THE JUST RATE OF DEPLETION OF A NATURAL RESOURCE*

by

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I. Intergenerational Equity

A major focus of the rapidly growing literature on natural resource use is the modification of the Ramsey-Koopmans, macro-growth model to account for the constraints imposed by the *exhaustibility* of production inputs.[1] There are now published papers which explore the implications of a wide variety of alternative assumptions about the underlying technology.

In stark contrast, however, is the absence of debate over the appropriate measure of intergenerational welfare.[2] Almost universally it is assumed that the objective of generation t is to maximize a function of the form,

$$V(t) = \int_t^{\infty} U(C(\tau))e^{-\rho(\tau-t)}d\tau, \tag{1}$$

where $C(\tau)$ is the consumption (vector) of generation τ.

As usually interpreted, the total utility of generation t, $V(t)$, is a discounted sum of the direct utility that each future generation derives from its own consumption. The parameter ρ is the *social* (rather than the private) rate of pure time preference.

A fundamental difficulty with this approach, is that selection of this social discount rate is essentially ad hoc. A value of ρ that seems reasonable when thinking of a simple growth model, is difficult to defend in the more extreme versions of the natural resource models.

Reacting to this difficulty, Koopmans, in his 1975 Nobel lecture, concluded that ..

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[1] A good sample of this work is to be found in the Review of Economic Studies Symposium on Natural Resources (1974). In particular see Solow, Stiglitz and Dasgupta and Heal. For a useful recent survey, see Blank, Anderson and d'Arge (1977).

[2] A rare exception is the discussion by Solow (op. cit.).

> .. how much to discount future utilities cannot be equitably resolved a priori and in the abstract. One needs to take into account the opportunities expected to be available to the various consumers now and later, for the given technology and resource base.

It seems to me, however, that there is a more satisfactory way of resolving this conundrum. In what follows I shall develop a concept of justice that avoids the need to determine a social discount rate.

To begin, the discount factor ρ in expression (1) will be interpreted as the *private* rate of pure time preference. Given the biologically and culturally induced concern a generation feels for the well-being of its descendants, it is natural to assume that ρ is strictly positive. Then $V(t)$ is a measure of the private preferences of generation t. Solving for the consumption plan that maximizes $V(t)$, subject to resource constraints, then becomes an exercise in positive rather than normative economics.

To address the equity issue we therefore require a concept of intergenerational justice. The idea that I wish to explore is closely related to the ideas of Rawls (1971) and the fairness principle described by Varian (1974). The basic notion is that the current generation is unfairly exploiting its earlier point in time if, *by choice*, it leaves some future generation t^* with assets that would make the current generation worse off, were it compelled to exchange places with generation t^*.

This definition, introduced in Phelps and Riley (1977) fails to take account of the possibility that, because of the nature of the technology, the future is inevitably worse in the above sense. Let A_t be the set of plans feasible for generation t. Suppose that under plan $a \in A_t$, generation t perceives that the consumption stream faced by generation t^* ($> t$) yields a strictly lower utility level. Suppose also that there is an alternative plan, a*, that is perceived to increase the welfare of generation t* without decreasing the perceived welfare of those generations that appear worse off than generation t*. Then the current generation is again unfairly exploiting its earlier point in time under plan a. The above ideas are formalized in the following definition.

INTERGENERATIONAL JUSTICE AS NONEXPLOITATION

A plan $a \in A_t$, the set of plans feasible for generation t, is just if EITHER

$$V^a(\tau) \geq V^a(t), \qquad \tau > t$$

OR

for each $t^* > t$ satisfying $V^a(t^*) < V^a(t)$ there is no $a^* \in A_t$ such that

$$V^{a^*}(t^*) > V^a(t^*)$$

and $$V^{a^*}(\tau) < V^a(t^*) \Rightarrow V^{a^*}(\tau) \geq V^a(\tau)$$

It will be assumed that in the just economy, each generation chooses the plan that maximizes its own utility, subject to the constraint that the plan should not be exploitative.

In contrast to the Rawlsian 'Maxi-min' criterion no comparison of the utility levels of different agents is required. Instead we require of the current generation that it uses its own preferences to determine whether its consumption plan is just.

This feature also distinguishes the concept from that of fairness. A necessary condition for fairness is that no agent wishes to switch consumption bundles with any other. Except for special cases in which the optimal plan leads to a constant utility stream, this condition will not be satisfied under justice as non-exploitation.

We now show that under certain circumstances the non exploitation constraint is never binding. Suppose that the unconstrained maximization of $V(t)$ yields a strictly increasing consumption stream, $C(\tau)$. Then if $U'(\cdot) > 0$ we have

$$(1) \qquad V'(t) = \rho \int_t^{\infty} U(C(\tau))e^{-\rho(\tau-t)}d\tau - U(C(t))$$

$$> \rho \int_t^{\infty} U(C(t))e^{-\rho(\tau-t)}d\tau - U(C(t))$$

$$= 0$$

Therefore a necessary condition for intervention to satisfy the constraints of justice is that the unconstrained consumption sequence be decreasing with τ, at least over some interval.[3] Since the latter is a common characteristic of natural resource constrained growth models, it is to an example of such a model that we now turn.

In the following section a model of natural resource use is described, and the optimal plan of the current generation is derived. Then in section III conditions are sought under which this plan is exploitative. The best plan for the current generation, among those satisfying the constraints of justice as non-exploitation, is also derived. Finally, in section IV, the implications for public intervention in a competitive market system are discussed.

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[3] In the Ramsey-Koopmans macro-growth model with initial capital stock lower than the golden-rule level, consumption rises monotonically with time. For such economies the unconstrained optimum therefore satisfies the constraints of justice as non-exploitation.

