THE VALUE OF LEARING

ABOUT CONSUMPTION HAZARDS*

by

Robert A. Jones

and

John G. Riley**

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**Department of Economics, UCLA

CONTENTS

INTRODUCTION

This report examines the implications of reducing uncertainty about the hazards associated with various forms of consumption. Chapter 1 focusses on the determinants of the dollar valuation of such a reduction in uncertainty, measured as the willingness to pay. The chapter begins with the simplest 'Marshallian' case and then successively generalizes the results at the cost of making Taylor's series approximations. It is shown that the value of reducing uncertainty is readily determined once estimates have been made of the ex-post shifts in demand associated with the information.

A major simplifying feature of the models in chapter 1 is that all prices are exogenous. While this is perhaps a reasonable first approximation for many applications it is surely inappropriate for non-produced commodities of uncertain quality. One important case is the adjustment of land prices to reflect differences in air quality in an urban environment. This case is the primary focus of chapter 2. First the equilibrium location of a population with different incomes is described. It is shown that there is only a mild presumption in favor of location in the less hazardous areas by the more wealth. Optimal location of an identical population is then examined. Finally it is shown that the expected value of research which reduces uncertainty about an environmental hazard may be fully reflected in land values.

Chapter 3 introduces time into the analysis, taking account of the fact that the *prospect* of future information will affect consumption decisions made prior to the receipt of the information. The central result is that if the possibly harmful effects of consuming a particular good depend on its accumulated consumption over the lifetime, then the prospect of receiving information about the maximum safe level of consumption reduces current consumption of that good.

1. THE VALUE OF INFORMATION

Consumption decisions are frequently made without perfect knowledge of the effects of these decisions on ultimate welfare. Individuals may be uncertain of the quality of a product, the effects it may have on health, or the extent to which its consumption extenuates effects of inescapable environmental hazards. More appropriate decisions could generally be made if these characteristics were known prior to making consumption commitments. This chapter seeks to determine the value of such information to an individual as measured by his willingness to pay.

The connection between this problem and policy issues lies in the twin facts that it is difficult for individuals to test products for health hazards without committing themselves to consumption, and that the output of testing -- information -- has the characteristics of a public good. The market system thus can not be relied upon to produce a socially efficient level of information about consumption hazards. Some benefit valuation measure is necessary if the government is to rationally engage in information production.

Formally, suppose a consumer is uncertain about the state of the world $s \varepsilon S$. His realized utility $u(x,s)$ depends on both s and the vector of goods $x = (x_1,\ldots,x_n)'$ he consumes. To focus on uncertainty about the characteristics of goods assume that neither the consumer's income M nor the price vector p that he faces are state dependent. Further assume that the individual has a prior subjective probability distribution over the set S of possible states and that his preferences satisfy the von Neumann-Morgenstern axioms.

If no further information is forthcoming the individual chooses a consumption vector x^o maximizing expected utility subject to his budget constraint, achieving an expected utility

(1) $$U^{\circ}(M) = \underset{x}{\text{Max}} \{E_s u(x,s) \mid p'x \leq M\} = E_s u(x^{\circ},s).$$

Alternatively, suppose the individual acquires at a cost V perfect information about s. Upon being informed that state s prevails he chooses consumption x^s achieving utility

(2) $$U^s = \underset{x}{\text{Max}} \{u(x,s) \mid p'x \leq M-V\} = u(x^s,s).$$

Prior to knowing the true state this utility level is a random variable; but the level of expected utility anticipated by the individual is

(3) $$U^*(M-V) = E_s u(x^s,s).$$

Since $x^s = x^{\circ}$ is feasible when $V = 0$, it follows from (2) that $U^*(M) \geq U^{\circ}(M)$. Hence there is some V^* for which the individual is just indifferent between purchasing the information and not: $U^*(M-V^*) = U^{\circ}(M)$. We term V^* the

The following sections derive expressions for V^* under progressively less restrictive assumptions about the utility function $u(x,s)$. Section 1.1 examines the simple Marshallian case in which s affects the marginal utility of just one good and the marginal utility of expenditures on other goods is constant. A particularly simple expression for the value of information emerges. Section 1.2 considers situations in which the marginal utility of more than one good varies, obtains V^* for a logarithmic utility function, and explores the accuracy of a first order approximation to V^* which might be more widely applicable.

Section 1.3 extends the analysis in two directions. A general utility function $u(x,s)$ is employed and the information structure offered may leave the consumer just <u>better</u> informed, rather than <u>perfectly</u> informed, about s. A first order approximation to V^* is derived which is related to the results of the previous sections.

