

LIQUIDITY AS FLEXIBILITY

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Discussion Paper #73

July 1976

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Current actions frequently influence the costs of following different courses of future action: a house might be less costly to build on land left unimproved than on land occupied by another structure; fewer additional resources might be required to increase output in a plant equipped with one type of machinery than another; lower "economic losses" might be sustained in liquidating a portfolio of government bonds than one of rare coins; shopkeepers might be unable to deliver on short notice goods not previously ordered and held in inventory; an act of consumption today reduces the range of consumption levels attainable tomorrow. Economic decisions will be influenced, in part, by the implications of current actions for the cost -- or even possibility -- of undertaking different actions in the future.

A current action leaves an agent in some "position." The position may refer to the state of development of a piece of land, a portfolio held, contract entered into -- whatever is appropriate to characterize the current decision. Future action will leave the agent in a future position. Let us call one current position more flexible than another if the range of alternative future positions attainable from it, at any given level of cost, is larger than that of the other. Put another way, the more flexible is the initial position, the lower is the cost of reaching any alternative position in the future.

Flexibility -- having more future options available at lower cost -- is clearly desirable from an individual agent's viewpoint. This paper explores those factors determining the value of flexibility, focussing especially on

what the agent expects learn in the future about uncertain events in the still more distant future. The conclusion we hope to convey is this: in a large number of fairly general situations, information and flexibility are complementary. The more one expects to learn in the near future (the more rapidly will ultimate uncertainties be dispelled), then the more valuable will be any increment in flexibility. The analysis makes clear that risk-aversion is in no way essential for the desirability of flexibility (indeed, its presence can even diminish the attractiveness of flexible positions); and that, although their values are highly dependent on each other, the prospect of further information must be sharply distinguished from keeping options open.

The notion of flexibility has arisen in various economic contexts. F. Lavington (1921, pp. 91-97) provides a superb early discussion of what he terms "risk arising from the immobility of invested resources." G. Stigler (1939) describes one plant as being more flexible than another if it has a flatter average cost curve; this is further elaborated by C. Tisdell (1968) and M. Merkhofer (1975). T. Marschak (1962) suggests maintaining flexibility in research and development strategies by running parallel programs. The possible importance of environmental preservation in keeping open future options (the "irreversibility effect") is brought out by Fisher, Krutilla and Cicchetti (1972), Arrow and Fisher (1974) and C. Henry (1974a, 1974b). That individuals might have a distinct preference for "postponement of choice" is explored by T. Koopmans (1964). T. Marschak and R. Nelson (1962) note the potential usefulness of flexibility as an economic concept and discuss possible general definitions.

Another, more familiar, term has been used in discussions of portfolio choice. Some assets are more liquid than others -- "more certainly realizable

at short notice without loss (Keynes, 1930, p. 67)." The term liquidity has been used to refer both to an asset's certainty of yield (including capital gains or losses) and to its difference between purchase and sale price (including all real costs of transacting). Current usage tends more toward the latter sense (Note the progression in Hicks' view of liquidity through 1962, 1967, 1974; see also Hirshleifer, 1972, and Cropper, 1976.), and that is the sense in which we shall use the term. Holding a larger quantity of liquid assets consequently leaves an agent in a more flexible position since his portfolio can be transformed into other assets or goods at lower cost. H. Makower and J. Marschak (1938) thus describe more liquid assets as being more "plastic." The role played by the prospect of additional information in determining the demand for liquid assets is developed by J. Hirshleifer (1972), S. Goldman (1974) and J. R. Hicks (1974).

In most of the above works, the belief that new information will emerge, that something more will be learned with the passage of time, is essential. The classic discussions of A. G. Hart (1942; 1947, pp. 421-22) warrant special mention for forcefully suggesting a connection between the value of remaining flexible and the accumulation of information over time.

The next section develops the concept of flexibility and introduces the notion of an information structure. Section II presents a simple portfolio choice problem in detail to illustrate the interaction between the two concepts. Section III contains general propositions, and provides counterexamples to seemingly plausible conjectures. Section IV applies these results to consumption-saving decisions and the choice of plant capacity. Conclusions, connections with general equilibrium and macroeconomic issues are pursued in section V.

I. Actions, Information and Payoffs

The issue of flexibility arises only in sequential decision problems: there must be more than one opportunity to act. Let us consider problems with time horizons of just three periods, the minimum necessary for our purposes: In the first period the agent must choose an initial position, a first period action, while uncertain about what future events might occur. In the intermediate period observation may change the agent's beliefs about the likelihood of various eventualities, and he has another opportunity to act, choosing a second position. In the final period all relevant uncertainties are resolved, and the consequences of these actions are revealed. What concerns us is how the prospect of learning in the second period about events to become known in the third period influences the agent's first period choice.

The consequences of this process accrue to the agent as a payoff, which depends both on the sequence of positions he chooses and on the sequence of events beyond his control which occurs. This payoff is designated by a function $f(a,b,s)$: a refers to the agent's first period position, b to his second period position, and s to the "state of the world" (sequence of events beyond the agent's control) as of the final period. The payoff may be measured in units of wealth, utility or any other appropriate objective. The agent is assumed to maximize his expected payoff.

Flexibility is a property of initial positions: it says something about the cost of moving to various second period positions. In order to rank initial positions according to their flexibility, some part of the total payoff $f(a,b,s)$ must be imputed to the move from a to b as distinct

from having been in positions a and b. In other words the payoff must be decomposed into a form

$$f(a,b,s) = r(a,s) + u(b,s) - c(a,b,s) \quad (1)$$

where $r(a,s)$ represents the return to the first period action, $u(b,s)$ represents the return to the second period action and $c(a,b,s)$ represents the cost of switching from a to b. If a and b were portfolios of assets, for example, $r(a,s)$ and $u(b,s)$ could be the yields on the two portfolios over the first and second time intervals respectively, including dividends, interest and capital gains (defined, say, as the difference between the costs of acquiring the portfolio at the beginning and end of a time interval); and $c(a,b,s)$ could be the cost of liquidating those assets in a which are not in b, including commissions, penalties and any bid-ask spread on those assets liquidated. However any such decomposition is, to some extent, arbitrary. Certain requirements must be imposed on the switching cost function to render it meaningful.

We wish the switching cost function to capture the notion that, in many situations, one can move automatically, without any explicit action or effort, from a first period position to certain second period positions. Such a move will be called staying in the same position, and the switching costs involved will be 0. The particulars of any given problem must be relied upon to determine the most natural, or automatic, second period positions following from each first period position. For problems in which the first and second period alternatives are similar in form this may pose no difficulties: for example, if the positions are portfolios of assets, then staying in the same position could mean continuing to hold the same portfolio; if the positions refer to the good being produced by a plant capable

of producing different goods (with suitable adjustments to equipment and retraining of employees), then staying in the same position could mean continuing to produce the same good; if the positions mean the presence or absence of a hydroelectric development on a particular river, then staying in the same position could mean leaving the river in its previous state of development. But for problems in which the first and second period alternatives are quite different in nature the association of first period positions with "most natural" second period positions becomes a more delicate issue: The first period position might be the choice of fixed technology to be embodied in a new plant, while the second period position is the level of output at which the plant is later operated. In such a situation one might wish to associate with each technology a level of future output at which its cost advantage over other technologies is at a maximum (the earlier choice is not regretted given the later choice), and call that level of output staying in the same position. By such a relabeling of the alternatives for choice open in the two periods we can thus think of the positions chosen as coming from the same set, and require that $c(a,a,s) = 0$ for any a chosen and s which occurs. That some moves are technically impossible, such as switching from completely destroying a species to recreating it, can be captured by an arbitrarily large $c(a,b,s)$; that some positions are technically impossible to occupy during one or the other periods can be captured by assigning an arbitrarily large negative value to either $r(a,s)$ or $u(b,s)$, so that a rational agent is effectively precluded from making such choices. We shall assume furthermore that $c(a,b,s) \geq 0$ for all a,b,s .

If an imputation of switching costs with the above characteristics is found, then position a is defined to be more flexible than position a' if $c(a', b, s) \geq c(a, b, s)$ for all $b \neq a'$ and all s . The extremes of this ordering are a perfectly flexible position, designated by a^* , for which $c(a^*, b, s) = 0$ for all b and s ; and a completely irreversible position, designated by \bar{a} , for which $c(\bar{a}, b, s)$ is unboundedly large for all $b \neq \bar{a}$. One position is consequently more flexible than another if the set of alternative second positions attainable from it, at any given level of switching costs, is at least as large as that of the other.

Two limitations of this concept of flexibility should be mentioned. First, the ordering it produces of initial positions is only partial. Some pairs of positions may remain unranked by flexibility; indeed, in some contexts, no position may be ranked as more flexible than any other. Thus flexibility may not be a useful concept for analyzing all types of sequential decision problems. Second, the ordering produced may depend on a somewhat arbitrary imputation of switching costs if they are not provided by the particulars of a problem. If this is the case, then the appropriate strategy of positions (which depends only on $f(a, b, s)$) will not be affected, but the explanation analysis provided for why the strategy is optimal will be affected. An unambiguous explanation of an agent's behaviour can only be provided from an unambiguous picture of the components of the problem faced.

Although it is not the purpose of this paper to explore why some positions are more flexible than others, a brief mention of possible sources of flexibility suggests the range of applications in which the concept might be useful. Some sources of flexibility would be available even to Robinson

Crusoe: The mere act of waiting, of postponing commitment to irreversible actions, provides one with flexibility if the option to undertake those actions at some future date remains; the acquisition or construction of a tool of more flexible design, one which can be used for many purposes rather than just one, leaves one with more flexibility than a specialized tool; commitment to a production technology with more constant average variable costs gives one flexibility to increase output with smaller increments to total costs than if average variable costs rapidly increased. The flexibility provided by postponement, design and choice of technology do not hinge on interactions between agents.

Additional sources of flexibility appear in economies with many agents. Perhaps the most important source is that provided by the presence of markets: a specialized tool may be transformed into another tool through sale and purchase; an asset which has not matured may be liquidated through sale to another individual. Such transformations will not be costless, of course. Real resources are used in transacting, and the price that will be realized from sale on short notice may differ substantially from the short notice purchase price of the same good or asset. Costs of the latter sort generally vary from good to good, depending on their degree of marketability or saleability (Menger, 1892), and from individual to individual, depending on their market expertise and haste to complete a transaction. Indeed, the switching costs faced by any given agent might themselves reflect his solution to a complex programming problem. Another source of flexibility lies in the nature of contracts between agents. The broader is the range of circumstances in which an agent, at his option, is explicitly or implicitly relieved of the obligations of a contract, the more flexible

is the contractual position he enters. Flexibility may also be useful in discussing the structure of organisations. The more that decision-making authority is delegated to those closest to the information sources relevant for those decisions, the lower will be the intra-organization communication costs involved in adapting behaviour to new circumstances.

Having discussed the payoff structure for the sequential decision problem, and the concept of flexibility based on it, we now turn to examine what an agent knows, and believes he might come to know, about the uncertain events beyond his control.

But first, why should any close link be suspected between uncertainty and flexibility? Flexibility certainly influences current decisions in a world of perfect foresight: irreversibility of physical investment alters the optimal path of capital accumulation even without risk (Arrow, Beckmann and Karlin, 1958, Arrow, 1968, Arrow and Kurz, 1970); asset liquidation costs influence optimal portfolio choice even with perfectly foreseen future cash needs (Baumol, 1952, Grossman, 1969). What is added by explicitly introducing uncertainty?