II. A Model of Natural Resource Use

To focus on essentials, the model to be examined is a relatively simple one. Supplies of a renewable resource L (labor) and an energy source, E, are used to produce a single consumption good, C, according to the concave, constant returns to scale production function,

$$C = G(E, L) \tag{2}$$

where $G(0, L) = G(E, 0) = 0$. The supply of the renewable resource not used in the production of C, is available for the production of energy. Energy output per unit of L is equal to β. The latter increases exogenously over time. There is also a stock N(0) of a natural energy source. The change over time of the total energy stock is then given by

$$\dot{N} = \beta(\tau)(\bar{L} - L) - E \tag{3}$$

where $\bar{L}$ is the total available supply of the renewable resource.

Initially $\beta(\tau)$ is sufficiently small that the cost of the alternative energy source outweights potential benefits. However, over time $\beta(\tau)$ increases thereby reducing the marginal cost of this source.

The preferences of the current generation ($t = 0$) are represented by the utility function

$$V(0) = V\{C(\tau)\,|\,\tau \geq 0\} = {}_0\!\int^{\infty} U(C(\tau))e^{-\rho\tau}d\tau \qquad \rho \geq 0 \tag{4}$$

U(C) is assumed to be a strictly increasing concave function such that $U'(0) = \infty$.

We begin by seeking the consumption and natural energy use plan that maximizes the utility of the current generation. Ignoring initially the non-negativity constraint,

$$N(\tau) \geq 0,$$

maximization of (4) subject to (2) and (3) becomes a standard control problem. We therefore form the Hamiltonian

(5) $$H = e^{-\rho\tau}\{U(C) + \lambda(\tau)[\beta(\tau)(\bar{L} - L) - E]\}$$

Necessary conditions for a maximum are that at each point in time the control variables, E and L, must be chosen to maximize H.

Moreover the auxiliary variable $\lambda(t)$ must follow a path satisfying

(6) $$\frac{d}{d\tau}(e^{-\rho\tau}\lambda(\tau)) = -\frac{\partial H}{\partial N}$$

Since the Hamiltonian (τ) is independent of N, expression (6) implies that

(7) $$\lambda(\tau) = \lambda(0)e^{\rho\tau}$$

Also

(8) $$\frac{\partial H}{\partial E} = e^{-\rho\tau}\{U'(C)\frac{\partial G}{\partial E} - \lambda(\tau)\}$$

$$= 0 \qquad \text{for a maximum.}$$[4]

Similarly

(9) $$\frac{\partial H}{\partial L} = e^{-\rho\tau}\{U'(C)\frac{\partial G}{\partial L} - \beta(\tau)\lambda(\tau)\}$$

$$\geq 0 \qquad \text{for a maximum}$$

where the strict inequality implies $L = \bar{L}$.

case 1: All energy is supplied from the natural stored source ($L(\tau) = \bar{L}$)

From (7) and (8)

$$U'(G(E, \bar{L}))\frac{\partial G}{\partial E}(E, L) = \lambda(0)e^{\rho\tau}$$

Since U(C) and $G(E, \bar{L})$ are concave functions it follows that both U'(C) and $\frac{\partial G}{\partial E}$ are decreasing functions of E. Thus $E(\tau)$ and hence $C(\tau)$ decrease over time.

case 2: Both energy sources are utilized ($L(\tau) < \bar{L}$). In this case inequality (9) becomes an equality. Combining this with (8) yields

[4] The assumption that $U'(0) = \infty$ and $G(0, L) = 0$ rules out the possibility of a corner solution. Similarly it must be the case that $L(\tau) > 0$ for all τ.

(10) $$\frac{\partial G/\partial E}{\partial G/\partial L} = \frac{1}{\beta(t)}$$

Expression (10) is the requirement that the marginal value of energy (or 'demand price') must be equal to the marginal cost (or 'supply price') of the alternative source if the latter is to be utilized.

Since G is homogeneous of degree 1, the fact that the right hand side of (10) is decreasing over time implies that L/E is decreasing over time. This in turn implies that $\partial G/\partial L$ is increasing and that $\partial G/\partial E$ is decreasing over time. Then from (8) $U'(C)$ is increasing over time. Hence the optimal consumption path $C(\tau)$ is again decreasing. Finally if output $G(E, L)$ and L/E are both decreasing it must be the case that $L(\tau)$ is decreasing.

We have therefore shown that whether or not $L(\tau) = \bar{L}$, the optimal consumption stream is decreasing. But the supply of the natural stored energy source is finite. Therefore, for sufficiently large τ, the economy is forced to rely almost entirely on the alternative energy source. Since $\beta(\tau)$ is increasing with τ the production possibilities steadily increase over time. Hence a declining consumption path implies that resources are eventually wasted. But such resources could have been used to increase $C(\tau)$ for some interval, and hence increase $V(0)$. We have therefore reached a contradiction.

It follows that the non-negativity constraint must eventually be binding. Thus there is some switching point s such that $N(\tau) > 0$, $\tau < s$ and $N(s) = 0$.

With a positive rate of pure time preference, and a net marginal productivity of zero, the current generation would, at time s wish to borrow from future generations. Since this is not feasible, $\dot{N}(s) = 0$. But this argument holds for all $\tau > s$. We therefore have

$$N(\tau) = 0, \qquad \tau \geq s$$

We shall call this phase 3.

In phase 3 the economy is entirely dependent upon its current production of energy from the alternative source. It therefore chooses an input level $L(\tau)$ such that

$$C(\tau) = \underset{L(\tau)}{\mathrm{Max}} \{G(\beta(\tau)(L - L(\tau)), L(\tau))\}$$

Clearly $C(\tau)$ is increasing throughout this phase. Furthermore the first order condition,

$$\frac{\partial G/\partial E}{\partial G/\partial L} = \frac{1}{\beta(\tau)}$$

combined with the fact that $C(\tau)$ is increasing, implies that $E(\tau)$ is increasing in phase 3. Finally it can be shown that whether or not $L(\tau)$ is increasing depends upon whether or not the elasticity of output with respect to energy supply is a decreasing function of E. These results may be summarized as follows.

Proposition 1:

There is a switching point s, *beyond which the current generation's utility maximizing plan relies solely on the alternative energy source. In this phase consumption and energy production increase over time. Moreover the amount of the renewable resource used in the production of* C *increases over time, if and only if the elasticity of output with respect to energy input is a decreasing function of* E.

