1}{U_3} \left\{ \frac{\partial}{\partial y} U(n, y, S(n, y)) \right\} &= S_2 + \frac{U_2}{U_3} \\ &= \left\{ \text{slope of } S(n, y) \right\} - \left\{ \text{slope of indifference curve for type } n \text{ through } y, S(n, y) \right\} \end{aligned}$$

From A3, the bracketed partial derivative on the left hand side is strictly decreasing in y , and negative for large y .

With these preliminaries we can now demonstrate:

Lemma 3: For each type n there is a unique, maximal level of the signal $y(n)$.

Proof:

Suppose $\langle y', w' \rangle$ is maximal for type n' . For this to be informationally consistent, condition (I) must be satisfied hence

$$w' = S(n', y')$$

From Lemma 2 there is at most one other offer $\langle y'', w'' \rangle$ satisfying (I) such that $\langle y'', w'' \rangle \sim_n \langle y', w' \rangle$. Without loss of generality suppose $y' < y''$ as depicted in Figure 2. From A4, for all $n < n'$ the indifference curve through $\langle y', w' \rangle$ is, as depicted, above the indifference curve of type n' for all $y > y'$. Moreover, since $S(n, y)$ is strictly increasing in n , the curve $w = S(n, y)$ lies below $w = S(n', y)$ for all y . Both curves vary continuously with n , therefore for n sufficiently close to n' they intersect at points A and B to the right of y' . From Lemma 2 these are the only two intersections. Therefore the only offers satisfying condition (I), that is, $w = S(n, y)$, which type n prefers to $\langle y', w' \rangle$, lie along the arc AB. But by assumption $n < n'$ so this contradicts Lemma 1.

Q.E.D.

Combining Lemmas 1 and 3, we have shown that $y(n)$, and hence $w(y(n))$, are strictly increasing functions of n . The continuity of these functions is established as follows.

Lemma 4: $y(n)$ takes on all values in $[y(\underline{n}), y(\underline{n})]$.

Proof:

Suppose the statement is false. Then for $\langle y'', w'' \rangle$, maximal for some n' , we have:

$$\langle y'', w'' \rangle > \langle y', w' \rangle = \sup_{n < n'} \{y(n), w(y(n))\}.$$

The functions $y(n)$ and $w(y(n))$ are strictly increasing in n , with least upper bound $\langle y', w' \rangle$. Then for any $\epsilon' > 0$, $\exists \delta' > 0$ such that $\forall n \in (n' - \delta', n')$

$$0 < y' - y(n) < \epsilon' \text{ and } 0 < w' - w(y(n)) < \epsilon'$$

Furthermore, U is uniformly continuous, hence for any $\epsilon < 0$ $\exists \delta' > 0$ such that $\forall n \in (n' - \delta', n')$

$$(6) \quad |U(n, y(n), w(y(n))) - U(n, y', w')| < \epsilon$$

Suppose further that $\langle y'', w'' \rangle > \langle y', w' \rangle$.

Since U is uniformly continuous, $\exists \delta'' > 0$ such that $\forall n \in (n' - \delta'', n')$

$$(7) \quad U(n, y'', w'') - U(n, y', w') > \epsilon$$

Combining (6) and (7) we then have

$$\forall n \in (n' - \delta, n') \text{ where } \delta = \min\{\delta', \delta''\}$$

$$U(n, y'', w'') - U(n, y(n), w(y(n))) > 0.$$

But this contradicts the definition of $y(n)$ as the maximizing value of the signal for type n .

Applying an almost identical argument we also can rule out the possibility that $\langle y'', w'' \rangle < \langle y', w' \rangle$. Then type n' is indifferent between these two offers, exactly as depicted in Figure 2. The proof of Lemma 3 can, therefore, be applied without change, to contradict our initial supposition. Q.E.D.

From Lemma 1, $y(n)$ is strictly increasing, hence Lemma 4 implies that $y(n)$ is also continuous on $[\underline{n}, \underline{n}]$. We can, therefore, invert and write

$n = n(y)$, where n is an increasing continuous function of y on $[y(\underline{n}), y(\bar{n})]$. From condition (I) it follows that

$$w = S(n(y), y) = w(y).$$

The differentiability of $S(n, y)$ and continuity of $n(y)$ then imply that $w(y)$ is a strictly increasing continuous function on $[y(\underline{n}), y(\bar{n})]$.

Suppose finally that $w(y)$ is not differentiable, that is, there is some level of the signal y' where the curve has a kink (see Figure 3). From Lemma 4 $\langle y', w' \rangle$ is maximal for some type n' therefore $w(y)$ must be on or below the indifference curve $\{\langle y, w \rangle \mid \langle y, w \rangle \sim_n \langle y', w' \rangle\}$. Since the latter is differentiable any 'upward' kink is inconsistent with the assumption that y' is maximal. Therefore the kink must be as depicted in the Figure. However, the slope of indifference curves through $\langle y', w' \rangle$ varies continuously with n . Therefore there is an interval (n'', n''') containing n' , over which $\langle y', w' \rangle$ is maximal. But this contradicts Lemma 1.

We have therefore proved:

Theorem 2: Given assumptions A1 - A5, if $w(y)$ is an informationally consistent price function, it is monotonically increasing and differentiable on $[y(\underline{n}), y(\bar{n})]$.

