A Theory of Exchange Rate Determination

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Introduction

The recent behavior of exchange rates in a flexible rate system has puzzled many economists. Exchange rates have exhibited considerable volatility and together with prices have failed to conform to the predictions of the purchasing power parity theory; frequently exchange rate changes have failed to resemble contemporaneous changes in relative price levels in either magnitude or direction. Exchange rates and their rates of change over time have been more volatile than relative price levels and rates of inflation.[1]

These features of exchange rate behavior have been regarded as anomalous to existing theories. Moreover, they have often been regarded as inconsistent with equilibrium, and several disequilibrium interpretations of this anomalous behavior have been suggested in the literature. These disequilibrium interpretations have resulted in many unfavorable reconsiderations of the case for flexible exchange rates. Aliber (1976) has summarized the situation with the question, "Why have floating exchange rates behaved differently from the way we were promised?" McKinnon (1976) is representative in asking, "...why has past theorizing failed to comprehend the present 'disordered' and high-cost state of markets for foreign exchange?" Mussa (1976) is not alone in modifying his views about the desirability of fixed exchange rates on the basis of the recent experience with flexible rates.

This paper proposes an alternative explanation of exchange rate behavior. It seems desirable to try to account for observed exchange rate behavior with an equilibrium explanation before resorting to a disequilibrium explanation. The equilibrium explanation presented in this paper involves the simultaneous determination of exchange rates and relative prices of different goods in international trade in an intertemporal framework with uncertainty and rational expectations. Relative price changes caused by real disturbances play an important role in the explanation.[2] These relative price changes were emphasized in the traditional literature on exchange rates but have been neglected in the recent exchange rate literature associated with the monetary approach.[3] This paper emphasizes relative price changes and integrates the important issues discussed by the traditional "elasticity theorists" into a full general equilibrium framework. By examining exchange rate determination in a model that includes relative price changes, this paper clearly establishes the distinction between the issues involved in international arbitrage of goods prices and the issues involved in the determination of exchange rates.

The model results in exchange rate behavior similar to that which is actually observed - exchange rates can be represented by stochastic processes with rather complicated properties: they may be highly volatile and will exhibit autocorrelated deviations from purchasing power parity. These results hold even though prices freely adjust to clear markets. International capital flows will also be volatile, reflecting changes in transitory incomes and expected future prices.

Second, the model explains why exchange rate changes are associated with terms-of-trade changes and yet why the relationship between the exchange rate and the terms of trade cannot be exploited by government exchange rate policies, that is, why government policies designed to influence the terms of trade or the balance of trade by actions directly affecting the exchange rate cannot succeed, even though the exchange rate and the terms of trade may be statistically related. Government commercial policies such as tariffs or quotas can, however, affect the exchange rate by changing the terms of trade.[4]

Third, the model shows why exchange rate fluctuations can easily be misinterpreted as sources of additional uncertainty in an economy and as causes of relative price changes among goods in international trade. Exchange rate changes will often appear to cause relative price changes and generate additional uncertainty even when all markets are in equilibrium. The source of misunderstanding is related to the common fallacy that relative price changes cause inflation: just as people may call an increase in the price of food the cause of inflation when it is actually just one way in which inflation occurs, so people may call a change in the exchange rate a cause of some relative price change when the exchange rate change is actually just one of the ways in which the relative price change occurs.

The model shows how a change in the terms of trade caused by some relative supply or demand shift is divided between nominal price changes in each country and an exchange rate change. If p_1 is the price of the export good of country one in terms of its own money and p_2 is the price of the export good of country two in terms of country two's money, then the relative price of country one's exports in terms of country two's exports is p_1/ep_2, where e is the money one price of money two. When relative supplies or demands change the consequent change in p_1/ep_2 generally occurs through changes in each of p_1, p_2, and e. This creates a correlation between the exchange rate and the terms of trade. The greater the changes in the terms of trade and the larger the role of changes in the exchange rate in effecting these terms of trade changes, the greater the variability of exchange rates. The more persistent the shifts in the supplies or demands for goods, the more persistent the deviations from purchasing power parity.

The division of terms of trade changes between changes in nominal prices and changes in the exchange rate depends upon the elasticities of supply and demand that were emphasized in the traditional exchange rate theories. In this sense the model resembles the traditional elasticities approach to exchange rates. But the division of the terms of trade changes into nominal price and exchange rate changes also depends upon the time series properties of exogenous variables through their effects on expectations of future exchange rates and future prices and upon other factors influencing asset demands. In this sense the model resembles the monetary or asset approach to exchange rates. The model can therefore be described as a synthesis of the elasticity and monetary approaches.

Besides rationalizing exchange rate volatility and autocorrelated deviations from purchasing power parity, the model has several other implications. The exchange rate will be correlated with the terms of trade and this correlation will be greater for countries with more homogeneous monetary policies. These implications are consistent with the evidence presented by Dornbusch and Krugman (1976). On the other hand, the model predicts that exchange rate changes caused by monetary factors do not affect the terms of trade, and this prediction is consistent with the evidence surveyed by Magee (1977). The model implies that deviations from purchasing power parity and changes in the terms of trade have roughly the same characteristics and bear approximately the same relationship to each other under both fixed and flexible exchange rate systems. The model is also useful for interpreting statements by bankers and foreign exchange dealers that, when analyzed with a straightforward monetary approach, appear to be either incorrect or beside-the-point.[5] Moreover, the model is consistent with the application of the standard tools of price theory to examinations of the demands and supplies of goods in international trade, in contrast to disequilibrium models which imply that prices of internationally traded goods do not adjust to clear markets but are instead systematically related to monetary policies.

Finally, Fischer[6] has argued that because in simple monetary models the exchange rate is unconnected with relative prices or with the real allocation of resources, the role played by the exchange rate in those models is inconsistent with the concern actually expressed by people about the level of the exchange rate. This paper explains why rational people may care about the level of the exchange rate - because for given levels of the exogenous variables such as money supplies, the equilibrium exchange rate is an indicator of relative prices.

Exchange Rate Behavior: Problems and Suggested Explanations

This paper examines some of the literature on exchange rates and outlines the basic economics of the formal model to be presented in the next chapter. While most large changes in exchange rates can be explained by differing rates of inflation across countries, many substantial changes in exchange rates remain unexplained. These "deviations from purchasing power parity" consist of changes in relative prices of either traded versus non-traded goods or of the export goods of different countries. The traditional elasticities approach emphasized changes in the relative prices of different countries' export goods - the terms of trade. The elasticity approach has two interpretations, only one of which has been presented in the modern exchange rate literature. That interpretation is a disequilibrium interpretation. But there is an alternative equilibrium interpretation which will be presented here. The equilibrium interpretation is useful both for understanding the observed behavior of exchange rates and for examining issues of government exchange rate policy.

The Derived Demand for Foreign Exchange

People demand foreign exchange because they want to purchase foreign goods or assets. Foreign exchange may also be demanded to repay debts in foreign currency, pay interest on foreign debts, make gifts, or to be held as an asset for later resale. Once foreign goods have been imported using foreign exchange, they may be resold at home for domestic money. If most importing is done by specialists rather than by individual consumers then people will not hold substantial balances of foreign exchange in their roles as consumers. Foreign exchange will then be held primarily by multinational trade and investment firms, banks and foreign exchange dealers, and other agencies engaged in direct international trade of goods or assets. Moreover, if there are economies of scale in obtaining monetary services from a stock of real balances, the total quantity of foreign exchange held will be less if there are specialized importers than if each consumer did his own importing.

Because foreign exchange is demanded to purchase foreign goods and assets it is useful to view the demand for foreign exchange as a derived demand. The derived demand for foreign exchange depends upon the foreign money prices of goods and assets that can be purchased with foreign money as well as the exchange rate and the prices of other goods. If the elasticity of demand for imports is greater in the long run than in the short run, the elasticity of demand for foreign exchange will also be greater in the long run. Shifts in the demand for foreign exchange occur with changes in the price of foreign goods or the domestic prices of other goods, or with shifts in the demand for imports due to changes in wealth, income, tastes, and so on.

While the derived characteristic of demand for foreign exchange was widely recognized in the traditional exchange rate literature, its implications were never thoroughly investigated. References to the derived characteristic of demand for foreign exchange can be found in Cassel (1922, p. 138), Machlup (1939, pp. 111, 115, 119), Robinson (1947, p. 83), Friedman (1953, pp. 159, 162), Friedman and Schwartz (1962, pp. 161, 590 note 35), Hodgson (1967, p. 250), Mickesell and Furth (1971, pp. 6-17, 57), Machlup (1972, pp. 29...), and Salop (1977, p. 66).[7] The traditional elasticities approach formalized the derived demand for foreign exchange for the case of a static model and developed specific formulas for certain cases. For example, given the foreign money price of imports and the domestic money price of exports, an increase in the price of foreign exchange raises the relative price of imports, and the quantity demanded of foreign exchange (derived in this static model from expenditures on imports) rises or falls as the elasticity of demand for imports is greater than or less than one. The Marshall-Lerner condition states that in such a case there will be a balance of trade surplus or deficit (starting from a zero balance of trade) as the sum of the elasticities of the domestic demand for imports and the foreign demand for imports is less than or greater than minus one. Other particular assumptions led to other specific formulas.[8] But the important unifying characteristic of the elasticity models was that they derived the demand for foreign exchange from the demand for foreign goods.

Although their formal models did not incorporate any aspects of intertemporal substitution or uncertainty, the economists associated with these traditional models often recognized that expectations and intertemporal substitution opportunities affect the demand supply of foreign exchange.[9] Intertemporal considerations are important for two reasons. First, expectations of an increase in the price of foreign exchange will induce people to purchase foreign exchange today. By similar reasoning, the current demand for foreign exchange also depends upon expected future prices of goods as well as the current prices of goods and the current and expected future exchange rate. Machlup (1939, pp. 155-6) notes the relation between intertemporal substitution in foreign exchange and intertemporal substitution in goods. Second, as Friedman (1953, pp. 182-3), Dornbusch (1976c, p. 238), and Dornbusch and Krugman (1976, p. 539) have noted, the dynamic path of the exchange rate in response to an exogenous shock is affected by the difference between the short-run and the long-run elasticities of demand for foreign exchange.

But these issues were never integrated into the formal theoretical models of these economists.

Purchasing Power Parity

Much of the progress in empirically explaining exchange rate behavior was achieved with models that disregarded the derived demand for foreign exchange. Such models can account for most of the variation in exchange rates because large changes in exchange rates are generally associated with different rates of inflation in the countries concerned. A model of the derived demand for foreign exchange is not required for the inference that a change in the stock of money will, other things the same, be associated with a corresponding increase in every nominal price including the nominal price of foreign exchange. This result is guaranteed by the general property of zero-degree homogeneity of demands and supplies with respect to all nominal prices, and is not tied to a specific model. Empirically other things are not always the same but most large changes in exchange rates can nevertheless be accounted for by differential growth rates of money or general price levels. Relationships like this fall under the general heading of purchasing power parity.

Appendix I summarizes the relation between exchange rates and ratios of price indexes for several countries and time periods. The first part of the table is taken from Gaillot's (1970) results which apply the purchasing power parity relation over a sixty year period. The other parts of the table summarize data for more recent time periods and shorter time intervals. Deviations from purchasing power parity have two characteristics. First, the deviations often persist over time in one direction - they do not appear as white noise. Second, exchange rates and their rates of change vary more than ratios of price indexes and their rates of change. Neither phenomenon is unique to the data sets in the table. For example, Friedman and Schwartz (1963) note that for the greenback period of 1861-79, the US - UK exchange rate varied by about 2 to 1, while the ratio of price levels varied by only about 1.3 to 1. They offer the judgment that "To some extent, this residual variation may reflect the crudeness of our calculations of purchasing power parity. It seems most unlikely, however, that it can be wholly accounted for by such statistical errors." So it appears that while purchasing power parity can account for most major changes in exchange rates, there are substantial deviations from purchasing power parity to be explained.

There are several versions of the purchasing power parity theory. In each of its forms, the theory claims that there is a proportional relationship between exchange rates and a ratio of prices or of price indexes. In a one-sector macroeconomic model, purchasing power parity is often justified by reference to arbitrage in the goods market, so that the purchasing power parity theory becomes just an extension of the law of one price. Some authors use the term in precisely this sense (e.g. Magee, 1976). Other authors use the term to refer to a proportionality between exchange rates and a ratio of price indexes that are computed using the same weights in each index for any price. As Bresciani-Turroni (1934, p. 121) argued, this proportionality is not very interesting because it is just another extension of the law of one price, whereas the essence of the purchasing power parity hypothesis is that exchange rates are an index of monetary conditions in the countries concerned.