By the definition of the switch point, s, $N(\tau)$ is positive for all $\tau < s$. Then the conditions derived above for the optimal use of the natural energy source, apply over the interval $[0, s]$. We have seen that $L(\tau)$ is non-increasing. Then in general we may distinguish two other phases.

Phase 1: $L(\tau) = \bar{L}$

Phase 2: $L(\tau) < \bar{L}$ and $L(\tau)$ is decreasing.

Moreover, given the strict concavity of $U(G(E, L))$ and the linearity of (3), it can be shown that there can be no discontinuity in the time paths of the control variables $E(\tau)$ and $L(\tau)$. In particular there is no discontinuity at $\tau = s$. Therefore, since $L(s) < \bar{L}$ the economy must pass through phase 2. Whether or not it passes through phase 1 depends upon the initial conditions. To summarize, we have:

Proposition 2:

> *The current generations maximizing plan can be divided into three phases. In the first only the natural stored source of energy is utilized. Energy use and output declines over time. In the second phase both energy sources are utilized. Output continues to decline and the amount of the renewable resource used to produce energy from the alternative source increases steadily. Eventually phase 3 is reached in which the stock of stored energy is zero.*

To obtain further qualitative results it is necessary to specify more precisely the nature of the technology and preferences. In what follows the following assumptions will be utilized.[5]

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[5] Generalization to the class of constant elasticity of utility functions is straightforward.

(11) $\beta(\tau) = \beta(0)e^{\gamma\tau}$

(12) $G(E, L) = E^{\alpha}L^{1-\alpha}$

(13) $U(C) = \ln C$

Denoting the time at which the economy switches from phase 1 to phase 2 as r, and utilizing the necessary conditions (7), (8) and (9), the optimal time profiles of the control variables can be shown to satisfy:

$$L(\tau) = \begin{cases} \bar{L} & 0 \leq \tau \leq s \\ \bar{L}e^{-(\rho+\gamma)(\tau-r)} & r \leq \tau \leq s \\ (1-\alpha)\bar{L} & s \leq \tau \end{cases} \tag{14}$$

$$E(\tau) = \begin{cases} \alpha L\beta(s)e^{\rho(s-\tau)} & 0 \leq \tau \leq s \\ \alpha L\beta(s)e^{\gamma(\tau-s)} & s \leq \tau \end{cases} \tag{15}$$

These profiles and the implied consumption and natural resource stock profiles are depicted in Figure 1.

Given the continuity of $L(\tau)$ we also require

$$\bar{L}e^{-(\rho+\gamma)(s-r)} = (1-\alpha)\bar{L}$$

Hence

$$\theta = s - r = -\frac{\ln(1-\alpha)}{\rho + \gamma} \tag{16}$$

Expression (16) tells us that the length of phase 2, θ, is independent of initial conditions. This greatly simplifies the following comparative statics analysis.

To determine the second switch point s we must equate the total energy demand over [0, s] with total supply.

$$\begin{aligned} \text{cumulative supply} &= N(0) + \int_r^s \beta(\tau)(L - \bar{L})d\tau \\ &= N(0) + \beta(s)\bar{L}\{\frac{1}{\gamma}(1-e^{-\gamma\theta}) - \frac{(1-\alpha)}{\rho}(e^{\rho\theta}-1)\} \end{aligned}$$