It follows immediately that $w(y)$ must satisfy the first order condition (3). Moreover, since $w(y) = S(n(y), y)$, $n(y)$ must be a differentiable function of y . Then (3) implies (2) which in turn can be rearranged to obtain (1). We have therefore proved:

Theorem 3: If $w(y)$ is informationally consistent, it is a member of the family of twice differentiable functions $w = w(y, k)$.

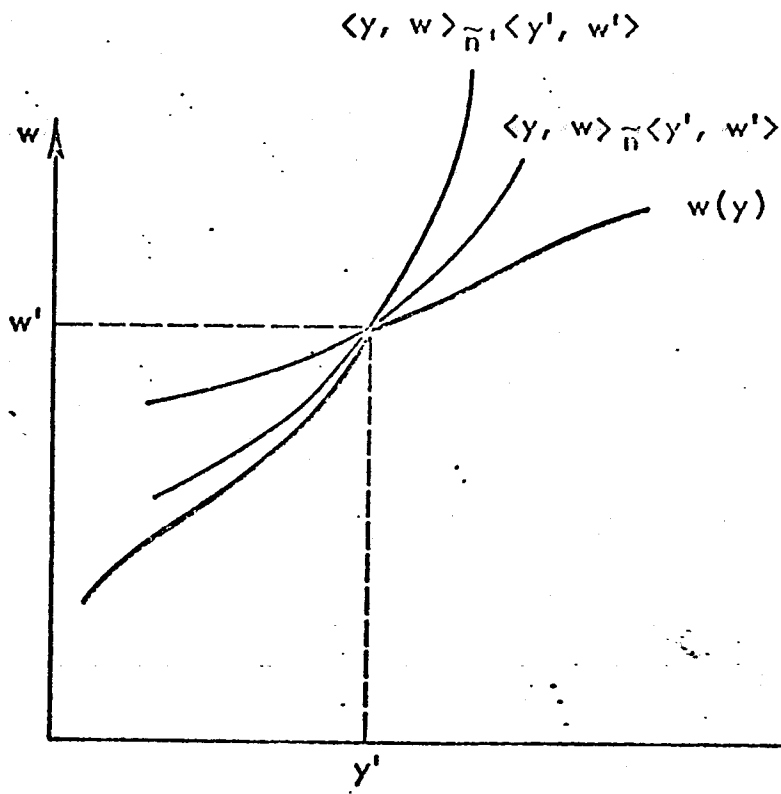


Fig. 3—A kinked price function

IV. INSTABILITY OF INFORMATIONALLY CONSISTENT PRICE FUNCTIONS

Spence in his initial exposition of signalling interpreted informational consistency as a necessary and sufficient condition for equilibrium. However, the ensuing discussion by Rothschild/Stiglitz and Riley has made it clear that informational consistency does not eliminate opportunities for potential gain, by price searching agents.

In the first part of this section, it is demonstrated that for all members of the family of INC price functions it is profitable for a single buyer, to offer a higher price to sellers who signal at the lowest level $y(\underline{n})$. Of course, the buyer makes losses on those products sold by type \underline{n} . However, the new offer is also attractive to agents selling higher quality products. We shall see that there is a price \bar{w} , which attracts agents selling products with an average value exceeding \bar{w} .⁴ First, it is necessary to strengthen our assumptions about the distribution of n . Assumption A5 is replaced by:

A5' n is distributed on $[\underline{n}, \bar{n}]$ according to the twice differentiable, strictly increasing function $F(n)$.

Consider the profile of welfare levels, $U^*(n) = \max_y U(n, y, w(y))$. This has a slope at the lower end point of the distribution, \underline{n} , given by:

$$(8) \quad \frac{dU^*}{dn} = U_1 + [U_2 + U_3 \frac{dw}{dy}] \frac{dy}{dn}$$

$$= U_1(\underline{n}, \underline{y}, w(\underline{y})),$$

since from (4) the bracketed expression is zero.

Next consider the average value, $\bar{S}(n)$, of products offered individuals in $[\underline{n}, n]$ assuming that they choose the same level of the signal, \underline{y} , as type \underline{n} ,

$$(9) \quad \bar{S}(n) = \frac{\int_{\underline{n}}^n S(x, \underline{y}) F'(x) dx}{F(n)}$$

Integrating by parts, this can be rewritten as

$$\bar{S}(n) = S(n, \underline{y}) - \frac{\int_{\underline{n}}^n S_1(x, \underline{y}) F(x) dx}{F(n)}$$

Then differentiating totally with respect to n we have:

$$\bar{S}'(n) = F'(n) \left\{ \frac{\int_n^n S_1(x, y) F(x) dx}{F(n)^2} \right\}$$

