LEADS, LAGS AND CAUSALITY

By

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Discussion Paper Number 81
October 1976

* I would like to thank Jack Carr, Michael Darby, and Edward Leamer for helpful criticism on an earlier draft. Financial support was provided by the Foundation for Research in Economics and Education.

I. INTRODUCTION

Granger has proposed a definition of causality between two time series in which the time series X is said to cause the time series Y relative to the universe U if predictions of Y based on U are better than predictions of Y based on U-X. Granger provides an explicit definition for two stationary time series with zero means. This causal model is

$$X_t = \sum_{j=1}^{m} a_j X_{t-j} + \sum_{j=0}^{m} b_j Y_{t-j} + \varepsilon_t \tag{1}$$

and

$$Y_t = \sum_{j=0}^{m} c_j X_{t-j} + \sum_{j=1}^{m} d_j Y_{t-j} + \eta_t \tag{2}$$

where ε_t and η_t are two uncorrelated white noise series. The Granger definition of causality implies that X_t is causing Y_t if some c_j is not zero. Similarly, Y_t is causing X_t if some b_j is not zero. If both some c_j and b_j are not zero there is a feedback relation between X and Y. Sims has suggested an alternative statement of Granger Causality (p. 545):

> ...Y can be expressed as a distributed lag function of current and past X with a residual which is not correlated with any values of X, past or future, if, and only if, Y does not cause X in Granger's sense.
>
> We can always estimate a regression of Y on current and past X. But only in the special case where causality runs from X to Y can we expect that no future values of X would enter the regression if we allowed them. Hence, we have a practical statistical test for unidirectional causality: Regress Y on past and future values of X, taking account by generalized least squares or prefiltering of the serial correlation.... Then if causality runs from X to Y only, future values of X in the regression should have coefficients insignificantly different from zero as a group.

Sims applies this test for causality to quarterly United States money and income data from 1947 to 1968 and concludes that (p. 547):

> These results allow firm rejection of the hypothesis that money is purely passive, responding to GNP without influencing it. They are consistent with the hypothesis that GNP is purely passive, responding to M according to a stable distributed lag but not influencing M.

Sims also remarks that (p. 543):

> ...the method is not easily fooled. Simple linear structures with reversed causality like the one put forth by Tobin cannot be constructed to give apparent money-to-GNP causality.

This paper illustrates that the Sims test can be easily fooled by simple lead systems with reversed causality in which turning points of the caused series lead those of the causal series. When the Sims test is applied to simulated data for a simple lead system, the pattern of coefficients obtained depends on the turning points of the two series' and not on the direction of causality. As a result, the Sims test incorrectly selects the leading series as the causal series. The simulation results are of particular interest because of similarities between the lead system data and money, price, and income data to which the Sims test has previously been applied.

II. SOME SIMULATION RESULTS

Consider the following system in which X causes Y in Granger's sense

(3) $$Y_t = \bar{Y}_t + u_t,$$

where u_t is a stochastic error with mean zero and variance σ_u^2 and $\bar{Y}_t$ is some function of current and past X. We will assume that $\bar{Y}_t$ is generated by the differential equation

(4) $$\frac{d\bar{Y}}{dt} + a\bar{Y} = b\,\frac{dX}{dt} + cX,$$

where a, b, and c are constants. This particular equation is assumed because $\bar{Y}_t$ can be made to either lead or lag a cyclical X_t by a proper choice of the coefficients b and c. To obtain a specific solution we assume that X_t is given by $\sin(\eta t)$, so that Y_t becomes

(5) $$Y_t = Ce^{-at} + \int_o^t e^{-a(t-\varepsilon)}[b\eta\cos\eta\varepsilon + c\sin\eta\varepsilon]d\varepsilon + u_t,$$

where C is determined from the initial conditions at time zero. Evaluating the integral of equation (5) gives

