A Theory of Exchange Rate Determination *

Alan C. Stockman

University of California, Los Angeles

June, 1978

Discussion Paper No. 122

June 1978

# I. INTRODUCTION

The recent behavior of exchange rates in a flexible rate system has puzzled many economists. Exchange rates have exhibited considerable volatility and together with prices have failed to conform to the predictions of the purchasing power parity theory; frequently exchange rate changes have failed to resemble contemporaneous changes in relative price levels in either magnitude or direction. Exchange rates and their rates of change over time have been more volatile than relative price levels and rates of inflation.[1] These features of exchange rate behavior have been regarded as anomalous to existing theories. Moreover, they have often been regarded as inconsistent with equilibrium, and several disequilibrium interpretations of this anomalous behavior have been suggested in the literature. These disequilibrium interpretations have resulted in many unfavorable reconsiderations of the case for flexible exchange rates.

This paper proposes an alternative explanation of exchange rate behavior. It seems desirable to try to account for observed exchange rate behavior with an equilibrium explanation before resorting to a disequilibrium explanation. The equilibrium explanation presented in this paper involves the simultaneous determination of exchange rates and relative prices of different goods in international trade in an intertemporal framework with uncertainty and rational expectations. Relative price changes caused by real disturbances play an important role in the explanation.[2] These relative price changes were emphasized in the traditional literature on exchange rates but have been neglected in the recent exchange rate literature associated with the monetary approach. This paper emphasizes relative price changes and integrates the important issues discussed by the traditional "elasticity theorists" into a full general equilibrium framework. By examining exchange rate determination in a model

that includes relative price changes, this paper clearly establishes the distinction between the issues involved in international arbitrage of goods prices and the issues involved in the determination of exchange rates.

The model results in exchange rate behavior similar to that which is actually observed—exchange rates can be represented by stochastic processes with rather complicated properties: they may be highly volatile and will exhibit autocorrelated deviations from purchasing power parity. These results hold even though prices freely adjust to clear markets. International capital flows will also be volatile, reflecting changes in transitory incomes and expected future prices.

Second, the model explains why exchange rate changes are associated with terms-of-trade changes and yet why <u>the relationship between the exchange rate and the terms of trade cannot be exploited by government exchange rate policies</u>, that is, why government policies designed to influence the terms of trade or the balance of trade by actions directly affecting the exchange rate cannot succeed, even though the exchange rate and the terms of trade may be statistically related. Government commercial policies such as tariffs or quotas can, however, effect the exchange rate by changing the terms of trade.[3] Third, the model shows why exchange rate fluctuations can easily be misinterpreted as sources of additional uncertainty in an economy and as causes of relative price changes among goods in international trade. Exchange rate changes will often appear to cause relative price changes and generate additional uncertainty even when all markets are in equilibrium. The source of misunderstanding is related to the common fallacy that relative price changes cause inflation: just as people may call an increase in the price of food the cause of inflation when it is actually just one way in which inflation occurs, so people may call a change in the exchange rate a cause of some relative price change when the exchange rate change is actually just one

of the ways in which the relative price change occurs.

The model shows how a change in the terms of trade caused by some relative supply or demand shift is divided between nominal price changes in each country and an exchange rate change. If $p_1$ is the price of the export good of country one in terms of its own money and $p_2$ is the price of the export good of country two in terms of country two's money, then the relative price of country one's exports in terms of country two's exports is $p_1/ep_2$, where e is the money one price of money two. When relative supplies or demands change the consequent change in $p_1/ep_2$ generally occurs through changes in each of $p_1$, $p_2$, and e. This creates a correlation between the exchange rate and the terms of trade. The greater the changes in the terms of trade and the larger the role of changes in the exchange rate in effecting these terms of trade changes, the greater the variability of exchange rates. The more persistent the shifts in the supplies or demands for goods, the more persistent the deviations from purchasing power parity. The division of terms of trade changes between changes in nominal prices and changes in the exchange rate depends upon the elasticities of supply and demand that were emphasized in the traditional exchange rate theories. In this sense the model resembles the traditional elasticities approach to exchange rates. But the division of the terms of trade changes into nominal price and exchange rate changes also depends upon the time series properties of exogenous variables through their effects on expectations of future exchange rates and future prices and upon other factors influencing asset demands. In this sense the model resembles the monetary or asset approach to exchange rates. The model can therefore be described as a synthesis of the elasticity and monetary approaches.