At n both the numerator and the denominator of the bracketed expression and their first derivatives disappear. Applying l'Hôpital's Rule twice we therefore have:

$$(10) \bar{S}'(n) = F'(n) \left\{ \frac{S_{11}(n, y) F(n) + S_1(n, y) F'(n)}{2F(n)F''(n) + 2F'(n)^2} \right\}$$

$$= 1/2 S_1(n, y), \text{ since } F(n) = 0 \text{ and } F'(n) > 0.$$

Suppose an agent offers the average value $\bar{S}(n)$, over some interval $[n, n]$, to anyone with the minimum level of the signal. The welfare of type n , if he accepts this new offer is

$$(11) \tilde{U}(n) = U(n, y, \bar{S}(n))$$

Consider now the class of offers of the form $\langle y, \bar{S}(n) \rangle$ for different values of n . Clearly, $\tilde{U}(n) = U^*(n)$. Moreover, differentiating (11) with respect to n , we have:

$$\begin{aligned} \frac{d}{dn} \tilde{U}(n) &= U_1 + U_3 \bar{S}'(n) \\ &= U_1 + 1/2 U_3 S_1(n, y) \quad \text{from (10)} \\ &> U_1(n, y, S(n, y)) \quad \text{since } S_1, U_3 > 0 \\ &= \frac{d}{dn} U^*(n) \quad \text{from (8).} \end{aligned}$$

The profile $\tilde{U}(n)$ is, therefore, steeper at n and hence lies above $U^*(n)$ for some interval $(n, n_1]$. Then suppose the new offer is of the form

$$\langle y, \bar{S}(\tilde{n}) \rangle \text{ where } \tilde{n} \in (n, n_1].$$

For those in $(\underline{n}, \tilde{n}]$, the welfare resulting from taking the new offer is

$$\begin{aligned} & U(\underline{n}, \underline{y}, \bar{S}(\tilde{n})) \\ & \geq U(\underline{n}, \underline{y}, \bar{S}(\underline{n})) \quad \text{since } \underline{n} \leq \tilde{n} \\ & = \bar{U}(\underline{n}) \quad \text{from (11)} \\ & > U^*(\underline{n}) \quad \text{since } \underline{n} \in (\underline{n}, \underline{n}_1]. \end{aligned}$$

Therefore, all those individuals with $\underline{n} \in [\underline{n}, \tilde{n}]$ prefer the new offer. Since the average value of the products offered by those in $[\underline{n}, \tilde{n}]$ is $\bar{S}(\tilde{n})$ the agent making the new offer would just break even were no one else to accept. However, $\tilde{n} \in (\underline{n}, \underline{n}_1]$ which implies that $\bar{U}(\tilde{n}) > U^*(\tilde{n})$. It follows that there is some interval $(\tilde{n}, \underline{n}_2)$ over which the new offer is also preferred. Sellers from this interval raise the average value of products to $\bar{S}(\underline{n}_2)$, thereby making the new offer strictly profitable.

We have, therefore, proved:

Theorem 4: Given an informationally consistent set of offers $\langle \underline{y}, w(\underline{y}) \rangle$, there is an alternative offer in the neighborhood of that accepted by those at the lower endpoint of the distribution, which, if made by a single agent, is strictly profitable.

To clarify the intuition behind this result, consider the welfare level of type \underline{n} and some other type \tilde{n} , given that the price function $w(y)$ is INC. Utilizing Taylor's expansion we have:

$$\begin{aligned} U^*(\tilde{n}) - U^*(\underline{n}) &= \frac{d}{dn} U(\underline{n}, \underline{y}(\underline{n}), w(\underline{y}(\underline{n}))) \Big|_{\underline{n}=\underline{n}} (\tilde{n} - \underline{n}) + O(\tilde{n} - \underline{n})^2 \\ &= U_1(\underline{n}, \underline{y}, w(\underline{y})) (\tilde{n} - \underline{n}) + O(\tilde{n} - \underline{n})^2 \quad \text{from (8)} \end{aligned}$$

Therefore, for types who are similar, differences in the level of the signal have a negligible effect on the levels of welfare achieved. Expressing this differently, to a first order approximation, type \tilde{n} would be as well off choosing the same level of the signal as type \underline{n} . This follows because $w(y)$ is the lower envelope, for all \underline{n} , of the sets of offers preferred to $\langle \underline{y}(\underline{n}), w(\underline{y}(\underline{n})) \rangle$.

If, however, types \underline{n} and \bar{n} are pooled, there is a first order impact upon the average value of the product. Hence, if the two types are paid the average, both are strictly better off. Then by paying a slightly lower price, the gains are shared by buyer and sellers.

This argument cannot, however, be extended to alternative offers at levels of the signal other than the minimum, y . The problem is that for any such "interior" offer the higher price attracts not only sellers of superior quality products, but also those with inferior products. Intuitively one might anticipate that any conclusions about the profitability of these offers will depend upon the distribution of the unobservable n . Indeed, this is the case. As long as the density function declines sufficiently rapidly with increasing n , the only alternative profitable offers are those in the neighborhood of the lower endpoint of the distribution. However, if the density function increases with n sufficiently rapidly, the losses incurred as a result of attracting inferior quality products are more than offset by the larger number offering superior quality products.

For a proof of these statements, we consider the profitability of an alternative offer at some level of the signal y , greater than the minimum, that is

$$\langle \tilde{y}, \tilde{w} \rangle = \langle y(\bar{n}), \tilde{w} | \bar{n} > \underline{n}, \tilde{w} > w(\tilde{y}) \rangle.$$

Clearly this offer is preferred by type \bar{n} , and for \tilde{w} sufficiently close to $w(\tilde{y})$ there exist types $n_1 (< \bar{n})$ and $n_2 (> \bar{n})$ who are just indifferent to the new offer. Moreover, if $w(y)$ is informationally consistent, type n_1 maximizes welfare by choosing the offer $\langle y(n_1), w(y(n_1)) \rangle$. Therefore, at all other observed levels of the signal $y(n)$, the offered price $w(y(n))$ must be less than that necessary to attract type n_1 . Formally, the indifference curve,

$$(12) \quad U(n_1, y(n), w) = U(n_1, y(n_1), w(y(n_1)))$$

which we can express as

$$(13) \quad w = w_1(n)$$

just touches the profile of informationally consistent offers $w_c(n) = w(y(n))$ at n_1 . Two such indifference curves are depicted in Figure 4. By construction n_1 and n_2 are the values of n for which these indifference curves pass through the new offer $\langle \tilde{y}, \tilde{w} \rangle$.