Finally, section 1.4 introduces the notion of an increment in information. The definition of <u>better</u> information utilized is that developed by

Blackwell (1953) and Marschak (1968). Examples with information parametrized by the power of a statistical test and by the reduction in subjective variance about a parameter are provided. The objective throughout is to express the value of information in terms of theoretically observable parameters such slopes of and shifts in usual demand functions.

1.1 THE MARSHALLIAN CASE

Suppose that the uncertain state of the world only influenced how one good entered the consumer's utility function and that the marginal utility of expenditure on other goods was known and constant. The utility function has the form

$$u(x,s) = w(x_1,s) + y \tag{4}$$

where $y = \sum_{2}^{n} p_i x_i$. Further suppose that $s = \{1,2\}$; s takes on only two possible values with probabilities π_1 and π_2 respectively. In the absence of further information the consumer facing a budget constraint $p_1x_1 + y = M$ anticipates an expected utility of

$$\pi_1 w(x_1,1) + \pi_2 w(x_1,2) + M - p_1x_1. \tag{5}$$

The subscripts on p_1 and x_1 are suppressed for the remainder of this section since there is only one good whose consumption characteristics are uncertain. The value of x^o maximizing (5) satisfies, at an interior optimum,

$$\pi_1 \frac{\partial w}{\partial x}(x^o,1) + \pi_2 \frac{\partial w}{\partial x}(x^o,2) = p. \tag{6}$$

Interpreted in Marshallian terms, the function $p^o(x)$ defined by the left-hand side of (6) is the price that would generate a demand of x if if no additional information is forthcoming.

Compare this outcome with the decisions made if perfect information was available. The consumer's utility function in state s is $w(x,s) + M - V - px$ and, at an interior optimum, x would be chosen to satisfy

$$\frac{\partial w}{\partial x}(x^s,s) = p. \qquad s = 1,2 \tag{7}$$

The functions $p^s(x)$ defined by the left-hand side of (7) are the perfect information Marshallian demand curves. Figure 1 depicts these demand curves. Notice that the incomplete information demand curve $p^o(x) = \Sigma\pi_s p^s(x)$ is simply the probability weighted average of the perfect information demand curves, and hence that $p^o(x^o) = \Sigma\pi_s p^s(x^o) = p$ from (6).

With imperfect information the expected utility realized is, from (5),

$$\begin{aligned} U^o(M) &= \sum_s \pi_s[w(x^o,s) - px^o] + M \\ &= \sum_s \pi_s\left[\int_o^{x^o}\frac{\partial w}{\partial x}(q,s)dq - px^o\right] + M \\ &= \sum_s \pi_s \int_o^{x^o}[p^s(q) - p]dq + M. \end{aligned} \tag{8}$$

If perfect information is acquired at cost V the utility realized in state s is

$$U^s = \int_o^{x^s}[p^s(q)-p]dq + M - V$$

and hence the prior expected utility would be

$$U^*(M-V) = \sum_s \pi_s \int_o^{x^s}[p^s(q)-p]dq + M - V \tag{10}$$

The value V^* of perfect information to the consumer, defined by $U^*(M-V^*) = U^o(M)$, is

