

ALTERNATIVE SIGNALLING  
EQUILIBRIUM CONCEPTS\*

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Among many recent developments in the economics of information, none has generated more controversy than the concept of "market signalling" introduced by Spence.

If buyers are less well-informed about product quality than sellers, and no additional information is available, market clearing prices must reflect some weighted average of product quality. Then if potential sellers of the highest quality products have the greatest opportunity costs and these costs exceed price, they will not enter the market.

This "adverse selection" phenomenon is, however, offset if sellers of higher quality products can adopt activities that operate as a "signal" to potential buyers. Intuitively, an activity is a potential signal if entering into it has a lower marginal cost for sellers of higher quality products. For example, the fly-by-night operator faces a higher advertizing cost per unit of sales than the new entrant who plans to build and then maintain his reputation. Or, in the labor market, productivity on the job is positively correlated with performance in school. Therefore, the higher productivity worker has on average a lower personal cost of obtaining a given set of educational credentials. Similarly, in purchasing insurance the marginal cost of accepting a higher coinsurance rate for a specific loss is lower for those with a lower probability of loss.

But on closer inspection, it turns out that "informational equilibria" -- that is, equilibria in which signalling is needed to distinguish product quality -- do not have the stability properties of classical Walrasian equilibria. In papers by Rothschild and Stiglitz (1976) and Riley (1975, 1977) it has been shown that unless the difference between quality levels is sufficiently large there is, for every set of signal-price pairs  $S$ , some alternative  $s' \neq S$

which, if offered by a single buyer would make the latter better off. That is, there is no Cournot-Nash signalling equilibrium.

One possible inference that might be drawn from this result is that signalling could not be an important phenomenon in a competitive economy. After all if any attempt at signalling were to result in interference, in the form of competitive responses, there would be no opportunity for individuals to identify the correlation between the level of the signal and the underlying quality of the product.

However this is too simplistic since the interference cannot take place until *after* signalling has been established. It is therefore more appropriate to suppose that an economy is initially in a state of informational equilibrium with signals reflecting product quality, and to ask whether the potential instability is likely to lead to a general collapse of the equilibrium.

Following the approach adopted by Wilson (1977) there have been several related attempts to introduce quasi-dynamic considerations in which every agent takes into account the *reaction* by other agents when contemplating a "defection" from the initial set of signal-price pairs  $S$ . It is then argued that the set  $S$  forms a stable equilibrium if, for every alternative  $s'$  that would make a single defector better off, this same signal-price pair would make the defector worse off after the reaction by other agents.

Where these attempts differ is in the assumptions made about the type of reactions that potential defectors anticipate. In this paper these differences are highlighted by considering a simple model in which there are two classes of agents who are not directly distinguishable. For concreteness the discussion is placed in the insurance context. A simple relabelling converts the model into one of educational signalling.

## I. A Simple Insurance Model

Consider an economy in which all agents face some risk of incurring a loss of  $L$  dollars. Agents are identical<sup>1</sup> except in the probability of loss  $\pi$ , which may take on the values  $\pi_h$  or  $\pi_\ell$  ( $\pi_h > \pi_\ell$ ). With costless information about  $\pi$  and a competitive insurance industry offering fair policies, all consumers would obtain full coverage. However with  $\pi$  unobservable insurance companies offer policies which include a coinsurance rate  $y$ . Then if a loss occurs a company pays out only  $(1-y)L$  dollars.

Associated with any coinsurance rate  $\hat{y}$  is a payout-premium ratio  $\hat{R}$ . Suppose only consumers in the  $i$ th risk class purchase the policy  $\hat{s} = \langle \hat{y}, \hat{R} \rangle$ . The expected payout divided by the premium is then  $\pi_i \hat{R}$ . Assuming that entry into the insurance industry continues until expected profit is zero, that is expected payout equals premium revenue, we require  $\pi_i \hat{R} = 1$ .