A connection arises since the prospect of learning enhances the value of remaining flexible; and for there to be a prospect of learning there must be prior lack of knowledge, or uncertainty. Returning to the three period problem, suppose the agent is initially uncertain about the ultimate state of the world s , but has given probabilistic beliefs about the likelihood of various states occurring. Learning entails changing these beliefs through observation. If there is no prospect of learning in the second period, then the agent knows with certainty what probabilistic beliefs he will hold at that time, and hence can plan with certainty what position he

will choose. Whether his initial position permits a wide or narrow range of alternative positions to be attained with low switching costs is irrelevant. But if there is a prospect of learning in the second period, then the agent is uncertain of the beliefs he will hold at that time, and hence is uncertain about which position will then appear most appropriate. If a relatively inflexible initial position is chosen, then the agent faces the possibilities either of incurring high switching costs or of forgoing opportunities to profit from what he learns. The more the agent expects to learn (the more uncertain he is about the beliefs he might hold in period two), then the more attractive are initial positions which keep many options open.

More formally, let S denote the (finite) set of possible states of the world, and p_s the agent's initial subjective estimate of the probability of s occurring. In the second period the agent receives an observation, or message, y from a (finite) set of possible observations Y . The agent believes that y will be received with probability q_y , and if it is received revises his beliefs about s occurring from p_s to π_{sy} . The column vector $p = (p_s)$ denotes his prior probability distribution over S , $\pi_y = (\pi_{sy})$ his distribution conditional on message y being received, and $q = (q_y)$ his estimates of the likelihood of receiving the various possible messages. Π is the entire $|S| \times |Y|$ matrix $[\pi_{sy}]$ of conditional probabilities. Consistency of the agent's beliefs requires that $\Pi q = p$, that his prior beliefs about s be a message-probability weighted average of his possible posterior beliefs.

The possible observations Y together with the probabilities (Π, q) constitute the agent's information structure. Information can come from a

multitude of sources both public and private. The message y might simply be an earlier observation of s , the unknown random variable of the third period, as would be the case if y were a firm's earnings one quarter and s its earnings the next. But there is no need for y to take the same form as s : y might be crop plantings in the spring and s the quantities harvested in the fall; y might be a vector of leading indicators and s the following year's level of demand for an industrial commodity; y might be the opinion of legal counsel and s the outcome of court action. In most risky situations events can and do occur which alter one's expectations about future events.

How does our agent profit from the prospect of emerging information? Suppose that the agent has committed himself to an initial position a -- and hence to an expected first period return of $\sum_{s \in S} p_s r(a,s)$ -- and wishes to evaluate his expected second period return. Also suppose, for the moment, that a is perfectly flexible so that switching costs may be ignored. Figure 1 depicts the determination of the expected second period return for a situation with two ultimate states and two possible messages. The horizontal axis is of length 1, each point on it representing a possible probability distribution over the two states. The distance of a point from the righthand end of the interval is the probability of state 1 occurring; the distance from the lefthand end is the probability of state 2 occurring. The point p represents the agent's prior distribution over the two states; π_1 and π_2 represents his beliefs conditional on message 1 (which favors state 1) and message 2 respectively being received. On the vertical axes are indicated the payoffs to taking each of the three possible second period actions $\{b_1, b_2, b_3\}$ if either state 1 or state 2 occurs. For any probability

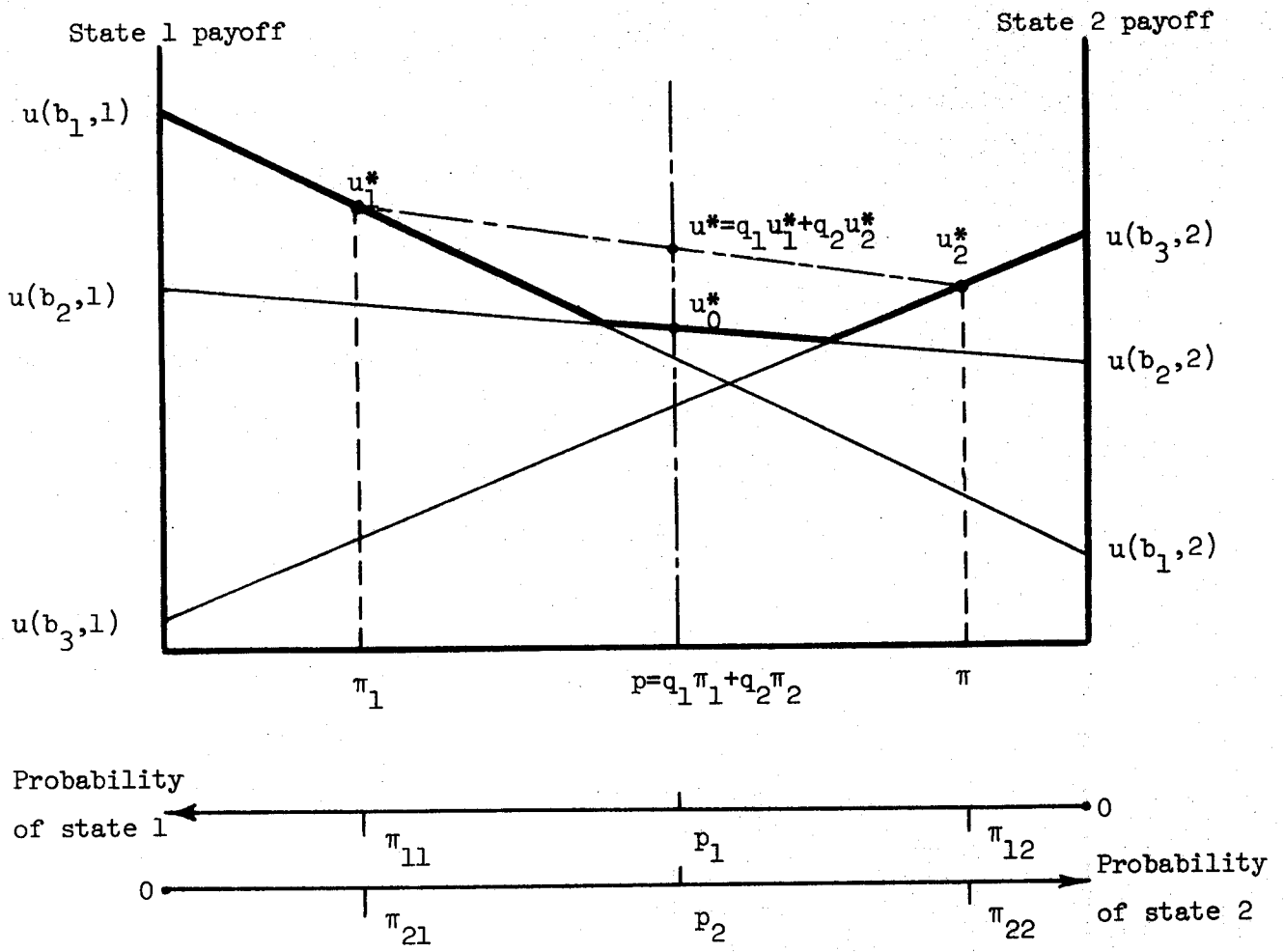


FIGURE 1

distribution over the two states the expected payoff to taking each position is given by the height of the straight line joining its payoff in state 1 to that in state 2. The convex upper boundary of these lines is the maximum expected second period return as a function of the probability distribution over S . If message 1 is received, the expected payoff is maximized by choosing position b_1 , and is u_1^* ; if message 2 is received, position b_3 is chosen, and the expected payoff is u_2^* . Since the agent's prior distribution must be a message-probability weighted average of his conditional distributions ($p = q_1\pi_1 + q_2\pi_2$), and since his expected payoff before it is known which message is to be received is the same weighted average of his conditional expected payoffs, his expected payoff is $u^* = q_1u_1^* + q_2u_2^*$ located at the point on the line joining u_1^* and u_2^* above p . If no additional information was to be received (the agent's distribution over S remaining at p when the second position must be chosen), then b_2 would be the optimal action and would yield an expected return u_0^* . Thus the value of this prospect of additional information to the individual is the difference $u^* - u_0^*$. Since the maximum expected payoff (being the maximum of a collection of linear functions) is always a convex function of the probability distribution over S , and since $p = E(\pi_y)$, Jensen's Inequality tells us that the value of information must be non-negative.

If the initial position a had not been perfectly flexible, we need only replace the payoff to each second period position $u(b,s)$ with its payoff net of switching costs, $u(b,s) - c(a,b,s)$, to similarly determine the expected second period return and value of information. This return will, however, vary with the choice of initial position a .

It remains to establish what it means to increase the amount an agent expects to learn in the second period. What does it mean to say that one information structure (Y, Π, q) conveys more information than another (Y', Π', q') ? J. Marschak and K. Miyasawa (1968) discuss three equivalent concepts of more information applicable to this context:

(E) (Y, Π, q) is more informative than (Y', Π', q') iff every rational decision-maker would pay at least as much for (Y, Π, q) as for (Y', Π', q') , regardless of his payoff structure.

(G) (Y, Π, q) is more informative than (Y', Π', q') iff there exists a $|Y| \times |Y'|$ non-negative matrix M with columns summing to 1 such that $\Pi' = \Pi M$ and $q = Mq'$.

(R) (Y, Π, q) is more informative than (Y', Π', q') iff the distribution of π_y is "riskier" than that of $\pi_{y'}$ (i.e.: for every convex function ϕ defined on the $(|S| - 1)$ -simplex to which π_y and $\pi_{y'}$ belong, $\sum_{y \in Y} q_y \phi(\pi_y) \geq \sum_{y' \in Y'} q_{y'} \phi(\pi_{y'})$). The definitions imply that both information structures have the same prior probability distribution $\Pi'q' = p = \Pi q$ over states.