If one rejects, with Bresciani-Turroni, Friedman and Schwartz (p. 62, note 66), Frenkel (1977), and others, the interpretation of purchasing power parity as an extension of the law of one price then purchasing power parity is a hypothesis about the relationship between the values taken by several endogenous variables, and not a theory of exchange rate determination. This conclusion was also reached by Dornbusch and Krugman (pp. 538, 542). The accuracy of the purchasing power parity hypothesis is then independent of the accuracy of any particular theory of exchange rate determination. The purchasing power parity hypothesis can be rephrased as stating that most changes in exchange rates are due to nominal shocks (defined in some particular way) and that very few are due to real shocks (again defined in some particular way). If this is true, then the accuracy of the purchasing power parity relation is largely independent of the particular price index or monetary aggregate used in calculations. If the hypothesis does not hold with much accuracy, then calculations with different price indexes may yield somewhat different results. Friedman and Schwartz (p. 62, note 66) argue that the price indexes used should be chosen with the criterion that the resulting number reflects the value that the exchange rate would take if it were affected only by nominal shocks and not by real shocks. For example, on the grounds that productivity growth should be classified as a real change, Friedman and Schwartz suggest that factor price indexes are preferred to product price indexes for purchasing power parity calculations because the ratio of factor price indexes may be roughly constant even though the product price index in the country with greater productivity growth falls, changing the purchasing power parity calcualted with product price indexes. A change in the exchange rate induced by this real shock would then show up as a deviation from purchasing power parity, as Friedman and Schwartz argue that it should. Cassel (1922, p. 144) also suggested using factor price indexes for purchasing power parity calculations. Alternatively, Friedman and Schwartz argue that prices of exports but not prices of imports should be included in calculating purchasing power parity, again to make it an index of what the exchange rate would be if it were only influenced by nominal shocks. The choice of appropriate price indexes clearly reflects the classification of shocks into real and nominal components and the presumptions one has about what real shocks might have important influences on exchange rates; an investigation of how various real shocks affect exchange rates requires a theory of exchange rate determination.

Monetary Models of the Exchange Rate

Monetary models of the exchange rate attempt to bypass the problem of finding an appropriate price index for purchasing power parity calculations by supplementing the purchasing power parity relation with money demand functions and equilibrium conditions in the money market. The exchange rate can then be expressed solely as a function of nominal money supplies and the variables assumed to affect money demands. This approach assumes that the appropriate price indexes for calculating purchasing power parity, whatever they may be, are the same as the indexes that are appropriate for deflating nominal money balances in a demand function for real money balances. Here "appropriateness" is judged by the criterion of providing a good fit in the purchasing power parity relation. (This is, of course, a different criterion than that suggested by Friedman and Schwartz.) The equation for the exchange rate resulting from the basic monetary model is

$$d \ln e = c_o \, d \ln(M^s/M^{*s}) - d \ln (m^d/m^{*d})$$

where the exchange rate, e, is the domestic price of foreign money, M^s and M^{*s} are domestic and foreign nominal money supplies, and m^d and m^{*d} are the demands for real balances of domestic and foreign moneys, typically taken to be functions of real income and nominal interest rates. The coefficient c_o should be unity, and the estimated equation should provide estimates of the income and interest elasticities of the demands for moneys.

The monetary models have not been very successful in explaining short-run movements in exchange rates.[10] The fraction of exchange rate variation explained (without the use of a lagged dependent variable) is typically small and the estimated elasticities of the exchange rate with respect to the ratio of money supplies, c_o, is typically closer to zero than one with monthly or quarterly data. These low coefficients may be due to measurement error on monthly or quarterly money supply and real income data, but constraining the coefficients to more reasonable values reduces the explanatory power of the equations still further. The monetary models explain deviations from purchasing power parity solely in terms of deficiencies in the price indexes employed and do not explain the observed correlation, discussed below, between the exchange rate and the terms of trade. The poor performance of the monetary models in accounting for short-run changes in exchange rates has led to proposals of alternative theories of these changes.

Other Explanations of Exchange Rate Behavior

Two other explanations of the observed behavior of exchange rates have been offered by Mussa (1976, 1977) and Dornbusch (1976a, 1976b). Mussa emphasizes the role of new information about the future in affecting the current exchange rate. An increase in the expected rate of domestic inflation reduces the demand for domestic money and causes all nominal prices, including the price of foreign exchange, to rise today. This can account for fluctuations in exchange rates that are associated with fluctuations in the ratio of domestic and foreign price levels, but this alone cannot account for deviations from purchasing power parity. If this theory is supplemented with the hypothesis that measured price indexes are poor proxies for actual price levels, then a theory of price level determination could be appended to this hypothesis as in the monetary models of the exchange rate. Alternatively, Mussa suggests that prices of goods do not adjust rapidly to clear markets: they adjust only slowly to changes in demands or supplies or to unanticipated changes in the money supply. As long as prices of goods remain out of equilibrium while the exchange rate changes, this hypothesis can account for deviations from purchasing power parity. The persistence of these deviations is explained by the slow adjustment of prices to their equilibrium values.

Dornbusch has suggested that this assumption of slow price adjustment could explain the observed volatility of exchange rates. He suggested that an unexpected increase in the domestic money supply reduces the domestic interest rate because with fixed prices (and, for simplicity, exogenous real income) the interest rate must fall to induce people to hold the increased supply of real money balances. But if the foreign nominal interest rate is constant, arbitrage in the bond market requires that the domestic currency be expected to appreciate at a rate equal to the difference between the foreign and domestic nominal interest rates. The attempt by people to obtain foreign exchange to purchase foreign bonds leads to an immediate depreciation of the domestic currency until the currency has depreciated so much that it is expected to appreciate, at the required rate, to its new long run equilibrium. At that point the real returns from holding foreign and domestic bonds are equal to any person, although the real return to a domestic resident differs from the real return to a foreign resident. The assumed slow adjustment of goods prices keeps these real returns from being equated. The exchange rate therefore depreciates more in the short run when goods prices are fixed than in the long run when goods prices have adjusted to clear markets. This overshooting hypothesis might therefore account for the observed volatility of exchange rates as well as the sometimes-observed movements of exchange rates in the opposite direction from that predicted by purchasing power parity. An empirical implication of this hypothesis is that depreciations greater than those predicted by purchasing power parity are associated with increases in the expected rate of appreciation and vice-versa. Empirically, however, there is apparently no relationship between these exchange rate changes and changes in the expected rate of appreciation, measured using the forward premium.[11]

Deviations from Purchasing Power Parity Involve Relative Price Changes

Deviations from purchasing power parity, whatever prices or price indexes are used, always involve relative price changes. The only questions are what relative prices are changing and why. In the explanations of exchange rate fluctuations proposed by Dornbusch or Mussa, the prices of goods available to people in one country change relative to prices of those same goods in another country because domestic nominal prices are temporarily fixed in each country and a monetary shock causes a change in the exchange rate. A nominal shock therefore causes a disequilibrium change in relative prices. Other economists (e.g. Balassa, 1964) have emphasized changes in the relative prices of traded and non-traded goods. But the relative price change that was emphasized most in the traditional literature on foreign exchange markets was the terms of trade.

Taussig (1917) and Graham (1922) discuss both changes in the terms of trade and the relative prices of traded and nontraded goods. Keynes, Machlup, Robinson, and Metzler all emphasized changes in the terms of trade. Keynes (1930, pp. 73-4) criticized the purchasing power parity hypothesis for assuming the terms of trade is constant, and suggested that variations in the terms of trade constitute "one of the greatest difficulties in the way of the maintenance of a country's external equilibrium." Machlup's 1939 paper and Robinson's 1947 paper derived the demand for foreign exchange from the demand for imports and examined the relationship between the terms of trade and the balance of payments or the exchange rate. Krueger (1969) noted that the traditional theory viewed the terms of trade as "the key variable," and that the relationship between the terms of trade and the exchange rate was one of the two major questions investigated in the literature (the other being the effect of devaluation on the trade balance). Friedman (1953, p.180) emphasised the role of changes in the relative price of a country's imports and exports. More recently, terms of trade changes have been discussed by Dornbusch and Krugman (1976, pp. 538-9) and in applied studies such as Da Silva (1977).

Newspaper reports about international economics often contain statements such as "Swiss products are expensive in the U.S. because of the strength of the Swiss Franc" and "Exchange dealers reported only moderate intervention by the central banks of Germany, Japan, and Switzerland, even though their exported goods could become more costly in the U.S. if their currencies continue to appreciate."[12] The terms of trade often plays a major role in government foreign exchange market policy.

The simple correlation coefficients between the monthly percentage changes in the exchange rate with the dollar and the monthly percentage change in the terms of trade (measured as the ratio of the domestic export price index divided by the import price index to the U.S. export price index divided by the U.S. import price index) from January, 1974, through July, 1977 are -.29 for the Canadian dollar, -.16 for the French franc, -.33 for the Deutschemark, -.15 for the lire, .21 for the yen, and -.24 for the guilder. The number of monthly observations is 43, so the implied t-statistics are -1.9, -1.0, -2.2, -.9, 1.4, and -1.6. These correlations measure only the contemporaneous relation between the exchange rate and the terms of trade and even these are likely to be biased toward zero by measurement error in the export and import price indexes. Dornbusch and Krugman have also presented evidence of this correlation, and Isard (1977) has presented evidence that the exchange rate is correlated with changes in the terms of trade even of disaggregated categories of goods.

Changes in the terms of trade also play an important role in the explanation presented by Friedman and Schwartz (1963) of deviations from purchasing power parity during the U.S. greenback era, from the Civil War to 1879. During the Civil War, U.S. cotton exports were cut off, resulting in a rise in the the price of gold (foreign exchange) relative to purchasing power parity by 20% and affecting the terms of trade (pp. 66-7, 75). After the Civil War, as the supply of goods for export rose again and reduced the terms of trade, the domestic currency appreciated from about 20% below purchasing power parity to about 10% above purchasing power parity (p. 76).

Later movements in the exchange rate may also have been related to changes in the terms of trade (pp. 77-8). According to the price indexes reported in Graham (192) (the U.S. price index reported by Graham is from Mitchell, 19), the simple correlation coefficient between the log deviation from purchasing power parity (measured with general price indexes in the U.S. and the U.K. and with the greenback price of gold, to which the pound sterling was pegged) and the log terms of trade (measured with export price indexes converted at the current exchange rate) is -.68, calculated with 13 annual observations from 1866 through 1878. The sign indicates that currency depreciations are associated with increases in the relative price of a country's exports. (This result does not seem to be due to spurious correlation induced by using the exchange rate in calculating the terms of trade, since the simple correlation between the log deviation from purchasing power parity and the log of the ratio of export price indexes, not converted at the exchange rate, is -.77, and the correlation between the log of the exchange rate and the log deviation from purchasing power parity, which would be negative if there is spurious correlation, is in fact .24.)[13]

The terms of trade is apparantly an important left-out-variable in the monetary models of the exchange rate. A simple monetary model for monthly changes in the Deutschmark price of U.S. dollars from January, 1974 through July, 1977 is[14]

$$d \ln e = \underset{(.0040)}{-.0053} + \underset{(.14)}{.24}\, d \ln (M_G/M_{us}) - \underset{(.24)}{.32}\, d \ln (y_G/y_{US})$$

$$+ \underset{(.003)}{.0009}\, d\,(i_G - i_{US}) \qquad R^2 = .12 \quad DW = 1.85$$

where M, y, and i are money supplies, real incomes measured by indexes of industrial production, and short-term nominal interest rates for Germany and the U.S. But adding a terms of trade variable measured by $TT_G = P_G^{ex} P_{US}^{im} / P_G^{im} P_{US}^{ex}$ where P^{ex} and P^{im} are export and import price indexes results in

$$d \ln e = \underset{(.0038)}{-.0037} + \underset{(.13)}{.21} d \ln (M_G/M_{US}) - \underset{(.23)}{.37} d \ln (y_G/y_{US})$$

$$- \underset{(.003)}{.0004} d (i_G - i_{US}) - \underset{(.15)}{.38} d \ln TT \qquad R^2 = .25 \quad DW = 2.00$$

and the implied t-statistic for the hypothesis that the coefficient on d ln TT is zero is -2.49.[15] Similar results hold for other countries. The estimated equation for the Canadian dollar - U.S. dollar exchange rate over the same period is

$$d \ln e = \underset{(.0022)}{.0041} + \underset{(.056)}{.0028} d \ln (M_C/M_{US}) + \underset{(.16)}{.15} d \ln (y_C/y_{US})$$

$$- \underset{(.0042)}{.0080} d (i_C - i_{US}) - \underset{(.06)}{.14} d \ln TT_C \qquad R^2 = .20 \quad DW = 2.22 .$$

The important role played by the terms of trade is clear; the interpretation is not.

Changes in the Terms of Trade Cause Changes in the Equilibrium Exchange Rate

There are two interpretations of the relationship between changes in the terms of trade and changes in the exchange rate. According to one interpretation, the forces that cause the change in the exchange rate also cause a change in the terms of trade because prices of goods do not adjust to clear markets. The change in relative prices is therefore a disequilibrium phenomenon. This interpretation can be found in Dornbusch and Krugman (1976), Dornbusch (1976c), and Isard (1977). Another version of this disequilibrium interpretation can be found in Kemp (1969, chapter 14) and Negishi (1968). This version begins with a two-country, two-good, two-money model with complete specialization in production and formally differentiates the system with respect to the exchange rate, allowing prices to change but not allowing asset stocks to change. That is, the exchange rate is assumed to change even though no policy variables have changed: there is no change in either money supply and no government foreign exchaage market actions. This "short run" analysis allows one to derive some of the formulas presented by earlier foreign exchange market theorists. The associated "long run analysis" involves changes in policy variables and hence money supplies, but then either the elasticities of demand and supply of goods have no effect on the final equilibrium or they affect it only insofar as shifts in demand cause changes in real incomes and hence changes in the demands for domestic moneys (see Mussa, 1974).