Besides rationalizing exchange rate volatility and autocorrelated deviations from purchasing power parity, the model has several other implications.

The exchange rate will be correlated with the terms of trade and this correlation will be greater for countries with more homogeneous monetary policies. These implications are consistent with the evidence presented by Dornbusch and Drugman (1976). On the other hand, the model predicts that exchange rate changes caused by monetary factors do not affect the terms of trade.[4] The model implies that deviations from purchasing power parity and changes in the terms of trade have roughly the same character-istics and bear approximately the same relationship to each other under both fixed and flexible exchange rate systems. The model is also useful for interpreting statements by bankers and foreign exchange dealers that, when analyzed with a straightforward monetary approach, appear to be either incorrect or beside-the-point.[5] Moreover, the model is consistent with the application of the standard tools of price theory to examinations of the demands and supplies of goods in international trade, in contrast to dis-equilibrium models which imply that prices of internationally traded goods do not adjust to clear markets but are instead systematically related to monetary policies.

Finally, Fischer[6] has argued that because in simple monetary models the exchange rate is unconnected with relative prices or with the real allocation of resources, the role played by the exchange rate in those models is inconsistent with the concern actually expressed by people about the level of the exchange rate. This paper explains why rational people may care about the level of the exchange rate--because for given levels of the exogenous variables such as money supplies, the equilibrium exchange rate is an indicator of relative prices.

## II. PURCHASING POWER PARITY

Much of the progress in empirically explaining exchange rate behavior was achieved by noting that large changes in exchange rates are generally associated with different rates of inflation in the countries concerned. A full model of the foreign exchange market is not required for the inference that a change in the stock of money will, other things the same, be associated with a corresponding increase in all nominal prices including the nominal price of foreign exchange. This result is guaranteed by the zero-degree homogeneity of demands and supplies with respect to all nominal prices and is not tied to a specific model. The purchasing power parity hypothesis, which states that there is a proportional relationship between the exchange rate and a ratio of foreign and domestic prices or price indexes, can be thought of as stating that other things are approximately the same; purchasing power parity is therefore a conjectured relationship between several endogenous variables and is not in itself a theory of exchange rate determination.[7] The accuracy of the purchasing power parity hypothesis is then independent of the accuracy of any particular theory of exchange rate determination, and the hypothesis can be rephrased as stating that most changes in exchange rates are due to nominal shocks (defined in some particular way) and that very few are due to real shocks (again defined in some particular way). If this is true, then the accuracy of the purchasing power parity relation is largely independent of the particular price index or monetary aggregate used in calculations.

Tables I and II summarize the relationship between exchange rates and ratios of price indexes for several countries. The results in Table I are taken from Gailliot's (1970) calculations and show that the purchasing power parity relation held fairly well for the countries examined over a sixty year

period. Table II shows plots of the ratios of the consumer price indexes of several countries to the United States consumer price index, using monthly data, and the corresponding exchange rates each month. Deviations from purchasing power parity appear to have two characteristics. First, the deviations often persist over time in one direction. Second, exchange rates and their rates of change vary more than ratios of price indexes and their rates of change. Neither phenomenon is unique to the data sets in the tables For example, Friedman and Schwartz (1963, p. 64) note that for the greenback period of 1861-79, the US-UK exchange rate varied by about 2 to 1, while the ratio of price levels varied by only about 1.3 to 1. They offer the judgment that "To some extent, this residual variation may reflect the crudeness of our calculations of purchasing power parity. It seems most unlikely, however, that it can be wholly accounted for by such statistical errors."