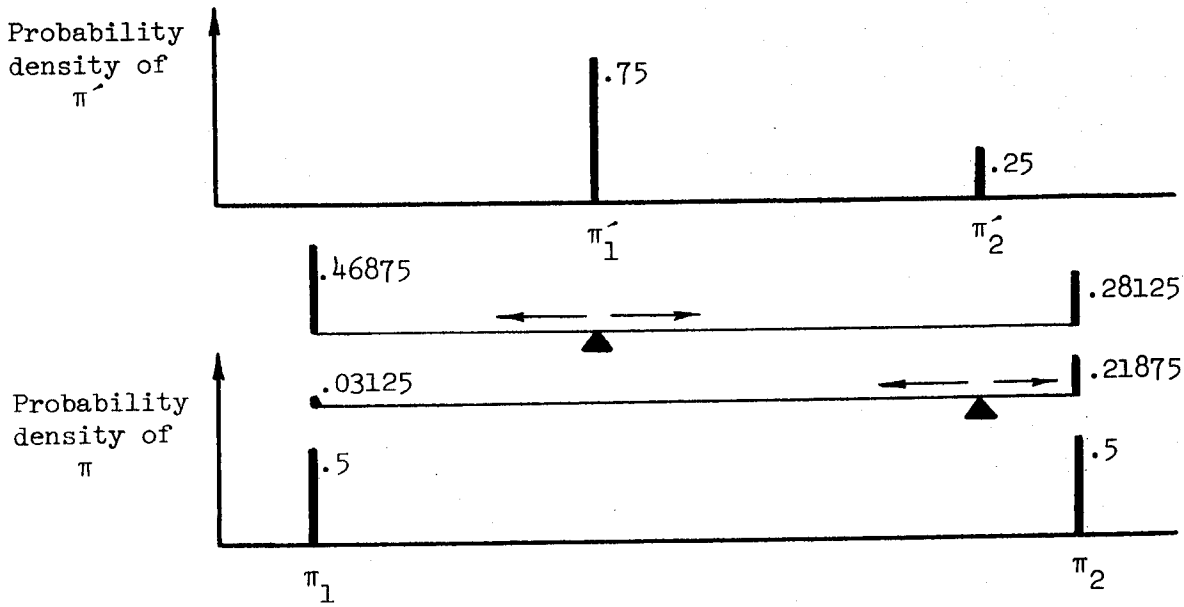
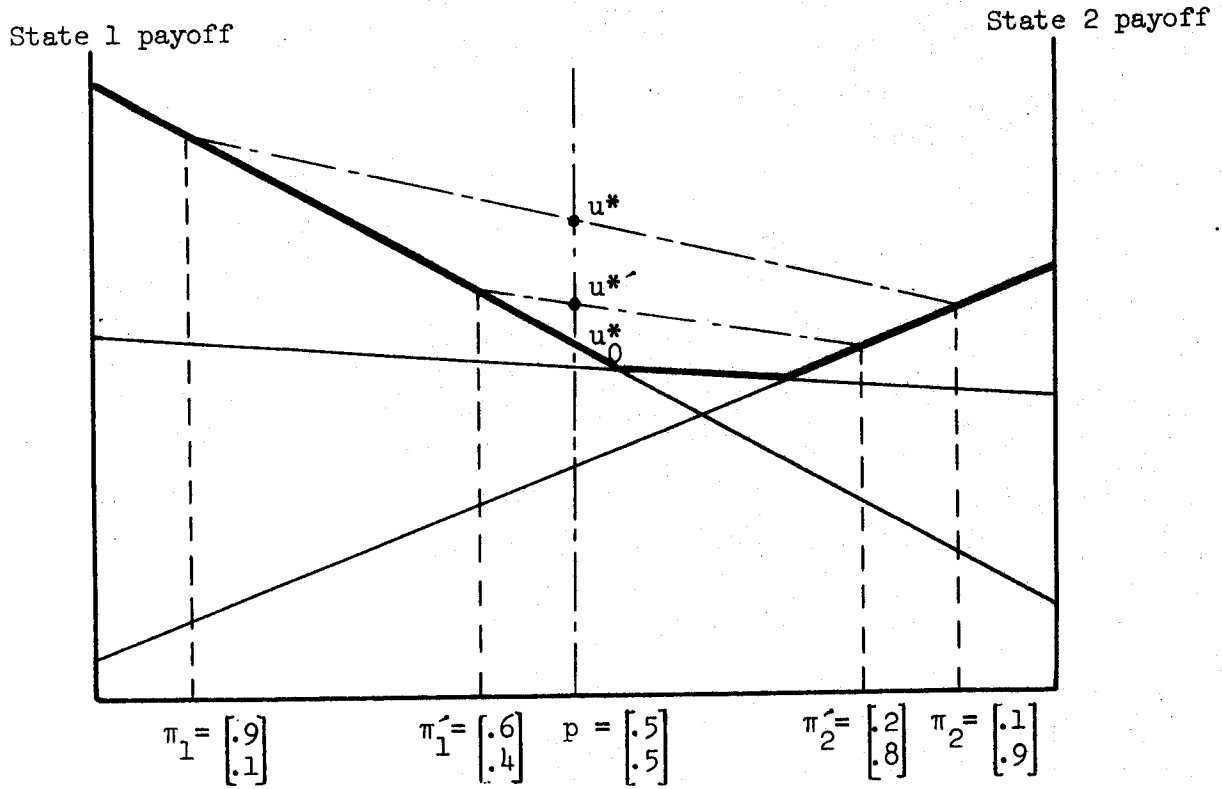
Definition (E) is clearly appealing from an economist's standpoint and needs no interpretation. Definition (G), based on the work of D. Blackwell (1951), states that (Y, Π, q) can be "garbled" into (Y', Π', q') . It means that one could construct a "black box" which accepts messages y as inputs and generates messages labelled as y' (with exactly the same joint distribution with states s as the real y') as outputs in the following manner: if y is fed in, then $y' \in Y'$ is sent out with probability $M_{yy'} q_y / q_y$. This garbling might just add "noise" to the messages through the random generation process, or it may completely obliterate distinctions between inputs by assigning

them the same output with certainty (dropping observations in a sample would be an example). Definition (R) states that the probability mass of the discrete random variable π (which takes on the value π_y with probability q_y for $y \in Y$) is more dispersed over the unit simplex than that of π' in the sense that $E(\phi(\pi)) \geq E(\phi(\pi'))$ for every convex function ϕ . Equivalently (Rothschild and Stiglitz, 1970), we could say that the distribution of π is obtained from that of π' by a sequence of "mean preserving spreads" of probability mass. This last definition is most useful for purposes of graphical exposition, and justifies using the phrase "being more uncertain about one's future beliefs" to describe a more informative message structure.

Figure 2 shows the expected (second period) returns associated with an information structure (Y, Π, q) and a less informative structure (Y', Π', q') , each having two possible messages and the same prior distribution p over the two states. Although the payoff structure is unchanged, the first information structure promises an expected return $u^* - u^{*'}$ higher than the second. The lower graphs depict the (discrete) probability density functions for the random variables π and π' and indicate the two "mean preserving spreads" through which one distribution can be transformed into the other. Specific values for all parameters are indicated for illustration.

The concept of more informative provides a partial ordering of information structures. At one extreme is the prospect of perfect information: Π consists solely of 0's and 1's; once y is received the agent knows for certain which s will occur. At the other extreme is the prospect of no information: each column of Π is identical and equals p ; regardless of which y is observed the agent retains his prior beliefs about s . Those pairs of

FIGURE 2



$$\Pi = \begin{bmatrix} .9 & .1 \\ .1 & .9 \end{bmatrix} \quad \Pi' = \begin{bmatrix} .6 & .2 \\ .4 & .8 \end{bmatrix} \quad M = \begin{bmatrix} .625 & .125 \\ .375 & .875 \end{bmatrix} \quad q = \begin{bmatrix} .5 \\ .5 \end{bmatrix} \quad q' = \begin{bmatrix} .75 \\ .25 \end{bmatrix}$$

information structures which cannot be ranked (as would have been the case if π_2 and π_2' were reversed in Figure 2) are those whose relative attractiveness depends on the payoff structure with which they will be used.

We thus have two partial orderings: one ranks initial positions according to their flexibility, the other ranks information structures according to the amount one expects to learn about s . Although one's information structure frequently is a matter of choice (more information can be gathered at some cost), we assume that the agent's information structure is given; it is treated as a parameter to be varied for our analysis. However the agent is free to choose the flexibility of his initial position, trading off expected first period returns $E(r(a,s))$ against flexibility which may increase expected second period returns if further information is forthcoming. We should like to be able to show that the more an agent expects to learn, the more flexible a position he should adopt. But, as will be seen in section III, that is not always true without further qualification. First, let us explore how the concepts developed so far apply to the demand for liquid assets.

II. Liquidity as Flexibility

It was mentioned previously that markets provide individuals with flexibility: assets may be transformed through sale and purchase into other assets. In a monetary (as opposed to barter) economy such transformations are effected in two stages: the asset held is sold for the medium of exchange, which is then used to purchase the desired good or asset. The liquidity (saleability) of an asset refers to the ease, or costlessness, with which the first stage can be accomplished -- that is, the ease with which an asset can be converted to money. In such a context the medium of exchange is the most liquid asset, since the costs associated with the first stage of transforming one's assets are completely avoided. The sequential asset choice problem presented in this section interprets the demand for liquid assets as a desire for flexibility, and illustrates the complementarity between information and flexibility asserted in the previous section.

Consider an individual who must invest all his wealth in one of three assets M , A_1 , A_2 (money, asset 1, asset 2) during period one. In period two some further information is received, and he must choose whether to hold M , A_1 or A_2 until period three. Suppose there are just two ultimate states $S = (s_1, s_2)$ which may occur, and two observations $Y = (y_1, y_2)$ which may be received.

The payoff to a sequence of actions and states is determined as follows: Holding M over the first period contributes 0 to the total return, while holding either A_1 or A_2 contributes \bar{r} with certainty. Holding M over the second period again returns 0, no matter which state ultimately occurs; but A_1 and A_2 yield 1 and 0 respectively if s_1 occurs, and 0 and 1 respectively if s_2 occurs. Furthermore, no switching costs are incurred if the individual changes from holding M in the first period to either A_1 or A_2 in the second,

or if he continues holding M, A_1 or A_2 over from the first period into the second. But if he switches from holding either A_1 or A_2 initially to holding some other asset later, then a "liquidation cost" of $\bar{c} > 0$ must be paid. According to the definition of section I, M is a more flexible first period position than either A_1 or A_2 . Table 1 summarizes this payoff structure.

Initial position <u>a</u>	Second position <u>b</u>	Payoff $f(a,b,s)$	
		state 1	state 2
A_1	A_1	$\bar{r} + 1$	\bar{r}
	M	$\bar{r} - \bar{c}$	$\bar{r} - \bar{c}$
	A_2	$\bar{r} - \bar{c}$	$\bar{r} - \bar{c} + 1$
M	A_1	1	0
	M	0	0
	A_2	0	1
A_2	A_1	$\bar{r} - \bar{c} + 1$	$\bar{r} - \bar{c}$
	M	$\bar{r} - \bar{c}$	$\bar{r} - \bar{c}$
	A_2	\bar{r}	$\bar{r} + 1$

TABLE 1

The information structure must also be specified: let the prior probability of s_1 occurring and of receiving observation y_1 be the same, denoted by $1 \geq \alpha \geq 0$. That is, $p = q = (\alpha, 1 - \alpha)$. A single parameter $1 \geq \rho \geq 0$ indicates the informativeness of the observations by defining the matrix of probabilities of

s_1 conditional on y_j as

$$\Pi = [\pi_{sy}] = \begin{bmatrix} \alpha(1 - \rho) + \rho & \alpha(1 - \rho) \\ (1 - \alpha)(1 - \rho) & (1 - \alpha)(1 - \rho) + \rho \end{bmatrix}$$

Parameter ρ is the correlation coefficient between y and s (viewing both as random variables taking on values 1 or 2); higher values of ρ indicate more informative information structures in the sense of section I. The value $\rho = 1$ corresponds to perfect information, the value $\rho = 0$ corresponds to no information being conveyed by y .

The individual is assumed to maximize his expected payoff. The optimal strategy hence yields an expected return

$$J(\Pi, q) = \text{Max}_{a \in D} \sum_{j=1}^2 q_j \text{Max}_{b \in D} \sum_{i=1}^2 \pi_{ij} f(a, b, s_i) \quad (2)$$

in which $D = (M, A_1, A_2)$ represents the set of positions from which the agent may choose, and $f(a, b, s_i)$ is the payoff function from Table 1. Solution of the problem is relatively straightforward but lengthy, involving many inequalities. Figure 3 presents those aspects of the solution with which we are concerned. The regions A_2, M, A_1 bounded by dotted lines indicate the values of the information structure parameters α, ρ for which holding initially those assets is the optimal strategy. Notice that region M vanishes if either $\bar{r} > \bar{c}/2$ or $\bar{r} > 1$: if the opportunity cost of remaining liquid overshadows either the alternative switching costs or the maximum second period yield at stake, then money is never held. Indeed, position M is valuable only because if either A_1 or A_2 is chosen initially, and subsequent observation indicates that the opposite position promises higher expected returns, then either the switching cost \bar{c} must be incurred or the agent must pass up the opportunity to profit from this new information.