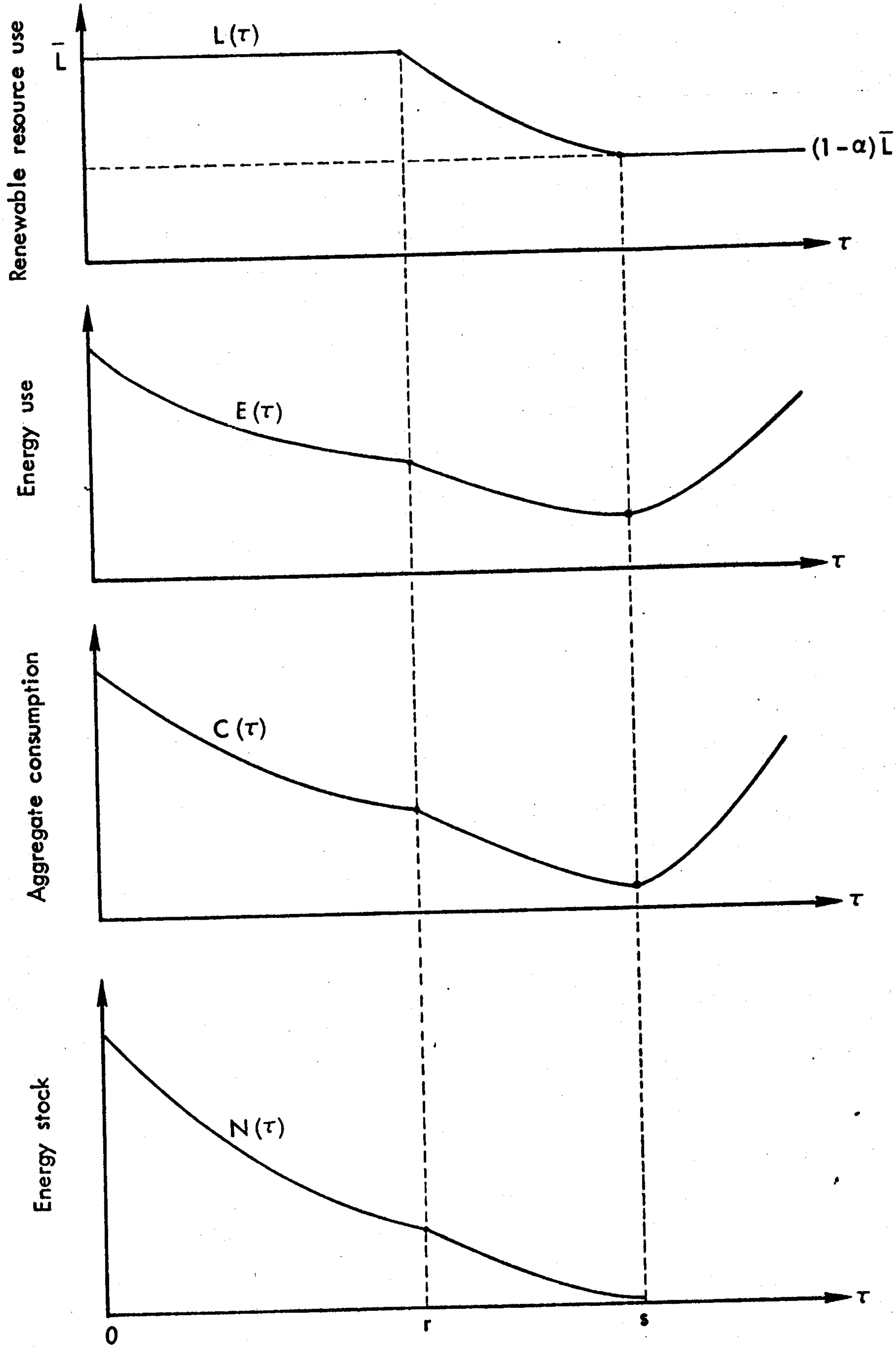

Fig. 1 — The Maximizing Plan for the current generation

$$\begin{array}{c}\text{cumulative}\\ \text{demand}\end{array} = {}_0\int^s E(\tau)\,d\tau = {}_0\int^s \alpha\bar{L}\beta(s)e^{\rho(s-\tau)}\,d\tau$$

$$= \frac{\alpha\beta(s)\bar{L}(e^{\rho s}-1)}{\rho}$$

But at $\tau = s$, cumulative demand and cumulative supply are equal. We therefore have,

$$e^{\gamma s}\left\{\frac{\alpha}{\rho}(e^{\rho s}-1) - \frac{1}{\gamma}(1-e^{-\gamma\theta}) + \frac{(1-\alpha)}{\rho}(e^{\rho\theta}-1)\right\} = \frac{N(0)}{\bar{L}\beta(0)}\ . \tag{17}$$

Noting that the left hand side of (17) is increasing in s, we have, as an immediate implication:

Proposition 3:

An increase in $N(0)/\beta(0)$, *that is, a higher initial level of naturally stored energy or higher initial unit cost of producing energy, lengthens the time until the alternative source is utilized and the time at which the natural resource is fully depleted.*

Comparative statics analysis of a change in the rate of technological progress γ, or the private rate of pure time preference, ρ, is not so straightforward. A difficulty arises because an increase in either ρ or γ, reduces θ, the length of phase 2. This works in opposition to other effects so that the net effect is ambiguous. However for sufficiently large ρ and γ the marginal effect of the change in θ is small relative to other effects. We therefore have

Proposition 4:

There exists a triple $(\widehat{\frac{N(0)}{\beta(0)}}, \hat{\rho}, \hat{\gamma})$ *such that for all larger* $(\frac{N(0)}{\beta(0)}, \rho, \gamma)$, *the switch point* s, *is decreasing in* ρ *and* γ.

The intuition behind this result is that an increase in ρ raises the marginal rate of substitution of earlier for later consumption. Similarly an increase in γ raises consumption in later periods and hence the marginal rate of substitution of earlier for later consumption. To offset this increase, the current generation raises its earlier consumption and hence its rate of natural energy use.

Lastly we compare the final depletion dates of two economies with the same technologies but different initial stocks of N. Suppose economy 1 has a current stock $N_1(0)$ and a planned final depletion date, s_1. Economy 2 has a larger current stock. Moreover at time t this will have depleted to the current level of the first economy, that is, $N_2(t) = N_1(0)$. Then from (17) the date of final depletion, s_2 must satisfy

$$(18) \qquad e^{\gamma(s_2-t)}\left\{\frac{\alpha}{\rho}(e^{\rho(s_2-t)}-1) - \frac{1}{\gamma}(1-e^{-\gamma\theta}) + \frac{1-\alpha}{\rho}(e^{\rho\theta}-1)\right\} = \frac{N_1(0)}{\bar{L}\beta(0)e^{\gamma t}}$$

Since the left hand side of (18) is strictly increasing in s_2, it follows from a comparison with (17), that $s_2 - t$ is less than s_1, and that s_2 exceeds s_1. To summarize, we have:

Proposition 4:

Two economies have the same technology and preferences but different levels of naturally stored energy. Their respective optimal policies imply that the natural resource stock will be depleted after an interval s_i; $i = 1, 2$, *and the second economy will have the current natural resource stock of the first after an interval of length* t. *Then* $s_1 < s_2 < s_1+t$.

This proposition tells us that the economy with the larger initial stock of energy maintains a larger stock until the date of complete exhaustion.

However the gap between the times the two economies reach any particular level of N decreases over time.

III. The Just Rate of Natural Resource Use

We have seen that maximizing the current generations utility yields a decreasing consumption profile over the first two phases. This raises the possibility that the constraints of justice as non-exploitation of future generations, are violated. To determine conditions under which this is the case, we must examine the slope of the profile

$$(19) \qquad V(t) = \int_t^{\infty} U(C(\tau))e^{-\rho(\tau-t)}d\tau$$

resulting from the maximization of V(0). For the economy defined by (11)-(13) we have

$$V(0) = \int_0^{\infty} e^{-\rho\tau}(\alpha \ln E(\tau) + (1-\alpha)\ln L(\tau))d\tau$$

Substituting for $L(\tau)$ and $E(\tau)$ from (14) and (15) and integrating by parts then yields,[6]

$$(20) \qquad V(0) = \frac{1}{\rho}\left\{\begin{array}{c} \alpha \ln \alpha\bar{L}\beta(s) + (1-\alpha)\ln \bar{L} + \alpha\rho s e^{-\rho s} - \alpha \\ + e^{-\rho s}(1 + \frac{\gamma}{\rho})(1 - (1-\alpha)e^{\rho\theta}) \end{array}\right\}$$

