DYNAMIC SYSTEMS, RULES OF CORRESPONDENCE AND LAGGED DEPENDENT VARIABLES

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I. Introduction

In all scientific endeavor there is a clear distinction between theoretical structures and the particular events (observations) which theories attempt to explain. In its purest form a theory is represented by a fully axiomatized system. Given the axioms, all implications of the theory can be explored by logical or mathematical transformations. Few theories are ever fully axiomatized; instead, the theory is represented by a model which is usually stated in mathematical form. The implications of the theory are then examined by mathematical manipulations of the model. Scientific theories are separated from logical structures by the fact that scientific theories are capable of being falsified. That is, the theories generate some statements which can be tested against observable phenomena. An important aspect of testing theories is the development of rules of correspondence which connect the theoretical statements and the observation statements. In many cases the rules of correspondence involve a mathematical structure that is as complex as the theoretical model.[1]

To be more explicit and to develop concepts to be used later, we adopt some terminology commonly employed in systems theory. The top half of Figure 1 illustrates a typical dynamic system composed of a difference equation which specifies the output Y_t as a function of the input X_t. The system is dynamic because lagged values of Y_t influence the current output. For a given input, the system equation could be solved to yield the theoretical output as a function of time. The theoretical model could then be evaluated by comparing the theoretical output with observations. In many cases, Y_t is not directly observed and some observation system (rules of correspondence) is required to transform Y_t into the theoretical observables

Z_t. The observation system is represented schematically in the lower half of Figure 1. The theory would then be evaluated by comparing Z_t with the actual observations $\hat{Z}_t$. This comparison would entail a nonlinear iterative adjustment of certain constants of the model in order to obtain the Z_t which best fits the $\hat{Z}_t$. The Z_t will not equal the $\hat{Z}_t$ for a number of reasons. The model solution might represent the mean value of the output of a stochastic system, the model might be an incorrect description of the actual dynamic process, or $\hat{Z}_t$ might be corrupted by observation errors which are caused by errors in the data collection process or an inexact correspondence between the data collected and the equivalent theoretical variables.

The distinction between theoretical models, theoretical observations, and actual observations which are to be explained by the model is often ignored in applied economics. This may be due to the fact that most economic models are formulated so that the output Y_t is directly observed. There is, therefore, no need for an observation system which transforms the theoretical output into theoretical observations. This still leaves an important distinction between the theoretical output Y_t and the observations of that output $\hat{Y}_t$. This distinction is also ignored especially in difference equation models where past values of the observations are substituted for past values of the theoretical output. This paper illustrates that the common practice of ignoring the difference between Y_t and $\hat{Y}_t$ has rendered most estimates of dynamic difference equation models virtually meaningless.

II. Dynamic Economic Models

Dynamic economic models are usually represented by a stochastic difference equation. We will examine a linear second order system which has been widely used in partial adjustment models such as the multiplier accelerator models of income determination. The theoretical model is

$$Y_t = aY_{t-1} - bY_{t-2} + cX_t + u_t, \tag{1}$$

where Y_t is the output, X_t is the input, and u_t is an error term which is assumed to be independently distributed with mean zero and variance σ_u^2. We will assume that the system output is directly observable so that Y_t represents the theoretical data and $\hat{Y}_t$ the actual data.

The dynamic properties of difference equation models can be described by considering the output response to a step input. That is, we assume the input jumps to the value X* at time zero and we examine how the output converges to the new equilibrium where the mean value of Y_t is given by $cX^*/(1-a+b)$. The second order system of equation (1) will converge (for $b < 1$) through the damped oscillation

$$Y_t = r^t[k \cos(\theta t - \varepsilon)] + \frac{cX^*}{1-a+b} + w_t, \tag{2}$$

where $r = \sqrt{b}$, $\theta = \cos^{-1}(a/2r)$ and ε and k are determined from the initial conditions. The error w_t represents the propagation of u_t through the dynamic system and has mean zero and the autocorrelated structure $w_t = u_t + aw_{t-1} - bw_{t-2}$. Suppose that we have data $\hat{Y}_t$ describing the response of an unknown system to a step input. How would we determine if the model of equation (1) describes the dynamics of that system? For an initial set of parameters a, b and c and initial conditions k and ε, we could solve the damped oscillation of equation (2) for the mean value of

Y_t (denoted by $\overline{Y}_t$) and compute the residuals $e_t = \hat{Y}_t - \overline{Y}_t$. The model parameters and initial conditions could then be adjusted to minimize Σe_t^2. Since $\overline{Y}_t$ is nonlinear in the parameters and initial conditions, this adjustment would involve some iterative scheme. The model would then be judged on the basis of how accurately the theoretical data series $\overline{Y}_t$ reproduces the actual data series $\hat{Y}_t$. Alternatively, we could start with the initial conditions $\overline{Y}_0$ and $\overline{Y}_1$ and generate $\overline{Y}_t$ recursively from

$$\overline{Y}_t = a\overline{Y}_{t-1} - b\overline{Y}_{t-2} + cX_t, \tag{3}$$

to obtain an expression which is equal to the damped oscillation of equation (2). The system output would then be given by

$$Y_t = a\overline{Y}_{t-1} - b\overline{Y}_{t-2} + cX_t + w_t. \tag{4}$$

The parameters a, b and c and the initial conditions $\overline{Y}_0$ and $\overline{Y}_1$ could then be iteratively adjusted to obtain the $\overline{Y}_t$ which best fits the data $\hat{Y}_t$.