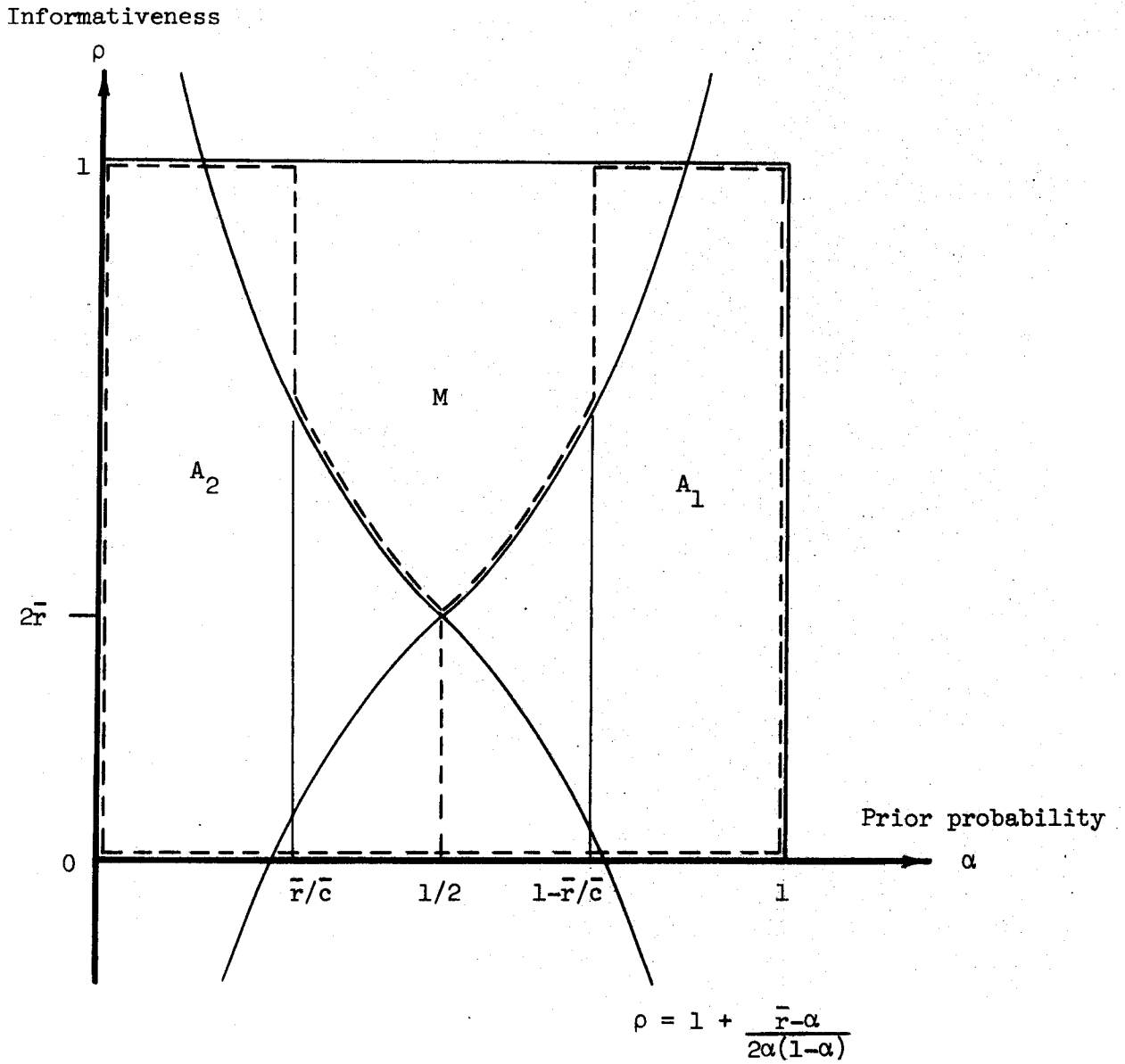


FIGURE 3

Varying the parameters has intuitively plausible effects on the attractiveness of M. Reducing \bar{r} , the yield available on alternative assets, moves outward the vertical boundaries and downward the lower boundaries of region M, enlarging the set of information structures for which M is the optimal initial position. Increasing \bar{c} has a similar effect. Moving α toward $1/2$, thereby increasing the prior uncertainty about which asset will have the highest return, can move one into region M but not out. Increasing ρ similarly can only move one into M. Thus a decrease in the opportunity cost of holding money, increase in the cost of switching out of alternative assets, increase in ultimate uncertainty or increase in the informativeness of forthcoming observations all enhance the relative attractiveness of staying liquid.

A more suggestive way of seeing the effect of emerging information on the demand for money is to ask at what \bar{r} is the individual indifferent between holding all three assets. Assuming $\alpha = 1/2$, so the individual is indifferent between A_1 and A_2 , the three regions intersect at $\rho = 2\bar{r}$. The yield \bar{r} supporting a portfolio of all three assets held in positive quantities is thus an increasing function $\bar{r} = \rho/2$ of the informativeness parameter up to $\bar{r} = \bar{c}/2$, at which point it stays constant to prevent region M from vanishing entirely.

This illustrative model is distinguished from existing explanations for the holding of money by those factors which are absent. First, although uncertainty is essential (M is not chosen if α is 0 or 1), risk aversion is not to rationalize holding money as an asset. In fact, introducing risk aversion can either enhance or diminish the value of flexibility: suppose the agent is extremely risk-averse, concerned only with maximizing his minimum payoff. If further observation promises less than perfect information ($\rho < 1$), Table 1 reveals he

must adopt a strategy of holding either A_1 or A_2 for both periods; only in that way is he guaranteed a return of at least \bar{r} . Alternatively, if further observation promises perfect information ($\rho = 1$), then he must adopt a strategy of holding M initially; only in that way is he guaranteed a return of 1. Since there are points in Figure 3 at which M is held although $\rho < 1$, and where A_1 is held although $\rho = 1$, risk-aversion has no unambiguous effect on the relative attractiveness of flexibility. Of course, an appeal to risk-aversion (or some type of decreasing marginal return to investing in each asset) may still be useful in explaining portfolio diversification.

Second, although differential liquidation costs are essential, the agent need never incur these costs. He is never compelled to liquidate a position (to meet, for example, some foreseen or unforeseen "cash requirement"); positions are liquidated only if the expected gain from doing so exceeds the switching costs. Such could never be the case if \bar{c} exceeds 1. Hence the agent is not trading off interest yield against expected transaction costs as in inventory approaches to money demand.

Finally, notice that money is dominated in terms of immediate (period one) and future (period two) yields by both alternative assets; none does worse than yield 0 over each period. The value to liquidity arises since the individual is uncertain about which alternative asset will be best, and he expects some of that uncertainty to be dispelled while there is still time to act. If no further information about the ultimate state is forthcoming until it occurs ($\rho = 0$), flexibility is valueless and M is not held. Flexibility is valuable only to the extent that it permits more profitable exploitation of future information and opportunities expected to arise.

III. Information and the Value of Flexibility

To what extent does the positive relationship between the amount one expects to learn and the value of flexibility carry over into more general contexts? A partial answer to this question is provided by the propositions which follow. The first two describe payoff structures permitting the optimal initial position to be determined just from prior beliefs about s , regardless of the information content of future observations. The last four give additional conditions on the payoff and information structures under which increased informativeness does imply increased flexibility of the optimal initial position. Finally, a counterexample demonstrates that some such additional conditions are indeed necessary.

An optimal strategy consists of a first period position a , and a set of second period positions $\{b_y\}$ to be taken depending on the observation $y \in Y$ received, which maximizes the expected total payoff. All positions are chosen from a given set of alternatives D . The expected payoff so obtained may be computed recursively according to the maximum principle of dynamic programming and depends on the information structure:

$$J(\Pi, q) = \underset{a \in D}{\text{Max}} \sum_y \sum_s q_y \pi_{sy} f(a, b_y, s) \quad (3)$$

$$\{b_y\} \in D^Y$$

$$= \underset{a \in D}{\text{Max}} \sum_y q_y \underset{b \in D}{\text{Max}} \sum_s \pi_{sy} f(a, b, s)$$

Substituting our assumed decomposition of $f(a, b, s)$ and separating terms, $J(\Pi, q)$ can be alternatively written

$$\begin{aligned}
 J(\Pi, q) &= \text{Max}_{a \in D} \left\{ \sum_s p_s r(a, s) + \sum_y q_y \text{Max}_{b \in D} \sum_s \pi_{sy} [u(b, s) - c(a, b, s)] \right\} \\
 &= \text{Max}_{a \in D} \{ \bar{r}(a) + v(a; \Pi, q) \} \quad (4)
 \end{aligned}$$

The expected first period return $\bar{r}(a)$ depends only on the first period position a and the prior beliefs p concerning s , which must be the same for any two information structures comparable in terms of informativeness. We also know from definition (E) of section I that the expected second period return (net of switching costs) $V(a; \Pi, q)$ increased with the informativeness of (Π, q) for any fixed a . Whether or not increased informativeness induces a more flexible a to be chosen, however, hinges on whether or not this increase in $V(a; \Pi, q)$ is greater for more flexible a . In other words, is it the case that if (Π, q) is more informative than (Π', q') and if a is more flexible than a' , then

$$V(a; \Pi, q) - V(a'; \Pi, q) \geq V(a; \Pi', q') - V(a'; \Pi', q') \quad (5)$$

If (5) is satisfied for all a more flexible than a' available to the decision-maker, then an increase in informativeness, since the positions' first period returns $\bar{r}(a)$ remain unchanged, can only move the optimal initial position in the direction of greater flexibility.

The following notation is adopted for this section: a, a', b, b', d, d' all are positions in D ; a^* denotes a perfectly flexible position; \bar{a} denotes a completely irreversible position; $a \succ_F a'$ indicates that a is more flexible than a' ; $(\Pi, q) \succ_I (\Pi', q')$ indicates that (Π, q) is more informative than (Π', q') , implying also that $\Pi q = \Pi' q' = p$. Reference is also made to a particular position in D : namely, that which promises the highest expected second period

return (ignoring switching costs) on the basis of prior beliefs. This position, denoted by \tilde{b} , satisfies $\sum_s p_s u(\tilde{b}, s) = \text{Max}_{b \in D} \sum_s p_s u(b, s)$. The value of any position refers to the total return $\bar{r}(a) + V(a; \Pi, q)$ expected if that position is initially adopted. The optimal initial position is that with the highest value.

Several inequalities following from these definitions and leading to the first four propositions are collected in the following lemma.

Lemma: Let $(\Pi, q) \succeq_I (\Pi', q')$. Then

- (L.1) $V(a^*; \Pi, q) \geq V(a^*; \Pi', q')$ for all $a^* \in D$
- (L.2) $V(a; \Pi, q) \geq V(a; \Pi', q')$ for all $a \in D$
- (L.3) $V(\bar{a}; \Pi, q) = V(\bar{a}; \Pi', q')$ for all irreversible $\bar{a} \in D$
- (L.4) $V(a^*; \Pi, q) = V(a^{*'}; \Pi, q)$ for all perfectly flexible $a^*, a^{*' \in D$

(L.1) follows from the definition of V in equation (4), the non-negativity of switching costs and the definition of perfect flexibility; a perfectly flexible initial position permits attainment of the highest possible expected second period return. (L.2) follows from definition (E) of more informative; more information is always better. (L.3) follow from the fact that since $c(\bar{a}, b, s)$ is unbounded for $b \neq \bar{a}$ the optimal response to any observation is necessarily $b = \bar{a}$. That $p_s = \sum_y q_y \pi_{sy} = \sum_{y'} q_{y'} \pi_{sy'}$ is the same for both information structures for all s implies the expected payoff is independent of the information content of later observations. (L.4) follows from (L.1) applied to both a^* and $a^{*'}$.

The first two propositions emphasize that for some payoff structures the amount one might learn in the future has no impact on the optimal initial position. The same points are made by J. R. Hicks (1974, p. 44).