Also $U(C(0)) = \alpha \ln E(0) + (1-\alpha)\ln L(0)$

$$= \alpha \ln \alpha\bar{L}\beta(s) + \alpha\rho s + (1-\alpha)\ln \bar{L}.$$

Differentiating (19), the rate at which V(t) changes with t is given by

$$(21) \qquad \frac{d}{dt}(V(t)) = \rho V(t) - U(C(t))$$

Then

$$\frac{d}{dt}(V(0)) = -\alpha\rho s(1 - e^{-\rho s}) - \alpha + e^{-\rho s}(1 + \frac{\gamma}{\rho})(1 - (1-\alpha)e^{\rho\theta})$$

It is readily verified that $\frac{d}{dt}(V(0))$ is strictly decreasing in s, and is negative for sufficiently large s. Moreover, from Proposition 3, s is

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[6] At one point in the integration use is also made of condition (16).

strictly increasing in $N(0)/\beta(0)$. Therefore for all current stocks of naturally stored energy that are sufficiently high relative to the current level of β, the plan that maximizes the welfare of the current generation yields a lower level of $V(t)$ to those generations immediately following.

Since for this economy it is always possible to lower the utility of the current generation and thereby raise $\underset{\tau \geq 0}{\text{Min}}\ V(\tau)$, such a plan violates the constraints of justice as non-exploitation.

From the discussion in section (I), a current energy stock of zero implies that the utility profile is strictly increasing. That is, $\frac{d}{dt}(V(0))$ is positive for $N(0) = 0$. Since $\frac{d}{dt}(V(0))$ is strictly decreasing with $N(0)/\beta(0)$ there must be some critical value, ξ, such that the constraints of justice are violated, if and only if[7]

$$N(0)/\beta(0) > \xi.$$

Since exactly the same argument can be applied to a future generation t, with natural resource stock $N(t)$, we therefore have:

Proposition 5:

There is some critical number $\xi > 0$ *such that the plan that maximizes the utility,* $V(t)$, *of the current generation* t, *violates the constraints of justice as non-exploitation, if and only if*

$$N(t) > \xi\beta(t).$$

Suppose the inequality in Proposition 5 is satisfied at $t = 0$. Then to satisfy the non-exploitation constraint, the current generation must choose a plan other than that which maximizes $V(0)$.

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[7] Strictly speaking we have only shown that $\frac{d}{dt}(V(0))$ is decreasing in $N(0)/\beta(0)$ for values sufficiently high to ensure that there is a phase 1. However the above analysis is readily extended to cases in which the economy begins in phase 2.

We are now in a position to derive

Proposition 6:

Suppose $N(0) > \xi\beta(0)$. *Then the current generation's best plan among those that are just, is to maintain a constant level of* $V(\tau)$ *until* $N(\tau) = \xi\beta(\tau)$. *For all larger* τ, *the constraints of justice are not binding and* $V(\tau)$ *increases continuously.*

Since $N(\tau)$ is non-increasing and $\beta(\tau)$ is strictly increasing with τ, there is a unique time q at which $N(q) = \xi\beta(q)$. From Proposition 5 the plan that maximizes $V(q)$ yields an increasing sequence $V(t)$, for all $t \geq q$. Moreover, for all $t < q$,

$$V(t) = \int_t^\infty e^{-\rho(\tau-t)}U(C(\tau))d\tau$$

$$= \int_t^q e^{-\rho(\tau-t)}U(C(\tau))d\tau + e^{-\rho(q-t)}\int_q^\infty e^{-\rho(\tau-q)}U(C(\tau)d\tau$$

$$= \int_t^q e^{-\rho(\tau-t)}U(C(\tau))d\tau + e^{-\rho(q-t)}V(q)$$

Thus for any predetermined $N(q)$, the plan that maximizes $V(q)$ also maximizes the utility of earlier generations. This establishes the second part of Proposition 6.

To establish the first part, let $E^*(\tau)$ and $L^*(\tau)$ be the time profiles of the control variables in the constrained optimum. These yield a consumption stream $C^*(\tau)$ and associated utility profile $V^*(\tau)$. One by one we shall eliminate all but the last of the following four possibilities

(i) $\frac{d}{d\tau}(V^*(\tau)) < 0$ at $\tau = 0$

(ii) $\frac{d}{d\tau}(V^*(\tau)) > 0$ for all $\tau < q$

(iii) $\frac{d}{d\tau}(V^*(\tau)) > 0$ for all $\tau < t < q$

(iv) $\frac{d}{d\tau}(V^*(\tau)) = 0$ at $\tau = 0$

Suppose (i) is true. Then by lowering $E(\tau)$ in the neighborhood of $\tau = 0$ $\underset{\tau>0}{\text{Min}}\, V^*(\tau)$ is unaffected. This leaves additional stored energy which can be used to raise the minimum utility. But this implies that the original plan $\{E^*(\tau), L^*(\tau)\}$ is not just.

Suppose instead that (ii) is true. From Proposition 5 (ii) is not true for the plan $\{E(\tau), L(\tau)\}$ that maximizes $V(0)$. Hence $V^*(0) < V(0)$. Also the linearity of the energy producing technology implies that for any $\lambda \in [0, 1]$ the convex combination

$$\{E_\lambda(\tau), L_\lambda(\tau)\} = \{\lambda E(\tau) + (1-\lambda)E^*(\tau), \lambda L(\tau) + (1-\lambda)L^*(\tau)\}$$

is another feasible plan. Furthermore, since $U(G(E, L))$ is strictly concave, it follows that

$$V_\lambda(\tau) > \lambda V(\tau) + (1-\lambda)V^*(\tau)$$

By assumption $V^*(\tau)$ is increasing in τ. Also $V(0) > V^*(0)$. Therefore, for λ sufficiently close to zero,

$$V_\lambda(\tau) > V^*(0) = \underset{\tau}{\text{Min}}\, V^*(\tau), \text{ for all } \tau \geq 0$$

But then, once again, $\{E^*(\tau), L^*(\tau)\}$ is not a just plan.