This approach has, in general, not been followed in the estimation and evaluation of dynamic economic models. Instead of using equation (4) to generate the theoretical output, past data values are substituted for past theoretical values in equation (1) to obtain the linear model

$$\overline{Y}_t = a\hat{Y}_{t-1} - b\hat{Y}_{t-2} + cX_t, \tag{5}$$

for which the system output becomes[2]

$$Y_t = a\hat{Y}_{t-1} - b\hat{Y}_{t-2} + cX_t + u_t. \tag{6}$$

Equation (3) is nonlinear in the model parameters because $\overline{Y}_{t-1}$ and $\overline{Y}_{t-2}$ are generated recursively and thus implicitly depend on the model parameters. Equation (5) is linear because the data $\hat{Y}_{t-1}$ and $\hat{Y}_{t-2}$ are independent of the model parameters. With equation (3), two degrees of freedom are lost by estimating the initial conditions $\overline{Y}_0$ and $\overline{Y}_1$. With equation (5), two degrees of freedom are lost because the first two data points $\hat{Y}_0$ and $\hat{Y}_1$ are needed to start the equation. The attraction of equation (5) is that all the tools of linear estimation theory can be used to determine the model parameters. However, this simplification was obtained by the questionable step of ignoring the difference between theory and observation. The remainder of this section illustrates that the practice of inserting data values for past theoretical values has rendered the estimation of dynamic economic models virtually meaningless.

The preceding paragraph contains a rather sweeping indictment of a good deal of applied econometrics which needs to be justified. To set the stage for this justification we will first examine a case where equation (1) describes the response of an actual system to a step input. We also assume that the output is observed without error so that $\hat{Y}_t = Y_t$. Starting from an initial equilibrium in which the mean value of Y equalled twenty, equation (1) was recursively solved fifteen different times, with u_t a normal random variable with zero mean and unit variance, to obtain the output response for the step input $X^* = 10.0$. Each simulation generated thirty five months of data for the parameters $a = 1.5$, $b = 0.75$ and $c = 1.0$. The simulations represent a model in which Y_t converges to the new mean value of forty through a damped oscillation which has a period of 12 months and whose amplitude declines by approximately 80% during each period.

The simulated data were then fit using the linear model of equation (5) which makes no distinction between theory and data and using the nonlinear model of equation (3) in which the theoretical mean value of Y_t is generated recursively. With equation (5) the regression residuals $\hat{Y}_t - \overline{Y}_t$ are equal to u_t and are a white noise process. Ordinary least squares (OLSQ) will, therefore, provide consistent and efficient estimates of the model parameters. With equation (3) the regression residuals are equal to w_t and have the autocorrelated structure $w_t = u_t + aw_{t-1} - bw_{t-2}$. The OLSQ iterative scheme used to minimize Σw_t^2 will, therefore, generate consistent but inefficient estimates of the model parameters. As a result, the parameters were also estimated using an iterative generalized least squares (GLS) scheme to minimize the summed square of the transformed residuals $e_t = w_t - aw_{t-1} + bw_{t-2}$ where e_t equals the white noise process u_t.[3] Estimation results for the fifteen simulations are summarized in Table 1 and illustrate that all three estimation techniques accurately recover the model parameters. The standard deviations for the nonlinear OLSQ and GLS estimates indicate that these estimates are more robust to the particular realization of u_t. The nonlinear OLSQ estimation produces residuals which are strongly autocorrelated because they are equal to w_t. The residuals for the other estimation procedures have the properties of u_t. This different residual pattern is illustrated in Figure 2 which graphs the data $\hat{Y}_t$ for the first simulation and the values of $\hat{Y}_t$ given by equations (3) and (5).

On the basis of the Monte Carlo results reported in Table 1 the reader might wonder what all the fuss is about. The linear model accurately recovers the model parameters, and, as illustrated in Figure 2, accurately reproduces the system output. Clearly the advantages of working with a linear estimation model greatly outweigh the slight edge in robustness obtained with the nonlinear model. However, lets proceed by considering the "data" graphed in Figure 3. We assume that these data represent the response of a hypothetical system to the step input X* = 10.0. The data were constructed with considerable malice aforethought because they obviously could not have been generated by a model which converges to a new equilibrium through the damped oscillation of equation (2). If we attempt to fit the data with the dynamic system described by equation (1), we should get a clear indication that the theoretical model does not describe the dynamic process illustrated in Figure 2.