More generally, if  $\bar{\pi}$  is the average probability of loss for those purchasing  $\hat{s}$ , the zero expected profit condition becomes

$$\bar{\pi} \hat{R} = 1 \text{ or } \hat{R} = 1/\bar{\pi}$$

In general companies will offer a different payout-premium ratio  $R$  for a different coinsurance rate  $y$ . Then each expected utility maximizing consumer selects a level of  $y$  yielding the solution of

$$\text{Max}_y U(\pi; y, R(y)) = (1-\pi)u(I-p(y)) + \pi u(I-yL-p(y))$$

$$\text{where } p(y) = (1-y)L/R(y).$$

Note that  $yL$  is the level of coinsurance so  $(1-y)L$  is the insurance coverage and  $p(y) = (1-y)L/R(y)$  is the insurance premium (assumed paid whether or not a loss occurs). Indifference curves in  $\langle y, R \rangle$  space are depicted in Figure 1 for the two risk classes. There are two important aspects of these curves. First, for every policy  $s = \langle y, R \rangle$  the slope of the indifference curve for the high risk

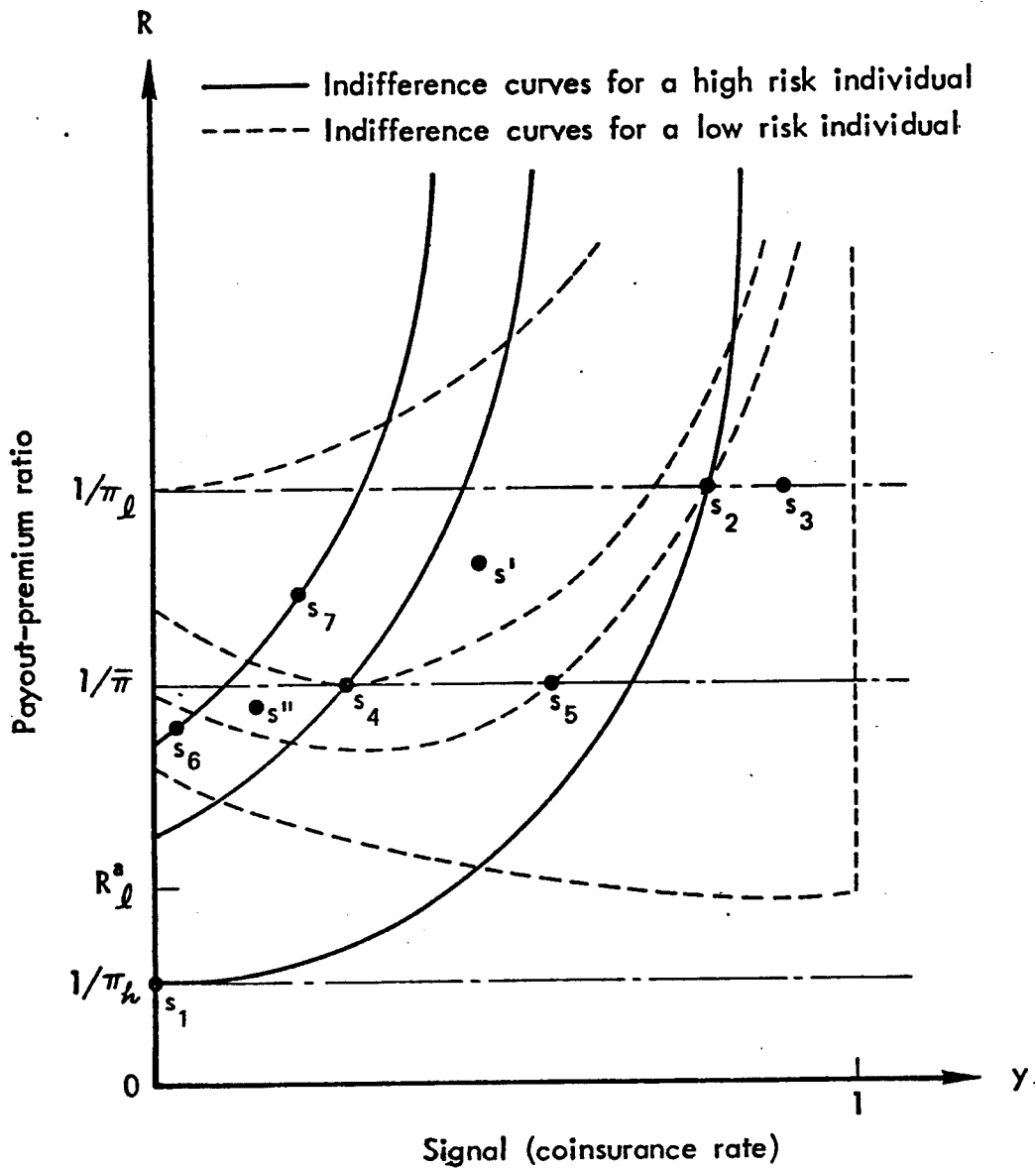


Fig. 1— Preferences of the two risk classes

class ( $\pi = \pi_h$ ) is greater. That is, the increase in the payout-premium ratio necessary to maintain expected utility in the face of a higher coinsurance rate is always greater. Second, suppose a consumer is offered a fixed payout-premium ratio  $\hat{R}$  but is free to select his coinsurance rate. Then if the coinsurance rate is 'unfair' that is  $\hat{R} < 1/\pi$ , the consumer will prefer some coinsurance<sup>2</sup>. The intuition is straightforward. With fair insurance a consumer will shed all risk. However a decline in the payout-premium ratio raises the cost of each unit of coverage so less than full coverage is preferred. At some point the payout-premium ratio drops so low that no insurance is purchased ( $y = 1$ ).

Consider for example the indifference curves of the low risk class in Figure 1. For  $R > 1/\pi_\ell$  the indifference curves are upward sloping everywhere so, holding  $R$  constant, the most preferred coinsurance rate is zero. For  $R < 1/\pi_\ell$  the indifference curves are first downward sloping so, holding  $R$  constant, some coinsurance is preferred. Finally for  $R \leq R_\ell^a$  the payout-premium ratio is so low that the low risk class prefer to purchase no insurance ( $y = 1$ )