There is an alternative equilibrium interpretation of the elasticity approach to the foreign exchange market and of the relation between the terms of trade and the exchange rate. Domestic money is demanded because it provides the particular services of allowing people to transact (cheaply) in domestic markets to purchase goods, and foreign exchange is demanded by importers becaase it is used to finance imports, purchase foreign assets, and so on. Other things the same, the demand for foreign exchange depends on the exchange rate. But as Friedman (1953, pp. 159-60) noted,

> The changes continuously taking place in the conditions of international trade alter the 'other things' and so the desirabilities of using the currencies of various countries for each of the purposes listed. The aggregate effect is at one time to increase, at another to decrease, the amount of a country's currency demanded at any given rate of exchange relative to the amount offered for sale at that rate.

Real supply and demand shocks affect both relative prices and the derived demand for foreign exchange. A shock that increases the demand for Japanese television sets may also increase the derived demand for yen to import those sets, so the derived demand for foreign exchange is affected as people substitute between domestic and foreign goods (Machlup, 1972, p.35). Friedman and Schwarts, in explaining why the U.S. dollar did not depreciate by even more than it did during the Greenback era, suggest that economic growth improved "the competitive position of the United States in exports more than it had expanded its demand for imports, which is to say, had increased the demand for U.S. dollars by foreigners (to buy U.S. exports) more than it had increased the demand for foreign currency by U.S. residents (to buy imports). The effect of such a shift in comparative advantage would be to raise the value of the U.S. currency in terms of foreign currencies at which trade would balance," i.e. relative to purchasing power parity. The changes in the demand for foreign exchange that result from real supply and demand shocks affect the equilibrium exchange rate. Therefore changes in the terms of trade are associated, in equilibrium, with changes in the exchange rate.

Exchange rate changes do not cause changes in the terms of trade; exchange rate changes are simply one of the ways in which the terms of trade change occurs. The equilibrium version of the elasticities approach therefore leads to an entirely different interpretation of the correlation between the exchange rate, or deviations from purchasing power parity, and the terms of trade than is suggested by the disequilibrium models.

While changes in the terms of trade occur partially through changes in the exchange rate, not all changes in the exchange rate are associated with changes in the terms of trade. A currency reform in one country that left unchanged the distribution of wealth would change the price of foreign exchange along with all other nominal prices. In this sense, changes in the exchange rate may be caused by either real or monetary factors.

It follows that government foreign exchange market policies will not be able to exploit the relationship between the exchange rate and the terms of trade in order to achieve a desired terms of or balance of trade. The relationship between the exchange rate and the terms of trade is due to shifts in the underlying real supplies and demands for foreign or domestic goods, which will not be substantially affected by government foreign exchange market transactions. When a change in the exchange rate is due to such changes in real conditions, government foreign exchange market policies can reverse the change in the exchange rate only by affecting general price levels - it cannot reverse the changes in real conditions that originally caused the exchange rate movement. Other policies such as tariffs, quotas, and controls on foreign exchange transactions may affect the exchange rate indirectly by directly affecting the terms of trade (Cassel, 1922, pp. 147-62, Friedman, 1953, pp. 167-9) but foreign exchange market transactions cannot be used as a tool by policymakers to exploit the exchange rate - terms of trade correlation.

If changes in the terms of trade do not substantially affect a ratio of foreign and domestic consumer or wholesale price indexes then the exchange rate will vary by more than the purchasing power parity calculated with these indexes. If changes in real supplies or demands persist over time then deviations from purchasing power parity will also persist over time. In fact, the next section shows how even transitory changes in real supplies and demands may affect the international allocation of wealth and hence cause deviations from purchasing power parity that persist over time. The next section presents a formal model of exchange rate determination in a two-country world with two goods and two moneys.

The Basic Model

Overview and Individual Optimization Problems

Consider a world with two countries, two goods, and two moneys. People in country one produce only good one but consume both goods one and two; people in country two produce only good two but consume both goods. Thus there is complete specialization in production and trade occurs so that people can consume both goods.

Let "individual one" be a representative individual in country one. (By utilizing the fiction of a representative individual one can abstract from differences in behavior of people within country one.) Individual one maximizes the quantity

(1) $E[\sum_{t=0}^{\infty}\beta^t U^1(c^1_{1t}, c^1_{2t})]$

where $\{c^1_1, c^1_2\}_t$ is a bivariate stochastic process representing individual one's consumption of goods one and two, $U^1(\cdot)$ is the current period utility function of individual one, $\beta \varepsilon (o,1)$ is a discount term, and E is an expected value operator.[16] The constraints on the behavior of individual one are discussed below.

Similarly there is a representative individual in country two who maximizes the quantity

(2) $E[\sum_{t=0}^{\infty}\beta^t U^2(c^2_{1t}, c^2_{2t})]$

where $\{c^2_1, c^2_2\}_t$ is the stochastic process describing individual two's consumption of goods one and two $U^2(\cdot)$ (which need not be the same function as $U^1(\cdot)$) gives current period utility of individual two, and β and E are as described before.

Production of goods one (in country one) and two (in country two) is exogenously given by the stochastic process $\{y_1, y_2\}_t$. Both goods are perishable after one period so there is no storage of goods. Assume the process $\{y_1, y_2\}_t$ is generated by independent realizations of a random vector y_t from a stationary probability distribution with cumulative distribution function $F_y(\cdot)$. Thus there is some randomness in production but it is independent across time periods. The assumptions that output is exogenous, that goods are nonstorable, that production is specialized, and that shocks to production are independent both across goods and over time could all be relaxed with no important change in the results - the mathematical solution to the model would differ but the main economic issues emphasized in the paper would be unchanged.

Let $\{M_1^s, M_2^s\}_t$ be the nominal quantities of moneys one and two which have been issued by the governments of countries one and two are used within those countries for domestic transactions. International transactions could in principle involve the use of either money for payments. Empirically, roughly two-thirds of international trade contracts appear to be denominated in the seller's currency.[17] The choice of a currency for payments in international trade may depend upon the costs, in terms of depreciation uncompensated by interest payments on money, of holding each money and differential transactions costs in handling alternative currencies.[18] I assume here that all international transactions are financed with the seller's currency. Thus when country one imports good two from country two, payments are made with money two. Similarly, country one receives money one in payment for its exports. This is consistent with the assumptions of the traditional elasticities approach discussed in chapter II.

I abstract from domestic resales of the imported good within the domestic country and in terms of the domestic money.

The demands for moneys are derived from the demands for goods by specifying a simple transactions technology that prevents individuals from engaging in barter. The formulation of the transactions technology is similar to that of Lucas (1977) and is one version of the formulation proposed by Clower (1967) and also used by Grandmont and Younes (1972), (1973) to study issues in the monetary theory of closed economies. The transactions technology involves a "liquidity constraint" on individual behavior that attempts to reflect the fact that money is held between transactions and that many transactions would be very costly without the use of money. The form of the liquidity constraint used in this paper requires that goods be purchased with money and that this money be held before it is spent.[19] Expenditures during any period must be financed out of money available at the beginning of the period. This ensures that an individual cannot sell his output for money and instantaneously spend that money for goods, i.e. he cannot barter.[20] He carries his receipts from current sales of output into the next period.

Since imports must be financed with foreign exchange (foreign money), the transactions technology applied to imports results in a demand for foreign exchange that is derived from the demand for imports. This again follows the approach of the traditional foreign exchange market theories. People, as importers, hold positive balances of foreign exchange; they procure this foreign money on the foreign exchange market at the price e, the price of money two in terms of money one. (I will refer to country one as the domestic country, so e is the price of foreign exchange.)[21]

Let the sequence of events each period be the following: the representative individual in country one enters each period with some domestic money, M_1^1, which he may use for domestic purchases, and some foreign exchange, M_2^1, for importing purposes. The superscripts denote the holder of the money (individual one or individual two); the subscripts denote money one or money two. Individual one then harvests his output, y_1, and takes it to market. (Individual two takes y_2 to market.) He observes the equilibrium prices at which all trades take place. (The word "prices" means both goods prices, p_1 and p_2, and the exchange rate, e.) He purchases goods one and two and sells his own output for money. He then goes to foreign exchange market to purchase (or sell) foreign exchange to carry into the next period.

Note that this individual receives payment for the sales of his own good after he has purchased goods this period; his current receipts are not available for financing current consumption. Note also that as in other highly aggregated models the individual is assumed to purchase on the market the goods he consumes. I assume that the fraction of his consumption that <u>he</u> produces is negligible. Thus, although individual differences are ignored for some purposes in the theoretical construction of a representative individual, not all differences are ignored. The goods produced by different people within country one (or two) differ in some characteristics, and each consumer demands a bundle of these goods with several characteristics.[22] Relative price changes between these goods are ignored so that they may be aggregated into a composite "good one" (or "good two").[23]

Each period individual one chooses consumption of good one, c_1^1, consumption of good two, c_2^1, end-of-period holdings of domestic money (one), $M_1^{1\prime}$, and end-of-period holdings of foreign exchange, $M_2^{1\prime}$, subject to the constraints

(3) (a) $p_1 y_1 + \tau_1 + eM_2^1 - p_1 c_1^1 - ep_2 c_2^1 - M_1^{1\prime} - eM_2^1 = 0$

(b) $p_1 c_1^1 \leq M_1^1 + \tau_1$

(c) $p_2 c_2^1 \leq M_2^1$

where M_1^1 and M_2^1 are predetermined (by last period's choices), y_1 is his output, which he sells at the price p_1, the price of good one in terms of money one. τ_1 and τ_2 are realizations of a stochastic process $\{\tau_1, \tau_2\}_t$ representing transfer payments of money one to individual one and of money two to individual two. (These are taxes if they take negative values.) These transfers occur overnight (between periods) and are available with other initial money holding to finance consumption this period. (3a) is a budget constraint which states that the intitial assets and current income of individual one may be allocated to current consumption and end-of-period balances. (3b) and (3c) are liquidity constraints imposed by the assumed transactions technology. They state that current purchases of domestic goods are limited by initial holdings of domestic money and current imports are limited by initial holdings of foreign exchange. The analogous constraints for individual two's optimization problem are

(4) (a) $ep_2 y_2 + M_1^2 + eM_2^2 + e\tau_2 - p_1 c_1^2 - ep_2 c_2^2 - M_1^{2\prime} - eM_2^{2\prime} = 0$

(b) $P_1 C_1^2 \leq M_1^2$

(c) $P_2 C_2^2 \leq M_2^2 + \tau_2$

The Role of the Governments

The governments of each country have two roles: they determine $\{\tau_1, \tau_2\}_t$, the money supply changes financed by transfers or taxes to (from) their residents, and they may intervene in the foreign exchange market by buying or selling foreign exchange. Let Θ_t denote purchases of money two with money one by the combined actions of the two governments on the foreign exchange market. The policies of the two governments can the be summarized by the stochastic process $\{\tau_1, \tau_2, \Theta\}_t$.

I assume that $\{\Theta\}_t$ is the result of a joint decision of the governments of countries one and two. I therefore avoid game-theoretic aspects of the decisions to intervene in the foreign exchange market. Further, it is a matter of indifference (to this model) which country conducts the foreign exchange market intervention. If one country has insufficient reserves (of foreign currency) to sell all the foreign money that the intervention decision requires, the other country can always conduct the intervention since it can print the asset to be sold on the foreign exchange market. That is, there cannot be an international liquidity or reserve problem within this model. Such problems presumably arise in the real world because countries are unable to agree upon a choice of $\{\tau_1, \tau_2, \Theta\}_t$ and are unwilling to cooperate in the foreign exchange market operations required to achieve a target Θ.

Let $M_1{}^s$ and $M_2{}^s$ denote the nominal quantities of moneys one and two outstanding at the beginning of the period. Then

(5) $M_1{}^s = M_1^1 + \tau_1 + M_1^2$

$M_2{}^s = M_2^1 + M_2^2 + \tau_2$

At the end of the period the nominal money supplies are

$$(6)\quad M_1^{s'} = M_1^{s} + \Theta$$

$$M_2^{s'} = M_2^{s} - \frac{1}{e}\Theta$$

where Θ is the foreign exchange market intervention undertaken by governments during the period. At the beginning of the following period nominal money supplies are

$$(7)\quad M_1^{s'} + \tau_1' \qquad \text{and} \qquad M_2^{s'} + \tau_2'$$

where the transfers τ_1' and τ_2' occur between periods.

Let γ_j^i, $(i,j) = 1,2$ denote the fraction of money j held by residents of country i. Notice that $\gamma_1^1 + \gamma_1^2 = 1 = \gamma_2^1 + \gamma_2^2$. These allocation parameters are economically (endogenously) determined.

<u>Prices</u>

At the beginning of any period the state of the world can be described completely by the state vector

$$(8)\qquad s \equiv \begin{pmatrix} y_1 \\ y_2 \\ \gamma_1^1 \\ \gamma_2^1 \\ M_1^s \\ M_2^s \\ \Theta \end{pmatrix}$$

and the probability distribution functions $F_{\underline{y}}(\cdot)$, $F_{\underline{t}}(\cdot)$, and $F_{\theta}(\cdot)$ which generate the stochastic processes on $\{y_1, y_2\}$, $\{t_1, t_2\}$, and Θ. Denote by $F(\cdot,\cdot,\cdot,\cdot,\cdot)$ the joint cumulative probability distribution function of these variables. The state vector s includes current outputs of each good, the nominal supplies of each money at the beginning of the period and their allocations, and the extent of government intervention in foreign exchange markets. A complete account of the state of the world includes both s and $F(\cdot)$, which individuals use to form their expectations about the future.