Proposition 1: If all positions are perfectly flexible, then the "myopic" policy of choosing the initial position offering the highest expected first period return is optimal.

The proof is immediate from (L.4) and the fact that the total value of a position is $\bar{r}(a) + V(a; \Pi, q)$.

Proposition 2: If all positions are "economically irreversible," then the optimal initial position depends only on the prior beliefs p .

By economically irreversible is meant that $u(b, s) - u(a, s) \leq c(a, b, s)$ for all a, b, s ; there is no circumstance in which switching position is profitable.

Consequently $V(a; \Pi, q) = \sum_y \sum_s q_y \pi_{sy} u(a, s) = \sum_s p_s u(a, s)$, and the value of any initial position is simply $\sum_s p_s [r(a, s) + u(a, s)]$.

The next four propositions suggest that if an increase in what one expects to learn does change the optimal initial position, then the change tends to be in the direction of greater flexibility.

Proposition 3: An increase in the informativeness of the information structure increases the value of any (even partially) flexible position relative to any irreversible position.

Combining (L.2) and (L.3) implies $V(a; \Pi, q) - V(\bar{a}; \Pi, q) \geq V(a; \Pi', q') - V(\bar{a}; \Pi', q')$ if $(\Pi, q) \succeq_I (\Pi', q')$. The first period returns $\bar{r}(a)$, $\bar{r}(\bar{a})$ are unchanged. This proposition states that the prospect of more information (including some as opposed to none) can induce an agent to change from an irreversible initial position to one which is at least somewhat flexible, but never the other way around. It can be considered an extension of the findings of Henry (1974b),

and Arrow and Fisher (1974), in that it points out that the disadvantage to irreversible positions increases monotonically with the amount of information expected.

Proposition 4: The prospect that some more information is forthcoming, as opposed to none, increases the value of any perfectly flexible position relative to that position \tilde{b} offering the highest expected second period return based on prior beliefs alone.

Let (Π', q') be a completely uninformative information structure ($\pi_{sy'} = p_s$ for all s, y') and let $(\Pi, q) \succeq_I (\Pi', q')$. Since $\sum_s p_s u(\tilde{b}, s) = \text{Max}_{b \in D} \sum_s p_s u(b, s)$, it follows that $V(\tilde{b}; \Pi', q') \geq V(a; \Pi', q')$ for all $a \in D$. If a^* is a perfectly flexible position, (L.1) then implies that $V(\tilde{b}; \Pi', q') = V(a^*; \Pi', q')$. Combining with (L.1) again yields $V(a^*; \Pi, q) - V(\tilde{b}; \Pi, q) \geq V(a^*; \Pi', q') - V(\tilde{b}; \Pi', q')$.

Since the last proposition says little about positions other than a^* or \tilde{b} which might be chosen, it is best applied in contexts where there are only two alternatives. Suppose, for example, a firm contemplates introducing a new product to be sold in the third period, is uncertain of its demand, but expects no further information until the good is produced and sold. It must choose whether to go ahead with production (\tilde{b}) or postpone the decision until some future date (a^*), realizing that planning can be costlessly resumed (postponement is perfectly flexible) but that losses will be incurred if production plans are aborted. Suppose further that current demand estimates indicate production to be profitable ($\sum_s p_s u(\tilde{b}, s) \geq \sum_s p_s u(a^*, s)$); and that there is some opportunity cost to postponing commitment ($\bar{r}(\tilde{b}) > \bar{r}(a^*)$), perhaps because material costs are expected to rise, perhaps because time is a factor

in the production process. The firm should clearly go ahead with production immediately. But suppose, now, the firm hears of a consumer survey, the results of which will be announced in period two and could change the firm's beliefs about the profitability of production. Proposition 4 states that the prospect of such a survey could induce the firm to rationally postpone any production commitment until the results are known.

The last two propositions indicate the effect of more information on the relative values of particular positions, one of which is less flexible than the other. The next proposition obtains a monotonic relationship between the amount one expects to learn the the flexibility of the optimal initial position for particular types of payoff structures.

Proposition 5: If the payoff structure satisfies

- (i) for each a there is a set $D_a \subset D$ such that $c(a,b,s) = \begin{cases} 0 & \text{for } b \in D_a \\ \infty & \text{for } b \notin D_a \end{cases}$
 and for all a, a' either $D_a \supset D_{a'}$ or $D_a \subset D_{a'}$.
- (ii) for each a and $d \in D$ there exists a $d' \in D_a$ such that for all probability distributions $\pi = (\pi_s)$ over S either $\text{Max}_{b \in D_a} \sum_s \pi_s u(b,s) \geq \sum_s \pi_s u(d,s)$ or $\text{Max}_{b \in D_a} \sum_s \pi_s u(b,s) = \sum_s \pi_s u(d',s)$.

Then an increase in informativeness of the information structure increases the flexibility of the optimal initial position.

Proof of the proposition is in the appendix, but its assumptions warrant explanation: Condition (i) says that each initial position carries with it some set of future "options" which are costlessly available; all other positions are unattainable. Moreover all initial positions are completely ordered according to their sets of options left open for future choice. $D_a \supset D_{a'}$ implies,

of course, that $a \succeq_F a'$; so all positions can be ranked in terms of flexibility. Condition (ii) is less transparent: it says that if the options D_a open from a are augmented by any one additional option d , then for all circumstances (beliefs π) in which option d would be chosen, the same position d' would be the next best choice; put another way, any additional option displaces at most one previously available option.

Figure 4 depicts payoff structures which do and do not satisfy condition (ii) with two possible states of the world. Suppose that from an initial position b_1 only b_1 is attainable in period two, from b_2 both b_2, b_1 are attainable, and from b_3 all of b_3, b_2, b_1 are attainable. Thus $b_3 \succeq_F b_2 \succeq_F b_1$. The heavy solid lines indicate the maximum expected payoff as a function of π if options b_2 and b_1 are open in the second period; the dotted lines indicate the payoff if all three options b_3, b_2, b_1 are available. With second period payoffs as in Figure 4A, condition (ii) is satisfied: position b_3 is chosen only when b_2 would otherwise have been chosen. But with payoffs as in Figure 4B, condition (ii) is not satisfied: if b_3 is chosen, one cannot say whether b_1 or b_2 would be the next best choice.

Proposition 5 promises considerable application to economic problems since the following frequently encountered payoff structure satisfies its requirements: Let the second period decision be the choice of level of some control variable b subject to an inequality constraint $b \leq z$. Let the first period decision be the choice of level of some control variable a , which not only contributes directly to the total payoff through $\bar{r}(a)$ but also affects the level of constraint on b : that is, $z = z(a)$. Initial positions can be completely ranked according to their levels of $z(a)$, with $z(a) \geq z(a')$ indicating that $a \succeq_F a'$, satisfying condition (i). Further suppose that the

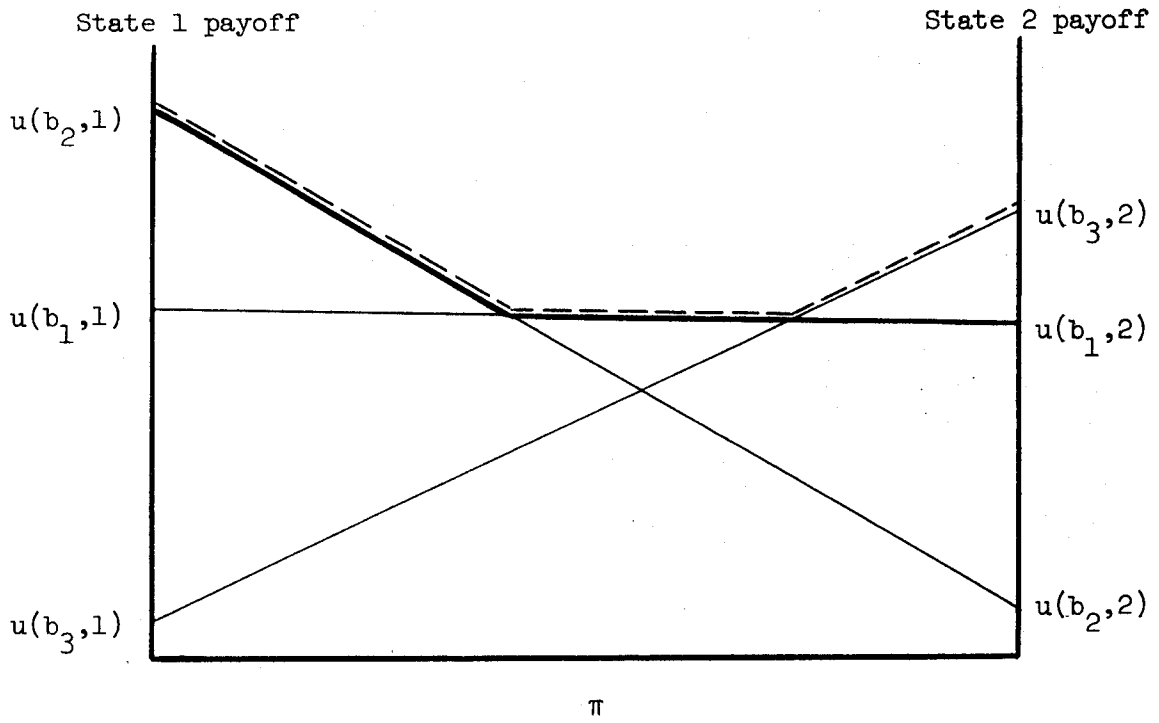


FIGURE 4A

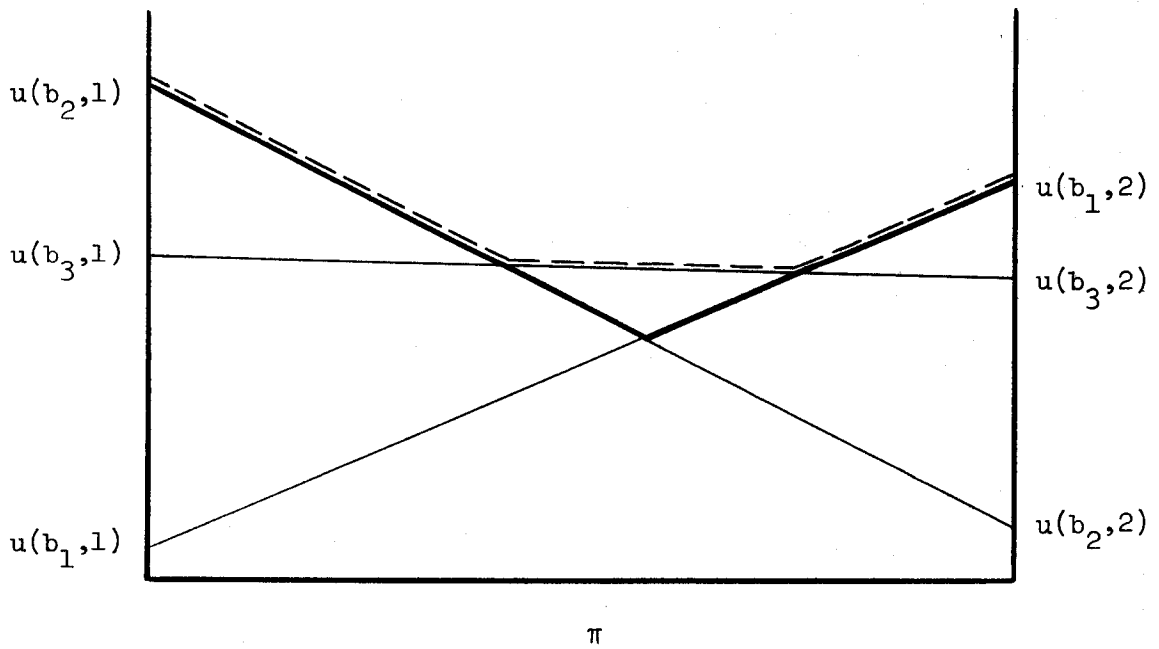


FIGURE 4B

objective maximized in period two is a concave function of the control variable b . Then, whenever the maximized objective would increase with removal of the constraint on b , $b = z(a)$ must be the optimal second period decision, satisfying condition (ii). Proposition 5 states that in such circumstances any increase in information expected in period two leads the decision-maker to choose a value for a associated with a higher $z(a)$. Section IV illustrates how this principle might apply to consumption-saving decisions.