## II. Alternative Equilibrium Concepts

Spence in his initial analysis (1973, 1974) describes a set of contracts  $S$  as an equilibrium set if, when agents select freely among these contracts, each such contract breaks even.

Here we shall call such a set *informationally consistent* if the different risk classes accept different contracts and *weakly informationally consistent* if one or more contract attracts both classes<sup>3</sup>. It is easy to see that there is a whole family of informationally consistent sets of contracts. For example in Figure 1  $\{s_1, s_2\}$  and  $\{s_1, s_3\}$  are both informationally consistent. In each case the high risk class purchases full coverage and the payout-premium

ratio  $(1/\pi_h)$  generates zero expected profits. The low risk class signal by purchasing a policy with a positive coinsurance rate. Another informationally consistent set of policies is the one element set  $\{s_1\}$ . Once again the high risk class purchases full coverage but now the low risk class are better off without any insurance. This is an illustration of Akerlof's adverse selection equilibrium with only the 'lemons' left in the insurance market.

There is also a family of weakly informationally consistent policies of which  $s_4$  and  $s_5$  are members. In each case the single policy attracts both risk classes and the zero expected profit condition is satisfied ( $R = 1/\bar{\pi}$ ).

It therefore appears as though multiple equilibria are the rule rather than the exception in a world of informational asymmetry. However when the stability of these equilibria are examined the problem is not whether there are too many, but whether there are any! To be precise, unless differences between risk classes are sufficiently large, none of the 'informational equilibria' satisfy the requirements of a Cournot-Nash equilibrium.

To see this consider first the weakly informationally consistent policy  $s_4$ . If a new firm enters and offers the alternative policy  $s' = \langle y', R' \rangle$  it attracts only the low risk class. Since the payout premium ratio  $R'$  is less than  $1/\pi_l$  the new policy is profitable.