Individuals choose consumptions and end-of-period asset holdings to maximize (1) or (2) subject to (3) or (4). The equilibrium conditions require that all markets clear:

$$c_1^1 + c_1^2 = y_1$$

$$c_2^1 + c_2^2 = y_2$$

(9)

$$M_1^{1\prime} + M_1^{2\prime} = M_1^{s\prime}$$

$$M_2^{1\prime} + M_2^{2\prime} = M_2^{s\prime}$$

Only three of these four markets are independent, as can be verified by adding the budget constraints of individuals one and two. There are three prices, p_1, p_2, and e (the money one price of good one, the money two price of good two, and the money one price of money two) which adjust each period to ensure equilibrium.

The demand functions of individual one for consumption and end-of-period money holdings depend upon the prices he faces, p_1, p_2, and e, his initial money holdings, M_1^1 and M_2^1, his current income, y, (in terms of good one), and his beliefs about future prices and incomes. Given these beliefs about the future (which enter through the expected value operator in (1) and (2)), individual one's behavior can be described by the optimal policy or demand functions.

$$c_1^1\ (p_1,\ p_2,\ e,\ M_1^1 + \tau_1,\ M_2^1,\ y_1)$$

$$c_2^1\ (p_1,\ p_2,\ e,\ M_1^1 + \tau_1,\ M_2^1,\ y_1)$$

(10)

$$M_1^{1\prime}(p_1,\ p_2,\ e,\ M_1^1 + \tau_1,\ M_2^1,\ y_1)$$

$$M_2^{1\prime}(p_1,\ p_2,\ e,\ M_1^1 + \tau_1,\ M_2^1,\ y_1)$$

(10′) or $d^1(p_1,\ p_2,\ e,\ M_1 + \tau_1,\ M_2,\ y_1)$

Similar demand functions describe individual two's behavior:

$$c_1^2\ (p_1,\ p_2,\ e,\ M_1^2,\ M_2^2 + \tau_2,\ y_2)$$

(11)

$$c_2^2\ (p_1,\ p_2,\ e,\ M_1^2,\ M_2^2 + \tau_2,\ y_2)$$

$$M_1^{2\prime}(p_1, p_2, e, M_1^2, M_2^2 + \tau_2, y_2)$$

$$M_2^{2\prime}(p_1, p_2, e, M_1^2, M_2^2 + \tau_2, y_2)$$

(11′) or $d^2(p_1, p_2, e, M_1^2, M_2^2 + \tau_2, y_2)$

A vector of prices

$$p \equiv \begin{pmatrix} p_1 \\ p_2 \\ e \end{pmatrix}$$

that assures equilibrium therefore depends on (from (9), (10), and (11)) $M_1^1 + \tau_1$, M_2^1, M_1^2, $M_2^2 + \tau_2$, y_1, y_2, $M_1^{s\prime}$, and $M_2^{s\prime}$. So, using (5), (6), and the definition of γ_j^i, the price vector p depends upon y_1, y_2, γ_1^1, γ_2^1, M_1^s, M_2^s, and θ, which are the elements of the state vector s. Let

$$p = \phi(s)$$

give prices as a fixed function of the state of the world, and let

$$P \equiv \{x \in R_t^3 | d^1(x, M_1^1 + \tau_1, M_2^1, y_1) + d^2(x, M_1^2, M_2^2 + \tau_2, y_2) = n,$$

$$n = (y_1, y_2, M_1^{s\prime}, M_2^{s\prime}) ; M_1^1, M_2^1, M_2^2, \tau_1, \tau_2, M_1^{s\prime}, M_2^{s\prime} > 0\}$$

be the set of possible values taken by the function $\phi(\cdot)$. The problem to be investigated involves the characterization of the function $\phi(\cdot)$ and the behavior of its values as the state vector changes over time. For example, in what sense (if any) will purchasing power parity hold? What disturbances cause deviations from purchasing power parity and what are the characteristics of these deviations (e.g. autocorrelated, correlated with other variables)?

The statement of the model will be completed by defining the expected value operators in (1) and (2). Given the expectations held by individuals about future values of the variables, individuals will be able to formulate demand functions, and prices will adjust to clear markets. These prices depend on the state of the world. On the other hand, the prices that occur in each state of the world affect expectations about future prices and therefore affect the demand functions today. The model will be completed with the assumption of rational expectations, to be defined in section 6.

Dynamics

Prices of goods and foreign exchange change over time as the state vector changes, and this relation is summarized by the function ϕ (s). The state vector changes for two reasons. First, new disturbances occur exogenously on initial money supplies, foreign exchange market intervention, and outputs (real income). Second, γ_1^1 and γ_2^1 change over time as people adjust optimally to past disturbances and to changes in expectations about the future. Given the expectations held by individuals about future variables the demand functions and resulting market prices determine (together with the exogenous transfers or taxes that will occur after the end of this period) next period's allocation parameters $\gamma_1^{1\prime}$ and $\gamma_2^{1\prime}$.

Note that

$$\underline{s}' \equiv \left(y_1', y_2', \frac{z_1^{1\prime}(s) + \tau_1'}{M_1^s + \tau_1' + \theta}, \frac{z_2^{1\prime}(s)}{M_2^s + \tau_2' - \frac{1}{e}\theta}, M_1^s + \tau_1' + \theta, M_2^s + \tau_2' - \frac{1}{e}\theta, \theta'\right)$$

where $z_2^{1\prime}(s) \equiv Z_2^{1\prime}(s, \phi(s))$, $s = 1,2$ and where $Z_2^{1\prime}(s, \phi(s))$ is the (average) aggregate choice by people in country one of end-of-period balances of money j given s and $p = \phi(s)$. So s′ depends upon y_1', y_2', τ_1', τ_2', θ', and s, given the function $\phi(\cdot)$. That is,

(12) $s' = G(s, w')$

where $w' \equiv (y_1', y_2', \tau_1', \tau_2', \theta')$.

So the time path of goods prices and the exchange rate are determined by

$$p' = \phi(s') = \phi(G(s, w')) = \text{function } (s, w').$$

A similar line of reasoning shows that the price that will prevail j periods into the future is a function of the current state vector and the shocks $w', w'', \ldots, w^{(j)}$.

<u>Reformulation of the Optimization Problems</u>

Before defining rational expectations it is useful to rewrite the individuals' optimization problems in dynamic programming form. Consider the indirect utility function $V^1: R_+^3 \times P \to R_+$ defined by the maximum value attained by the objective function in the solution to the problem,

(13) $$V^1(M_1^1 + \tau_1, M_2^{1\prime}, y_1, p) = \text{MAX } \{U^1(c_1^1, c_2^1) + \beta \int V^1(m_1^{1\prime} + \tau_1', m_2^{1\prime}, y_1', \phi^e(s'))\, dF(w')\}$$

where maximization is with respect to $(c_1^1, c_2^1, M_1^{1\prime}, M_2^{1\prime})$ and subject to the constraints (3), where $\phi^e(\cdot)$ maps the space of state vectors S into P. Equation (13) says that individual one maximizes current period utility plus the discounted expected value of future utilities given that he knows he will continue to behave optimally in the future. The optimization problem of individual two can be similarly reformulated. His indirect utility function will be

$$(14)\ V^2\,(M_1^2, M_2^2 + \tau_2, y_2, p) = \text{MAX}\ \{U^2(c_1^2, c_2^2) + \beta\int V^2(M_1^{2\prime}, M_2^{2\prime}, \tau_2^{\prime}, y_2^{\prime}, \phi^e\,(s^{\prime}))\ dF(w^{\prime})\}$$

where maximization is with respect to $(c_1^2, c_2^2, M_1^{2\prime}, M_2^{2\prime})$ and subject to the constraints (4).

Expectations

Assume each individual has rational expectations in the following sense:

1. The function F $(\cdot)$ in (13) and (14) is the cumulative probability distribution function describing the behavior of $w \equiv (y_1, y_2, \tau_1, \tau_2, \theta)$, defined earlier.
2. The function $\phi^e\ (\cdot)$ in (13) and (14) is the same function $\phi\ (\cdot)$ that guarantees market-clearing each period.
3. The individual knows that $s^{\prime}$ is determined by (12).

I assume that the information available to each individual includes the current state vector s. These individuals observe three market prices. At one extreme these prices may constitute all of the individual's information about aggregate variables, and these three prices would not provide sufficient information to exactly infer the current state vector. On the other hand, assumptions about the information available to people seem rather arbitrary in this exercise, and while limitations on information may yield some interesting results, they are not required for the purposes of this paper. Therefore, to avoid the impression that incomplete information about current variables are necessary for the results and to avoid the complications associated with such incomplete information, I assume that the current state of the world is known to both representative individuals. This amounts to knowledge of current outputs and incomes, aggregate money supplies, the world distribution of wealth, and the extent to which governments are buying or selling foreign exchange.

Each individual, since he knows s and F (w'), knows the induced probability distribution function of s' and therefore the induced probability distribution on $p' = \phi(s')$. His current behavior is based upon these expectations.

Each individual takes next period's state vector s' as exogenous to his own decisions (and random). Included in s' are

$$z_1^{1\prime}(s) = Z_1^{1\prime}(s, p(s))$$

and

$$z_2^{1\prime}(s) = Z_2^{1\prime}(s, p(s))$$

on which the individual has, through his knowledge of s and G $(\cdot)$, perfect foresight. This one-step-ahead perfect foresight on $z_1^{1\prime}$ and $z_2^{1\prime}$ is a result of the individual's knowledge of the aggregate decisions that are made today in state of the world s. Now each individual chooses his own end-of-period balances optimally given $z_1^{1\prime}$ and $z_2^{1\prime}$. But $z_1^{1\prime}$ and $z_2^{1\prime}$ are just the (average) aggregates of the choices of all these individuals. It can be verified that, by construction of the Markov process $G(\cdot)$, the market clearing prices $p(s)$ ensure that the consistency requirements

$$(15)\quad M_1^{1\prime}(\underline{s}, \phi(\underline{s})) = Z_1^{1\prime}(\underline{s}, \phi(\underline{s}))$$

and

$$M_2^{1\prime}(\underline{s}, \phi(\underline{s})) = Z_2^{1\prime}(\underline{s}, \phi(\underline{s}))$$

are met.

Equilibrium

An equilibrium is a set of functions

$V^1(\cdot)$, from $R_+^3 \times P$ to R_+, bounded and continuous, such that (13) holds given $\phi(\cdot)$ and $G(\cdot)$;

$V^2(\cdot)$, from $R_+^3 \times P$ to R_+, bounded and continuous, such that (14) holds given $p(\cdot)$ and $G(\cdot)$;

$d^1(\cdot)$, from $R_+^3 \times P$ to R_+^4, continuous, and such that (13) is solved;

$d^2(\cdot)$, from $R_+^3 \times P$ to R_+^4, continuous, and such that (14) is solved;

$\phi(\cdot)$, from S to P, continuous, and such that (9) holds when $d^1(\cdot)$ and $d^2(\cdot)$ are used; and

$G(\cdot)$, from $S \times R_+^2 \times R^2$ to S, such that (15) holds.

The equilibrium thus requires that people maximize expected utility given current prices and expected future prices, that prices adjust to clear markets and that people know that prices will also adjust to clear markets in the future, and that the expectations of individuals about the future state of the world are rational in the sense defined earlier.

Construction of Equilibrium

Consumer Optimization

Consider first the optimization problem of the representative individual in country one, who takes s, $F(\cdot)$, $\phi(\cdot)$, and $G(\cdot)$ as given. (The state vector s is a given vector of numbers, the others are given functions.)

Claim one: For each s and $F(\cdot)$ and for all $\phi(\cdot)$ and $G(\cdot)$, there is a unique, bounded, continuous, non-negative function $V^1(\cdot)$ satisfying (13) where $p = \phi(s)$, and V^1 is increasing and concave in its first three arguments, and differentiable with respect to the first two arguments. Furthermore, there is a unique set of continuous optimal policy (demand) functions with properties to be described below.

By assumption, $U^1(\cdot)$ is a continuously differentiable, bounded, and strictly concave non-negative function and $\beta \in (0, 1)$. Note that the constraints (3) form a convex set constraining c_1^1, c_2^1, $M_1^{1\prime}$, and $M_2^{1\prime}$. Equation (13) defines an operator $T = V \to V$ where V is the space of continuous bounded functions. (T takes values of the space of bounded functions into the space of bounded functions because the sum of two bounded functions is bounded. A proof that T has values in the space of continuous bounded functions can be found in Berge, pp. 115-116.) It is easily verified that T is monotone. If $V(s) > W(s)\ \forall\ s \equiv (M_1^1, M_2^1, y_1, p)$

$$\text{then } (TV)(s) \geq U(c_{1w}^1, c_{2w}^1) + \beta\int V(M_{1w}^{1\prime}, M_{2w}^{1\prime}, y_1{}', \phi(s')) \, dF(w')$$

where $(c_{1w}^1, c_{2w}^1, M_{1w}^{1\prime}, M_{2w}^{1\prime})$ solves the problem,

$$\text{MAX } \{U(c_1^1, c_2^1) + \beta\int W(M_1^{1\prime}, M_2^{1\prime}, y_1{}', \phi(s')) \, dF(w')\}$$

So $(TV)(s) \geq U(c^1_{1w}, c^1_{2w}) + \beta\int W(M^{1\prime}_{1w}, M^{1\prime}_{2w}, y_1{}', \phi(s')) \, dF(w') \equiv (TW)(s)$.