$$V^* = \sum_s \pi_s \int_{x^o}^{x^s}[p^s(q) - p]dq. \tag{11}$$

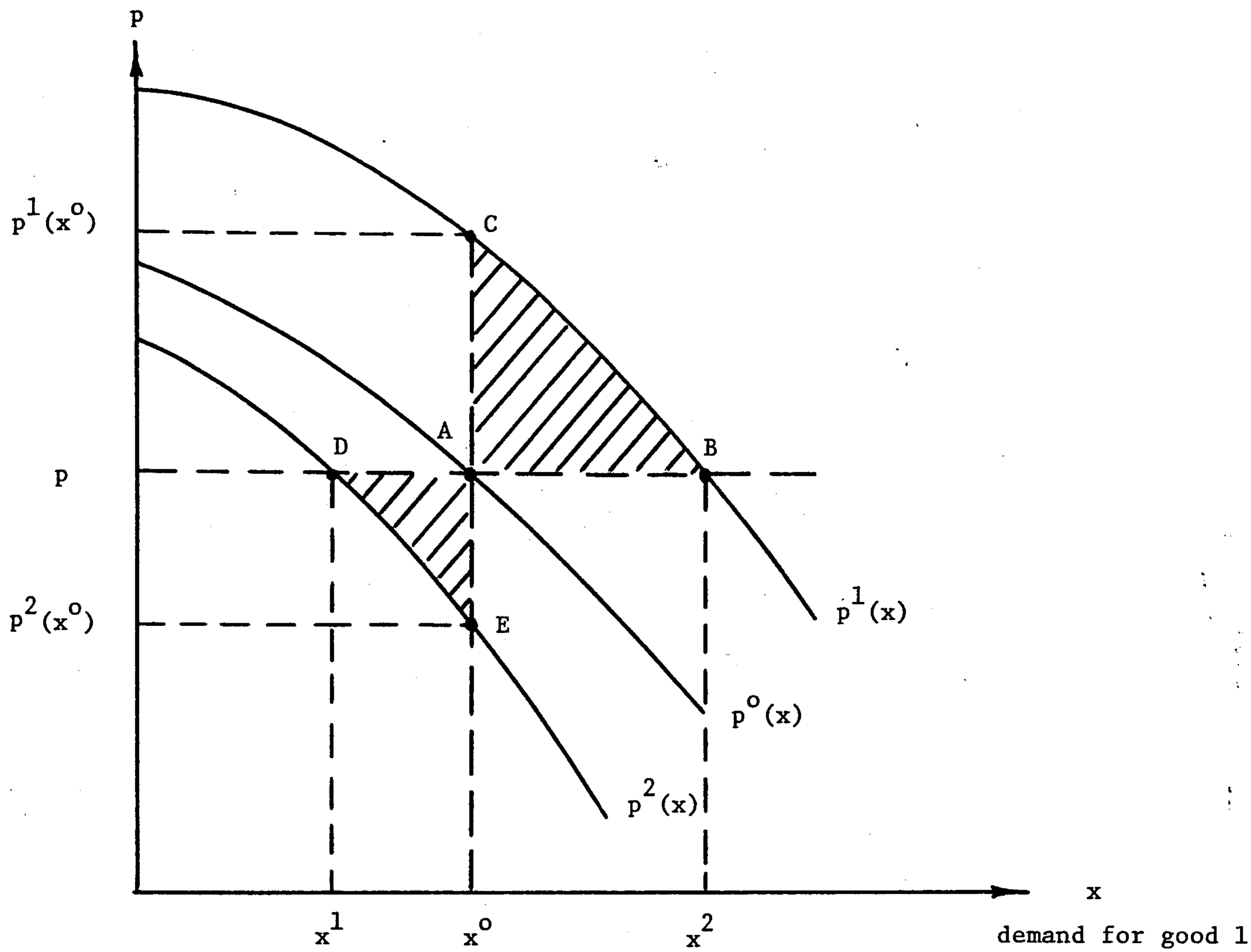

FIGURE 1: Value of Information

For the case where $S = \{1,2\}$ depicted in Figure 1, V^* may be rewritten as

$$V^* = \pi_1 \int_{x^o}^{x^1} [p^1(q)-p]dq + \pi_2 \int_{x^2}^{x^o} [p-p^2(q)]dq$$

$$= \pi_1(\text{area ABC}) + \pi_2(\text{area ADE}).$$

Area ABC is the potential consumer surplus lost if x^o is consumed when the true state is s=1; area ADE when s=2. The value of information is equal to the expected consumer surplus increase its use provides.

Expression (11) for V^* remains valid if more than two states are possible. Let us approximate the demand curves $p^s(x)$ by <u>parallel</u> linear demand curves of slope $dp^o(x^o)/dx = \Sigma\pi_s dp^s(x^o)/dx$. Substitution into (11) yields

$$(12) \qquad V^* \approx \frac{\Sigma\pi_s[p^s(x^o) - p^o(x^0)]^2}{2|dp^o/dx|} = \frac{\text{var}[p^s(x^o)]}{2|dp^o/dx|}$$

in which $\text{var}[p^s(x^o)]$ is the variance of the perfect information demand price for the imperfect information quantity demanded, and dp^o/dx is the slope of the Marshallian demand curve for x. The value of perfect information increases both with the sensitivity of demand prices to the state of the world and with the sensitivity of quantity demanded to price.

1.2 LOGARITHMIC UTILITY FUNCTIONS

Expression (12) for the value of information in the Marshallian case has considerable appeal since it depends on the slope of and shifts in a potentially observable demand curve. A generalization of (12) to non-Marshallian situations would be of great help in valuing prospective research on product characteristics or on environmental hazards whose effects may be extenuated by altering consumption patterns. This section determines V* for a non-Marshallian logarithmic utility function, obtains a first order approximation to V* which generalizes (12), and numerically assesses the reliability of the approximation. The results suggest that the approximation technique, which is extended to general utility functions in section 1.3, is sufficiently reliable to be useful.