Monetary models of the exchange rate supplement the purchasing power parity relation with money demand functions and equilibrium conditions in the money markets. The exchange rate can then be expressed solely as a function of nominal money supplies and the variables assumed to affect money demands. The equation for the exchange rate resulting from the basic monetary model is

$$d \ln e \; = \; d \ln (M^s/M^{*s}) \; - \; d \ln (m^d/m^{*d})$$

where the exchange rate $e$ is the domestic price of foreign money, $M^s$ and $M^{*s}$ are domestic and foreign nominal money supplies, and $m^d$ and $m^{*d}$ are the demands for real balances of domestic and foreign moneys, typically taken to be functions of real income and nominal interest rates. Upon substituting the assumed money demand functions into the above expression, the exchange rate is a function of both real incomes and nominal interest rates, and estimates of the equation should recover estimates of the parameters of the

money demand functions. The success of the monetary models in explaining actual exchange rate behavior has been, perhaps not surprisingly, similar to the success of purchasing power parity. There remain substantial short-run variations in exchange rates unexplained by the monetary models. Moreover, while the monetary models avoid possible measurement error in price indexes used for calculating purchasing power parity, the models face the problems of errors in the money supply and income data and of the correct specification of the money demand functions. These problems, which are also present whenever money demand functions are estimated with monthly or quarterly data, may explain why the estimated elasticity of the exchange rate with respect to the money supply ratio often differs substantially from unity and the income and interest rate coefficients fail to resemble "reasonable" estimates of income and interest elasticities of the demand for money.

If exchange rate movements cannot be fully accounted for by general movements in the level of all nominal variables, that is, if real changes are important in influencing exchange rate behavior, then purchasing power parity calculations with different price indexes may yield somewhat different results. Friedman and Schwartz (1963, p. 62, n. 66) argue that the price indexes used should be chosen with the criterion that the resulting number reflects the value that the exchange rate would take if it were affected only by nominal shocks and not by real shocks. For example, on the grounds that productivity growth should be classified as a real change, Friedman and Schwartz suggest that factor price indexes are preferred to product price indexes for purchasing power parity calculations because the ratio of factor price indexes may be roughly constant even though the product price index in the country with greater productivity growth falls, changing the purchasing power parity calculated with product price indexes. A change

in the exchange rate induced by this real shock would then show up as a deviation from purchasing power parity, as Friedman and Schwartz argue that it should. Cassel (1922, p. 144) also suggested using factor price indexes for purchasing power parity calculations. Alternatively, Friedman and Schwartz argue that prices of exports but not prices of imports should be included in calculating purchasing power parity, again to make it an index of what the exchange rate would be if it were only influenced by nominal shocks. The choice of appropriate price indexes clearly reflects the classification of shocks into real and nominal components and the presumptions one has about what real shocks might have important influences on exchange rates; an investigation of how various real shocks affect exchange rates requires a theory of exchange rate determination.


III.  THE TERMS OF TRADE

Deviations from purchasing power parity (whatever prices or price indexes are used) always involve relative price changes. In the explanations of exchange rate fluctuations proposed by Dornbusch (1976a, 1976b) and Mussa (1976b) the prices of goods available to people in one country change relative to prices of those same goods in another country because domestic nominal prices are temporarily fixed in each country and a monetary shock causes a change in the exchange rate. A nominal shock therefore causes a disequilibrium change in relative prices in those models. Other economists (e.g. Balassa, 1964) have emphasized changes in the relative prices of traded and nontraded goods. But the relative price change that was emphasized most in the traditional literature on foreign exchange markets was the terms of trade.[8] Krueger (1969) noted that the traditional

theory viewed the terms of trade as "the key variable," and the terms of trade also plays an important role in the explanation presented by Friedman and Schwartz (1963) of deviations from purchasing power parity during the U.S. greenback era, from the Civil War to 1879. During the Civil War U.S. cotton exports were cut off, resulting in a rise in the price of gold (foreign exchange) relative to purchasing power parity by 20 percent and affecting the terms of trade (pp. 66-67, 75). After the Civil War, as the supply of goods for export rose again and reduced the terms of trade, the domestic currency appreciated from about 20 percent below purchasing power parity to about 10 percent above purchasing power parity (p. 76). Later movements in the exchange rate may also have been related to changes in the terms of trade (pp. 77-78). According to the price indexes reported in Graham (1922), the simple correlation coefficient between the log deviation from purchasing power parity (measured with general price indexes in the U.S. and the U.K. and with the greenback price of gold, to which the pound sterling was pegged) and the log terms of trade (measured with export price indexes converted at the current exchange rate) is -.68, calculated with thirteen annual observations from 1866 through 1878. The sign indicates that currency depreciations are associated with increases in the relative price of a country's exports.[9]