Equation (5) was estimated using the data of Figure 2, and the results are presented in the first row of Table 2. The estimation results are startling. The second order system couldn't possibly produce this data yet the estimate gives a high value of R^2 and indicates that the theoretical model is very good. The coefficients are "reasonable" and indicate a stable model, the standard errors in brackets are small, and there is no evidence of serial correlation of the residuals.[4] An investigator fitting the data with equation (5) would have no indication that the model was incapable of describing the actual dynamic process. The inadequacies of the model are clearly established by the nonlinear estimates of equation (3) presented in last two rows of Table 2. The very low R^2 clearly indicates that the theoretical data generated by the model cannot reproduce the actual data. Although confidence intervals for the nonlinear estimates of each parameter were not derived, a few significance tests were performed. In the first test, b was set equal to zero and the remaining parameters and one initial condition were estimated. The sum square of residuals increased by only ten percent and the appropriate F-test indicated that b was not significantly different from zero. The damped oscillation of the second order system does not do a significantly better job of reproducing the data than the damped exponential of the first order system. In fact, the entire model is not significantly better than the assumption that $\bar{Y}_t$ is a constant for the entire data interval. The low standard errors and the high R^2 reported in the first row of Table 2 are thus entirely spurious.

The disparity between the two estimation techniques is further illustrated in Figure 4 which compares the actual data with the theoretical data generated from equation (5) and from equation (3) using the OLSQ parameters. The theoretical data generated by equation (3) does not reproduce the variations in $\hat{Y}_t$ because this data cannot be replicated by a damped oscillation. If this is the case, why does equation (5) do such a good job? Equation (5) works because the substitution of $\hat{Y}_{t-1}$ and $\hat{Y}_{t-2}$ for lagged theoretical values prevents the model from deviating significantly from the actual data. Consider instances in Figure 4 where $\bar{Y}_t$ computed from equation (5) departs significantly from the path of $\hat{Y}_t$ (points eight and eleven for example). In period t+1 this discrepancy in the model is removed when $\hat{Y}_t$ is substituted for the lagged theoretical value $\bar{Y}_t$. In this way the model is continuously corrected back to the data series even if the dynamic specification of the model is completely incorrect. It would appear to be more difficult to construct a model which doesn't seem acceptable when estimated using the equivalent of equation (5) than to construct a model which does seem acceptable.[5]

In most applications the input X_t will vary with time and the output will represent the cumulative response of the system to the past X_t. A time varying X_t does not present any logical problems for the nonlinear estimation discussed above. It does become more difficult to write down the analytical solution which is the analog of equation (2), but no additional complications are introduced if we recursively generate the model solution from equation (3). The explicit analytical solution would, however, emphasize several important points. It is common place to include a constant term in regression equations such as (3) and (5). This implies that the system will have an equilibrium output equal to the constant times c/(1-a+b) even if there is zero input. In many cases it would be difficult to provide a theoretical argument for including such a constant. The constant might improve the residuals and be statistically significant, but its inclusion would be as ad-hoc as adding a time trend to the regression. Such ad-hoc procedures make it more difficult to evaluate how accurately a model describes the dynamics of an actual process since they improve the fit and mask the inability of the model to reproduce the data. The analytical solution would also illustrate that the output Y_t can be written as a distributed lag on the input X_t. Essentially each period's input represents an impulse which is propagated forward as a damped oscillation. The output at any time is the cumulative effect of all the past inputs. The pattern of lag coefficients would be completely determined by the dynamic specification of the model. If models are presented as arbitrary distributed lags, the implication is that the investigator is unable to or has not attempted to specify the dynamic structure of the model. The estimation of arbitrary distributed lag models is usually complicated by multicollinearity of the lagged data values. A number of procedures, such as the Almon technique, have been developed to deal with multicollinearity by constraining the lag coefficients. If half as much energy were expended in specifying the dynamic model and its equivalent lag structure, our understanding of economic processes might proceed at a more rapid pace. Finally, one often encounters difference equations which contain both lagged Y_t and lagged X_t. If this is not to be taken as confusion on the part of the model builder -- a mixture of a difference equation and its solution -- there should be a clear explanation of why the system response depends not only on the input but also on its time derivatives.

The reader may think that the problems encountered with equation (5) result from special characteristics of the hypothetical data series of Figure 3 and would never occur when estimating actual time series data. To dispell this idea we want to consider a simple model for net exports of the United States. We assume that changes in real net exports (Y_t) depend on the desired level Y_t^* according to the partial adjustment rule

$$Y_t - Y_{t-1} = \alpha(Y_t^* - Y_{t-1}) + u_t. \tag{7}$$

We further assume that desired real net exports equal βX_t where X_t is the deviation of real GNP from its trend. Equation (7) can then be written as the first order difference equation

$$Y_t = aY_{t-1} + bX_t + u_t \tag{8}$$

The dynamic model of equation (8) could be solved for the mean value of Y_t by substituting past data values for past theoretical values to obtain an equation analogous to equation (5)

$$\overline{Y}_t = a\hat{Y}_{t-1} + bX_t, \tag{9}$$

or by recursively generating the theoretical output to obtain an equation analogous to equation (3)