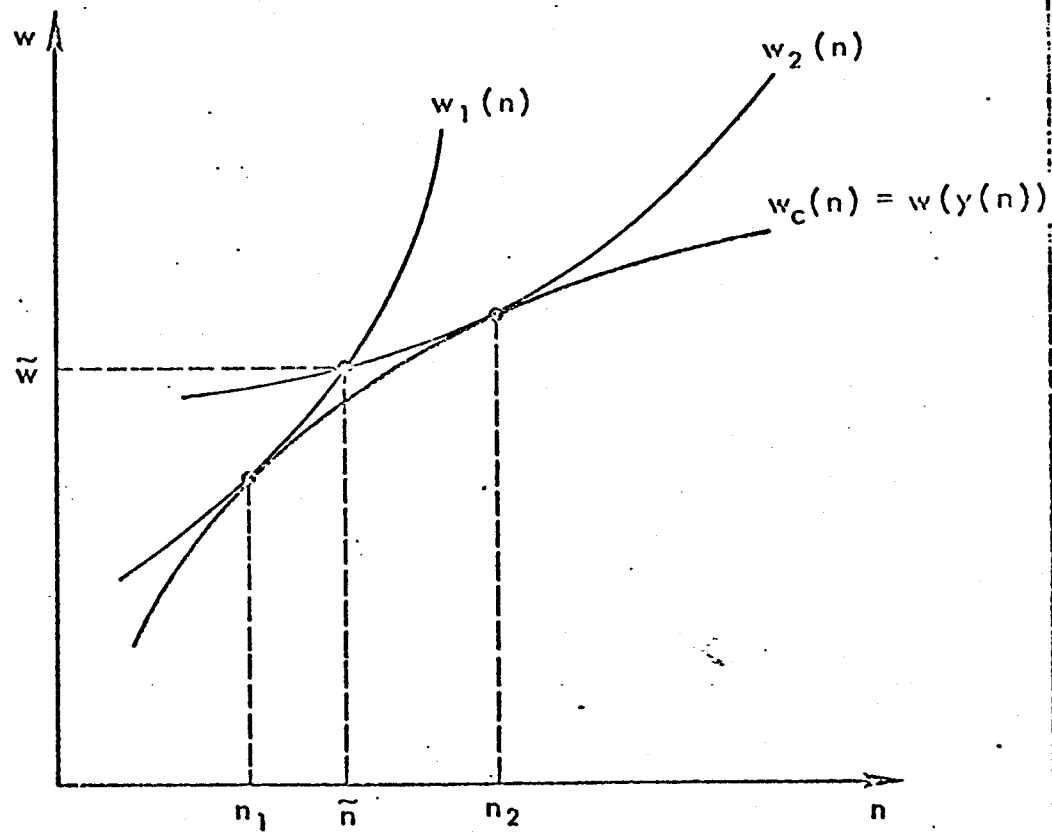


Fig. 4 — Interior Unraveling

Assumption A4 implies that at any intersection of two indifference curves satisfying (12), the steeper curve belongs to the individual with the lower value of n . It follows that the new offer is preferred by type n if and only if $n \in (n_1, n_2)$.

Before discussing the profitability of the new offer for the general case, consider the following simple example first discussed by Spence (1974) in his discussion of labor market signalling.⁵

$$U(n, y, w) = w - \frac{y}{n}, \quad S(n, y) = n$$

w is to be interpreted as the wage, and $\frac{y}{n}$ the cost of an education y for an individual of type n . Spence shows that for informational consistency,

$$w(y) = (2y + k)^{1/2}.$$

Since $w(y) = S(n, y) = n$, we can substitute for $w(y)$ to obtain:

$$y(n) = \frac{1}{2}n^2 - k.$$

Therefore, $U(n, y(n), w) = w - \frac{1(n^2 - k)}{2n_1}$ and the indifference curve

(12) becomes

$$w = \frac{1}{2}(n_1 + \frac{n^2}{n_1}).$$

For types n_1 and n_2 this curve must pass through (\bar{n}, \bar{w}) . We can, therefore, solve for \bar{w} , obtaining:

$$\bar{w} = \frac{1}{2}(n_1 + n_2)$$

The profit resulting from the new offer is therefore:

$$\int_{n_1}^{n_2} [s(n, \bar{y}) - \bar{w}] F'(n) dn = \int_{n_1}^{n_2} [n - \frac{1}{2}(n_1 + n_2)] F'(n) dn.$$

It follows immediately that the new offer is strictly profitable if and only if

the distribution function $F(n)$ is strictly convex in the neighborhood of \tilde{n} .

Of course, in general it is not possible to solve analytically for $w(y)$. However, we now show that if the density function is sufficiently steep, its sign determines the profitability of any new offer in the neighborhood of $w(y)$.