Finally, suppose (iii) is true. Noting again that $\frac{d}{d\tau}(V^*(\tau)) = \rho V^*(\tau) - U(C^*(\tau))$, we have

$$\rho V^*(0) - U(C^*(0)) > 0$$

and $$\rho V^*(t) - U(C^*(t)) \leq 0$$

Hence $$U(C^*(t)) \geq \rho V^*(t) > \rho V^*(0) > U(C^*(0))$$

Since $U(C)$ is an increasing function, consumption at time t exceeds consumption at time zero.

Consider the alternative plan $\{\tilde{E}(\tau), \tilde{L}(\tau)\}$ that maximizes $V(0)$ subject to the constraint that $N(t) = N^*(t)$. From the previous section we know that for such a plan $C(\tau)$ is strictly decreasing until the stored energy source is exhausted, and hence over $[0, t]$. Therefore $C^*(\tau)$ and $\tilde{C}(\tau)$ differ over $[0, t]$ and $\tilde{V}(0) > V^*(0)$. Then we can apply the argument made for case (ii) to show once again that $\{E^*(\tau), L^*(\tau)\}$ is not just.

This leaves (iv) as the only remaining possibility. Since an identical argument holds for any $\tau < q$ we therefore have

$$\frac{d}{d\tau}\,(V^*(\tau)) = 0; \qquad \tau \in [0,q]$$

This completes the proof of Proposition 6.

Solving for the optimal profiles of the control variables is then straightforward. Since

$$\frac{d}{d\tau}\,(V^*(\tau)) = \rho V^*(\tau) - U(C^*(\tau)) = 0 \qquad \tau \in [0, q]$$

it follows that $C^*(\tau)$ is constant over $[0, q]$. For expositional ease we shall consider only cases in which time q occurs before it is optimal to introduce the alternative energy producing technology. Then $L(\tau) = \bar{L}$ over $[0, q]$. Since

$$C(\tau) = G(E(\tau), L(\tau))$$

it follows also that $E^*(\tau)$ is constant over the interval $[0, q]$.

Furthermore, under the plan that maximizes $V(0)$ we have,

$$\frac{d}{d\tau}\,(V'(0)) = \rho V(0) - U(C(0)) < 0$$

Hence $$U(C(0)) > \rho V(0) > \rho V^*(0) = U(C^*(0))$$

Therefore initial consumption and energy use is *lower* when the initial generation chooses a plan that is non-exploitative.

It is also the case that at time q the stock of N is lower when the constraints of justice are ignored. Suppose not, that is, $N(q) > N^*(q)$. Since $N^*(q) = \xi\beta(q)$ there is some $q' > q$ such that $N(q') = \xi\beta(q')$. From Proposition 5,

$$(22) \quad V(q') = \underset{\tau \geq 0}{\text{Min}}\ V(\tau)$$

Furthermore $N(q') > N^*(q)$ and $\beta(q') > \beta(q)$. Then $V(q') > V^*(q)$. Combining this with (22) yields

$$\underset{\tau \geq 0}{\text{Min}}\ V(\tau) > V^*(q) = \underset{\tau \geq 0}{\text{Min}}\ V^*(\tau)$$

But this contradicts our assumption that $\{E^*(\tau), L^*(\tau)\}$ is maximal for the current generation, among those plans that are just.

Therefore

$$(23) \quad N(q) < N^*(q)$$

and there is some $q' < q$ such that $N(q') = N^*(q)$.

An immediate implication of Proposition 4 is that the date of complete exhaustion of the natural energy source is delayed when the current generation accepts the constraints of intergenerational justice. Since the length of time in which both technologies are utilized is the same for the constrained and unconstrained plans, the introduction of the alternative technology is also delayed. To summarize, we have demonstrated,

Proposition 7:

Suppose the current cost of energy production using the alternative technology is sufficiently high relative to the current stock of naturally stored energy, that maximizing current utility would be exploitative. Then among those plans that are just, the one that is *maximal for the current generation involves cutting back initial energy use. The rate of energy use is then maintained at this level for some interval. Moreover, as a result of the change in energy use, the date at which the alternative energy source is utilized is delayed.*

Figure 2 illustrates the essential difference between constrained and unconstrained plans.

IV. Competitive Markets

Before considering prices in the just economy we first describe how the plan that maximizes the utility of the current generation is achieved in a competitive market system. Let $p(\tau)$ be the price of energy and $w(\tau)$ be the price of L, both denominated in units of consumption at time τ. Also let $\sigma(\tau)$ be the return on a dollar invested currently for an interval of length τ.

Profit at time τ to the producers of the consumption good is then

$$\Pi_C(\tau) = G(E(\tau),L(\tau)) - p(\tau)\, E(\tau) - w(\tau)\, L(\tau)$$

Since G is concave, necessary and sufficient conditions for the maximization of $\Pi_C(\tau)$ are

$$\frac{\partial G}{\partial E} = p(\tau) \tag{24}$$

and

$$\frac{\partial G}{\partial L} = w(\tau) \tag{25}$$

Then by choosing $p(\tau)$ and $w(\tau)$ to satisfy (24) and (25) along the path, $\{E(\tau), L(\tau)\}$, that maximizes current utility, the optimal production of C is achieved competitively.