Next consider a set of contracts that are informationally consistent, for example,  $S = \{s_1, s_2\}$ . If a firm defects from this set offering the alternative policy  $s'' = \langle y'', R'' \rangle$  it attracts both risk classes and yields expected profits since  $R''$  is less than  $1/\bar{\pi}$ .

However note that while the defector offers  $s''$  it is always possible for a second insurance company to react with an offer such as  $s'$ . The defector, instead of making expected profits finds that the low risk types have been skimmed off by the reactor. The latter now makes profits while the defector

loses money. Note furthermore that since the reactor makes profits on all those accepting his policy the worst that could result from additional policy offerings by other firms is that these profits are eliminated. The reactor therefore stands only to gain from his new offering.

But if each agent recognizes that these opportunities for profitable reaction exist it seems reasonable that they will effectively deter a contemplated defection. Elsewhere I have shown, in a more general context, that under the Spencian assumptions there is always a unique "Reactive Equilibrium" in which every potential additional policy is open to this threat of reaction. For the two class case depicted in Figure 1 this is the pair  $S_R = \{s_1, s_2\}$ . Note that any pair of informationally consistent policies must lie respectively on the horizontal lines  $R = 1/\pi_\ell$ ,  $R = 1/\pi_h$ . Policy  $s_1$  is therefore best for the high risk group. Given  $s_1$ , the best policy for the low risk class that distinguishes the two classes is  $s_2$ . Thus the set  $S_R$  is a Pareto optimal informationally consistent set of policies.

However returning to Figure 1 it can be seen that the single policy  $s_4 = \langle y_4, R_4 \rangle$  is strictly preferred to the reactive equilibrium set of policies by both risk classes. Moreover  $s_4$  lies on the horizontal line  $R = 1/\bar{\pi}$  so it is a weakly informationally consistent policy. Since any other policy on this horizontal line must yield a lower expected utility to at least one risk class  $s_4$  is Pareto Optimal among the set of weakly informationally consistent policies.

Wilson (1977) has shown that if firms respond to a defection from some set of policies  $S$  by dropping just enough policies so that those remaining at least break even, the resulting equilibrium is unique and is Pareto Optimal among weakly informationally consistent sets of policies. In the simple 2 class model examined here this equilibrium is the single policy  $s_4$ .



As we have already seen, if insurance companies begin only offering  $s_4$  it is profitable for one company to drop  $s_4$  and offer instead a policy like  $s'$ . This skims off the profitable lower risk class and generates expected losses for all the other insurance companies. But if the latter all respond by dropping their initial policy,  $s'$  will be chosen by both risk classes. Since  $R' > 1/\bar{\pi}$  the defector now has expected losses.<sup>4</sup>

Wilson also considered, but did not pursue the possibility that insurance companies might subsidize one kind of policy with the profits from another policy. This idea has since been followed up by Miyazaki for the two class case and developed more fully by Spence (1977).

For our illustrative model it can be shown that there are policy pairs such as  $\{s_6, s_7\}$  in Figure 1 with the low risk class subsidizing the high risk class and the two policies breaking even in the aggregate. It can be seen that both risk classes would prefer the pair  $\{s_6, s_7\}$  over  $s_4$ . Spence has shown that for a general  $n$ -class model, where firms react by dropping loss making policies, there is a unique equilibrium which is weakly informationally consistent in the aggregate. He has also established that this equilibrium is Pareto optimal among all sets of contracts satisfying aggregate consistency.

In Figure 1 the difference between the two risk classes is such that all three equilibria are distinct. This is not necessarily the case. Indeed if differences among risk classes are sufficiently large the three equilibria will coincide. However the converse is also true. Whenever these differences are sufficiently small the equilibrium achieved is necessarily sensitive to the assumption made about how firms will respond to a defection. Since the three equilibria are Pareto optimal over successively larger sets of insurance policies, the equilibria are themselves Pareto-ranked. It is therefore of considerable

importance to focus more closely on the differences in assumptions, in order to better understand how well the marketplace allocates resources when there is informational asymmetry. My own admittedly interested view is that the more favorable the equilibrium in the Pareto sense the harder it is to justify it as being 'competitive.'