Also note that for any constant k, $(T(V + k))(\partial) = (TV)(\partial) + \beta k$. Therefore by Blackwell's sufficiency theorum[22] T is a contraction with modulus β. The Banach fixed point theorum then implies that (13) has a unique, bounded, continuous solution $V^1(\cdot)$ and $V^1(\cdot)$ is nonnegative because $U^1(\cdot)$ is nonnegative.

$V^1(\cdot)$ is increasing in its first three arguments because $T\psi$ is an increasing function of these arguments for <u>any</u> function ψ hence in particular for $\psi(\cdot) \equiv V^1(\cdot)$, so $V^1 = TV^1$ is increasing in its first three arguments. $V^1(\cdot)$ is also strictly concave in its first three arguments. Write $V^1(z, p)$ and let z^a and z^b be two different values of the three-dimensional vector z. Let $\lambda \in [0, 1]$ and $z^\lambda \equiv \lambda z^a + (1 - \lambda)z^b$. Let (c^a, M'^a) and (c^b, M'^b) be the policies that attain $(T\psi)(z^a, p)$ and $(T\psi)(z^b, p)$. Let $(c^\lambda, M'^\lambda) \equiv (\lambda c^a + (1 - \lambda)c^b, \lambda M'^a + (1 - \lambda)M'^b)$ and note that strict concavity of $U^1(\cdot)$ implies that if $V^1(\cdot)$ is concave

$$\text{then } (T\psi)(z^\lambda, p) \geq U^1(c^\lambda) + \beta\int V^1(M'^\lambda, y_1, p(\underline{s}')) \, dF(w^1)$$

$$> \lambda(TV^1)(z^a, p) + (1 - \lambda)(TV^1)(z^b, p).$$

So if V^1 is concave then TV^1 is strictly concave. Since $V^1 = \lim_{n\to\infty} T^n\psi$ and ψ is an arbitrary function from R^7_+ to R_+, choose ψ to be concave in its first three arguments. Then $V^1 = TV^1$ is concave in these arguments, M^1_1, M^1_2, and y_1.

$V^1(\cdot)$ is differentiable with respect to M_1^1 and M_2^1. For fixed p and sufficiently small $h > 0$, define $h_1 \equiv \begin{pmatrix} h \\ 0 \\ 0 \end{pmatrix}$, $h_2 \equiv \begin{pmatrix} 0 \\ h \\ 0 \end{pmatrix}$, and $x \equiv \begin{pmatrix} M_1^1 \\ M_2^1 \\ y_1 \end{pmatrix}$. Then if initial asset holdings are $x + h_1$, one feasible policy is to choose

$$\left(c_1(x) + \frac{h_1}{p_1},\ c_2(x),\ M_1'(x),\ M_2'(x)\right).$$ Similarly,

$$\left(c_1(x + h_1) - \frac{h_1}{p_1},\ c_2(x + h_1),\ M_1'(x + h_1),\ M_2'(x + h_1)\right)$$ is feasible at x.

(If at x consumption of good two is equal to initial balances of money two divided by the money two price of good two, then $c_2(x + h_1) = c_2(x)$.)

$$\text{So } V(x + h_1, p) \geq U\left(c_1(x) + \frac{h_1}{p_1},\ c_2(x)\right) + \beta\int V(M_1'(x),\ M_2'(x),\ y',\ p(s'))dF(w')$$

$$= U\left(c_1(x) + \frac{h_1}{p_1},\ c_2(x)\right) + V(x, p) - U(c_1(x),\ c_2(x))$$

and

$$V(x, p) \geq U\left(c_1(x + h_1) - \frac{h_1}{p_1},\ c_2(x + h_1)\right) + \beta\int V(M_1'(x + h_1),\ M_2'(x + h_1),$$
$$y',\ p(s'))\ dF(w')$$

$$= U\left(c_1(x + h_1) - \frac{h_1}{p_1},\ c_2(x + h_1)\right) + V(x + h_1,\ p) - U(c_1(x + h_1),$$
$$c_2(x + h_1))$$

therefore

$$U(c_1(x) + \frac{h_1}{p_1}, c_2(x)) - U(c_1(x), c_2(x))$$

$$\leq V(x + h_1, p) - V(x, p)$$

$$\leq U(c_1(x + h_1) - \frac{h_1}{p_1}, c_2(x + h_1)) - U(c_1(x + h_1), c_2(x + h_1))$$

Note that p_1 is constant and that $c_1(\cdot)$ and $c_2(\cdot)$ are continuous and divide by h and let h go to zero. Thus the partial derivative of V^1 w.r.t. M_1^1 is

$$V_1^1(M_1^1, M_2^1, y_1, p) = U_1^1(c_1(M_1^1, M_2^1, y_1, p), c^2(M_1^1, M_2^1, y_1, p))$$

Similarly it can be shown that the partial derivative w.r.t. M_2^1 is

$$V_2^1(M_1^1, M_2^1, y_1, p) = U_2^1(c_1(M_1^1, M_2^1, y_1, p), c_2(M_1^1, M_2^1, y_1, p))$$

The demand functions are continuous (see Berge pp. 115-6) and their properties can be established by examining the properties of the demand functions in a model with n periods and letting n go to infinity. Monotonicity properties carry over to the limiting functions because the demand functions can be shown to converge pointwise as $n \to \infty$. (For a proof of this pointwise convergence, see Lucas, 1976, pp. 10-11.) To examine the properties of the demand functions, consider first a two-period model, that is, a model in which the world ends after next period. Because the liquidity constraints require that moneys be held at the beginning of next period if any consumption at all is planned for next period, the assumption that the marginal utility of consumption of each good approaches infinity as consumption of that good approaches zero ensures that moneys will be demanded this period. Next period (the final period) moneys become useless except for current consumption, hence each individual will spend all his initial balances. (If one tried to give an economic interpretation to this two-period model, he would have to make some assumption guaranteeing that people would be willing to sell their output for money in the final period.) Prices are then given by

$$P_1(\underline{s};\ 0) = \frac{M_1^s\ (0)}{y_1\ (0)} \qquad \text{and} \qquad P_2(\underline{s};\ 0) = \frac{M_2^s\ (0)}{y_2\ (0)}$$

where the index "0" denotes the final period (zero periods to go). The indirect utility function in the final period is

$$V^1(M_1^1,\ M_2^1,\ y_1,\ p;\ 0) = U^1\left(\frac{M_1^1}{p_1},\ \frac{M_2^1}{p_2}\right). \tag{16}$$

The first order conditions for the consumer one maximization problem in the first period (indexed "1" for one period to go) are then

$$\lambda_0^1 = \frac{U_1^1(c_1^1,\ c_2^1)}{p_1} - \lambda_1^1 = \frac{U_2^1(c_1^1,\ c_2^1)}{ep_2} - \frac{\lambda_2^1}{e}$$

$$= \beta\int V_1^1(M_1^{1\prime},\ M_2^{1\prime},\ y_1{}',\ p(\underline{s}';\ 0);\ 0)dF(\underline{w}')$$

$$= \frac{1}{e}\beta\int V_2^1(M_1^{1\prime}, M_2^{1\prime}, y_1{}', p(\underline{s}'; 0); 0)dF(\underline{v}')$$

and

$$p_1y_1 + M_1^1 + \tau_1 + eM_2^1 - p_1c_1^1 - ep_2c_2^1 - M_1^{1\prime} - eM_2^{1\prime} = 0$$

$$\lambda_1^1(M_1^1 + \tau_1 - p_1c_1^1) = 0 \qquad \lambda_1^1 \geq 0,\ M_1^1 + \tau_1 - p_1c_1^1 \geq 0$$

$$\lambda_2^1(M_2^1 - p_2c_2^1) = 0 \qquad \lambda_2^1 \geq 0,\ M_2 - p_2c_2^1 \geq 0$$

where $\overline{\lambda}_0^1$ is the expected value of the (random) Lagrange multiplier for the budget constraint, and λ_1 and λ_2 are the (nonrandom) multipliers for the liquidity constraints. These conditions state simply that the individual equates the marginal value of additional income today (which is in the form of money that can be spent tomorrow) to the (price deflated) marginal utility of consumption of goods one and two. If the individual decides not to hold precautionary cash balances, i.e. if the liquidity constraints hold as equalities then λ_1 and λ_2 are nonzero. In this case the marginal value of an additional dollar of income this period differs from the marginal value of an additional unit of cash balances held at the beginning of this period, since the cash balances can be spent immediately while this period's income cannot be spent until after it is received, i.e. next period. Then the (price deflated) marginal utility of consumption is equated to the marginal value of an additional unit of initial money balances, $\overline{\lambda}_0 + \lambda_1$ or $\overline{\lambda}_0 + \frac{\lambda_2}{e}$. The marginal utility of additional income today is equated to the discounted expected value of a marginal unit of domestic money or a marginal unit of foreign exchange held at the beginning of next period. Because of (16), this is easy to compute in the two-period model. The demand functions that arise from these first order conditions have some ambiguous properties for the usual reasons. For example, an increase in p_1 given p_2, e, and y and initial money holdings induces substitution out of good one and into good two and both moneys (hence future goods); but since individual one has an income given in terms of y_1 his real income in terms of y_2 or in terms of a consumption bundle rises, and this income effect increases his demand, generally, for both goods today and both moneys (both goods tomorrow). I assume that the substitution effect dominates the income effect so that the net effect on the demand for good one is negative. Appendix II investigates the other properties of the demand functions in more detail. Assuming substitution effects generally dominate income effects, demands respond to prices, assets, and income according to the signs

$$c_1^1(\underset{-}{p_1},\ \underset{+}{p_2},\ \underset{+}{e},\ \underset{+}{M_1^1 + \tau_1},\ \underset{+}{M_2^1},\ \underset{+}{y_1})$$

$$c_2^1(\underset{+}{p_1},\ \underset{-}{p_2},\ \underset{-}{e},\ \underset{+}{M_1^1 + \tau_1},\ \underset{+}{M_2^1},\ \underset{+}{y_1})$$

$$M_1^{1\prime}(\underset{+}{p_1},\ \underset{+}{p_2},\ \underset{+}{e},\ \underset{+}{M_1^1 + \tau_1},\ \underset{+}{M_2^1},\ \underset{+}{y_1})$$

$$M_2^{1\prime}(\underset{+}{p_1},\ \underset{+}{p_2},\ \underset{-}{e},\ \underset{+}{M_1^1 + \tau_1},\ \underset{+}{M_2^1},\ \underset{+}{y_1})$$

and there are similar demand functions for individual two. (The signs may be interpreted as showing the direction of demand response to small finite changes in prices, assets, and income, since differentiability of the demand functions has not been established here.) The monotonicity properties of the demand functions follow from $V_1(\cdot; 0) > 0$, $V_2(\cdot; 0) > 0$, $V_{11}(\cdot; 0) < 0$, $V_{22}(\cdot; 0) < 0$. (The second derivative exists because of (16).) But

$$V_1(\cdot; 1) = U_1 c_{1_{M_1}}(\cdot; 1) + U_2 c_{2_{M_1}}(\cdot; 1)$$

$$+ \beta\int [V_1(\cdot; 0)\, M_{1\,M_1}'(\cdot; 1) + V_2(\cdot; 0) M_{2\,M_1}'(\cdot; 1)]dF(w')$$

which is positive since $U_1 > 0$, $U_2 > 0$, $V_1(\cdot; 0) > 0$, $V_2(\cdot; 0) > 0$

and since it has just been shown that $c_{1_{M_1}}(\cdot; 1)$, $c_{2_{M_1}}(\cdot; 1)$, $M_{1\,M_1}'(\cdot; 1)$,

and $M_{2\,M_1}'(\cdot; 1)$ are all positive. Similarly,

$$V_2(\cdot; 1) = U_1 c_{1_{M_2}}(\cdot; 1) + U_2 c_{2_{M_2}}(\cdot; 1)$$

$$+ \beta\int [V_1(\cdot; 0) M_{1\,M_2}'(\cdot; 1) + V_2(\cdot; 0) M_{2\,M_2}'(\cdot; 1)]dF(\underline{w}') > 0.$$