First our assumptions must be strengthened as follows:

A1'. $U(n,y,w)$ and $S(n,y)$ are thrice differentiable functions

A5". The density function of the unobservable, n , is thrice differentiable.

Given A1', the indifference curves $w = w(n)$ for each type and the lower envelope $w = w_c(n)$ are thrice differentiable functions.⁶

Since the two are tangential at n_1 we can apply Taylor's expansion to obtain,

$$(15) \quad w_1(n) - w_c(n) = [w_1''(n_1) - w_c''(n_1)] (n - n_1)^2 + 0(n - n_1)^3.$$

Moreover the two indifference curves $w_1(n)$, $w_2(n)$ intersect at $\langle \tilde{n}, \tilde{w} \rangle$.

Then

$$\begin{aligned} [w_1''(n_1) - w_c''(n_1)] (\tilde{n} - n_1)^2 + 0(\tilde{n} - n_1)^3 &= [w_2''(n_2) - w_c''(n_2)] (\tilde{n} - n_2)^2 \\ &+ 0(\tilde{n} - n_2)^3 \end{aligned}$$

Taking the square root of both sides it follows that to a first order of approximation, \tilde{n} lies midway between n_1 and n_2 , that is

$$(16) \quad \tilde{n} = \frac{1}{2} (n_1 + n_2) + 0(n_2 - n_1)^2$$

Also $w_c(\tilde{n}) = S(\tilde{n}, \tilde{y})$. Then from (15) we have:

$$(17) \quad \tilde{w} - S(\tilde{n}, \tilde{y}) = 0(n_2 - n_1)^2.$$

In addition, we shall require the following mathematical lemma.

Lemma 5: If $\bar{n} = \frac{1}{2}(n_1 + n_2)$ $\tilde{n} \in [n_1, n_2]$ and $\pi(n)$ is thrice differentiable on $[n_1, n_2]$, the integral $\int_{n_1}^{n_2} \pi(n)dn$ can be approximated as follows:

$$\begin{aligned} \int_{n_1}^{n_2} \pi(n)dn &= \pi(\bar{n}) (n_2 - n_1) + \pi'(\tilde{n}) (\bar{n} - \tilde{n}) (n_2 - n_1) \\ &+ \frac{1}{2} \pi''(\tilde{n}) \left[\frac{1}{12} (n_2 - n_1)^3 + (\bar{n} - \tilde{n})^2 (n_2 - n_1) \right] + O(n_2 - n_1)^4 \end{aligned}$$

Proof:

Taking a Taylor's series expansion of $\pi(n)$ around \bar{n} , we have $\forall n \in [n_1, n_2]$.

$$\begin{aligned} (18) \quad \frac{1}{6} \{ \min_{n \in [n_1, n_2]} \pi'''(n) \} (n - \bar{n})^3 &\leq \pi(n) - \pi(\bar{n}) - \pi'(\bar{n}) (n - \bar{n}) - \frac{1}{2} \pi''(\bar{n}) (n - \bar{n})^2 \\ &\leq \frac{1}{6} \{ \max_{n \in [n_1, n_2]} \pi'''(n) \} (n - \bar{n})^3 \end{aligned}$$

$$\text{Also } \int_{n_1}^{n_2} (n - \bar{n}) dn = (\bar{n} - \bar{n}) (n_2 - n_1)$$

$$\text{and } \int_{n_1}^{n_2} (n - \bar{n})^2 dn = \frac{1}{6} (n_2 - n_1)^3 + (\bar{n} - \bar{n})^2 (n_2 - n_1)$$

Finally, since $(n - \bar{n})^3 \leq (n - n_1)^3$, $\forall \bar{n} \in [n_1, n_2]$

$$\int_{n_1}^{n_2} (n - \bar{n})^3 dn \leq \frac{1}{4} (n_2 - n_1)^4$$

Integrating over the inequalities in (18) then yields the desired result.

Q.E.D.

We can now demonstrate:

Theorem 5 If the slope of the density function of the unobservable n is sufficiently large at \bar{n} , there are alternative offers $\langle y(\bar{n}), \tilde{w} \rangle$ close to $\langle y(\bar{n}), w(y, (\bar{n})) \rangle$, which are strictly profitable. If the slope is sufficiently large and negative, all such offers yield losses.

Proof:

Since the new offer $\langle \tilde{y}, \tilde{w} \rangle$ is preferred by all those in (n_1, n_2) , the resulting profit is

$$\int_{n_1}^{n_2} \pi(n) dn = \int_{n_1}^{n_2} [S(n, \tilde{y}) - \tilde{w}] F'(n) dn$$