(6) $$Y_t = Ke^{-at} + \gamma\sin(\eta t + \phi) + u_t,$$

where

(7) $$K = C - \frac{\eta(ab - c)}{a^2 + \eta^2}$$

(8) $$\gamma = \frac{[(ab\eta - c\eta)^2 + (b\eta^2 + ca)^2]^{1/2}}{a^2 + \eta^2}$$

and

(9) $$\phi = \tan^{-1}\left(\frac{ab\eta - c\eta}{b\eta^2 + ca}\right)$$

If b equals zero we have a simple lag system in which the turning points of $\bar{Y}_t$ lag those of X_t by the phase angle $\phi = \tan^{-1}(-\eta/a)$. If c equals zero we have a simple lead system in which the turning points of $\bar{Y}_t$ lead those of X_t by the phase angle $\phi = \tan^{-1}(a/\eta)$.

Although equation (6) illustrates the lead/lag structure of the model it does not reveal the distributed lag relationship between Y and X. To specify this relationship we assume that dX/dt is constant over each interval i to i+1 and is equal to $X_{i+1} - X_i$. The differential equation can then be solved for each interval and the result summed over all intervals to obtain Y as a distributed lag of current and past X. For the lead system this solution is given by[1]

$$(10) \qquad Y_t = \frac{b(1-e^{-a})}{a}X_t - \frac{b(1-e^{-a})^2}{a}\sum_{i=1}^{t-1} X_i e^{-a(t-i-1)} + u_t.$$

We can also compute X_t as a distributed lag on past X from the formula[2]

$$(11) \qquad X_t = 2.72X_{t-1} - 2.72X_{t-2} + 1.22X_{t-3} - 0.38X_{t-4} + \ldots.$$

The simple lead/lag system is thus a Granger type causal model in which X causes Y and in which there is no influence running from Y to X. Even though X causes Y, the turning points of Y for the lead system precede those of X. The remainder of this section will demonstrate that if this system is corrupted by observation errors the Sims test will select the leading series as the causal series and incorrectly indicate causality from Y to X.

To check the Sims test of causality, simulated data were generated for forty-five "months" using the two sets of coefficients listed in Table 1. The coefficients were selected so that $\bar{Y}_t$ has the same amplitude in both cases (twice the amplitude of X) and either leads or lags X_t by the same phase angle (approximately three months). The error terms u_t were selected from a table of normal errors with mean zero and variance 0.04. It was also assumed that X_t was observed with some error so that the "data" are actually $X_t + v_t$ where v_t has mean zero and variance 0.01. Observation errors on Y_t are, of course, incorporated in the error u_t.

To test for causality running from X to Y we run the regression

(12) $$X_t = \sum_{i=6}^{-4} \alpha_i Y_{t-i}$$

which includes four future, the current, and six past values of Y. In addition, the regression includes a constant, a linear trend, and a correction for first order serial correlation.[3] If the coefficients of the four future values of Y are significantly different from zero as a group, we infer causality from X to Y (actually we reject the hypothesis that there is no influence running from X to Y). To test for causality running from Y to X we run the regression

(13) $$Y_t = \sum_{i=6}^{-4} \alpha_i X_{t-i}$$

and infer causality from Y to X if the coefficients of the future values of X are significantly different from zero as a group.

For case 1, where Y leads X by approximately three months, the regression results are presented in Table 2. The regression of X_t on Y_t has no coefficients of future values of Y_t which are significantly greater than zero. The F-test gives no indication of causality running from X to Y. The regression of Y_t on X_t has the coefficients of X_{t+3} and X_{t+4} significantly different from zero. The F-test would lead us to reject the hypothesis of no influence running from Y to X. On the basis of these tests Sims would infer causality from Y to X when in fact causality runs from X to Y.[4] The test has merely picked the leading series as the causal series. The regression results for case 2, in which X leads Y by approximately three months, are presented in Table 3. The regression of X_t on Y_t has the coefficient of Y_t three months in the future significantly different from zero. The F-test indicates causality from X to Y. The F-test for the regression of Y_t on X_t indicates no causality from Y to X. Again the leading series is picked as the causal series. In this case the test results are consistent with the causal structure of the model. The validity of the F-tests depends on the assumption that the regression residuals are a white noise process. This assumption was tested by computing the cumulative periodogram of the residuals. In all instances the test falls below the maximum value given by Durbin. The difficulty is that the large number of regression parameters creates a wide indeterminate range which includes all the test results.