Also

$$V_{11}(\cdot; 1) = U_{11}(c_{1_{M_1}}(\ ; 1))^2 + U_{22}(c_{2_{M_1}}(\ ; 1))^2$$

$$+ \beta\int V_{11}(\ ; 0)dF(w')\ (M_{1\,M_1}'(\ ; 1))^2$$

$$+ \beta\int V_{22}(\ ; 0)dF(w')\ (M_{2_{M_1}}(\ ; 1))^2$$

$$+ U_1 c_{1_{M_1M_1}} + U_2 c_{2_{M_1M_1}}$$

$$+ \beta\int V_1(\; ; 0)dF(w')M_1'{}_{M_1M_1}(\; ; 1)$$

$$+ \beta\int V_2(\; ; 0)dF(w')M_2'{}_{M_1M_1}(\; ; 1)$$

$$+ 2\beta\int V_{12}(\; ; 0)dF(w')M_1'{}_{M_1}(\; ; 1)M_2'{}_{M_1}(\; ; 1).$$

The first four terms are negative, and the last term is zero since $U_{12} = 0$. (This expression may be thought of as a finite-difference approximation to the second derivative, since the differentiality of the demand function was not established.) The middle four terms sum to zero because the first order conditions and the budget constraint imply, for the case of $\lambda_1 = 0 = \lambda_2$,

$$U_1 c_{1_{M_1M_1}} + U_2 c_{2_{M_1M_1}} + \beta\int V_1(\cdot)dF(w')M_1'{}_{M_1M_1} + \beta\int V_2(\cdot)dF(w')M_2'{}_{M_1M_1} = 0.$$

In the case with $\lambda_1 \neq 0$ and $\lambda_2 \neq 0$, so the middle four terms reduce to $U_1 c_{1_{M_1M_1}}$ since $c_{2_{M_1}} = 0 = M_1'{}_{M_1} = M_2'{}_{M_1}$. But $c_{1_{M_1}} = \frac{1}{p_1}$ in this case so

$c_{1_{M_1M_1}} = 0$. Therefore the middle four terms still sum to zero. A similar

argument can be applied to the cases $\lambda_1 > 0$, $\lambda_2 = 0$ and $\lambda_1 = 0$, $\lambda_2 > 0$. Thus the second derivative of the value function in period 1 with respect to M_1 exists and is negative. Similarly one can verify that $V_{22}(\; ; 1)$ and $V_{12}(\; ; 1)$ are negative. This argument can be applied recursively to show that

$$V_1(\; ; n) > 0, \; V_2(\; ; n) > 0$$

$$V_{11}(\; ; n) < 0, \; V_{12}(\; ; n) < 0, \; V_{22}(\; ; n) < 0$$

for any n. (The only difference in the argument is that the last term in $V_{11}(\; ; n-1)$ will be negative instead of zero.) Since these were the properties that (together with the assumption that substitution effects outweigh income effects) ensured the monotonicity properties of the demand functions, these properties hold for any n. Therefore, since the demand functions converge pointwise, these monotonicity properties of the demand functions hold for the infinite horizon problem.

Market Equilibrium

A collection of results analogous to those stated in Claim One hold for individual two's optimization problem. One would now like to examine the market equilibrium that results from the behavior of each individual as described in Claim One and that results in a price function and a Markov process $G(\cdot)$. That is, one would like to show that for each s and $F(\cdot)$ there is a set of functions

$$(17) \qquad (v^1(\cdot), v^2(\cdot), d^1(\cdot), d^2(\cdot), \phi(\cdot), G(\cdot))$$

such that Claim one holds for each $v^i(\cdot)$, $d^i(\cdot)$ pair, (9) is satisfied by $d^1(\cdot)$ and $d^2(\cdot)$, and (15) holds. This would mean that individuals behave optimally given current market prices and anticipated future prices and that prices are determined so that markets clear, and individuals act as if they know this. While anticipations about the random part of the state vector are rational in the sense that the probability on the exogenous variables and the induced probability distribution on prices are known, anticipations about the elements of next period's state vector that are the result of (aggregate) individual choices made today are rational in the sense that the individual knows with certainty these aggregate choices and makes his own plans accordingly. As all individuals do this, their own choices <u>form</u> the aggregate choice that each takes as given.

Since Claim One and its counterpart for individual two guarantee that there are functions $v^1(\cdot)$, $v^2(\cdot)$, $d^1(\cdot)$, and $d^2(\cdot)$ with the desired properties for <u>each</u> $\phi(\cdot)$ and $G(\cdot)$, and since there is some $\phi(\cdot)$ that satisfies (9) (the equilibrium conditions) for each $d^1(\cdot)$, $d^2(\cdot)$, and $G(\cdot)$, then the set of functions (17) with the desired properties exists if there is a function $G(\cdot)$ satisfying (15). Denote $z' \equiv G^*(s)$, a subvector of $s' = G(s, w')$ since z' does not depend upon w', but only upon choices made today, prior to the realization of new shocks. Current prices for each $F(\cdot)$ depend upon both the state of the world s and the function $G^*(\cdot)$. Denote this correspondence by $\rho(s, G^*(\cdot))$. The problem is to find a function $\overline{G}^*(\cdot)$ from the space of the state vector to R_+^2 such that

$$\begin{pmatrix} M_1^{1\prime}(s,\ \rho(s,\ \overline{G}^*(s));\ \overline{G}^*(s)\ ) \\ M_2^{1\prime}(s,\ \rho(s,\ \overline{G}^*(s));\ \overline{G}^*(s)\ ) \end{pmatrix} = \overline{G}^*(s) .$$

Then (15) will hold with $\phi(s) \equiv \rho(s, \overline{G}^*(s))$. Unfortunately, the above equation that implicitly gives $\overline{G}^*(\cdot)$ is a fixed point problem in a space of functions and little is known (by me) about its solution. As a consequence, I cannot investigate a full steady-state equilibrium in this model. However, the equilibrium can be described for an n period version of the model for arbitrary n. It seems unlikely that the properties of the steady state equilibrium, if it exists, will be different from the equilibrium of an n period version of the model.

Properties and Extensions of the Model

A Special Case: Certainty

Suppose y, τ, and θ are perfectly predictable; in particular let y be a constant vector and $\tau_1 = \tau_2 = \theta = 0$. Then the optimization problem of representative individual one can be represented by

$$V^1(M_1^1, M_2^1, y_1, \phi(s)) = \text{MAX } \{U^1(c_1^1, c_2^1) + \beta\, V^1(M_1^{1\prime}, M_2^{1\prime}, y_1^{\prime}, \phi(s^{\prime}))\}$$

subject to (3). The path of the state vector is determined by $s_{t+1} = G(s_t, \bar{y}, 0) \equiv H(s_t)$, where $\bar{y}$ is the fixed value of the output vector. This is a special case of (12). If this difference equation has a solution $\bar{s}$ then this vector characterizes the steady state of this special case of the model. The issue is what determines the steady state level of goods prices and the exchange rate.

In this special case neither individual will find it useful to hold precautionary money balances. When each individual spends his entire initial money holdings on goods, the prices of goods are determined by a simple form of the quantity theory: $M_1^s = p_1 y_1$ and $M_2^s = p_2 y_2$. So the exchange rate can be written as

$$e = \frac{M_1^s}{M_2^s} \frac{y_2}{y_1} \frac{ep_2}{p_1} .$$

The first order conditions for the representative individuals' optimization problems are (with superscripts omitted)

$$\lambda_0 = \frac{U_1(c_1, c_2)}{p_1} - \lambda_1 = \frac{U_2(c_1, c_2)}{ep_2} - \lambda_2 \frac{1}{e}$$

$$= \beta V_1(M_1', M_2', \bar{y}, \phi(\bar{s})) = \frac{1}{e} \beta V_2(M_1', M_2', \bar{y}, \phi(\bar{s})) .$$

The last equality shows that the exchange rate is equal to the ratio of the marginal values of each money for purchasing goods next period. The exchange rate can also be expressed in terms of current-period variables in the following way. The first order conditions and the definition of of $V(\cdot)$ imply that

$$V_1(M_1, M_2, \bar{y}, \bar{p}) = U_1 c_{1_{M_1}} + U_2 c_{2_{M_1}} + \beta V_1(M_1', M_2', \bar{y}, \bar{p}) M_{1_{M_1}}'$$

$$+ \beta V_2(M_1', M_2', \bar{y}, \bar{p}) M_{2_{M_1}}'$$

$$= (\lambda_0 + \lambda_1) p_1 c_{1_{M_1}} + (\lambda_o + \frac{\lambda_2}{e}) c_{2_{M_1}} ep_2$$

$$+ \lambda_0 (M_{1_{M_1}}' + M_{2_{M_1}}' e)$$

since the budget constraint implies $1 = p_1 c_{1_{M_1}} + ep_2 c_{2_{M_1}} + M_{1_{M_1}}' + eM_{2_{M_1}}'$.

So $\lambda_o = \beta\lambda_1 p_1 c_{1_{M_1}} + \beta\lambda_2 p_2 c_{2_{M_1}} + \beta\lambda_o$. But $p_1 c_{1_{M_1}} = 1$ and $p_2 c_{2_{M_1}} = 0$ in this special case of the model, so

$$\lambda_o = \frac{\beta}{1 - \beta} \lambda_1 .$$

Similarly, $\lambda_o = \frac{\beta}{1-\beta} \frac{\lambda_2}{e}$ so $\lambda_1 = \frac{\lambda_2}{e}$.

Therefore $\frac{U_1}{p_1} = \frac{U_2}{ep_2}$ (which makes this special case of the model resemble the conditions in an economy without money) and so the exchange rate can be written as $$e = \frac{M_1^s}{M_2^s}\frac{y_1'}{y_2'}\frac{U_2}{U_1} = \frac{p_1}{p_2}\frac{U_1}{U_2} .$$

That is, the exchange rate is related to nominal money supplies, real outputs, and the marginal rate of substitution in consumption between foreign and domestic goods. Both real and nominal variables affect the exchange rate. If a ratio of production price indexes is used to calculate purchasing power parity, then deviations of the exchange rate from this value can occur through changes in the marginal rate of substitution between the goods.

A change in the terms of trade will generally occur through changes in each of p_1, p_2, and e. Suppose that equilibrium initially occurs at point A in Figure 1. Then let production conditions (endowments) change so that the new production point is B. Given nominal money supplies, the new values of p_1 and p_2 are determined by M_1^s/y_1' and M_2^s/y_2'. Suppose that at the old exchange rate this results in a relative price p_1/ep_2 shown by the slope of line ℓ through the equilibrium point B. The highest indifference curve that can be attained at B is $\bar{\bar{U}}$ (I am assuming for simplicity that the utility functions of the two representative individuals are identical and homothetic). If the exchange rate were unchanged, people would attempt to move along a budget line ℓ to a preferred position by purchasing less of good one and more of good two. Individuals in country one therefore increase their demand for foreign exchange to purchase these imports; individuals in country two supply less foreign exchange because of their reduced demand for country one's good at this relative price. Consequently, the price of money two in terms of money one will rise until the relative price of good one, p_1/ep_2, has fallen to a point where ℓ is tangent to $\bar{\bar{U}}$ at B. Then the foreign exchange market (and each goods market) clears. The change in the

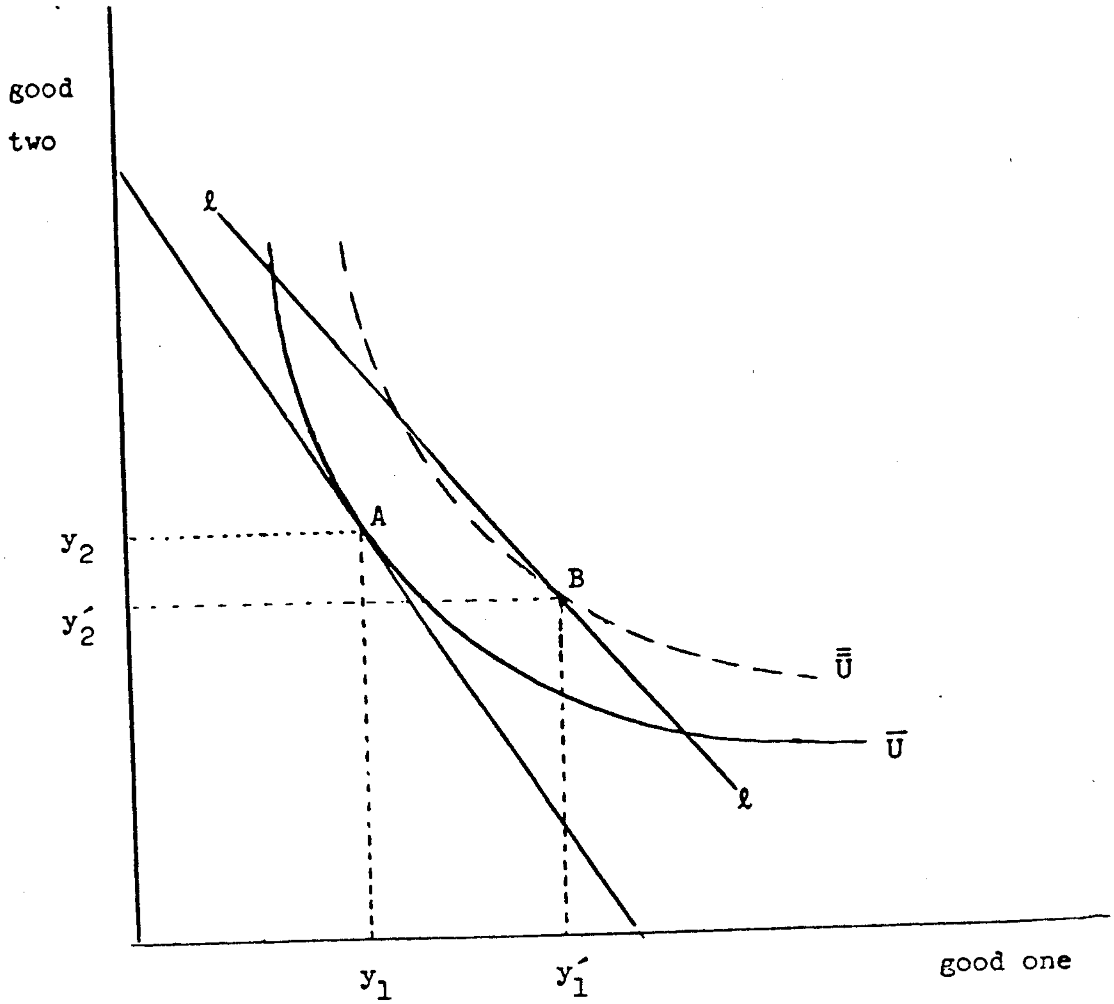

Figure I

terms of trade has been divided between changes in nominal export prices and the exchange rate.