The simple correlation coefficients between the monthly percentage changes in the exchange rate with the dollar and the monthly percentage change in the terms of trade (measured as the ratio of the domestic export price index divided by the import price index to the U.S. export price index divided by the U.S. import price index) from January, 1974, through July, 1977 are -.29 for the Canadian dollar, -.16 for the French franc, -.33 for the Deutschemark, -.15 for the lira, .21 for the yen, and -.24

for the guilder.[10] These correlations measure only the contemporaneous monthly relation between the exchange rate and the terms of trade and even these are likely to be biased toward zero by measurement error in the export and import price indexes. Dornbusch and Krugman have also presented evidence of this correlation, and Isard (1977) has presented evidence that the exchange rate is correlated with changes in the terms of trade of even disaggregated categories of goods.[11]

The important role played by the terms of trade is clear: the interpretation is not. In order to examine the equilibrium relationship between the exchange rate and the terms of trade, the next section presents a model in which both are endogenous.

## IV. A MODEL

### Overview and Individual Optimization Problems

Consider a world with two countries, two goods, and two moneys. People in country one produce only good one but consume both goods one and two; people in country two produce only good two but consume both goods. Thus there is complete specialization in production and trade occurs so that people can consume both goods.

Let "individual one" be a representative individual in country one. He maximizes the quantity

$$E[\sum_{t=0}^{\infty} \beta^t U^1 (c_{1t}^1, c_{2t}^1)] \tag{1}$$

where $\{c_1^1, c_2^1\}_t$ is a bivariate stochastic process representing individual one's consumption of goods one and two, $U^1(\cdot)$ is the current period utility function of individual one, $\beta \in (0, 1)$ is a discount term, and E

is an expected value operator. The constraints on the behavior of individual one are discussed below.

Similarly there is a representative individual in country two who maximizes the quantity

$$E \left[ \sum_{t=0}^{\infty} \beta^t U^2 (c_{1t}^2, c_{2t}^2) \right]$$ (2)

where $\{c_1^2, c_2^2\}_t$ is the stochastic process describing individual two's consumption of goods one and two, $U^2 (\cdot)$ (which need not be the same function as $U^1 (\cdot)$) gives current period utility of individual two, and $\beta$ and $E$ are as described before.

Production of goods one (in country one) and two (in country two) is exogenously given by the stochastic process $\{y_1, y_2\}_t$. Neither good is storable. Assume the process $\{y_1, y_2\}_t$ is generated by independent realizations of a random vector $y_t$ from a stationary probability distribution with cumulative distribution function $F_y(\cdot)$, so the randomness in production is independent over time. The assumptions that output is exogenous, that goods are nonstorable, that production is specialized, and that shocks to production are independent both across goods and over time could all be relaxed with no important change in the results—the mathematics would differ but the main economic issues emphasized in the paper would be unchanged.

Let $\{M_1^s, M_2^s\}_t$ be the nominal quantities of moneys one and two that have been issued by the governments of countries one and two and are used within those countries for domestic transactions. International transactions could in principle involve the use of either money for payments. Empirically, roughly two-thirds of international trade contracts appear to be denominated in the seller's currency.[12] The choice of a currency for payments in international trade may depend upon the costs,,in terms of depreciation uncompensated

by interest payments on money, of holding each money and differential transactions costs in handling alternative currencies.[13] I assume here that all international transactions are financed with the seller's currency. Thus when country one imports good two from country two, payments are made with money two. Similarly, country one receives money one in payment for its exports. This is consistent with the assumptions of the traditional elasticities approach to the foreign exchange market.

Since people demand foreign exchange because they want to purchase foreign goods or assets, it is useful to view the demand for foreign exchange as a derived demand. This approach was taken in the traditional exchange rate literature, although its implications were never fully investigated. References to the derived characteristic of demand for foreign exchange can be found in Cassel (1922, p. 138), Machlup (1939, pp. 111, 115, 119), Robinson (1949, p. 83), Haberler (1949), Friedman (1953, pp. 159, 162), Friedman and Schwartz (1963, pp. 161, 590, n. 35), Hodgson (1972, p. 250), Mikesell and Furth (1974, pp. 6-17, 57), and Machlup (1972, pp. 29ff). The traditional elasticities approach formalized the derived demand for foreign exchange in a static model and developed specific formulas for certain cases (e.g., the Marshall-Lerner condition). The formulas obtained depended on the particular assumptions (Mundell, 1971, pp. 94-97), but the important unifying characteristic of the elasticity models was that they derived the demand for foreign exchange from the demand for foreign goods.