$$\bar{Y}_t = a\bar{Y}_{t-1} + bX_t. \tag{10}$$

The model parameters could then be estimated by minimizing the sum square of residuals $\hat{Y}_t - \bar{Y}_t$. Equation (9) would yield a linear regression with white noise residuals u_t while equation (10) would yield a nonlinear OLSQ estimation with residuals $w_t = u_t + aw_{t-1}$. Estimation results for United States quarterly data for the period 1964-I to 1974-IV are presented in Table 3. The linear estimation results indicate that the model of equation (8) accurately describes the dynamic process determining the level of net exports. The nonlinear results, however, indicate that this conclusion is illusory. This point is further emphasized by Figure 5 which contains a graph of the net export data and the theoretical output generated by equations (9) and (10). The graph of equation (10) clearly indicates the poor performance of the dynamic model. Equation (9) obscures the inadequacies of the model because the substitution of $\hat{Y}_{t-1}$ for past theoretical values forces the output to track the data series. An investigator using the standard estimation techniques would never realize how badly the model actually performs. One could, I suppose, argue that the model was correct and that differences between equations (9) and (10) merely represent the propagation of the unpredictable stochastic disturbance. This argument is particularly distasteful because it allows the acceptance of almost any difference equation and inhibits the development of more accurate dynamic models.

III. Observation Errors

In the previous section we analyzed a case where Y_t differed from $\hat{Y}_t$ because the dynamic model was incapable of describing the actual system response to a step input. In this section we consider cases where the model of equation (1) describes the actual dynamic process but where the process is observed with some error. We will initially assume that the input is observed without error but that the output data are given by

$$\hat{Y}_t = Y_t + v_t \tag{11}$$

where v_t is independently distributed with zero mean and unit variance. The data from the first simulation were corrupted by the errors v_t and estimated using equations (3) and (5) to generate the theoretical data $\overline{Y}_t$. The estimation results are summarized in Table 4. If past data values are used to generate $\overline{Y}_t$ from equation (5), the observation errors will produce autocorrelated residuals. Consequently, least squares applied to equation (5) will yield biased estimates of the model parameters. The simulation results summarized in the first row of Table 4 confirm this fact. The direction of the biases is determined by the way in which the data error v_t is propagated through equation (5). The data error creates a residual component e_t^* with the autocorrelated structure $e_t^* = v_t + ae_{t-1}^* - be_{t-2}^*$. In the estimation, the values of a and b would be biased downward in an attempt to reduce the variance of this autocorrelated error. However, as a and b are biased downward an additional residual component is created which represents the incorrect propagation of the stochastic disturbance u_t through the model. The values of a and b are selected which minimize the combined variance of these two residual components. Since the equilibrium mean value of Y_t derived from the model is given by $cX^*/(1-a+b)$, c will be biased in order to maintain this multiplier close to a value of four. For the fifteen simulations the average value of $c/(1-a+b)$ was 4.07.

Of course the careful econometrician would recognize that the estimates of equation (5) were biased even if he hád no knowledge of the data errors. Although the Durbin-Watson statistic falls in the indeterminate range, the Durbin test for serial correlation in the presence of a lagged dependent variable clearly indicates a correlated error structure. The standard remedy would be to replace the lagged dependent variables with instrumental variables or to transform the data in order to eliminate the autocorrelated error term. Of course instrumental variables methods would be completely ineffective because the specification of X_t precludes using its lagged values as instruments. Transformations of the data entail an iterative process to determine the autocorrelated structure which minimizes the sum square of residuals. These computations are not less burdensome than those required for the nonlinear estimates of equation (3).

Results for the nonlinear estimates of equation (3) are presented in the second and third rows of Table 4. Comparing these results with those of Table 1 illustrates that the nonlinear OLSQ estimation results are very robust to observation errors. This is because the model parameters imply a particular dynamic path for the mean value of Y_t; i.e., they specify the period of the oscillation, how rapidly it damps to zero, and the final equilibrium value of Y. As a result we would not expect random variations in $\hat{Y}_t$ to have a significant impact on the parameter values which best represent the underlying dynamic process. The nonlinear GLS results are not as robust to the observation errors. The data transformation required for the GLS estimates appears to amplify the effect of these errors.[6]

In most cases the input variable X_t will not be measured without error, but we will observe some $\hat{X}_t$ which differs from the true input. In that case the OLSQ estimates obtained from equation (3) might be sensitive to the input errors. We therefore want to investigate the case where the output observations are given by equation (11) and where

$$\hat{X}_t = X^* + w_t, \tag{12}$$

where w_t has zero mean and unit variance. To the investigator it will appear that the system is driven by a random input while the input is actually constant but corrupted by observation errors. The input and output of the first Monte Carlo study were corrupted by w_t and v_t respectively and the model parameters were estimated using equation (3) to generate the theoretical data. Estimation results for the nonlinear OLSQ estimates are summarized in Table 5. Comparing these results with the second row of Table 1 shows that the estimation process is not significantly affected by the errors in X_t. In any dynamic model there is some systematic output response for the nonstochastic portion of the input. In this case, the parameters and the input determine the damped oscillation and final equilibrium value which best fits the data. Corrupting the input and output by random errors does not significantly alter the coefficients which provide the best representation of the underlying dynamic process.