Suppose the utility function of each individual is $U = 5(c_1 + c_2) - 2(c_1^2 + c_2^2) + 5c_1c_2$. Suppose that inital production is at $y_1 = 2$, $y_2 = 3$. Now let production change to $y_2 = 3$, $y_2 = 2$. Initially the relative price of the two goods is 16/9. Now, since each nominal price is determined by a simple quantity theory with unit velocity, the exchange rate is

$$e = \frac{M_1^S}{M_2^S} \frac{3}{2} \frac{9}{16} \propto \frac{27}{32} .$$

With the production change and constant money supplies, the exchange rate becomes proportional to 2/3 times 16/9, or 32/27. The new relative price of good one in terms of good two is 9/16. The money one price of good one is reduced by about one-half, while the money two price of good two is increased by about one-third. The increase in the exchange rate of about two-fifths accounts for the remainder of the reduction in the relative price of good one by about two-thirds. The depreciation of money one is associated in this example with a "worsening" of country one's terms of trade, i.e. a decrease in the relative price of its export good.

Other things the same, the lower the marginal rate of substitution between goods one and two the larger the depreciation of money one relative to money two required for equilibrium. If the marginal rate of substitution between the goods is greater in the long run than in the short run then the exchange rate will depreciate more in the short run than in the long run even if production remains at point B. This "overshooting" by the exchange rate of its long run value resembles a conclusion reached by Dornbusch (and discussed above), but here it occurs as an <u>equilibrium</u> phenomenon.

While a change in the terms of trade is associated with a change in the exchange rate. government exchange market transactions cannot succeed at affecting the terms of trade. If the government of country one were to attempt to depreciate domestic money by purchasing foreign money on the foreign exchange market, M_2^s would fall and M_1^s would rise. As a result, p_1 and e would rise and p_2 would fall, but p_1/ep_2 would remain approximately unchanged. (The qualifier is necessary because of the distribution effects associated with changes in the values of moneys caused by the government transactions.) The reason that government exchange market transactions cannot exploit the relationship between the exchange rate and the terms of trade is that the exchange rate change did not "cause" the terms of trade change (although it may appear that way to some people living in this hypothetical world), but was merely one way in which the terms of trade change occurred.

A producer of good one (individual one) may reasonably regard the exchange rate increase as undesirable in the sense that he would prefer to be producing a relatively more valuable good. These same individuals would be roughly indifferent to a change in the exchange rate that was accompanied by changes in all other nominal prices. In an extended model in which people are uncertain about whether an exchange rate change is associated with a "real" or "nominal" disturbance, people may reasonably be concerned about <u>any</u> exchange rate change, since people will rationally impute some part of that change to real factors and some part to nominal factors.

If the government of country one were to peg the exchange rate, then figure 1 would be unchanged but nominal money supplies would change by $dM_1^s + e\, dM_2^s = 0$, M_1^s rising in the example above. Thus p_1 and p_2 would change proportionally to the money supply changes in addition to the changes due to the real disturbance. So p_1 will rise more than if the exchange rate had been flexible and money supplies constant, and p_2 will fall more than in that case. The deviation from purchasing power parity will therefore be roughly the same under either exchange rate system, although in the flexible exchange rate case it will occur partially through exchange rate changes while in the pegged exchange rate case it will occur through changes in nominal export prices.

International Borrowing and Lending

The basic results of the model are unchanged if individuals can purchase or issue financial assets denominated in either currency. Assume that a bond denominated in currency j is always sold for money j, that the bonds are claims on one-period loans with nominal interest rates i_1 and i_2, and that bonds are bought and sold immediately after goods are bought and sold each period and before foreign exchange market transactions are completed. Therefore an individual who is short of cash cannot borrow by issuing bonds to finance current consumption; he can only borrow in order to obtain money which he can spend in the future. This assumption prevents individuals from, in effect, trading bonds directly for goods without holding money, and therefore prevents bonds from playing the role reserved for money. These assumptions therefore distinguish (albeit in a rigid manner) between money and bonds by giving money a "liquidity" property not shared by bonds.

Each individual enters each period with a portfoloi of both foreign and domestic bonds. Let B_j^i denote the bonds denominated in money j held by individual i at the beginning of the period. B_j^i may be either positive or negative. Each individual chooses end-of-period bond holdings simultaneously with current consumption and end-of-period money holdings. The initial level of bond holdings enters the indirect utility function $V^i(\cdot)$. Denoting end-of-period holdings by $B_j^{i\prime}$, the budget constraint for individual one can be rewritten

$$p_1 y_1 + M_1^1 + eM_2^1 + B_1^1 + eB_2^1 - p_1 c_1^1 - ep_2 c_2^1$$
$$- M_1^{1\prime} - eM_2^{1\prime} - \frac{B_1^{1\prime}}{1 + i_1} - \frac{B_2^{1\prime}}{1 + i_2} = 0 ,$$

and the first order conditions are unchanged although two new conditions are added:

$$\bar{\lambda}_0 = (1 + i_1)\, \beta\, E \frac{\partial V^1}{\partial B_1^{1\prime}} = \frac{1}{e} (1 + i_2)\, \beta\, E \frac{\partial V^1}{\partial B_2^{1\prime}} ,$$

and there are two new equilibrium conditions to match the two additional prices i_1 and i_2 :

$$B_1^{1\prime} + B_1^{2\prime} = 0 = B_2^{1\prime} + B_2^{2\prime} .$$

The value of an additional bond denominated in money one is

$$V_{B_1} = U_1\, c_{1_{B_1}} + U_2\, c_{2_{B_1}} + EV_1^{\prime}\, M_{1_{B_1}}^{\prime} + EV_2^{\prime}\, M_{2_{B_1}}^{\prime}$$
$$+ EV_{B_1^{\prime}}^{\prime}\, B_{1_{B_1}}^{\prime} + EV_{B_2^{\prime}}^{\prime}\, B_{2_{B_1}}^{\prime}$$
$$= (\bar{\lambda}_0 p_1 + \lambda_1 p_1)\, c_{1_{B_1}} + (\bar{\lambda}_0 e p_2 + \bar{\lambda}_1 e p_2)\, c_{2_{B_1}}$$
$$+ \bar{\lambda}_0 (M_{1_{B_1}}^{\prime} + eM_{2_{B_1}}^{\prime} + (1 + i_1)\, B_{1_{B_1}}^{\prime} + e(1 + i_2)\, B_{2_{B_1}}^{\prime})$$

which, after differentiating the budget constraint with respect to B_1 and substituting, becomes

$$V_{B_1} = \bar{\lambda}_0 + \lambda_1 p_1 c_{1_{B_1}} + \lambda_2 ep_2 c_{2_{B_1}}$$

$$= \bar{\lambda}_0$$

since either $\lambda_1 = 0$ or $p_1 c_{1_{M_1}} = 1$ and $c_{1_{B_1}} = 0$, and similarly either $\lambda_2 = 0$ or $c_{2_{B_1}} = 0$.

An analogous argument implies that $V_{B_2} = e\,\bar{\lambda}_0$. Using these results in the first order conditions, one obtains

$$E\,V_1' = (1 + i_1)\,E\,V_{B_1}'$$

or

$$E\,\bar{\lambda}_o' + E\,\bar{\lambda}_1' = (1 + i_1)\,E\,\bar{\lambda}_o'$$

or

$$i_1 = \frac{E\,\lambda_1'}{E\,\bar{\lambda}_0'} \qquad \text{and similarly} \qquad i_2 = \frac{E\,\lambda_2'}{E\,\bar{\lambda}_0'} .$$

Individuals allocate to their portfolios quantities of money and bonds that make the ratio of expected marginal values of money and bonds equal to the nominal interest rate. The nominal interest rate adjusts to equate borrowing and lending.

Note that $e = \dfrac{1 + i_2}{1 + i_1} \dfrac{E\,(e'\lambda_0')}{E\,(\lambda_0')}$ which resembles the interest-parity condition $e = f\,(1 + i_2)/(1 + i_1)$ where f is the forward exchange rate in units of money one per unit of money two. A forward foreign exchange market is redundant in this model since forward market transactions can be duplicated in bond markets and the spot foreign exchange market. The implicit forward exchange rate in these transactions, and the rate at which individuals would be indifferent to facing in the market is in fact $f = E(e'\lambda_0')/E(\lambda_0')$. This generalizes the result in Stockman (1978) to the case in which purchasing power parity does not hold due to equilibrium relative price changes. (The term $f - E(e') = \text{cov}(e', \lambda_0')/E(\lambda_0')$ is discussed and empirically investigated in that paper.)

IV.3. Implications of the Model

The model was constructed so that deviations from purchasing power parity involved changes in the terms of trade. The change in the terms of trade is associated with an equilibrium change in the exchange rate and not simply changes in nominal export prices at a given exchange rate. In an n-period version of the model the comparative statics effect of a real supply shock can be obtained by differentiating the equilibrium conditions (3). If the output of good one is increased, holding everything else constant including individuals' expectations of the probability distributions on future exogenous variables, then one obtains the exchange rate change

$$
\begin{aligned}
d\,e = \frac{1}{\Delta}\{ & (c_{2_{p_1}} M'_{1_{p_2}} - c_{2_{p_2}} M'_{1_{p_1}})(1 - c^1_{1_{y_1}}) \\
& - (c_{1_{p_2}} M'_{1_{p_1}} - c_{1_{p_1}} M'_{1_{p_2}})\, c^1_{2_{y_1}} \\
& - (c_{1_{p_1}} c_{2_{p_2}} - c_{1_{p_2}} c_{2_{p_1}})\, M^{1\prime}_{1_{y_1}} \}\; dy_1
\end{aligned}
$$

where $c_{1_{p_1}} \equiv \dfrac{\partial c^1_1}{\partial p_1} + \dfrac{\partial c^2_1}{\partial p_1}$ and so on, and

$$\Delta \equiv c_{1_{p_1}} c_{2_{p_2}} M'_{1_e} + c_{1_{p_2}} c_{2_e} M'_{1_{p_1}} + c_{1_e} c_{2_{p_1}} M'_{1_{p_2}} - c_{1_{p_2}} c_{2_{p_1}} M'_{1_e}$$

$$- c_{1_e} c_{2_{p_2}} M'_{1_e} - c_{2_e} c_{1_{p_1}} M'_{1_{p_2}}$$

or

$$\frac{de}{dy_1} = a_1 + a_2 c^1_{1_{y_1}} + a_3 c^1_{2_{y_1}} + a_4 M^{1'}_{1_{y_1}} .$$

The effect of a change in income on the demand for domestic money, given p_1, p_2, and e, is captured in the fourth term, which gives the consequent appreciation of the exchange rate. The first term gives the effect of the increase in the supply of good one on the exchange rate operating through the implied changes in p_1 and p_2 and the induced effects on the demands for each money. The second and third terms give the effect on the exchange rate of changes in the demands for each good induced by the change in income represented by the supply shock. These effects operate, as the supply effect does, through changes in relative prices required to clear markets and the induced changes in the demands for each money. If an increase in current income has a negligible effect on the demands for goods one and two (a condition that is more closely approximated in the current model than it would be in a model that was extended to include durable goods) then the second and third terms will be negligible. Note that the magnitude of the exchange rate change, which is associated with the relative price change caused by the supply shock, depends (in a somewhat complicated way) on the elasticities of demand for both goods and moneys.

A similar expression can be derived for the change in the exchange rate due to an increase in y_2. These expressions give the changes in the exchange rate and prices of goods given the other elements of the state vector. They may be regarded as expressions for the change in prices, from one period to the next as a new realization of y is realized, that would occur if the only change in s were the change in y. But this will not generally be the case: aside from changes in money supplies through transfer payments and the extent of government transactions on the foreign exchange market, γ_1^1 and γ_2^1 will generally change over time, reflecting changes in the international distribution of wealth occuring through international capital flows (money in the basic model without bonds). Windfall gains in income will be dissipated slowly over time and will cause s to change over time even in the absence of new shocks (in a manner somewhat analogous to the process described in Dornbusch, 1976d). The terms of trade will change slowly over time and the deviation from purchasing power parity will persist, though diminish, over time. So deviations from purchasing power parity may be autocorrelated over time even when the underlying shocks are serially independent.