The demands for moneys can be derived from the demands for goods by specifying a simple transactions technology that prevents individuals from engaging in barter. The formulation of the transactions technology used here is similar to that of Lucas (1977) and is one version of the formulation proposed by Clower (1967) and also used by Grandmont and Younes (1972, 1973)

to study issues in the monetary theory of closed economies. The transactions technology involves a "liquidity constraint" on individual behavior that attempts to reflect the fact that money is held between transactions and that many transactions would be very costly without the use of money. The form of the liquidity constraint used in this paper requires that goods be purchased with money and that this money be held before it is spent.[14] Expenditures during any period must be financed out of money available at the beginning of the period. This ensures that an individual cannot sell his output for money and instantaneously spend that money for goods, i.e., he cannot barter.[15] He carries his receipts from current sales of output into the next period.

Since imports must be financed with foreign exchange (foreign money), the transactions technology applied to imports results in a demand for foreign exchange that is derived from the demand for imports. People, as importers, hold positive balances of foreign exchange; they procure this foreign money on the foreign exchange market at the price e, the price of money two in terms of money one. (I will refer to country one as the domestic country, so e is the price of foreign exchange.)[16]

Let the sequence of events each period be the following: the representative individual in country one enters each period with some domestic money, $M_1^1$, which he may use for domestic purchases, and some foreign exchange, $M_2^1$, for importing purposes. The superscripts denote the holder of the money (individual one or individual two); the subscripts denote money one or money two. Individual one then harvests his output, $y_1$, and takes it to market. (Individual two takes $y_2$ to market.) He observes the equilibrium prices at which all trades take place. (The word "prices" means both goods prices, $p_1$ and $p_2$, and the exchange rate, e.) He purchases

goods one and two and sells his own output for money. He then goes to foreign exchange market to purchase (or sell) foreign exchange to carry into the next period.[17]

Each period individual one chooses consumption of good one, $c_1^1$, consumption of good two, $c_2^1$, end-of-period holdings of domestic money (one), $M_1^1{}'$, and end-of-period holdings of foreign exchange, $M_2^1{}'$, subject to the constraints

(a) $\quad p_1 y_1 + M_1^1 + \tau_1 + e M_2^1 - p_1 c_1^1 - e p_2 c_2^1 - M_1^1{}' - e M_2^1{}' = 0 \qquad$ (3)

(b) $\qquad\qquad\qquad p_1 c_1^1 \leq M_1^1 + \tau_1$

(c) $\qquad\qquad\qquad p_2 c_2^1 \leq M_2^1$

where $M_1^1$ and $M_2^1$ are predetermined (by last period's choices), $y_1$ is his output, which he sells at the price $p_1$, the price of good one in terms of money one. $\tau_1$ and $\tau_2$ are realizations of a stochastic process $\{\tau_1, \tau_2\}_t$ representing transfer payments of money one to individual one and of money two to individual two. (These are taxes if they take negative values.) These transfers occur overnight (between periods) and are available with other initial money holding to finance consumption this period. Equation (3a) is a budget constraint which states that the initial assets and current income of individual one may be allocated to current consumption and end-of-period balances; (3b) and (3c) are liquidity constraints imposed by the assumed transactions technology. They state that current purchases of domestic goods are limited by initial holdings of domestic money and current imports are limited by initial holdings of foreign exchange. The analogous constraints for individual two's optimization problem are

(a) $\quad ep_2y_2 + M_1^2 + eM_2^2 + e\tau_2 - p_1c_1^2 - ep_2c_2^2 - M_1^{2\prime} - eM_2^{2\prime} = 0$ $\qquad$ (4)