By merely changing coefficients of our simple model we are able to conclude either that Y causes X or that X causes Y. In both cases the direction of causality depends on which series leads the other. With case 1, where Y leads X, merely sliding the graph of Y forward by approximately three months causes its turning points to "match up" with those of X. This indicates why the coefficients of X_{t+3} and X_{t+4} are the most significant in explaining Y. Conversely, sliding X backward by three months causes it to match up with Y and past values of Y accurately predict X. For case 2 where X leads Y the opposite is true. Sliding X forward by three months causes it to match up with Y and the coefficient of Y_{t+3} is the most significant in explaining X. Conversely, sliding Y backward by three months has it match up with X and past X (particularly X_{t-3}) are the most significant in explaining Y. The pattern of coefficients obtained in the regressions thus depends on which is the leading series and not on the direction of causality.

Since Y can be computed as a distributed lag on current and past X for both the lead and lag systems, we would not expect the Sims test to incorrectly indicate causality from Y to X. The incorrect results obtained with the lead system are apparently due to the data errors added to X_t. With data errors the model becomes

(14) $$Y_t = \Sigma\alpha_i X_i + \Sigma\alpha_i v_i + u_t.$$

The coefficients α_i which minimize the variance of the combined error $\Sigma\alpha_i v_i + u_t$ will not be equal to the coefficients which minimize the variance of the error of equation (10) which does not include data errors. To evaluate the effect of data errors we can increase u_t relative to v_t and observe what happens to the test for causality.[5] The results of this investigation are presented in Figure 1 which gives F-test values for the lead system regressions as a function of σ_u^2/σ_v^2. The regression of Y_t on X_t ceases to erroneously indicate causality from Y to X for ratios of $\sigma_u^2/\sigma_v^2 > 10$. However, as σ_u^2/σ_v^2 increases the pattern of coefficients for the regression of Y on X does not change much. The coefficient on X_{t+4} remains approximately twice as large as any of the other coefficients. Sims would incorrectly take this as evidence of influence running from Y to X (p. 545):

> In applying the F-tests for causal direction suggested in the previous section, one should bear in mind that the absolute size of the coefficients is important regardless of the F value.... If the estimated coefficients on future values are as large or larger than those on past values, bidirectional causality may be very important in practice, despite insignificant F's.

Not shown on the graph is the fact that the regression of X_t on Y_t begins to indicate causality from X to Y at values of $\sigma_u^2/\sigma_v^2 > 300$.

It does not seem unreasonable to expect data errors of the size needed to fool the Sims test to be present in most time series. The simulation results are, however, not limited to the case of data errors in X_t. If the X_t were stochastic and observed without error but Y_t depended on smoothed or permanent values of X_t we would still obtain the same type of simulation results. That is, if $X_t = \sin(\eta t) + v_t$ and is observed without error but $\bar{Y}_t$ is a function of the permanent X_t given by $\sin(\eta t)$ the simulation is unchanged. It is also not clear what would happen if the system were strongly non-linear and the distributed lag on past values did not accurately represent the dependence of one variable on the other. In this instance the test might also tend to select the leading series as the causal one.

III. SOME ECONOMIC APPLICATIONS

The simple lead/lag model used in the previous section has several economic applications. For instance, in Cagan's model of hyperinflation equation (6) describes the dependence of the rate of inflation on a cyclical rate of money issue. To illustrate this fact we consider the Cagan model given by

(15) $\ln(M/P) = \varepsilon - \alpha E,$

where M is the money stock, P is the price level, and E is the expected rate of inflation generated from the adaptive rule

(16) $\frac{dE}{dt} = \beta(\dot{P}-E),$

where $\dot{P}$ is the rate of inflation. The model can be solved for $\dot{P}$ as a function of the rate of monetary growth $\dot{M}$ by differentiating equation (15)