Given assumptions A1' and A5', we can apply Lemma 5 to obtain a third order approximation. Noting from (16) and (17) that both $[S(\tilde{n}, \tilde{y}) - \tilde{w}]$ and $\bar{n} - \tilde{n}$ are of second order in $(n_2 - n_1)$, we have:

$$\pi(\tilde{n}) (n_2 - n_1) = [S(\tilde{n}, \tilde{y}) - \tilde{w}] F'(\tilde{n})$$

$$\pi'(n) (n_2 - n_1) (\bar{n} - \tilde{n}) = S_1(\tilde{n}, \tilde{y}) F'(\tilde{n}) (n_2 - n_1) (\bar{n} - \tilde{n}) + O(n_2 - n_1)^4$$

$$\pi''(\tilde{n}) \left[\frac{1}{6} (n_2 - n_1)^3 + (\bar{n} - \tilde{n})^2 (n_2 - n_1) \right] = \frac{1}{6} [S_{11}(\tilde{n}, \tilde{y}) F'(\tilde{n})$$

$$+ S_1(\tilde{n}, \tilde{y}) F''(\tilde{n})] (n_2 - n_1)^3 + O(n_2 - n_1)^4.$$

Combining these expressions, the profit resulting from the new offer can be expressed as

$$\int_{n_1}^{n_2} \pi(n) dn = F'(n) \left\{ \begin{aligned} & [S(\tilde{n}, \tilde{y}) - \tilde{w}] + S_1(\tilde{n}, \tilde{y}) (n_2 - n_1) (\bar{n} - \tilde{n}) + \frac{1}{6} S_{11}(\tilde{n}, \tilde{y}) (n_2 - n_1)^3 \\ & + \frac{1}{6} S_1(\tilde{n}, \tilde{y}) (n_2 - n_1)^3 \left(\frac{F''(\tilde{n})}{F'(\tilde{n})} \right) + O(n_2 - n_1)^4 \end{aligned} \right\}$$

Since n is a qualitative variable there is no theoretical presumption about the sign of the third term inside the large brackets. Moreover, the sign of the second term does not seem amenable to further analysis. However, none of the first three terms inside the large bracket depend upon the distribution of n . Since from assumption A2, S_1 is strictly positive, it follows that if $\left| \frac{F''(n)}{F'(n)} \right|$ is sufficiently large the fourth term dominates.

Q.E.D.

This last result is important, because it indicates that in order to ensure the viability of equilibrium, it is not sufficient to impose some special assumption about the lower endpoint. However, given a density function which declines sufficiently rapidly to make all new "local" interior

offers unprofitable, what assumptions would eliminate the endpoint problem?

The simplest approach is to assume that there is a mass point at the lower end of the distribution (in the labor market, the army of the proletariat!). Then the greater value of some of the products for sale at any higher bid price has a negligible effect upon average value. As a result, any agent making such a bid suffers losses.

A more subtle approach is to modify the assumptions of the model in such a way that the marginal cost of signalling increases without bound as n approaches \underline{n} from above. Returning to the special labor market example, we have from equation (14)

$$y(n) = \frac{1}{2}n^2 - k$$

The highest possible value of k is $\frac{1}{2}\underline{n}^2$ since $y(n) \geq 0$. In this case, the net welfare associated with the informationally consistent price function is

$$\begin{aligned} U^*(n) &= w(y(n)) - \frac{y(n)}{n} \\ &= \frac{1}{2}n + \frac{n}{n} \end{aligned}$$

Note that for $\underline{n} > 0$, $\frac{d}{dn}U^*(\underline{n}) = 0$. That is, those types with slightly greater n are no better off than type \underline{n} . Clearly, all would prefer to be offered a price equal to the average value of their products.

In contrast, for the limiting case $\underline{n} = 0$, $U^*(n) = \frac{1}{2}n$. Moreover, if the distribution function is strictly concave, the average value over any interval $[0, n]$ is less than the median value, $\frac{1}{2}n$. Then if $\underline{n} = 0$, a new pooling offer, at the lower endpoint of the distribution, is not profitable. We have already seen that for this simple model, concavity rules out profitable interior offers. Therefore, if $\underline{n} = 0$ the INC price function $w(y) = (2y)^{1/2}$ is viable.⁷

Whether this result is generalizable, remains an open question. However, while it indicates the possibility of a "competitive" informational equilibrium, the special assumptions required are hardly satisfactory. We now explore the viability issue, in the absence of such assumptions.

V. ALTERNATIVE EQUILIBRIUM CONCEPTS

In the last section, we examined the nature of informationally consistent (INC) price functions, that is, price schedules for which the signal provided accurate information as to the value of the product offered for sale. Here it is useful to extend the notion of informational consistency. A set of contracts will be described as weakly informationally consistent (WINC), if the price paid to agents with a given signal is equal to the average value of the products traded.

For example, if all agents choose the same level of the signal, \tilde{y} , and the single price offered, \tilde{w} , is equal to the average value of all traded products, $\int_{\underline{n}}^{\bar{n}} S(n, \tilde{y}) dF(n)$, $\langle \tilde{y}, \tilde{w} \rangle$ is WINC. Alternatively, a WINC set of contracts may separate out some subset of the agents, while the others are in one or more heterogeneous pools.

We now analyze the viability of informationally consistent sets of contracts. First, consider the usual Nash or "perfectly competitive" equilibrium concept. In our context, this can be summarized as follows.

Nash Equilibrium: A set of WINC contracts is a Nash equilibrium if and only if there exists no alternative offer which, if made by a single agent would raise that agent's profits.

From Theorem 4 it follows immediately that there is no INC Nash equilibrium. Moreover, it is easy to show that, whenever heterogeneous types are pooled, there exist alternative profitable offers, which draw away the sellers of higher valued products.