(b) $\qquad\qquad\qquad p_1c_1^2 \leq M_1^2$

(c) $\qquad\qquad\qquad p_2c_2^2 \leq M_2^2 + \tau_2.$

## The Role of the Governments

The governments of each country have two roles: they determine $\{\tau_1, \tau_2\}_t$, the money supply changes financed by transfers or taxes to (from) their residents, and they may intervene in the foreign exchange market by buying or selling foreign exchange. Let $\theta_t$ denote purchases of money two with money one by the combined actions of the two governments on the foreign exchange market. The policies of the two governments can then be summarized by the stochastic process $\{\tau_1, \tau_2, \theta\}_t$.[18]

Let $M_1^s$ and $M_2^s$ denote the nominal quantities of moneys one and two outstanding at the beginning of the period. Then

$$M_1^s = M_1^1 + \tau_1 + M_1^2 \qquad (5)$$

$$M_2^s = M_2^1 + M_2^2 + \tau_2 .$$

At the end of the period the nominal money supplies are

$$M_1^{s\prime} = M_1^s + \theta \qquad (6)$$

$$M_2^{s\prime} = M_2^s - \frac{1}{e}\theta$$

where $\theta$ is the foreign exchange market intervention undertaken by governments during the period. At the beginning of the following period nominal money supplies are

$$M_1^{s\prime} + \tau_1' \quad \text{and} \quad M_2^{s\prime} + \tau_2' \qquad (7)$$

where the transfers $\tau_1'$ and $\tau_2'$ occur between periods.

Let $\gamma_j^i$, $(i, j) = 1,2$ denote the fraction of money $j$ held by residents of country $i$. Notice that $\gamma_1^1 + \gamma_1^2 = 1 = \gamma_2^1 + \gamma_2^2$. These allocation parameters are endogenously determined.

## Prices

At the beginning of any period the state of the world can be described completely by the state vector

$$s = (y_1, y_2, \gamma_1^1, \gamma_2^1, M_1^s, M_2^s, \theta) \tag{8}$$

and the probability distribution functions $F_y(\cdot)$, $F_t(\cdot)$, and $F_\theta(\cdot)$ which generate the stochastic processes on $\{y_1, y_2\}$, $\{t_1, t_2\}$, and $\theta$. Let $F(.,.,.,.,.)$ denote the joint cumulative probability distribution function of these variables. The state vector $s$ includes current outputs of each good, the nominal supplies of each money at the beginning of the period and their allocations, and the extent of government intervention in foreign exchange markets. A complete account of the state of the world includes both $s$ and $F( )$, which individuals use to form their expectations about the future.

Individuals choose consumptions and end-of-period asset holdings to maximize (1) or (2) subject to (3) or (4). The equilibrium conditions require that all markets clear:

$$
\begin{aligned}
c_1^1 + c_1^2 &= y_1 \\
c_2^1 + c_2^2 &= y_2 \\
M_1^{1'} + M_1^{2'} &= M_1^{s'} \\
M_2^{1'} + M_2^{2'} &= M_2^{s'}.
\end{aligned}
\tag{9}
$$

Only three of these four markets are independent, as can be verified by adding the budget constraints of individuals one and two. There are three prices, $p_1$, $p_2$, and e (the money one price of good one, the money two price of good two, and the money one price of money two) which adjust each period to ensure equilibrium.

The demand functions of individual one for consumption and end-of-period money holdings depend upon the prices he faces, $p_1$, $p_2$, and e, his initial money holdings, $M_1^1$ and $M_2^1$, his current income, $y_1$ (in terms of good one), and his beliefs about future prices and incomes. Given these beliefs about the future [which enter through the expected value operator in (1) and (2)], individual one's behavior can be described by the optimal policy or demand functions

$$c_1^1(p_1, p_2, e, M_1^1 + \tau_1, M_2^1, y_1) \tag{10}$$

$$c_2^1(p_1, p_2, e, M_1^1 + \tau_1, M_2^1, y_1)$$

$$M_1^{1'}(p_1, p_2, e, M_1^1 + \tau_1, M_2^1, y_1)$$

$$M_2^{1'}(p_1, p_2, e, M_1^1 + \tau_1, M_2^1, y_1)$$

Similar demand functions describe individual two's behavior:

$$c_1^2(p_1, p_2, e, M_1^2, M_2^2 + \tau_2, y_2) \tag{11}$$

$$c_2^2(p_1, p_2, e, M_1^2, M_2^2 + \tau_2, y_2)$$

$$M_1^{2'}(p_1, p_2, e, M_1^2, M_2^2 + \tau_2, y_2)$$

$$M_2^{2'}(p_1, p_2, e, M_1^2, M_2^2 + \tau_2, y_2)$$

A vector of prices

$$p = (p_1, p_2, e)$$

that assures equilibrium therefore depends on [from (9), (10), and (11)]
$M_1^1 + \tau_1$, $M_2^1$, $M_1^2$, $M_2^2 + \tau_2$, $y_1$, $y_2$, $M_1^s{}'$, and $M_2^s$. So, using (5), (6), and
the definition of $\gamma_j^1$, the price vector p depends upon $y_1$, $y_2$, $\gamma_1^1$, $\gamma_2^1$, $M_1^s$,
$M_2^s$, and $\theta$, which are the elements of the state vector s. Let

$$p = \phi(s)$$

give prices as a fixed function of the state of the world. The problem
is now to investigate the function $\phi(\cdot)$ and the behavior of prices as the
state vector changes over time.

## Dynamics

Prices of goods and foreign exchange change over time as the state
vector changes, and this relation is summarized by the function $\phi(s)$.
The state vector changes for two reasons. First, new disturbances occur
exogenously on initial money supplies, foreign exchange market intervention,
and outputs (real income). Second, $\gamma_1^1$ and $\gamma_2^1$ change over time as people
adjust optimally to past disturbances and to changes in expectations about
the future. Given the expectations held by individuals about future vari-
ables the demand functions and resulting market prices determine (together
with the exogenous transfers or taxes that will occur after the end of this
period) next period's allocation parameters $\gamma_1^1{}'$ and $\gamma_2^1{}'$.

Note that

$$\underline{s}' = \left(y_1{}',\, y_2{}',\, \frac{z_1^1{}'(s) + \tau_1{}'}{M_1^s + \tau_1{}' + \theta},\, \frac{z_2^1{}'(s)}{M_2^s + \tau_2{}' - \frac{1}{e}\theta},\, M_1^s + \tau_1{}' + \theta,\, M_2^s + \tau_2{}' - \frac{1}{e}\theta,\, \theta'\right)$$

where $z_2^1{}'(s) = Z_2^1{}'(s, \phi(s))$, s = 1, 2 and where $Z_2^1{}'(s, \phi(s))$ is the (average)
<u>aggregate</u> choice by people in country one of end-of-period balances of money j
given s and $p = \phi(s)$. So s' depends upon $y_1{}'$, $y_2{}'$, $\tau_1{}'$, $\tau_2{}'$, $\theta'$, and s, given

the function $\phi(\cdot)$. That is,

$$s' = G(s, w') \tag{12}$$

where $w' = (y_1', y_2', \tau_1', \tau_2', \Theta')$. So the time path of goods prices and the exchange rate are determined by

$$p' = \phi(s') = \phi(G(s, w')) = \text{function } (s, w').$$

A similar line of reasoning shows that the price that will prevail $j$ periods into the future is a function of the current state vector and the shocks $w', w'', \ldots, w^{(j)}$.

## Expectations

The model can be completed with rational expectations imposed through the expected value operators in (1) and (2). Given the expectations held by individuals about the future values of the variables, individuals will be able to formulate demand functions, and prices will adjust to clear markets. These prices depend on the state of the world. On the other hand, the prices that occur in each state of the world affect expectations about future prices and therefore affect the demand functions today.

Before defining rational expectations it is useful to rewrite the individuals' optimization problems. Define an indirect utility function $V^1(\cdot)$ by the maximum value attained by the objective function in the solution to the problem

$$V^1(M_1^1 + \tau_1, M_2^1{}', y_1, p) = \text{MAX } \{U^1(c_1^1, c_2^1) + \beta \int V^1(M_1^1{}' + \tau_1', M_2^1{}', y_1'$$

$$\phi^e(s')) \, dF(w')\} \tag{13}$$

where maximization is with respect to $(c_1^1, c_2^1, M_1^1{}', M_2^1{}')$ and subject to the constraints (3), and where $\phi^e(\cdot)$ maps the space of state vectors into the

space of price vectors. Equation (13) says that individual one maximizes current period utility plus the discounted expected value of future utilities given that he knows he will continue to behave optimally in the future. The optimization problem of individual two can be similarly reformulated. His indirect utility function will be

$$V^2(M_1^2, M_2^2 + \tau_2, y_2, p) = MAX \{U^2(c_1^2, c_2^2) +$$

$$\beta \int V^2(M_1^2{}', M_2^2{}', \tau_2{}', y_2{}', \phi^e(s')) \, dF(w')\} \tag{14}$$

where maximization is with respect to $(c_1^2, c_2^2, M_1^2{}', M_2^2{}')$ and subject to the constraints (4).