Also, if only two positions are available, one of which is irreversible, then conditions (i) and (ii) are met, and the proposition applies; this amounts to a restatement of Proposition 3.

The final proposition returns to comparing the relative values of a perfectly flexible position and that position promising the highest expected second period return on the basis of prior beliefs -- most usefully thought of as going ahead with what seems best at the moment versus waiting for more information. It relies on a more restrictive notion of more informative: we will say that information structure (Π, q) is a star-shaped spreading of (Π', q') if $Y = Y'$, $q = q'$, $\Pi q = \Pi' q' = p$, and there exists a set of numbers $0 \leq \lambda_y \leq 1$ such that $\pi_{sy} = \lambda_y \pi_{sy'} + (1 - \lambda_y) p_s$ for all $y = y'$ and s . This relation, denoted by $(\Pi, q) \succeq_S (\Pi', q')$, implies $(\Pi, q) \succeq_I (\Pi', q')$ but not vice versa.

Proposition 6: Suppose switching costs satisfy $c(a, b, s) = \begin{cases} c(a) & \text{for } b \neq a \\ 0 & \text{for } b = a \end{cases}$.

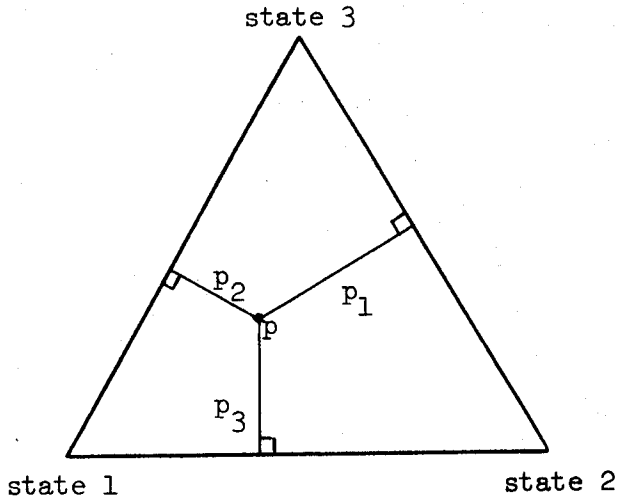
Then a star-shaped spreading of the information structure increases the value of any perfectly flexible position relative to that position \tilde{b} offering the highest expected second period return based on prior beliefs alone.

Proof is relegated to the appendix. The proposition requires that the cost of "reversing" any position be independent of the position switched to and of the ultimate state of the world. Using a more restricted notion of more information, a monotonic relationship between informativeness and the relative values of a^* and \bar{b} obtains, which was absent from Proposition 4.

Is star-shaped spreading likely to be encountered in practice? The situation is not as unlikely as might seem. First, if (Π', a') conveys no information, then any more informative structure (Π, a) is a star-shaped spreading of it. Second, if there are but two possible observations (say, the occurrence or non-occurrence of some event) , then \succeq_S is equivalent to \succeq_I . The asset choice problem of section II is thus an application of Proposition 6. Third, that the observations to be received have any predictive power may be contingent on the validity of some theory, which may be in doubt. Letting $\lambda = \lambda_y$ be the subjective probability that the theory is true, any increase in belief in the theory increases informativeness in the required manner. Or there may be some likelihood of information channel malfunction (The secretary misplaced the message; the research analyst made up the data). The more fanciful are invited to devise further situations.

Figure 5A displays two information structures satisfying $(\Pi, q) \succeq_S (\Pi', q')$ with three possible states of the world. In 5B, although $(\Pi, q) \succeq_I (\Pi', q')$, one structure is not a star-shaped spreading of the other.

This section concludes with a counterexample refuting the seemingly plausible conjecture that the prospect of more information always induces a more flexible position to be taken. Suppose the payoff structure satisfies condition (i) of Proposition 5 -- positions are ordered in terms of the number of future options left open. But suppose condition (ii) is not met, that the payoffs



Coordinates of a point in the unit simplex: $p_1 + p_2 + p_3 = 1$

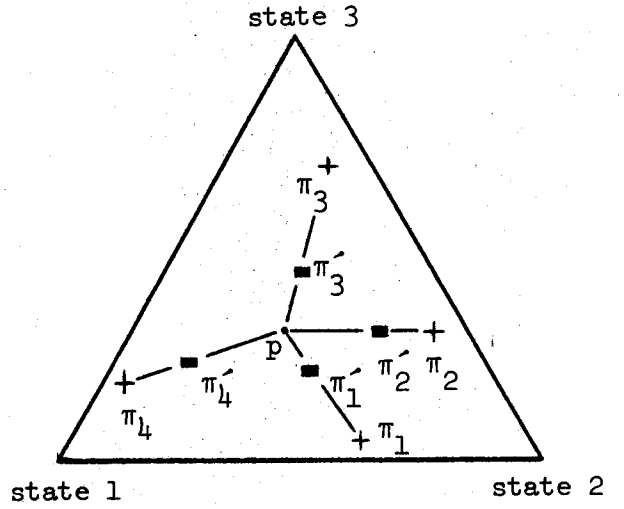


FIGURE 5A

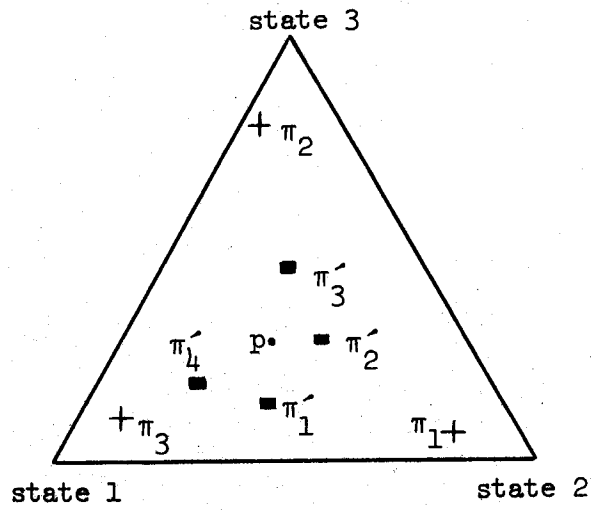


FIGURE 5B

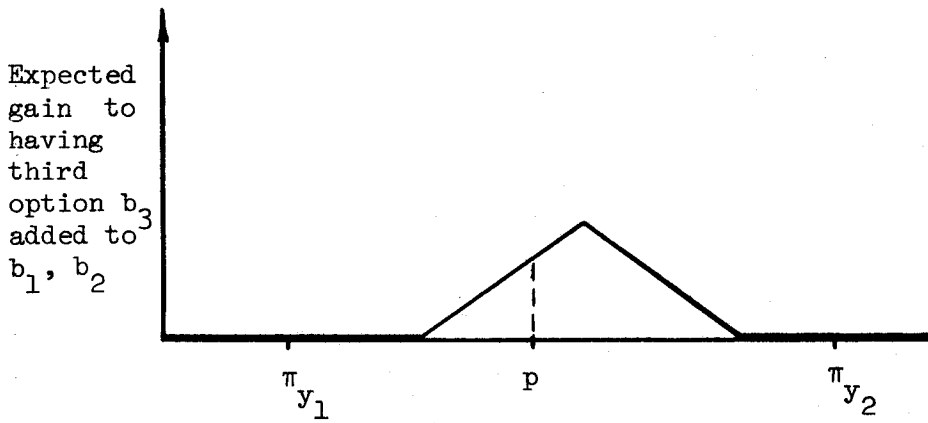


FIGURE 6

are as in Figure 4B. Compare the expected second period returns from initially choosing position b_3 , which leaves open b_1, b_2, b_3 as options, with choosing position b_2 , which leaves open just b_1, b_2 . The difference between these expected returns as a function the probabilities of various states occurring (i.e., the difference between the dotted and solid lines in 4B) is plotted in Figure 6. If there is no prospect of further information, then, with prior beliefs p as indicated in the figure, there is a positive expected value to having the additional option b_3 available; but if the prospect is to receive one of the two messages indicated by π_{v_1} and π_{v_2} , the additional option has no value. The promise of additional information has decreased the relative attractiveness of the more flexible position.

Finally, it should be pointed out that confining our attention to three period problems does not severely limit the general applicability of these results. One need only interpret the second period payoff $u(b,s)$ as the next period's value function for an ongoing dynamic program, with b as the decision state variable and s as the "nature" state variable (including any exogenously given information which has accumulated as of period three).

IV. Further Illustrations

The results of the previous section are in rather abstract form. The two examples of this section illustrate more concretely what maintaining flexibility entails in particular circumstances. The models are necessarily simplistic in the interests of brevity.

1. Plant capacity as flexibility:

Suppose a firm produces some good using two factors -- workbenches and artisans -- in equal fixed proportions. It must choose its plant capacity (number of workbenches to install) in the current period. In the second period it chooses the number of artisans to employ, producing output which is available for sale in the third period. The "positions" a and b chosen in each period are thus both levels of output: a is output capacity and b is output produced. Further assume that the firm faces fixed prices for workbenches (P_K), artisans (P_L) and its output (P_Q) sold in period three (it has already mailed its price lists), but is uncertain about the quantity of orders s that will materialize. Orders beyond actual production go unfilled; unsold output perishes. The firm's average variable cost and total cost (including capacity) curves are presented in Figures 7A and 7B for two different choices of plant size.

That part of the firm's profit directly imputable to its first period decision is the cost of capacity: $\bar{r}(a) = -aP_K$. Its profit from actually producing b if orders s materialize is $u(b,s) = P_Q \text{Min}\{b,s\} - bP_L$. That production must be chosen subject to the capacity constraint $0 \leq b \leq a$ can be captured by saying that switching costs $c(a,b,s)$ are 0 for $b \leq a$ and infinite for $b > a$. The firm's net profit is thus $\bar{r}(a) + u(b,s) - c(a,b,s)$.