Consider a pooling contract, $\langle \tilde{y}, \tilde{w} \rangle$, as depicted in Figure 5. Given assumption A4, the types for whom this is maximal, must be the elements of a closed interval $[n_1, n_2]$. Moreover, for the contract to be WINC, the price paid, w , must be equal to the average value, $\bar{S}(y) = \int_{n_1}^{n_2} S(n, \tilde{y}) dF(n)$, of the products sold by types $n \in [n_1, n_2]$. Then the curve $w = S(n_2, y)$ lies strictly above $\langle \tilde{y}, \tilde{w} \rangle$. Type n_2 are just indifferent between the pooling contract and a new offer C . Hence, if the latter is offered, some will accept. Since all those with lower n strictly prefer $\langle \tilde{y}, \tilde{w} \rangle$, everyone accepting the new offer yields a net gain CD to the agent making the new offer. Then no pool of different types is viable in a Nash equi-

librium. Combining this with the previous result, we have therefore proved:

Theorem 6: There is no Nash equilibrium set of (weakly) informationally consistent contracts.

As a possible alternative equilibrium concept, Rothschild/Stiglitz have suggested that, in a world of imperfect information, agents might only consider new offers "close to" the initial set of contracts. However, the results summarized in Theorem 4 apply in any arbitrarily small neighborhood of the lower endpoint of the distribution. Moreover, by shifting the new offer C around the indifference curve AB in the direction of A, pooling contracts are again unstable. Hence, for a continuously distributed unobservable, there is no WINC local Nash equilibrium.

Given this negative conclusion, how might such a market in fact behave? Presumably, in the absence of collusion, each agent would eventually learn to expect some reaction by other agents, to changes in his own list of offers. The question we shall now consider is whether there are relatively simple reaction functions which, if followed by all agents, support an equilibrium.⁸

Consider again the method by which an INC price function is "unraveled". The agent making the new offer profits by attracting a pool of heterogeneous types. But, as we have seen (Figure 5), it is then easy for a second agent to profitably attract the sellers of the highest quality products from this pool. As a result, the first agent suffers losses.⁹ Then if agents learn to anticipate such reactions it is surely plausible that they will choose not to make this type of alternative offer. These ideas suggest the following strategic equilibrium concept.

Reactive Equilibrium: A set of offers is a Reactive equilibrium if, for any profitable enlargement of the set of offers by one agent, there is a further enlargement of the set, which yields profits to a second agent and losses to the first. Moreover, a third enlargement of the set of offers does not result in losses to the second agent.

Note first that, in contrast to a game theoretic solution with strategic threats, the reaction envisaged here is restricted to be strictly profitable. Second, the reaction is not a response by all agents in the market, but simply by a second agent who "undercuts" the new offer as long as it is offered. This seems sensible in an imperfectly competitive world of many agents. Third, the reacting agent, unlike the initiator cannot suffer losses as a result of further undercutting. At worst, no one accepts his new offer.

From the discussion at the beginning of this section, it is clear that no INC price function can be broken by a pooling offer in a Reactive Equilibrium. Furthermore, from Theorem 1, the family of INC price functions can be written as $\{\langle y, w(y, k) \rangle \mid k \leq \bar{k}, w_k > 0\}$

Since $w(y, k)$ is larger for higher k , it follows that the family of INC price functions are Pareto ranked according to k . Then the Pareto dominating member $w(y, \bar{k})$ must be a Reactive equilibrium.

Now suppose the initial set of contracts is some other INC price function $w(y, k)$, $k < \bar{k}$. Since the complete schedule $w(y, \bar{k})$ is Pareto dominating, it would, if offered, attract all levels of n . Moreover, if an agent were to restrict its new offers to the set

$\{\langle y, w(y, \bar{k}) \rangle \mid y \leq y(n_1), n \in (n_1, n_2)\}$, he would just break even on all offers at levels of the signal in $[y, y(n_1))$. However, the offer $\langle y(n_1), w(y(n_1), \bar{k}) \rangle$ would be strictly preferred by types in some sub-interval $[n_1, n_2)$. The average value of the products traded at this level of the signal would then exceed $w(y(n_1), \bar{k})$, making the new set of offers strictly profitable. Since the buyer either breaks even or makes profits on all contracts, there is no reaction which can induce losses. We have therefore proved:

Theorem 7: The Pareto dominating member of the family of INC price functions is a Reactive Equilibrium.

While this theorem is somewhat reassuring, there is an obvious objection which merits further attention. Any attempt to describe simple decision rules that result in an equilibrium in which expectations based on signals are fulfilled, no matter how plausible, is to some degree ad hoc. We

must therefore ask whether there are other decision rules which also imply the viability of an informational equilibrium. And if so, do these alternative rules lead to different equilibria?

Wilson's discussion of insurance markets, with a finite number of risk classes, suggests that the answer to both questions is in the affirmative. His suggestion is that instead of one or more agents aggressively reacting to new opportunities for profit, all buyers develop a defence mechanism. Each agent is assumed to be able to anticipate the implications of a new offer, upon the profitability of all his own offers. Those offers which become unprofitable, as a result of the new offer, are simply withdrawn. Given this behavior, the agent contemplating the introduction of a new offer must ask whether it will remain profitable in the face of such a withdrawal. We can summarize this equilibrium concept as follows.

Wilson Anticipatory Equilibrium: A set of WINC offers is an anticipatory equilibrium, if there is no new set of offers which earns non-negative profits for each offer, and strictly positive profits for some new offer, after those offers in the original set, which have become unprofitable, have been withdrawn.