IV. Summary and Conclusion

In the philosophy of science there is a clear distinction between theory and observation. A theory is usually represented by a mathematical model, and implications of the theory are explored through mathematical manipulations of the model. In particular, the model can be used to generate a set of theoretical observations which can be compared with actual observations in order to evaluate the theory.

In dynamic economic models represented by difference equations, the distinction between theory and observation is usually ignored. Past data values are substituted for past theoretical values in order to generate the theoretical output path implied by the model. This substitution creates an equation which is linear in the model parameters and greatly simplifies the estimation process. However, the substitution of data for theoretical values makes it difficult to evaluate any model. It prevents the theoretical solution from deviating from the data even if the dynamic model is incapable of describing the actual process. The high values of R^2 and low standard errors for the model coefficients obtained in the estimation of dynamic models may, as a result, be entirely spurious.

If a clear distinction between theory and observation is maintained, then dynamic economic models are nonlinear in the model parameters. The estimation of the model becomes more difficult but there are several advantages which go with the added difficulty. It is possible to evaluate the models on the basis of how well the theoretical data series reproduces the actual data series. The problem of spuriously high values of R^2 is eliminated. The nonlinear OLSQ estimation process is also robust to observation errors in contrast to the linear estimation results.

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FOOTNOTES

[1]A comprehensive discussion of theories, models and rules of correspondence can be found in most books on the philosophy of science such as Nagel. Application of these concepts to the theory of dynamic systems is discussed by McGarty.

[2]This relationship is only true if the output is observed without error so that $\hat{Y}_t = Y_t$. If there are observation errors, the error term of equation (6) will be u_t plus the contribution of the observation errors. This point is discussed in Section III.

[3]The nonlinear least squares algorithm of Marquardt was used for the OLSQ and GLS estimates of equation (3). For the OLSQ estimates the initial conditions $\overline{Y}_0$ and $\overline{Y}_1$ were estimated but are not reported in Table 1. For the GLS estimates the residual transformation $e_t = w_t - aw_{t-1} + bw_{t-2}$ tends to make the model solution insensitive to the initial conditions thus making it difficult to estimate $\overline{Y}_0$ and $\overline{Y}_1$. Consequently $\overline{Y}_0$ and $\overline{Y}_1$ were set equal to $\hat{Y}_0$ and $\hat{Y}_1$ and were not estimated. The GLS Durbin-Watson statistic is computed from the transformed residuals but the value of R^2 measures how accurately $\overline{Y}_t$ reproduces the data series $\hat{Y}_t$.

[4]Since there are lagged dependent variables the Durbin-Watson statistic will be biased towards two and is not a good test for serial correlation. Durbin's test for serial correlation in the presence of lagged dependent variables gives no indication of serial correlation.

[5]Since the residuals show no evidence of serial correlation the estimate of equation (5) should at least be consistent. If the parameters of

equation (3) are restricted to those obtained from the linear estimate there is only an insignificant increase in the sum square of residuals. However obtaining consistent estimates of the parameters of a completely erroneous model is no great accomplishment especially if the estimation technique conceals the inadequacies of the model.

[6] A discussion of how data differencing amplifies the effect of observation errors is contained in Jacobs.

Table 1

Simulation Results for Dynamic Model

Estimation Model	$\bar{a}$	$\bar{b}$	$\bar{c}$	$\bar{R}^2$	$\overline{D\text{-}w}$
Equation (5) (Linear Model)	1.488 (0.067)	0.742 (0.053)	1.020 (0.114)	0.978	2.056
Equation (3) OLSQ (Nonlinear Model)	1.497 (0.028)	0.741 (0.021)	0.982 (0.099)	0.860	0.381
Equation (3) GLS (Nonlinear Model)	1.501 (0.026)	0.751 (0.018)	1.005 (0.079)	0.876	2.038

NOTE: The parameters are mean values obtained from the fifteen simulations; i.e., $\bar{a} = \Sigma a_i/15$. The values in brackets are standard deviations computed as $[(\Sigma a_i - \bar{a})^2/15]^{\frac{1}{2}}$.

Table 2

Results for Data of Figure 3

Estimation Method	a	b	c	R^2	D-w
Equation (5) (Linear Model)	1.530 (0.103)	0.799 (0.101)	1.124 (0.242)	0.911	2.029
Equation (3) OLSQ (Nonlinear Model)	1.381	0.825	1.862	0.201	0.267
Equation (3) GLS (Nonlinear Model)	1.543	0.813	1.117	-0.088	2.045

Table 3

Results for Model of Real Net Exports

Estimation Method	a	b	R^2	D-w
Linear Model	1.040 (0.050)	0.036 (0.014)	0.882	1.997
Nonlinear Model OLSQ	1.026	0.048	0.193	0.149

Table 4

Simulation Results for Output Data Errors

Estimation Method	$\bar{a}$	$\bar{b}$	$\bar{c}$	$\bar{R}^2$	$\overline{D\text{-}W}$
Equation (5) (Linear Model)	1.243 (0.123)	0.533 (0.103)	1.193 (0.146)	0.901	2.655
Equation (3) OLSQ (Nonlinear Model)	1.497 (0.027)	0.740 (0.021)	0.983 (0.104)	0.844	0.698
Equation (3) GLS (Nonlinear Model)	1.470 (0.029)	0.718 (0.016)	1.016 (0.079)	0.845	3.033