(17) $\dot{P} = \dot{M} + \alpha \frac{dE}{dt},$

and substituting into equation (16) to obtain

(18) $\frac{dE}{dt} = k(\dot{M}-E),$

where $k = \beta/(1-\alpha\beta)$. We can solve differential equation (18) for E_t as a function of $\dot{M}$ and substitute into equation (17) to obtain $\dot{P}$ as a function of $\dot{M}$. If we assume that $\dot{M} = \sin(\eta t)$, the solution for $\dot{P}$ becomes

(19) $\dot{P} = Ke^{-kt} + \gamma \sin(\eta t+\phi),$

where

(20) $\gamma = \frac{[\alpha^2k^4\eta^2 + (\alpha k\eta^2+k^2+\eta^2)^2]^{1/2}}{k^2 + \eta^2}$

and

(21) $\phi = \tan^{-1}\left(\frac{\alpha k^2\eta}{\alpha k\eta^2 + k^2 + \eta^2}\right)$

Note that $\phi > 0$ so the Cagan model is a simple lead system in which the turning points of $\dot{P}$ lead those of $\dot{M}$.

The results of Table 2 can thus be viewed as a simulation of Cagan's model of hyperinflation in which $\dot{P}(Y)$ leads the causal variable $\dot{M}(X)$. The Sims test thus incorrectly indicates causality from $\dot{P}$ to $\dot{M}$ for the simulated hyperinflation data. This point is of considerable interest when we compare the similarities of the $\dot{P}$ and $\dot{M}$ data for the Austrian hyperinflation with the X and Y data for the lead system graphed in Figure 2. The cycles of $\dot{P}$ are roughly twice the amplitude of the $\dot{M}$ cycles and $\dot{P}$ leads $\dot{M}$ by approximately one month. Based on the simulation results, we would expect a regression of $\dot{P}$ on $\dot{M}$ to have a large coefficient on $\dot{M}_{t+1}$ and thus indicate causality from $\dot{P}$ to $\dot{M}$. This is exactly what happens when Sargent and Wallace apply the Sim's test to the hyperinflation data. They concluded that there is a strong indication of causality running from inflation to money issue rather than vise versa. However, it appears that the Sims test has merely picked the leading series as the causal series. The Sargent and Wallace regression results are perfectly consistent with a model of hyperinflation in which causality runs from $\dot{M}$ to $\dot{P}$ but in which $\dot{P}$ leads $\dot{M}$.

The solution of the lead/lag system is also identical to the solution of a macroeconomic model discussed by James Tobin. Tobin constructed an ultra-Keynesian model in which income was determined by the investment multiplier and in which money passively responded to the needs of trade. By altering coefficients of the model, money can be made to either lead or lag the income response to cyclical investment. The simulations of Tables 2 and 3 can be viewed as two separate simulations of the Tobin model. In case 1 money (Y) leads income (X) and in case 2 income leads money. One could infer causality in either direction depending on which case was selected. Case 1 represents the sequence of turning points for the United States money and income data used by Sims in his test for causality. His conclusion that money causes income may merely reflect the fact that the leading series was picked as the causal series. Again the leading series may not be the causal series.

SUMMARY

Several simulations have illustrated that the Sims test for causality can be easily fooled by a simple linear model in which turning points of the dependent variable lead those of the causal variable. The Sims test selected the leading series as the causal series. This choice was not consistent with the causal structure of the lead system. The simulation was not some bizarre case but closely duplicated aspects of Cagan's model of hyperinflation.

Data errors in the causal series were what fooled the test. However the results would also apply to instances where the dependent variable was a function of smoothed or permanent values of the causal variable. If the test can be so easily fooled by simple linear models it seems unlikely to work for more complex non-linear structures.

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FOOTNOTES

[1]We have assumed that t is large enough so that the initial condition term Ce^{-at} has decayed to zero and can be ignored. The only error involved in the distributed lag approximation is the difference between the actual dX/dt and $X_{i+1} - X_i$ over each interval i to i+1. This error is negligible compared to u_t. An equation similar to equation (10) could also be derived for the lag system.