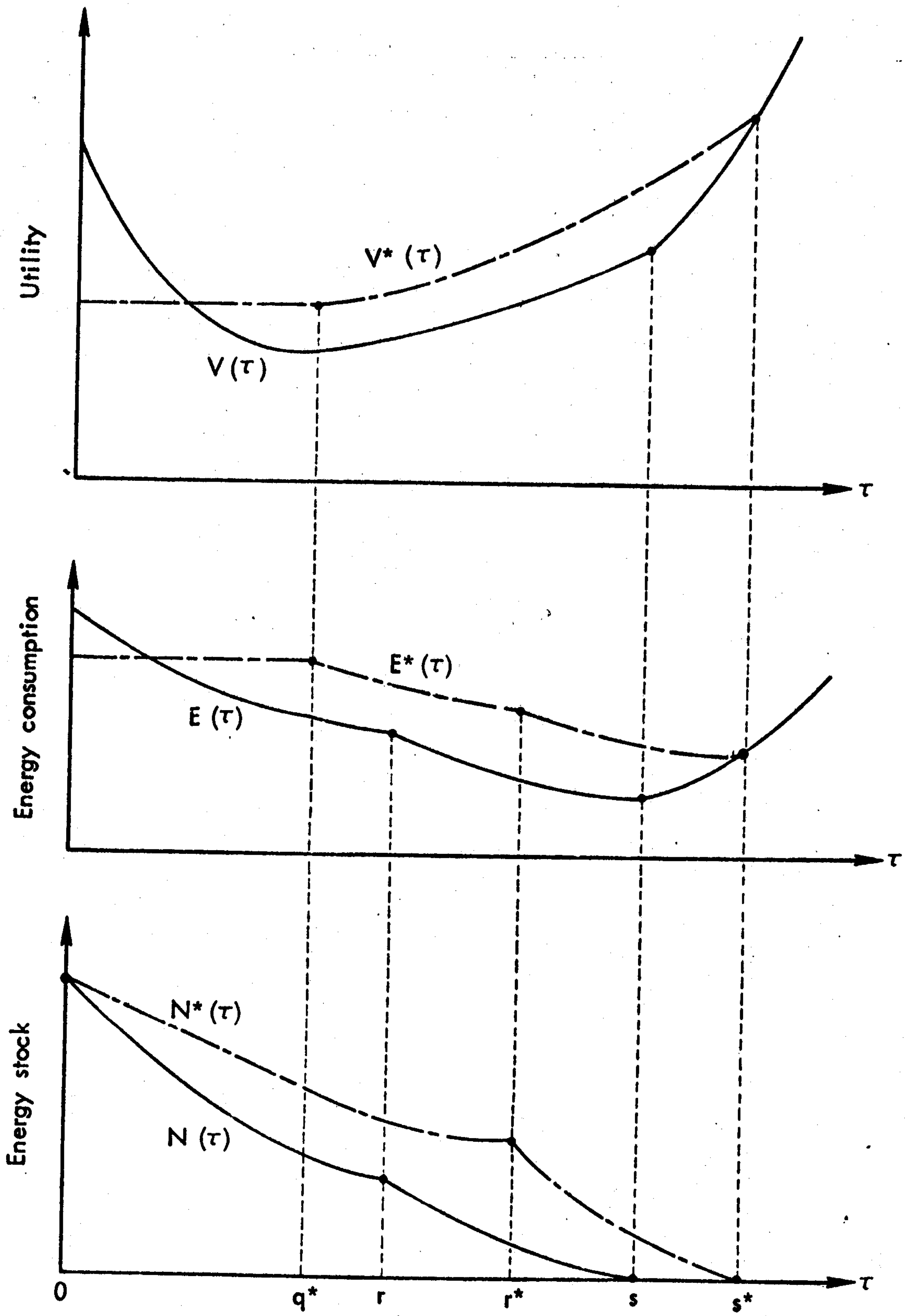

Fig. 2 — Comparison of the Just Plan with the plan which is maximal for the current generation

Next consider profit in the alternative energy producing industry. We have,

$$\Pi_E(\tau) = \{p(\tau)\ \beta(\tau) - w(\tau)\}\ L_E(\tau)$$

From Equations (8) and (9) we also have

$$p(\tau)\ \beta(\tau) < w(\tau), \quad 0 \leq \tau < r$$

$$p(\tau)\ \beta(\tau) = w(\tau), \quad r \leq \tau$$

Then the alternative source of energy will be introduced at time r in the competitive economy.

For the suppliers of the naturally stored energy source, the return to withholding supplies momentarily is the instantaneous capital gain $\dot{p}/p$. In the competitive equilibrium this riskless rate of return must equal the market rate of interest. We therefore choose $\sigma(\tau)$ so that over phases 1 and 2,

$$\dot{\sigma}/\sigma = \dot{p}/p \tag{26}$$

Then over these phases there is some number A such that

$$\sigma(\tau) = Ap(\tau) \tag{27}$$

Finally consider the consumption decision of the current generation. It faces a budget constraint

$$\int_0^\infty \frac{C(\tau)}{\sigma(\tau)} = p(0)\ N(0)$$

Necessary conditions for the maximization of

$$V(0) = \int_0^\infty e^{-\rho\tau}\ U(C(\tau))\ d\tau$$

are then

$$\frac{e^{-\rho\tau}\ U'(C(\tau))}{1/\sigma(\tau)} = \text{constant} \tag{28}$$

Rearranging utilizing (24) and (27) this condition becomes

$$e^{-\rho\tau}\ U'(C(\tau))\ \frac{\partial G}{\partial E} = \text{constant} \tag{29}$$

But this is exactly condition (9). Therefore if the prices of energy and the renewable resource satisfy (24) and (25), and the market rate of return satisfies (26), competitive decision making by consumers and firms results in the maximization of V(0).

For the special logarithmic, Cobb-Douglas case, the profile of energy prices

$$(30) \qquad p(\tau) = \frac{\partial G}{\partial E}(E(\tau), L(\tau))$$

follows a simple pattern. Differentiating (30) and utilizing (14) we have

$$\frac{\dot{p}}{p} = -(1-\alpha)\frac{\dot{E}}{E} + (1-\alpha)\frac{\dot{L}}{L}$$

Then from (14) and (15), the price of energy rises at a rate $(1-\alpha)\rho$ in phase 1 and falls thereafter at a rate $(1-\alpha)\gamma$. This is depicted in Figure 3.

At first sight it may seem surprising that the price of energy should be falling in phase 2 when the natural energy source is still being utilized. However in this phase the marginal cost of energy is the marginal cost of using the alternative technology. Since the latter steadily declines, so must the price of energy.