Table 5

Simulation Results for Output and Input
Data Errors from Nonlinear OLSQ Estimates of Equation (3)

$\bar{a}$	$\bar{b}$	$\bar{c}$	$\bar{R}^2$	$\overline{D\text{-}w}$
1.508	0.748	0.966	0.800	0.499
(0.036)	(0.025)	(0.115)		

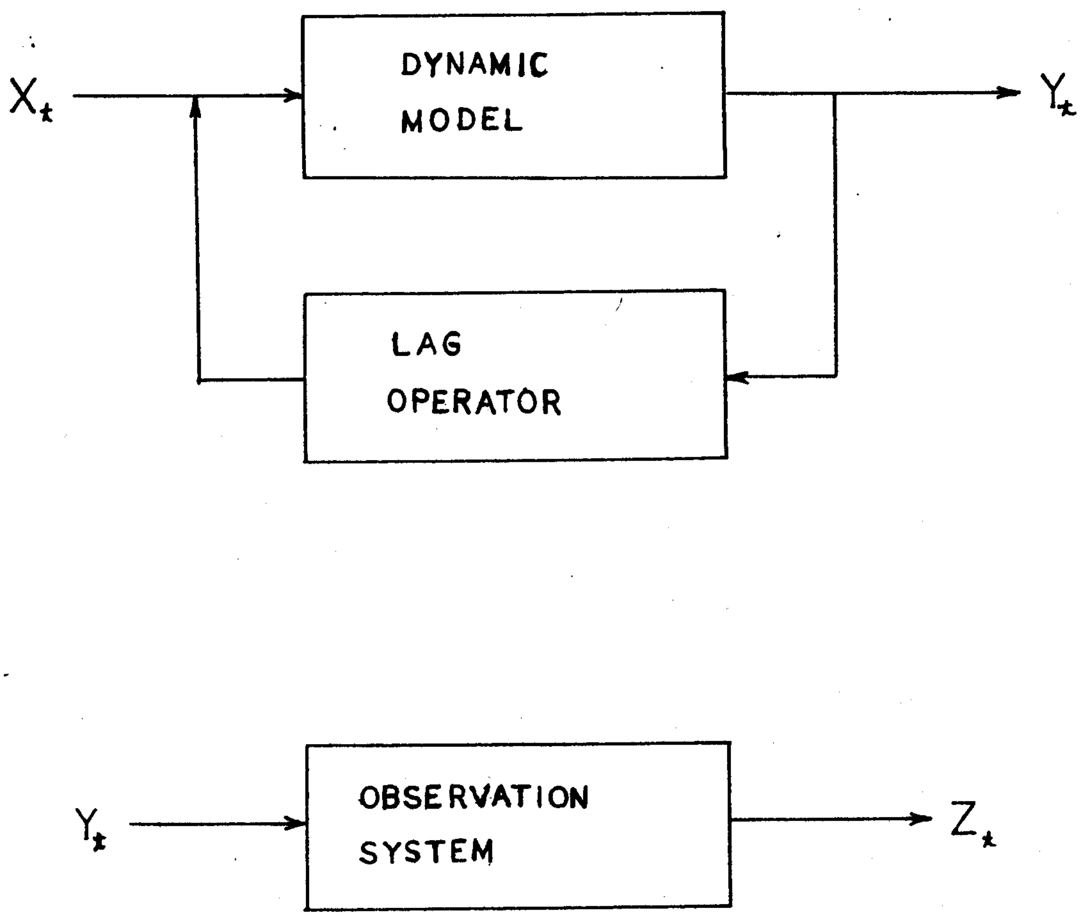

Figure 1

Theoretical and Observation Systems

Figure 2

Typical Simulation Results

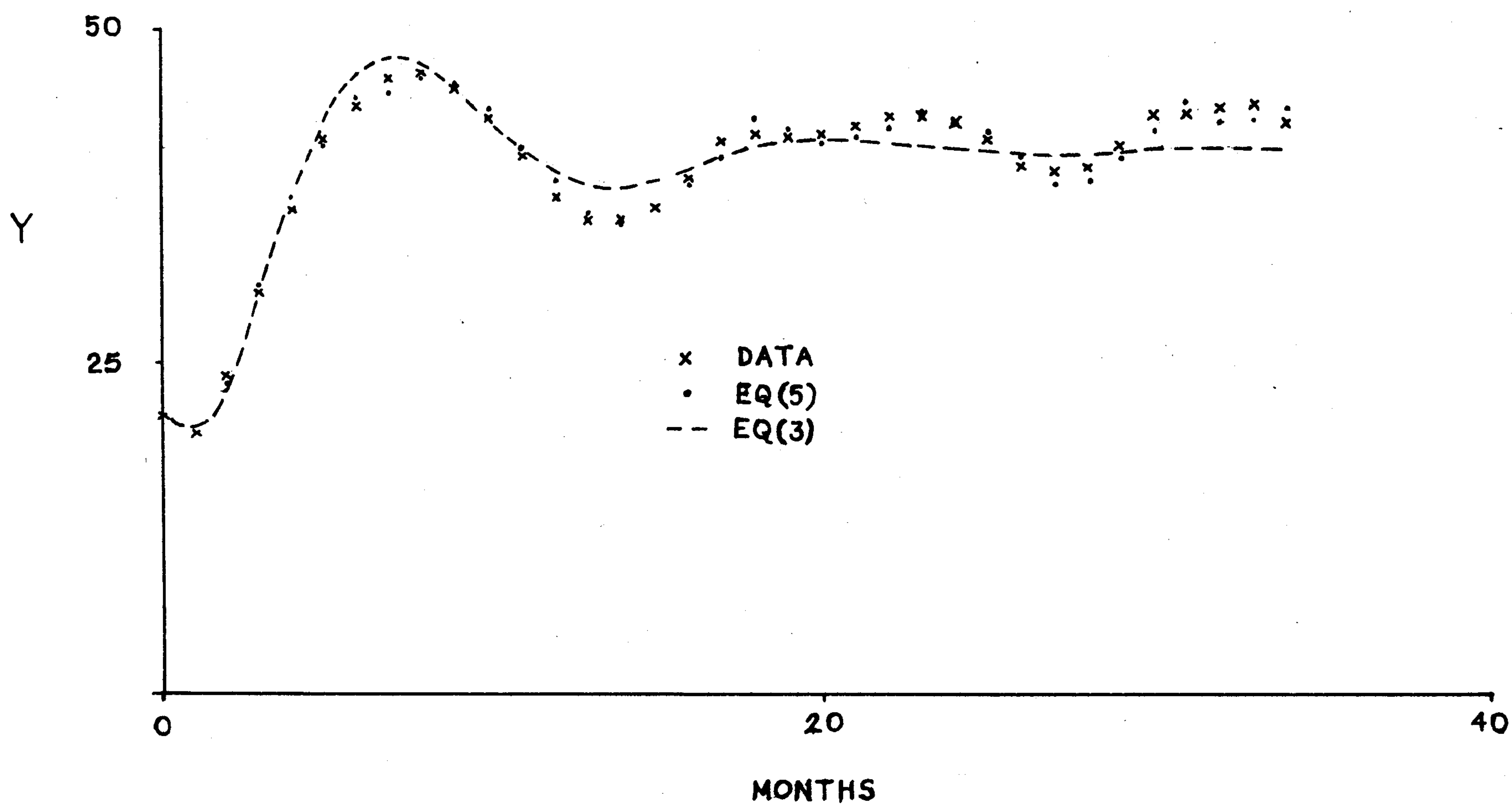

Figure 3