[2]We expand X_t in a Taylor series about X_{t-1} so that

$$X_t = X_{t-1} + (\frac{dX}{dt})_{t-1} + (\frac{d^2X}{dt^2})_{t-1}/2! + \ldots.$$

We then approximate the derivatives as $(dX/dt)_{t-1} = X_{t-1} - X_{t-2}$, $(d^2X/dt^2)_{t-1} = X_{t-1} - 2X_{t-2} + X_{t-3}$, ... to obtain equation (11).

[3]The regression parameters are those used by Sargent and Wallace to test for causality between money and prices during hyperinflation. The model parameters (primarily the fact that $\bar{Y}$ is twice as large as X) were also selected to replicate some aspects of the hyperinflation data. In any case, the simulation results are not sensitive to the number of past and future values used in the regressions.

[4]Ten separate simulations were performed using different error terms for each simulation. The Sims test indicated causality from Y to X in all but one simulation with an average F statistic of 5.49. The results presented in Table 2 are for the first simulation and are typical of the remaining cases. The incorrect results are not due to the fact that Sims has altered Grangers test for causality. If we regress X on past X and Y then we again

infer causality from Y to X. Since Y leads X, the past values of Y (particularly Y_{t-4}) are the most significant in explaining X. According to Granger, this implies causality from Y to X.

[5] The error terms v_t eliminates the high degree of multicollinearity between the values of X_t and makes it possible to estimate all of the coefficients of X_t in the regression of Y on X. As a result, v_t can't be reduced toward zero to investigate the effect of data errors and we must increase u relative to v. The results obtained are not very sensitive to the assumed value of σ_v^2.

Table 1

Parameters For Data Generation

Parameter	Case 1 Simple Lead System $\Phi > 0$	Case 2 Simple Lag System $\Phi < 0$
η	0.25	0.25
a	0.25	0.25
b	2.828	0.0
c	0.0	0.707
γ	2.0	2.0
Φ	0.785	-0.785

Table 2

Simple Lead System

Value of i	$(1-\rho L)X_t = (1-\rho L)\Sigma\alpha_i Y_{t-i}$		$(1-\rho L)Y_t = (1-\rho L)\Sigma\alpha_i X_{t-i}$	
	Future Values	No Future Values	Future Values	No Future Values
-4	-0.030		0.721	
-3	0.252		0.492	
-2	-0.149		0.049	
-1	-0.009		0.428	
0	0.011	0.010	0.395	0.682
1	0.127	0.201	-0.211	0.816
2	-0.050	-0.030	0.024	0.250
3	0.129	0.114	0.295	0.253
4	0.241	0.146	-0.237	-0.333
5	0.041	0.054	-0.359	-1.019
6	0.099	0.074	0.064	-0.649
Largest S.E.	0.228	0.178	0.270	
Smallest S.E.	0.171	0.133	0.214	
SSR	0.315	0.347	0.445	0.836
ρ	-0.206	-0.215	0.221	0.455
D.W.	2.028	2.010	2.142	2.166
F Test	0.533		4.613	

NOTE: The F-test is for all future coefficients equal to zero and is distributed as (4, 21). The 95% significance level is 2.84. All coefficient standard errors were approximately the same so only the largest and smallest are listed to indicate the range of values.

Table 3

Simple Lag System

Value of i	$(1-\rho L)X_t = (1-\rho L)\Sigma\alpha_i Y_{t-i}$		$(1-\rho L)Y_t = (1-\rho L)\Sigma\alpha_i X_{t-i}$	
	Future Values	No Future Values	Future Values	No Future Values
-4	0.187		0.390	
-3	0.378		-0.095	
-2	-0.094		-0.299	
-1	-0.106		0.268	
0	0.038	0.238	0.344	0.303
1	0.154	0.320	-0.015	0.251
2	-0.207	-0.107	0.387	0.245
3	0.111	0.014	0.645	0.653
4	0.188	-0.062	0.299	0.450
5	-0.076	-0.219	0.302	0.198
6	-0.079	-0.106	0.467	0.242
Largest S.E.	0.219	0.190	0.276	0.210
Smallest S.E.	0.150	0.156	0.217	0.161
SSR	0.297	0.497	0.461	0.539
ρ	-0.270	-0.013	0.160	0.152
D.W.	1.989	2.022	1.918	1.946
F Test	3.535		0.888	

NOTE: The F-test is for all future coefficients equal to zero and is distributed as (4, 21). The 95% significance level is 2.84. All coefficient standard errors were approximately the same so only the largest and smallest are listed to indicate the range of values.