To satisfy the constraints of justice it is necessary to reduce initial energy consumption, and then maintain it over an interval $[0,q^*]$. The marginal product of energy must therefore be constant over this interval. From (30) it follows that the future spot price of energy purchases $p^*(\tau)$, must also be constant. This is depicted in Figure 3. Since $E^*(0)$ is less than $E(0)$ it also follows from (30) that $p^*(0) > p(0)$. Further, from Figure 2, $E^*(q^*) > E(q^*)$. Hence the purchase price of energy is lower at the end of the justice constrained phase. *Acceptance of justice as non-exploitation therefore reduces peak energy prices at the expense of a higher current purchase price.*

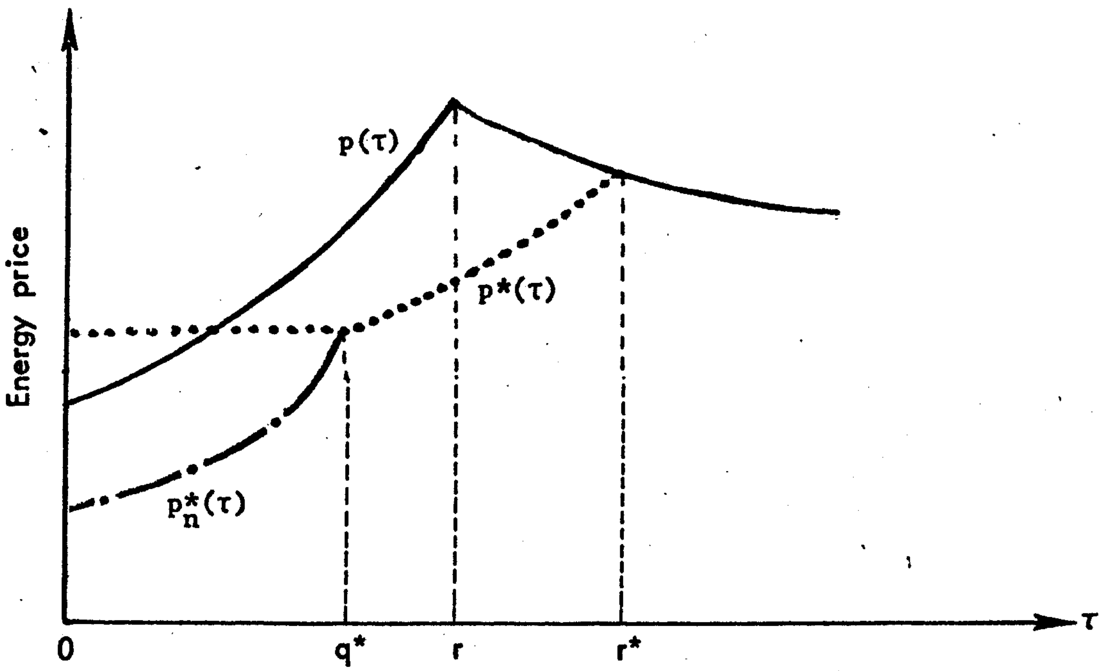

Fig.3 — Optimal energy pricing

With constant energy production, over the initial phase, final output is also constant. For equilibrium in the commodity market we therefore require, using (28),

$$\text{(31)} \qquad \frac{e^{-\rho\tau}U'(C^*(0))}{1/\sigma(\tau)} = \text{constant} \quad \tau \in [0,q^*]$$

Differentiating with respect to τ yields

$$\text{(32)} \qquad \frac{\dot{\sigma}}{\sigma} = \rho$$

Hence in the justice constrained phase, the market rate of interest must be equal to the private rate of pure time preference.

Finally we consider the supplies of naturally stored energy. Since production costs are assumed to be negligible, it is necessary in a competitive equilibrium, that the present value of naturally stored energy remains unchanged until the moment of complete exhaustion. Given the positive interest rate and constant purchase price of energy, it is therefore necessary to introduce a wedge between the selling and buying prices of energy during the justice constrained phase.

In order to maintain a constant present value of energy, the future spot selling price must rise at the market rate of interest. Therefore equilibrium in the energy market is achieved by maintaining a net price to sellers of

$$p_n^*(\tau) = \begin{cases} p(\tau)e^{-\rho(q^*-t)}, & 0 \leq \tau \leq q^* \\ p(\tau) & , q^* < \tau \end{cases}$$

The current generation is therefore able to attain the justice constrained optimum by introducing a declining ad valorem energy sales tax

$$v^*(\tau) = 1-e^{\rho(q^*-\tau)}, \; 0 \leq \tau \leq q^*.$$

The budget surplus is then redistributed in a lump sum manner.

Finally note that over the initial phase, $p_n^*(\tau)$ rises at a rate of ρ which is greater than the rate of increase in energy prices in the unconstrained market economy. Therefore, as depicted in Figure 3, the initial selling price of energy is lower than the initial unconstrained price.

V. Concluding Remark

In this paper we have examined the implications for energy consumption and pricing, of acceptance by the current generation, of the constraint that it should not voluntarily leave a later generation so resource poor that it is perceived to be worse off.

In the model considered each generation has a choice of using a naturally stored energy source or producing additional (solar?) energy. A crucial assumption is that the cost of production using the alternative technology declines over time. The 'energy crunch' is therefore alleviated in the very long run.

It has been shown that if this cost of production is sufficiently high, relative to the stock of naturally stored energy, the constraints of justice result in a lower rate of energy use initially, but a higher rate during some later interval. In this way the impact of the 'crunch' is spread more evenly across generations.

It has also been shown that such an adjustment can be achieved in a perfectly competitive market economy, by introducing a steadily declining ad valorem tax on energy sales.

It might be argued that if all the members of the current generation really accepted the non-exploitation principle, there would be no need for such intervention. Each individual would automatically take the principle into account when deciding upon his own consumption plan.

However, it is one thing for a group to agree on an objective and another to have them observe the agreement in the absence of any policing. Once enough members of the populations 'backslide' to affect the prices of energy and the renewable resource, the interests of the entire group are affected. Intervention to change the return on saving has an automatic policing feature. Once adopted, there is no need for any agent to take further account of the non-exploitation constraint. The utility maximizing plan becomes the just plan.

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