Assume each individual has rational expectations in the following sense:

1. The function $F(\cdot)$ in (13) and (14) is the cumulative probability distribution function describing the behavior of $w \equiv (y_1, y_2, \tau_1, \tau_2, \theta)$, defined earlier.

2. The function $\phi^e(\cdot)$ in (13) and (14) is the same function $\phi(\cdot)$ that guarantees market-clearing each period.

3. The individual knows that $s'$ is determined by (12).

I assume that the information available to each individual includes the current state vector $s$.[19] Each individual, since he knows $s$ and $F(w')$, knows the induced probability distribution function of $s'$ and therefore the induced probability distribution on $p' = \phi(s')$. His current behavior is based upon these expectations.

Each individual takes next period's state vector $s'$ as exogenous to his own decisions (and random). Included in $s'$ are $z_1^1{}'(s)$ and $z_2^1{}'(s)$ on which the individual has, through his knowledge of $s$ and $G(\cdot)$, perfect foresight. This is a result of the individual's knowledge of the aggregate decisions that are made today in state of the world $s$. Now each individual chooses his own

end-of-period balances optimally <u>given</u> $z_1^{1}{}'$ and $z_2^{1}{}'$. But $z_1^{1}{}'$ and $z_2^{1}{}'$ are just the (average) aggregates of the choices of all these individuals. It can be verified that, by construction of the Markov process $G(\cdot)$, the market clearing prices $p(s)$ ensure that the consistency requirements

$$M_1^{1}{}'(\underline{s}, \phi(\underline{s})) = Z_1^{1}{}'(\underline{s}, \phi(\underline{s})) \tag{15}$$

and

$$M_2^{1}{}'(\underline{s}, \phi(\underline{s})) = Z_2^{1}{}'(\underline{s}, \phi(\underline{s}))$$

are met.

## Equilibrium

An equilibrium requires both that people maximize expected utility given rational expectations, i.e., that the demand functions solve (13) and (14) when $\phi^{e}(\cdot)$ is replaced by $\phi(\cdot)$ and $s'$ by $G(s, w')$, where $G(\cdot)$ is such that (15) holds, and that prices clear markets, i.e., that the equilibrium conditions (9) hold when the demand functions are inserted. It is straightforward to examine the consumer optimization problem <u>given</u> the behavior of prices as a function of the state vector (summarized by the function $\phi$), the process generating the dynamic behavior of the state vector (summarized by the function G and the probability distribution function F), and, of course, the current state vector (see Stockman, 1978(b)). The demand functions obtained from the maximization problem have some ambiguous signs for the usual reasons -- wealth and substitution effects are not always reinforcing. But if substitution effects generally dominate wealth effects and wealth effects are positive then increases in initial holdings of either money or in current income result in increases in the demand for both goods and both moneys. Increases in $p_1$ result in a decreased

demand for good one but increases in the demand for the other good (in the absence of strong complementary) and increases in the demand for both moneys. Increases in $p_2$ increase the demand for both moneys and the demand for good one while decreasing the demand for good two. Increases in the exchange rate, e, induce increases in the demand for good one and money one and decreases in the demand for good two and money two.[20] As each individual chooses consumption and money holdings taking as _given_ the relation between prices and the state of the world and the process generating changes in the state of the world, the aggregate behavior of these individuals affects the things that each individual takes as given. While anticipations about the random part of the state vector are rational in the sense that the probability distribution on the exogenous variables is known, anticipations about the elements of next period's state vector that are the result of (aggregate) individual choices made today are rational in the sense that the individual knows with certainty these aggregate choices and makes his own plans accordingly. As all individuals do this, their choices _form_ the aggregate choice that each takes as given. Market equilibrium therefore requires that both (9) and (15) hold.[21]