The model presented in this paper shows that deviations from purchasing power parity and exchange rate volatility can be explained in an equilibrium framework with strong roots in traditional economic theory of foreign exchange markets (e.g. Machlup and Friedman). That theory also accounts for a correlation between the exchange rate and the terms of trade. In contrast to pure monetary models of the exchange rate, the theory provides a rationale behind the frequently encountered popular statements that appreciation of a currency is related to a fall in the country's import prices and a rise in the foreign price of its exports, and that a balance of trade deficit or the anticipation of a balance of trade deficit may be associated with a currency depreciation. Since changes in relative prices occur partially through changes in exchange rates, people may care about the level of the exchange rate in the sense that they care about the relative price of domestic and foreign export goods. People may blame a relative price change on the exchange rate for the same reason they may blame inflation on whatever good happened to suffer the greatest relative price increase during the inflation.

There appear to be several types of empirical evidence that could be used to discriminate between the equilibrium explanation of exchange rate determination presented in this paper and the disequilibrium explanations that were discussed above, short of estimating an entire general equilibrium structural model. First, the equilibrium theory implies that deviations from purchasing power parity and changes in the terms of trade are essentially real phenomena that will not be systematically related to the exchange rate system (except insofar as different exchange rate systems are associated with different characteristics of monetary policy - e.g. greater variability in the unanticipated component of the money supply might be associated with greater variability of relative prices along the lines of Barro, 1976). Ignoring the distribution effects of money supply changes and the consequent effects on relative prices, a change in the money supply affects the exchange rate only by affecting the general level of nominal variables and cannot reverse the change in real factors that caused the changes in the terms of trade and the exchange rate. Government monetary or exchange rate policy can, therefore, only add a nominally-induced change in the exchange rate to a relative-price-induced change in the exchange rate and hence cannot affect the terms of trade or the deviation from purchasing power parity. In pairs of countries with relatively greater differences in monetary policies and inflation rates, a greater fraction of exchange rate changes will be due to monetary rather than real changes and the correlation between the exchange rate and the terms of trade will be less pronounced, but the terms of trade and the deviations from purchasing power parity will be unaffected. A related implication is that the direction of causation in a simultaneous equations model is primarily from the terms of trade, or factors affecting the terms of trade, to the exchange rate, and not from the exchange rate to the terms of trade. (Again, this ignores the monetary affects on relative prices operating through distribution affects or other channels as in Barro (1976).) Second, the expected rate of change of the exchange rate, as revealed on the forward foreign exchange market (Stockman, 1978) should be related to anticipated changes in the terms of trade or factors associated with the terms of trade as well as to the anticipated inflation differential. (This may explain the widely-discussed role of the 1977 U.S. trade deficit in affecting the performance of the dollar on foreign exchange markets.) Third, applied work on the "real side of international trade" should, according to the equilibrium theory, be able to explain relative prices of goods in international trade without making important reference to monetary variables or to the exchange rate system. The exchange rate should enter such studies only as part of measured relative prices.

Further work on the theory presented here might focus on a more detailed characterization of the properties of the equilibrium exchange rate. Another extension would be to include more goods or introduce information, search, or transportation costs that prevent perfect arbitrage in the markets for each good. Other extensions might involve an explicit consideration of prior contracting in international trade, or the separation of individual consumers and firms that import foreign goods.

If the theory presented here is true, then government foreign exchange market and monetary policy cannot exploit the relationship between the exchange rate and the terms of trade. Government policies should therefore be directed at other goals not discussed in this paper, such as minimizing the amount of noise in the signals carried by market prices. The choice of a pegged versus flexible exchange rate system can then be based upon the classic arguments for each system, such as disciplining the monetary authorities or minimizing adjustment costs (Friedman's "daylight-savings-time" argument), or choosing some rate of inflation that may differ from the foreign rate. A persuasive argument for flexible rates might be to eliminate a constraint on monetary policy in order to pursue an easily predictable monetary policy. Although people may quite rationally care about the level of the exchange rate, its changes are only associated with, not causes of, the relative price changes which are really important.

Footnotes

1. By the purchasing power parity theory I mean a conjectured relationship of proportionality between the exchange rate and a ratio of some domestic and foreign price indexes.

2. Modigliani (1976) has objected to models of exchange rate determination that make small country assumptions or assume that there is only one traded good, making purchasing power parity equivalent to arbitrage in the goods market.

3. This literature is discussed below.

4. Cassel (1914) discussed the role of commercial policies in causing deviations from purchasing power parity. Mussa (1974) examined the effects of commercial policies on the balance of payments, and his argument could be applied to a flexible exchange rate case; the effect he emphasizes is the change in real income and hence the domestic demand for domestic money due to a tariff.

5. For example, the statement that a country's exports will become more costly in real terms if its currency appreciates is simply incorrect from the viewpoint of the monetary approach; the references to the U.S. balance of trade deficit in 1977 in discussions of the depreciation of the dollar are irrelevant from the viewpoint of the monetary approach.

6. The comments were made in discussion at the third annual conference on international economic policy at Wingspread, Wisconsin, July, 1977.

7. Cassell (1922, pp. 147-62) notes that the exchange rate is "an expression for the value set upon a means of procuring foreign commodities."

8. See Mundell (1971, pp. 94-7).

9. See for example Machlup (1939, pp. 154-6) and Krueger (1969, p. 2).

10. See, for example, Sargen (1977) and Magee (1976).

11. See Karplus and Stockman (1978, forthcoming).

12. Wall Street Journal, May 18, 1977 and January 31, 1977.

13. The U.S. import price index calculated by Mitchell and reproduced by Graham is uncorrelated with the U.K. export price index converted into dollars, and the correlation between deviations from purchasing power parity and the ratio of U.S. import and export prices is virtually zero.

14. All data is from <u>International Financial Statistics</u>.

15. The constraint that the coefficients on each variable are the same for both Germany and the U.S. was tested and easily not rejected. Allowing different coefficients results in a coefficient on d ln TT of -.29 with a standard error of .16 (t-statistic of -1.84). Similarly, adding seasonal dummies, which turn out to be jointly zero, makes virtually no difference to the results. If the coefficient on the rate of change of the money supply ratio is constrained to be unity on the

grounds that it differs from unity only because of measurement error in monthly money supply data, the residuals become autocorrelated. After quasi-differencing with an estimated first order autocorrelation coefficient of -.32, the coefficient on d ln TT becomes -.38 with a standard error of .21.

16. The utility functions are assumed to be continuously differentiable increasing strictly concave bounded functions from R_+^2 to R_+.

17. See Grassman (1973)

18. If there are freely tradeable international bonds with permissible short-selling then portfolio risks of currencies are irrelevant for money demands because the risk of each currency can be bought and sold in the bond market thereby separating the decision to hold the money from the decision to hold the risk of the money. Risk elements then only affect bond holdings, not money holdings. See Fama (1977) and Stockman (1978).

19. A more general formulation, used by Grandmont and Younes, is to allow some fraction of current income or other assets, as well as initial money holdings, to be spendt in the current period.

20. An alternative would be to put money directly into the individuals' utility functions. But as Lucas (1972) has argued,

> "All motives for holding money require that it be held for a positive time interval before being spent: there is no reason to use money (as opposed to barter) if it is to be received for goods and then instantaneously exchanged for other goods. There is also the question of whether money 'yields utility.' Certainly the answer in this context is yes, in the sense that if one imposes on an individual the constraint that he cannot hold cash, his utility under an optimal policy is lower than if the constraint were removed. It should be equally clear, however, that this argument does not imply that real or nominal balances should be included as an argument in the individual preference functions. The distinction is the familiar one between the utility function and the value of this function under a particular set of choices."

Of course, one can always derive an indirect utility function that includes money, but by starting from a more direct utility function, we gain restrictions on the indirect utility function that are otherwise lost and that permit investigation of issues that otherwise would be precluded from the analysis.

21. I assume there are no specialists who import foreign goods and resell them domestically. Import specialists would purchase non-storable goods from the producers with foreign money and resell them at home for domestic money, then take the domestic money to the foreign exchange market to acquire foreign money for next period's imports.

22. See Lucas (1977).

23. One set of sufficient conditions for no relative price changes is that the units of good one with differing characteristics are both demanded and supplied in fixed proportions, with each individual producing only

one type of the good. If the number of different types of the good is large then the proportion of a person's consumption that could be obtained from his own production is small.

24. Blackwell (1965)

Aliber, Robert Z. 1976 The Firm under Pegged and Flexibel Exchange Rates. Scandinavian Journal of Economics vol. 78 no. 2

Balassa, Bela. 1964 The Purchasing Power Parity Doctrine: A Reappraisal. Journal of Political Economy, vol 72

Barro, Robert J. 1977. A Simple Flexible Exchange Rate Model with Uncertainty and Rational Expectations. Quarterly Journal of Economics

Bresciani-Turroni, C. 1961 The Purchasing Power Parity Doctrine. reprinted in Saggi di Economia, Milano: Dott. A. Giuffre.

Cassel, Gustav. 1922. Money and Foreign Exchange After 1914. New York: MacMillan

1928. Postwar Monetary Stabilization. New York: Columbia University Press

Clower, Robert 1967. A Reconsideration of the Microfoundations of Monetary Theory. Western Economic Journal. vol. 6.

Da Silva Adroaldo Moura. 1977. The Basis of Minidevaluation Policy. Explorations in Economic Research V0l. 4 no. 1.

Dornbusch, Rudiger. 1976. The Theory of Flexible Exchange Rate Regimes and Macroeconomic Policy. Scandinavian Journal of Economics vol 78 no. 2 (a)

1976. Exchange Rate Dynamics. Journal of Political Economy vol 84.(b)

1976. Capital Mobility, Flexible Exchange Rates, and Macroeconomic Equilibrium. Recent Issues in International Monetary Economics. Claasen and Salin (eds.) Amsterdam: North-Holland (c)

1976. Exchange Rate Expectations and Monetary Policy. Journal of International Economics vol. 6. (d)

Dornbusch, Rudiger, and Krugman, Paul. 1977. Flexible Exchange Rates in the Short Run. Brookings Papers on Economic Activity

Dreze, J. and Modigliani, Franco. 1972. Consumption Decisions under Uncertainty. Journal of Economic Theory no. 5.

Fama, Eugene, and Farber, André. 1977. Money, Exchange Rates, and Capital Market Equilibrium. University of Chicago working paper.

Frenkel, Jacob. 1976. A Monetary Approach to the Exchange Rate. Scandinavian Journal of Economics vol. 78 no. 2.

Frenkel, Jacob, and Johnson, H.G. 1976. The Monetary Approach to the Balance of Payments. Toronto: Toronto University Press

Friedman, Milton. 1953 The Case for Flexible Exchange Rates, in Essays in Positive Economics. Chicago: University of Chicago Press

Friedman, Milton, and Schwartz, Anna. 1963. A Monetary History of the United States, 1867-1960. Princeton: Princeton University Press.

Gailliot, Henry J. 1970. Purchasing Power Parity as an Explanation of Long-Term Changes in Exchange Rates. Journal of Money, Credit, and Banking.

Girton, Lance, and Roper, David. 1977. A Monetary Model of Exchange Market Pressure applied to the Postwar Canadian Experience. American Economic Review.

Graham, Frank. 1922. International Trade under Depreciated Paper, the United States, 1862-1879. Quarterly Journal of Economics vol. XXXVI

Grandmont, J. M. and Younes, Y. 1972. On the Role of Money and the Existence of a Monetary Equilibrium. Review of Economic Studies, vol. 39.

1973. On the Efficiency of a Monetary Equilibrium. Review of Economic Studies. vol. 40.

Grassman, S. 1973. A Fundamental Symmetry in International Payment Patterns. Journal of International Economics, vol. 3, no. 2.

Hodgson, J.S. 1967. An Analysis of Floating Exchange Rates: the Dollar-Sterling Rate, 1919-25. Southern Economic Journal. vol. 39.

Holmes, J. 1967. The Purchasing Power Parity Theory: In Defense of Gustav Cassel as a Modern Theorist. Journal of Political Economy.

Isard, Peter. 1977. How Far Can We Push the "Law of One Price"? American Economic Review.

Karplus, Richard, and Stockman, Alan. 1978. Empirical Evidence on Alternative Theories of the Exchange Rate.(forthcoming)

Kemp, M. 1969. The Pure Theory of International Trade and Investment Englewood Cliffs: Prentice-Hall.

Keynes, J.M. 1930. A Treatise on Money.

Krueger, Anne. 1969. Balance of Payments Theory. Journal of Economic Literature.

Lucas, Robert. E., Jr. 1972. Expectations and the Neutrality of Money. Journal of Economic Theory. no. 5.

1976. Econometric Policy Evaluation: A Critique. The Phillips Curve and Labor Markets, Brunner, Karl, and Meltzer, Allan (eds.). Amsterdam: North-Holland.

Barro, Robert J. 1976. Rational Expectations and the Role of Monetary Policy. _Journal of Monetary Economics_ vol 2 no. 1.

Blackwell, David. 1965. Discounted Dynamic Programming. _Annals of Mathematical Statistics_ vol. 36 no. 1.

Frenkel, Jacob. 1978. The Purchasing Power Parity: Doctrinal Perspective and Evidence from the 1920's. _Journal of International Economics_ vol. 8 no. 2.

Mikesell, Raymond F., and Furth, J. Herbert. 1974. _Foreign Dollar Balances and the International Role of the Dollar_. New York: National Bureau of Economic Research (Columbia University Press).