Focussing upon the insurance market model, with a finite number of risk classes, Wilson has shown that in general there is a WINC anticipatory equilibrium. Unfortunately, his method of proof does not extend to the case, discussed in this paper, in which an unobservable characteristic is continuously distributed. However, after applying the well-known counter-example criterion, it is my conjecture that the theorem continues to hold.

From Theorem 4 we know that for any INC price function there is an alternative profitable offer which does not result in losses to any agents. Instead, there are simply no takers for other offers close to the bottom of the distribution. Then withdrawing these offers has no effect on the profitability of those remaining. We therefore have:

Theorem 8: Given assumptions A1 - A5', there is no INC Wilson anticipatory equilibrium.

Moreover, the above argument establishes that for any WINC Wilson equilibrium, there must be some interval of types $n \in [\underline{n}, \underline{n}_1]$ at the lower tail of the distribution who will accept the same "pooling" offer.

The result summarized in Theorem 5 is also revealing. Once again, the new offer does not create losses for any of those transacting at prices on the INC price function. Then, if the distribution function is sufficiently convex over $[\underline{n}, \underline{n}_1]$, there can be no subinterval over which the Wilson equilibrium is informationally consistent. It follows that for such cases the Wilson equilibrium set of contracts consists of one or more contracts, accepted by pools of heterogeneous types.

VI. FINAL REMARKS

The question posed in the introduction was whether equilibria in which agents' actions reveal valuable information to other agents, are viable. The first stage in the analysis involved a characterization of a set of informationally consistent contracts, that is, contracts for which each agent's expectations are realized. It was then shown that, for a fairly general class of economies, there is no set of informationally consistent contracts with the property that every additional contract at best breaks even.

It should be noted that the lack of a Nash equilibrium is not a phenomenon specific to this class of models. Recently, Grossman (1975) and Grossman and Stiglitz (1976) have explored the implications of agents using commodity futures and asset price movements, as signals of "insider" information. It is shown that if these signals are "consistent" in the sense that they provide outsiders with all useful information, there is no competitive "rational expectations" equilibrium. Related results can also be found in Green (1973) and Hart (1975).

Given the unsatisfactory nature of the Nash equilibrium concept, two attempts to characterize an imperfectly competitive equilibrium were then examined. In each, the assumption that agents ignore any response by other agents was relaxed. Since in both cases the modifications were relatively modest in scope, it could be argued that it was not necessary to stray far from the security of the neo-Walrasian fold, in the search for a market equilibrium. However, the results of the previous section indicate that the nature of the equilibrium is very sensitive to the type of response function built into the model. The Reactive equilibrium set of contracts is an informationally consistent differentiable price function. In contrast, Wilson's anticipatory equilibrium is never informationally consistent across all types, and under strong assumptions, may involve trading at only a single price.

The contrast between the imperfectly competitive equilibria analyzed above and the Arrow-Debreu, complete market equilibrium, is further sharpened by a consideration of welfare implications. From Theorem 4, there is always a set of contracts which would be preferred by all agents to the Reactive equilibrium set, and strictly preferred by those near the lower tail of the distribution. Moreover, Wilson has shown that

it is relatively easy to construct examples in which an alternative set of contracts is strictly preferred by all agents, to the Anticipatory equilibrium set.¹⁰

It therefore seems fair to conclude that the imperfectly competitive market structures described above are a major departure from the standard neo-Walrasian model.

This paper has indicated that the transmission of information via markets can be explained as an equilibrium phenomenon. However, the extent to which information is transferred even in our highly-structured model, remains unclear. Further resolution must await an exploration of the robustness of alternative non-myopic equilibrium concepts.

Footnotes

* The helpful comments and suggestions of M. Rothschild, J. A. Mirrlees, J. E. Stiglitz, M. A. Walker and C. A. Wilson are gratefully acknowledged.

¹In neither the discussion by Spence (1974) nor Riley (1975) is this formally established, although in the latter, strong necessary conditions are obtained.

²This corrects an error in the proof of non-existence for the labor market model, appearing in my earlier paper, Riley (1975).

³This alternative model was suggested originally by Mirrlees. For another human capital interpretation see Riley (1976).

⁴The method of proof follows closely that suggested by Stiglitz early in 1974. A later and apparently more general proof appearing in Riley (1975) is, unfortunately, flawed.

⁵I am grateful to Mirrlees for this example.

⁶Given Lemma 1 and Assumption A1, equation (1) can be inverted and differentiated twice. Then $y(n)$ and hence $w_c(n) = S(n, y(n))$ are thrice differentiable.

⁷For $\underline{n} = 0$, the welfare function $U(n, y, w) = w - y/n$ is undefined, thereby violating Assumption A1.

⁸The remainder of this section has been influenced by Wilson's elegant discussion of the viability of equilibrium in insurance markets.

⁹Strictly speaking, the alternative offer C, attracting as it does, a set of agents of zero measure, does not lead to finite losses for the first agent. However one can construct more complicated offer schedules that (i) yield finite profits to the second firm, (ii) impose finite losses on the first firm, and (iii) at worst have no takers, if additional contracts are introduced.

¹⁰However, it is the case that Wilson's Anticipatory Equilibrium is Pareto efficient, with respect to sets of weakly informationally consistent contracts.

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