For example, in the Miyasaki-Spence equilibrium insurance companies offer a menu of policies in which the higher risk classes are subsidized by the lower risk classes. It is therefore possible for an insurance company to make money simply by turning away some applicants for the policies accepted by the higher risk classes. Apart from the difficulty of detecting this form of rationing I find it hard to swallow the assumption that *all* other firms would respond with a threat to drop policies.

In the equilibrium proposed by Wilson each insurance policy generates zero expected profits so any 'cheating' must take the form of an announcement of a new type of policy. The detection issue is therefore not so serious. However there remains a distinct flavor of collusiveness about the envisaged response. For example in the two class model all insurance companies respond to any defection by dropping out of the insurance business. More generally, *all* companies drop policies until those remaining at least break even.

In contrast the 'reactive equilibrium' relies on a threat not from the market participants as a whole but from only one other firm (reactions by more than one firm only serve to strengthen the equilibrium). The key assumption is that every firm believes that at least one competitor is maintaining a close watch on its 'product line' and will exploit any sure gain.

### III. Concluding Remarks

In the Walrasian or Arrow-Debreu equilibrium under uncertainty traders sign contracts which are contingent upon the eventual state of the world. The central point of this paper is that serious difficulties arise when an attempt is made to extend the Walrasian equilibrium to a world of uncertainty without markets for every contingency.

It is important to recognize that the concept of an informationally consistent set of contracts, as first suggested by Spence, is a natural extension of the Walrasian approach. Only the consumer initially knows the probability of loss and hence the value of the risk that he is trading to an insurance company. Insurance policies are then characterized (labeled) by the level of the signal (the coinsurance rate)  $y$ . Each consumer, facing the same set of parametric payouts per dollar of premium  $R(y)$ , chooses to trade that risk which maximizes his own expected utility. On the other side of the market insurance companies, acting as price takers, have beliefs  $\tilde{\pi}(y)$  as to the probability of loss associated with each level of  $y$ . Then if the set of policies is informationally consistent these beliefs are correct and expected profits are zero ( $\tilde{\pi}(y)R(y) = 1$  for all  $y$ ).

The recent research summarized above has demonstrated that informational consistency is not sufficient to eliminate opportunities for potential gain. Indeed in general there is no Cournot-Nash equilibrium. However it has been argued that the implied potential instability is not, after all, devastating. Instead, by building into the equilibrium concept a recognition of possible reactions by other agents, stability is achieved. The transfer of information via markets can therefore be explained as a non-cooperative equilibrium phenomenon.

It will be of considerable interest to see whether the new equilibrium concepts also prove useful in other cases where the Cournot-Nash approach fails.

## FOOTNOTES

1. With observable differences the only change in the conclusions is that the results are contingent upon an agent being in some known class. Adding unobservable differences generates noise but again does not change the conclusions.
2. The first statement follows directly from the concavity of  $u(\cdot)$ . To derive the second statement note that if  $R(y) = \hat{R}$

$$\frac{dU}{dy} \Big|_{y=0} = (1-\pi) \frac{L}{\hat{R}} u' \left( I - \frac{L}{\hat{R}} \right) - \pi L \left( 1 - \frac{1}{\hat{R}} \right) u' \left( I - \frac{L}{\hat{R}} \right)$$

$$> 0 \text{ if and only if } \hat{R} < \frac{1}{\pi}$$

3. Whenever the high risk class is indifferent between two insurance policies we assume that it selects the lower coinsurance rate. This is a natural assumption since any significant amount of switching would lower the break-even payout-premium ratio on the policy with higher coinsurance and so make it strictly less desirable to the high risk class.
4. H. Grossman (1977) has pointed out that, in principle, Wilson's equilibrium is achievable via the reactions of *buyers* rather than *sellers* of insurance. When the new policy  $s'$  is offered, the high risk class recognize that the departure of the low risk class will force insurance companies to lower the initial payout-premium ratio to  $1/\pi_h$ . The new policy  $s'$  is therefore preferred by both risk classes.

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