Figure 1

F-Test for Lead System

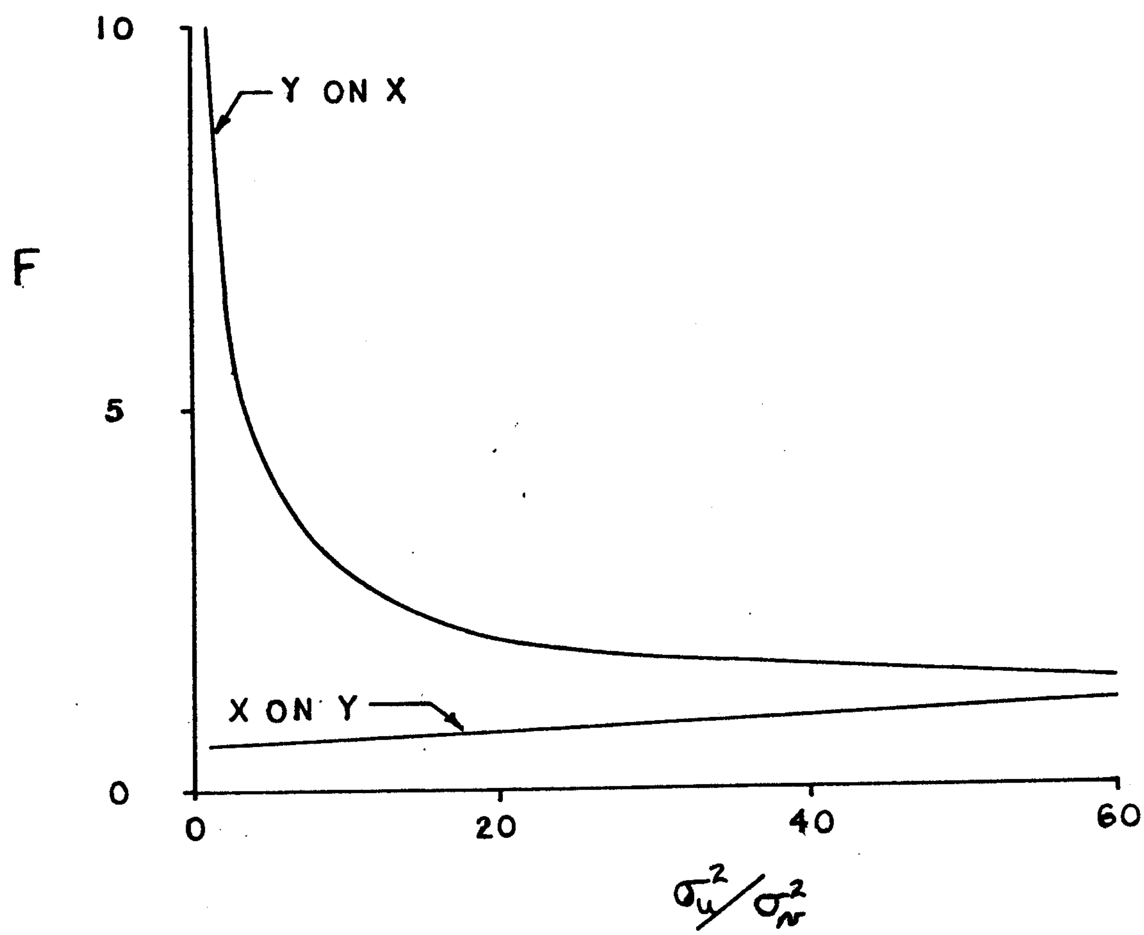

Figure 2

Lead System and Austrian Data