Average Variable Cost

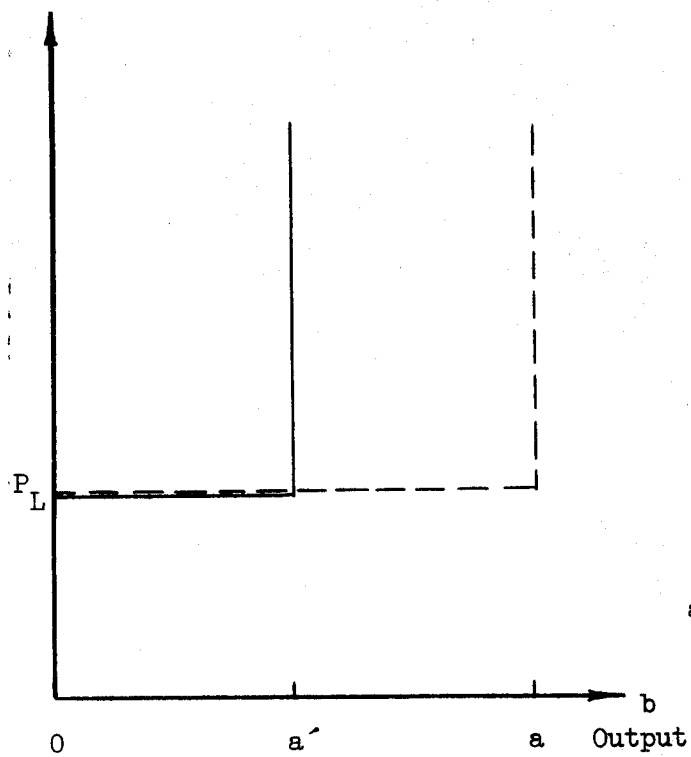


FIGURE 7A

Total Cost

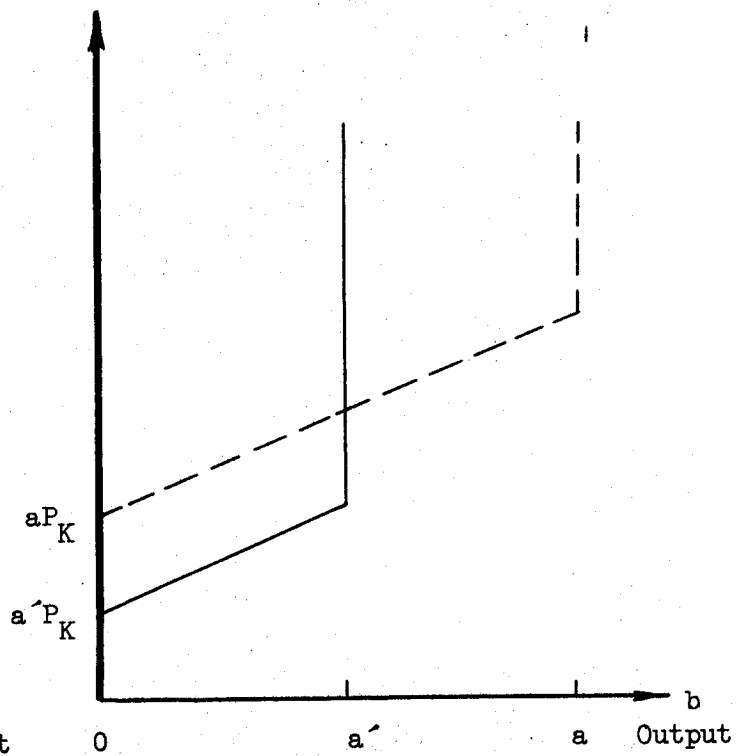


FIGURE 7B

The expected second period return from producing b , conditional on the probability distribution function $F(s) = \Pr\{\text{orders} \leq s\}$ representing beliefs about future orders, is

$$E[u(b,s)] = P_Q \left[\int_0^b s \, dF(s) + \int_b^\infty b \, dF(s) \right] - bP_L \quad (6)$$

It is readily verified that the above expression is concave in b . The b^* maximizing (6) subject to the constraint $0 \leq b \leq a$ satisfies

$$\begin{aligned} b^* &= 0 && \text{if } P_Q(1 - F(0)) < P_L \\ P_L &= P_Q(1 - F(b^*)) && \text{if } P_Q(1 - F(a)) \leq P_L \leq P_Q(1 - F(0)) \\ b^* &= a && \text{if } P_Q(1 - F(a)) > P_L \end{aligned} \quad (7)$$

In effect, the revenue the firm expects from an additional unit of output is P_Q times the probability $1 - F(b)$ that the unit will be sold; it then equates expected marginal revenue with marginal cost.

The structure of switching costs satisfy condition (i) of Proposition 5: higher initial choice of capacity strictly increases the levels of actual output attainable. Moreover if the firm were given the additional option of producing at a level $b = d > a$, and if $F(s)$ was such that it expected greater profits from producing at $b = d$ than at $0 < b < a$, then the concavity of (6) implies that $b^* = a$ is the next best choice. Thus condition (ii) is satisfied, and the conclusions of Proposition 5 apply: for any given prior beliefs about the level of future sales, the prospect of information leading to better sales estimates before the actual production decision is made leads the firm to optimally install higher initial capacity.

2. Saving as flexibility:

Different notation is used for the payoff structure of this illustration from previous parts of the paper. Consider an individual with a

lifespan of three periods and whose lifetime utility depends additively on consumption in those three periods: $U = u_1(c_1) + u_2(c_2) + u_3(c_3)$. Suppose he has no initial wealth, receives incomes w_1, w_2, w_3 in the three periods, and that the savings s_1, s_2 he carries forward from the first period to the second and the second to the third are constrained to be non-negative (human capital is unacceptable collateral, or Robinson Crusoe is storing yams). For simplicity, assume there is no interest paid on savings or rate of time discount on consumption.

More concretely, suppose that $u_i(c_i) = 8c_i - c_i^2$, and (for the moment) that the individual expects to receive incomes $w_1 = w_2 = w_3 = 1$ with certainty. His lifetime utility maximizing consumption plan will clearly entail consuming all income as it accrues and saving nothing: $s_1^* = s_2^* = 0$.

Now suppose uncertainty is introduced concerning his third period income, but maintain its expected value. Let $w_3 = 0$ with probability 1/2 and $w_3 = 2$ with probability 1/2, keeping $w_1 = w_2 = 1$ with certainty. What happens to the optimal consumption plan? It remains unchanged. This can be seen by noting that the optimal strategy requires the individual, in the second period, to equate his marginal utility of (period two) consumption with the expected marginal utility of period three consumption. Since the utility function is quadratic, marginal utility is linear in consumption, and expected marginal utility depends solely on expected consumption. By setting $s_1^* = s_2^* = 0$ the individual equates his expected marginal utilities of consumption in all periods as viewed from either period one or two. Expected utility has dropped, of course, with the introduction of income risk, but the optimal consumption plan is unchanged.

But suppose that some information might be received in period two regarding period three's uncertain income. In particular, assume that observations y_1 or y_2 might be received, each with probability $1/2$, that the probability of $w_3 = 2$ conditional on y_1 being received is $(1+\rho)/2$, and that the probability of $w_3 = 2$ conditional on y_2 being received is $(1-\rho)/2$. This is the information structure of section II with $\alpha = 1/2$. A higher value of $0 \leq \rho \leq 1$ indicates a more informative information structure.

Derivation of the optimal first period saving will be briefly sketched. The individual's expected period three income is $1+\rho$ if y_1 is received, $1-\rho$ if y_2 is received. The consequence of having identical quadratic utility functions for the last two periods is that period two consumption should optimally be as close as possible to half of the individual's expected remaining lifetime wealth. If savings s_1 were carried over from the first period and income $w_2 = 1$ is received, this means $c_2^* = 1 + (s_1+\rho)/2$ if y_1 is received, $c_2^* = 1 + (s_1-\rho)/2$ if y_2 . But (as it turns out) the individual cannot consume that level c_2^* conditional on y_1 without violating the non-negativity constraint on second period savings. So he gets as close as possible by choosing $c_2^* = 1 + s_1$ when y_1 occurs. The associated levels of savings carried into period three, contingent on y_1 and y_2 respectively, are $s_1^* = 0$ and $s_1^* = (s_1+\rho)/2$. Substituting the parameters of the information structure, the above conditional values of the period two choice variables, and $c_1 = 1-s_1$ into an expression for expected lifetime utility gives the first period objective as a function s_1 alone (the subscript on s_1 is dropped):

$$\begin{aligned}
 E(U) = & u_1(1-s) + \frac{1}{2}[u_2(1+s) + \left(\frac{1+\rho}{2}\right)u_3(2) + \left(\frac{1-\rho}{2}\right)u_3(0)] \\
 & + \frac{1}{2}[u_2\left(1+\frac{s-\rho}{2}\right) + \left(\frac{1-\rho}{2}\right)u_3\left(2+\frac{s-\rho}{2}\right) + \left(\frac{1+\rho}{2}\right)u_3\left(\frac{s+\rho}{2}\right)] \quad (8)
 \end{aligned}$$

Maximization of (8) delivers the optimal initial level of saving $s_1^* = 2\rho/7$. Thus the more rapidly will future income uncertainties be dispelled, the higher is the optimal initial rate of saving.

What has saving to do with flexibility? The connection arises from the non-negativity constraint on savings: since c_2 is constrained to be less than or equal to $1 + s_1$, increasing saving strictly enlarges the set from which second period consumption is chosen. In that sense, increased saving leads to greater flexibility. Indeed, the problem would have been another direct application of Proposition 5 were it not that u_3 depends on both c_1 and c_2 rather than just c_2 .

V. Concluding Remarks

Our basic point is a simple one: individuals will prefer to "wait and see" if there is something to be learned with the passage of time. Such a policy might entail rejecting what, on the basis of current estimate, is the seemingly most profitable course of action; but it is rational if that action significantly reduces the number, or increases the cost, of options available for future choice. Moreover this tendency can be expected to pervade a wide variety of economic phenomena, from a worker's choice of skills to acquire to the nature (and absence) of contractual agreements in industry.

The concept of flexibility also leads to information demand, general equilibrium and macroeconomic issues. Although the relationships are not clear, something can be glimpsed from what we already know.

The relationship between information and the value of flexibility can be turned around: rearranging equation (5), if it is satisfied, implies that the more flexible is an agent's position, the more valuable is any increment in information to him. Thus flexibility is a parameter relevant to study of the demand for information.

In a general equilibrium context, it appears that the gains to individuals from introducing flexible positions are similar to those from introducing contingent claims markets. Holding a liquid asset, for example, is viewed by the individual as equivalent to holding a bundle of contingent claims to the goods he might acquire in the future, where the contingencies are his future information states. Moreover the costs of state-verification and negotiation with other agents are avoided. But for this very reason the individual provides no information to the market about his future plans and intentions, thereby increasing the amount of "strategic uncertainty" (Radner, 1968) faced by other agents.

The connection of flexibility with macroeconomic issues is indicated by our suggestion that the demand for money and near-monies, saving and the postponement of investment in illiquid physical capital (consumer durables and business investment) can all be viewed, in part, as efforts by economic agents to retain flexibility. The empirical relevance of these considerations, however, hinges on the changeability of the information structure in the economy. If, in fact, the information structure is quite stable, then one might expect to explain changes in these economic aggregates quite well by examining only changes in non-informational variables such as interest rates, income and mean expectations about the levels of future variables. But if the information structure is unstable, either as a consequence of natural cyclical forces or the result of unpredictable government policy, then one might expect substantial "inexplicable" changes in the relationships between non-informational variables, and poor predictive performance by macroeconomic models based on them alone.

Having followed us this far, it will no doubt have occurred to the reader that we have nowhere said how to measure either flexibility, or information structures, or changes in either. Obviously, these matters require development for tests of practical significance to be made. Even without such developments, the viewpoint may be worth considering.