Hypothetical Response to Step Input

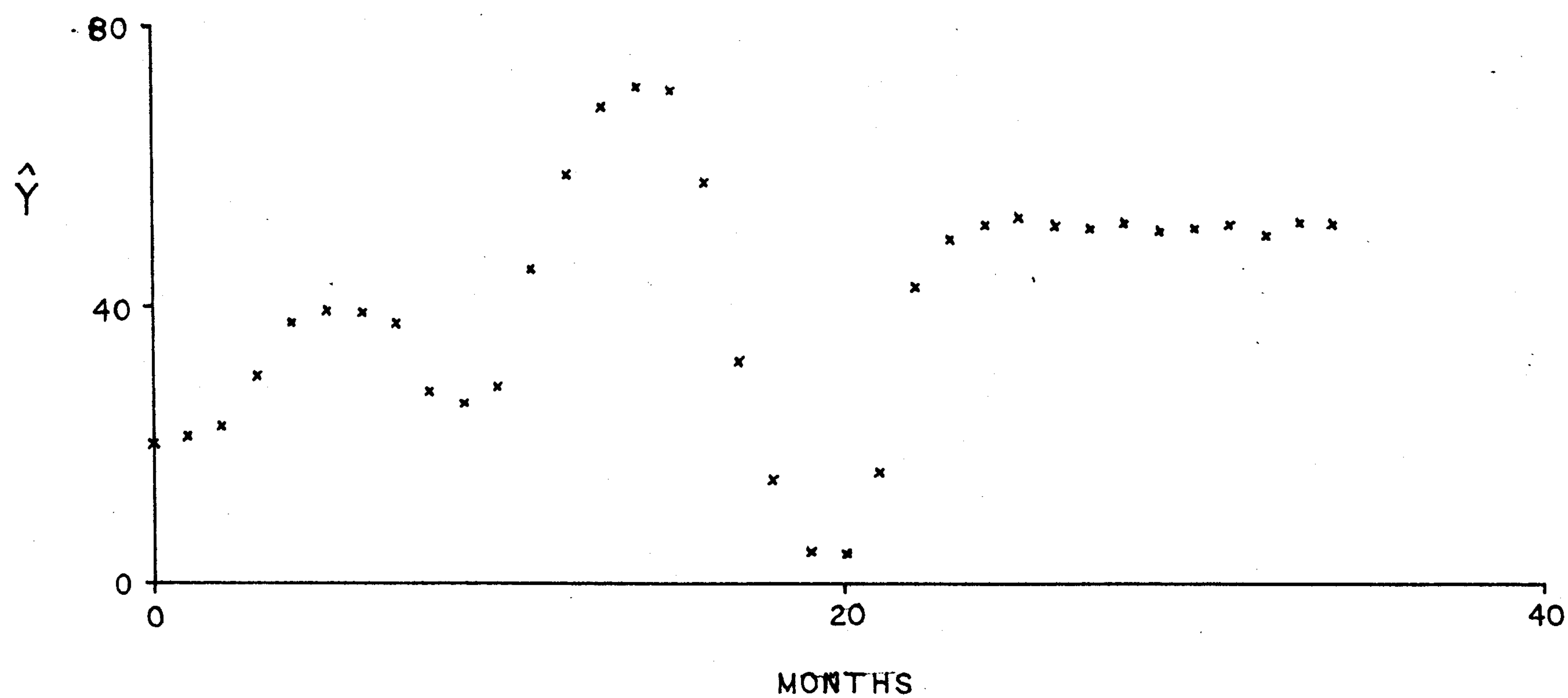

Figure 4

Hypothetical Data and Theoretical Output

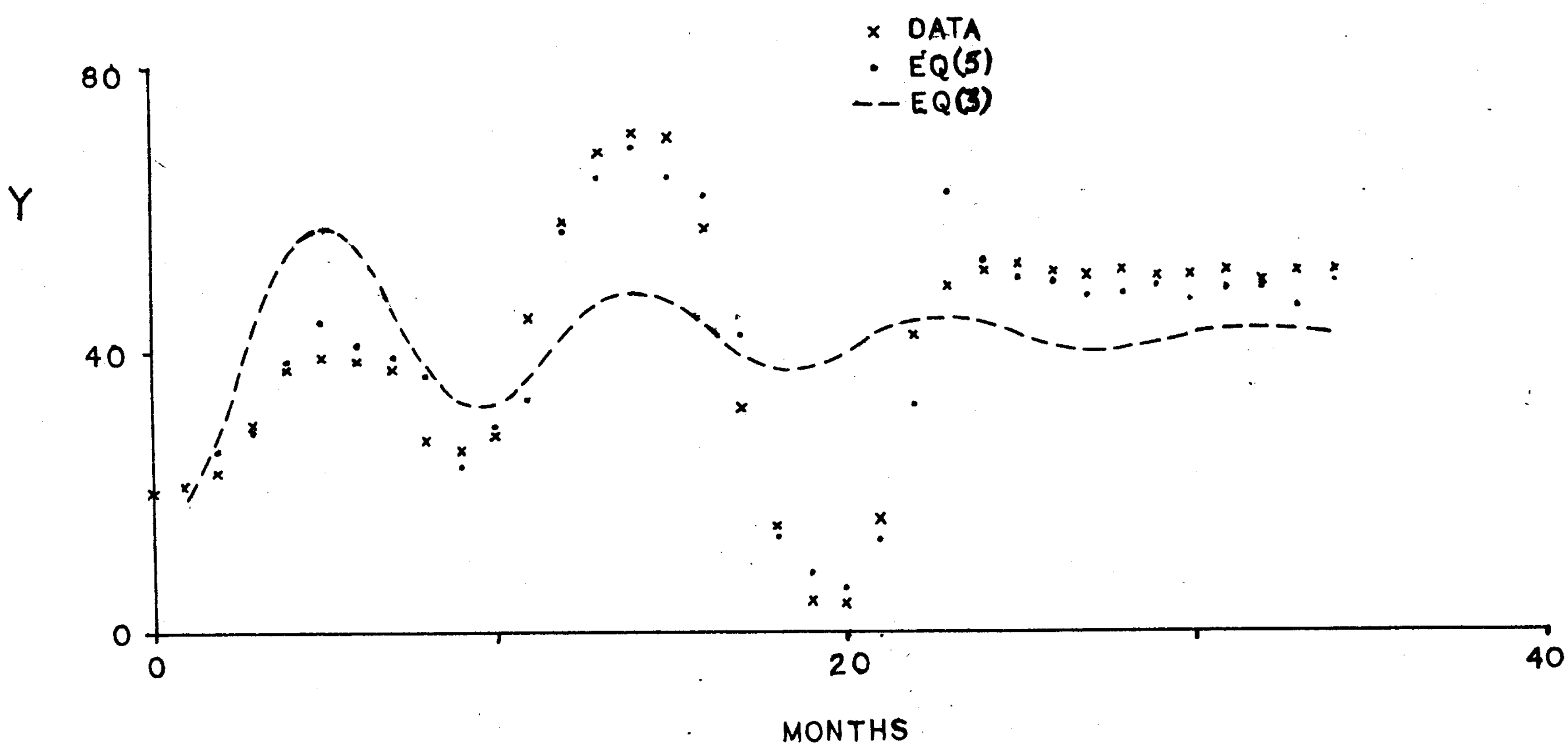

Figure 5

Net Exports Model