APPENDIX

Proof of Proposition 5:

Condition (i) implies that the expected second period return, having initially been in position a and holding beliefs π , is

$$\text{Max}_{b \in D} \sum_s \pi_s [u(b,s) - c(a,b,s)] = \text{Max}_{b \in D_a} \sum_s \pi_s u(b,s). \quad (1')$$

The conditional expected gain from having the larger set of positions $D_a \supset D_{a'}$, costlessly attainable is thus

$$\begin{aligned} G(\pi; a, a') &\equiv \text{Max}_{d \in D_a} \sum_s \pi_s u(d,s) - \text{Max}_{b' \in D_{a'}} \sum_s \pi_s u(b',s) \\ &= \text{Max}_{d \in D_a} \left\{ \text{Max}_{b \in D_{a'} \cup d} \sum_s \pi_s u(b,s) - \text{Max}_{b' \in D_{a'}} \sum_s \pi_s u(b',s) \right\} \\ &= \text{Max}_{d \in D_a} \left\{ \text{Max} \left\{ 0, \sum_s \pi_s u(d,s) - \sum_s \pi_s u(d',s) \right\} \right\} \end{aligned} \quad (2')$$

The position d' in the last expression is the fixed $d' \in D_{a'}$, asserted to exist for each $d \in D$ in condition (ii). The last expression follows from the second by condition (ii); the second from the first since $D_a \supset D_{a'}$. Since the innermost Maximum of the last expression in (2') is between two linear functions of the vector π , it is a convex function of π . The outer Maximum over $d \in D_a$ is thus a maximum of convex functions, and hence $G(\pi; a, a')$ is a convex function of π . The definitions of $G(\pi; a, a')$ and of the expected second period return to an initial position given an information structure (Π, q) reveal that

$$V(a; \Pi, q) - V(a'; \Pi, q) = \sum_y q_y G(\pi_y; a, a') \quad (3')$$

Knowing that $G(\pi; a, a')$ is convex, definition (R) of more informative tells us that $(\Pi, q) \succeq_I (\Pi', q')$ implies

$$\begin{aligned} V(a; \Pi, q) - V(a'; \Pi, q) &= \sum_y q_y G(\pi_y; a, a') \\ &\geq \sum_y q'_y G(\pi_y; a, a') = V(a; \Pi', q') - V(a'; \Pi', q') \end{aligned} \quad (4')$$

Finally, since $D_a \supset D_{a'}$ implies a $\succeq_F a'$, condition (i) implies that all positions are completely ordered by flexibility; since $(\Pi, q) \succeq_I (\Pi', q')$ implies $\Pi q = \Pi' q' = p$, it follows that expected first period returns are unchanged. Thus the value of any position increases more than every less flexible position, and the optimal initial position chosen after an increase in informativeness must be more flexible than the previous.

Proof of Proposition 6:

The definitions of a^* and \tilde{b} imply

$$\sum_s p_s u(\tilde{b}, s) = \text{Max}_{b \in D} \sum_s p_s u(b, s) \quad (5')$$

$$\text{Max}_{b \in D} \sum_s \pi_{ys} [u(b, s) - c(a^*, b, s)] = \text{Max}_{b \in D} \sum_s \pi_{ys} u(b, s) \quad (6')$$

The expected gain from having been initially in position a^* rather than \tilde{b} , conditional on beliefs $\pi_y = (\pi_{sy})$ being held, is

$$\begin{aligned} G(\pi_y; a^*, b) &\equiv \text{Max}_{b \in D} \sum_s \pi_{sy} u(b, s) - \text{Max}_{b \in D} \sum_s \pi_{sy} [u(b, s) - c(\tilde{b}, b, s)] \\ &= \text{Min} \left\{ c(\tilde{b}), \text{Max}_{b \in D} \sum_s \pi_{sy} [u(b, s) - u(\tilde{b}, s)] \right\} \end{aligned} \quad (7')$$

The latter expression derives from the assumed form of switching costs: if initially in \tilde{b} , one either stays in \tilde{b} or incurs the cost $c(\tilde{b})$ and moves to the optimal position. Since (Π, q) is a star-shaped spreading of (Π', q') ,

$$\begin{aligned} \text{Max}_{b \in D} \sum_s \pi_{sy} [u(b, s) - u(\tilde{b}, s)] &\leq \lambda_y \text{Max}_{b \in D} \sum_s \pi_{sy} [u(b, s) - u(\tilde{b}, s)] \\ &\quad + (1 - \lambda_y) \text{Max}_{b \in D} \sum_s p_s [u(b, s) - u(\tilde{b}, s)] \\ &\leq \text{Max}_{b \in D} \sum_s \pi_{sy} [u(b, s) - u(\tilde{b}, s)] \end{aligned} \quad (8')$$

The first inequality follows from the fact that, since the expressions over which the Maximum is taken are linear (and hence convex) in π_y , the Maximum must also be a convex function of π_y ; the second follows from $\lambda_y \leq 1$,

$\text{Max}_{b \in D} \sum_s \pi_{sy} [u(b,s) - u(\tilde{b},s)] \geq 0$, and (5'). (7') and (8') imply

$$G(\pi_y; a^*, \tilde{b}) \geq G(\pi_y; a^*, \tilde{b}) \quad (9')$$

Finally, since $q = q'$ from the definition of star-shaped spreading,

$$\begin{aligned} V(a^*; \Pi, q) - V(\tilde{b}; \Pi, q) &= \sum_y q_y G(\pi_y; a^*, b) \\ &\geq \sum_y q'_y G(\pi_y; a^*, b) = V(a^*; \Pi', q') - V(\tilde{b}; \Pi', q') \end{aligned} \quad (10)$$

That is, star-shaped spreading of the information structure increases the value of a^* relative to \tilde{b} .

REFERENCES

- Arrow, K. J., "Optimal Capital Policy with Irreversible Investment," Value, Capital and Growth, J. N. Wolfe (ed.), Chicago: Aldine, 1968.
- _____, Beckmann, M., and Karlin, S., "Optimal Expansion of the Capacity of the Firm," Studies in the Mathematical Theory of Inventory and Production, K. J. Arrow, S. Karlin and H. Scarf (eds.), Stanford: 1958.
- Arrow, K. J., and Fisher, A. C., "Preservation, Uncertainty and Irreversibility," Quarterly Journal of Economics, 89, May 1974, 312-319.
- _____, and Kurz, M., "Optimal Growth with Irreversible Investment in a Ramsey Model," Econometrica, 38, March 1970, 331-344.
- Baumol, W. J., "The Transactions Demand for Cash: An Inventory Theoretic Approach," Quarterly Journal of Economics, 66, November 1952, 545-566.
- Blackwell, D., "Equivalent Comparison of Experiments," Annals of Mathematics and Statistics, 24, June 1953, 265-272.
- Cropper, M. L., "A State-Preference Approach to the Precautionary Demand for Money," American Economic Review, 66, June 1976, 388-394.
- Fisher, A.C., Krutilla, J.V., and Cicchetti, C.J., "The Economics of Environmental Preservation: A Theoretical and Empirical Analysis," American Economic Review, 62, September 1972, 605-619.
- Goldman, S. M., "Flexibility and the Demand for Money," Journal of Economic Theory, 9, October 1974, 203-222.
- Grossman, H. I., "Expectations, Transaction Costs, and Asset Demands," Journal of Finance, 24, June 1969, 491-506.
- Hart, A. G., "Risk, Uncertainty, and the Unprofitability of Compounding Probabilities," Studies in Mathematical Economics and Econometrics, O. Lange, F. McIntyre, T. Yntema (eds.), Chicago: University of Chicago Press, 1942.
- _____, "Keynes' Analysis of Expectations and Uncertainty," The New Economics, S. Harris (ed.), London: Dobson, 1947.
- Henry, C., "Option Values in the Economics of Irreplaceable Assets," Review of Economic Studies, 41, January 1974, 89-104.
- _____, "Investment Decisions Under Uncertainty: The 'Irreversibility Effect'," American Economic Review, 64, December 1974, 1006-1012.
- Hicks, J. R., "Liquidity," Economic Journal, 72, December 1962, 787-802.

- Hicks, J. R., "The Two Triads," Critical Essays in Monetary Theory, Oxford: Clarendon Press, 1967.
- _____, The Crisis in Keynesian Economics, New York: Basic Books, 1974
- Hirshleifer, J., "Liquidity, Uncertainty and the Accumulation of Information," Uncertainty and Expectations in Economics, C. F. Carter and J. L. Ford (eds.), Oxford: Basil Blackwell, 1972.
- Keynes, J. M., A Treatise on Money, vol. II, London: MacMillan, 1930.
- Koopmans, T., "On Flexibility of Future Preference," Human Judgements and Optimality, M. W. Shelley and G. L. Bryan (eds.), New York: 1964.
- Lavington, F., The English Capital Market, London: Methuen, 1921.
- Makower, H., and Marschak, J., "Assets, Prices and Monetary Theory," Economica, N.S., 5, August 1938, 261-288.
- Marschak, J., "Money and the Theory of Assets," Econometrica, 6, October 1938, 311-325.
- _____, and Miyasawa, K., "Economic Comparability of Information Systems," International Economic Review, 9, June 1968, 137-174.
- Marschak, T., "Strategy and Organisation in a System Development Project," The Rate and Direction of Inventive Activity, Universities-National Committee for Economic Research, Princeton: Princeton University Press, 1962.
- _____, and Nelson, R., "Flexibility, Uncertainty and Economic Theory," Metroeconomica, 14, April-December 1962, 42-58.
- Menger, K., "On the Origins of Money," Economic Journal, 2, June 1892, 239-255.
- Merkhofer, M. W., Flexibility and Decision Analysis, PhD. dissertation, Department of Engineering-Economic Systems, Stanford University, June 1975.
- Radner, R., "Competitive Equilibrium Under Uncertainty," Econometrica, 36, January 1968, 31-58.
- Rothschild, M., and Stiglitz, J., "Increasing Risk: I. A Definition," Journal of Economic Theory, 2, September 1970, 225-243.
- Stigler, G., "Production and Distribution in the Short Run," Journal of Political Economy, 47, June 1939, 305-328.
- Tisdell, C. A., The Theory of Price Uncertainty, Production and Profit, Princeton: Princeton University Press, 1968.

- Hicks, J. R., "The Two Triads," Critical Essays in Monetary Theory, Oxford: Clarendon Press, 1967.
- _____, The Crisis in Keynesian Economics, New York: Basic Books, 1974
- Hirshleifer, J., "Liquidity, Uncertainty and the Accumulation of Information," Uncertainty and Expectations in Economics, C. F. Carter and J. L. Ford (eds.), Oxford: Basil Blackwell, 1972.
- Keynes, J. M., A Treatise on Money, vol. II, London: MacMillan, 1930.
- Koopmans, T., "On Flexibility of Future Preference," Human Judgements and Optimality, M. W. Shelley and G. L. Bryan (eds.), New York: 1964.
- Lavington, F., The English Capital Market, London: Methuen, 1921.
- Makower, H., and Marschak, J., "Assets, Prices and Monetary Theory," Economica, N.S., 5, August 1938, 261-288.
- Marschak, J., "Money and the Theory of Assets," Econometrica, 6, October 1938, 311-325.
- _____, and Miyasawa, K., "Economic Comparability of Information Systems," International Economic Review, 9, June 1968, 137-174.
- Marschak, T., "Strategy and Organisation in a System Development Project," The Rate and Direction of Inventive Activity, Universities-National Committee for Economic Research, Princeton: Princeton University Press, 1962.
- _____, and Nelson, R., "Flexibility, Uncertainty and Economic Theory," Metroeconomica, 14, April-December 1962, 42-58.
- Menger, K., "On the Origins of Money," Economic Journal, 2, June 1892, 239-255.
- Merkhofer, M. W., Flexibility and Decision Analysis, PhD. dissertation, Department of Engineering-Economic Systems, Stanford University, June 1975.
- Radner, R., "Competitive Equilibrium Under Uncertainty," Econometrica, 36, January 1968, 31-58.
- Rothschild, M., and Stiglitz, J., "Increasing Risk: I. A Definition," Journal of Economic Theory, 2, September 1970, 225-243.
- Stigler, G., "Production and Distribution in the Short Run," Journal of Political Economy, 47, June 1939, 305-328.
- Tisdell, C. A., The Theory of Price Uncertainty, Production and Profit, Princeton: Princeton University Press, 1968.