Suppose the consumer's utility function has the logarithmic form

$$(13) \qquad u(x,s) = \sum_{i=1}^{n} \theta_i^s \ln \beta_i^s x_i \qquad \theta_i^s \geq 0,\ \beta_i^s > 0$$

$$= \sum_i \theta_i^s \ln \beta_i^s + \sum_i \theta_i^s \ln x_i$$

whose parameters may all vary with s. In the absence of further information about s the consumer chooses consumption bundle x^o on the basis of prior beliefs to achieve

$$U^o(M) = E_s u(x^o,s) = \operatorname*{Max}_x \{u(x,s) \mid p'x \leq M\}.$$

But since u(x,s) is additively separable in β_i^s and x_i, x^o must also be the solution to

$$\operatorname*{Max}_x \{E_s \Sigma\theta_i^s \ln x_i \mid p'x \leq M\} = E_s \Sigma\theta_i^s \ln x_i^o.$$

The individual is clearly not indifferent to the values β_i take on, or to uncertainty about their values. But his consumption decision is completely independent of beliefs about their values. This implies that information altering his beliefs about the vector β but not about θ has no effect on his optimal consumption bundle, and hence that his ex ante expected utility level is unaffected by the prospective receipt of such information. In other words the information is valueless.[1] Any information of value, therefore, must concern the true values of θ_i. Without loss of generality we may set $\beta_i^s = 1$ for all i and s.

Let us further restrict our attention to the special case in which

$$\theta_i^s = \begin{cases} s & \text{if } i = 1 \\ (1-s)\gamma_i & \text{if } i=2,\ldots,n \end{cases}$$

where $\sum_2^n \gamma_i = 1$. Such a consumer is uncertain about his marginal valuation of commodity 1 <u>relative</u> to all other commodities but spends his income on commodities 2, ..., n in the same relative proportions. Given constant prices, Hick's aggregation theorem may be applied to reduce the individual's objective to

$$\text{(14)} \qquad \max_{x_1, y} \{E_s(s\ln x_1 + (1-s)\ln y) \mid p_1 x_1 + y \leq M\}$$

in which y denotes expenditures on goods 2,..., n.[2]

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[1] For instance, the only uncertain parameter may be β_1. β_1 could be the true quantity of some desirable characteristic per unit of x_1 purchased (e.g.: the true weight of the contents of a box of x_1). Although the consumer would prefer β_1 to be large (the box to be full), he would not pay anything to learn its true value.

[2] The expression in braces differs from the individual's expected utility by a function of prices and the distribution of s alone, and hence is independent of any variables within the individual's control. This constant is suppressed to maintain notational simplicity.

In the absence of further information about s the problem reduces to the certainty equivalent problem

(15) $$\max_{x_1,y} \{\bar{s} \ln x_1 + (1-\bar{s}) \ln y \mid p_1 x_1 \leq M\}$$

with $\bar{s}$ denoting the mean of s based on the consumer's current beliefs. The solution to the problem is $x_1^o = \bar{s}M/p_1$, $y^o = (1-\bar{s})M$, providing an expected utility

(16) $$U^o(M) = \bar{s}\ln\bar{s} + (1-\bar{s})\ln(1-\bar{s}) - \bar{s}\ln p_1 + \ln M.$$

If the individual pays V to acquire perfect information about s, then the problem faced once s is revealed is

(17) $$\max_{x_1,y} \{s \ln x_1 + (1-s)\ln y \mid p_1 x_1 + y \leq M-V\}$$

The utility level attained is

(18) $$U^s = s\ln s + (1-s)\ln(1-s) - s\ln p_1 + \ln(M-V).$$

The agent's expected utility prior to receipt of the perfect information is thus

(19) $$U^*(M-V) = E_s U^s = E_s\{s\ln s + (1-s)\ln(1-s)\} - \bar{s}\ln p_1 + \ln(M-V).$$

The value of the information is V* such that $U^o(M) = U^*(M-V^*)$. Equating (16) and (19) and rearranging produces

(20) $$-\ln(1-\frac{V^*}{M}) = E_s[s\ln s + (1-s)\ln(1-s)] - [\bar{s}\ln\bar{s} + (1-\bar{s})\ln(1-\bar{s})].$$

The first bracketed term is a strictly concave function of s and the second is this function evaluated at the expected value of s. Jensen's inequality implies (20) is necessarily positive if s has positive variance, implying in turn that V* is positive.

To facilitate comparison with the results of section 1.1, we find the first order approximation to V*. Expanding both sides of (20) in a Taylor series and equating the lowest order non-zero terms gives us

$$(21) \qquad \frac{V^*}{M} \approx \frac{V^*_a}{M} = \frac{\mathrm{var}(s)}{2\bar{s}(1-\bar{s})}$$

Comparison of (21) with (22) indicates that the Marshallian estimate of V* is biased downward by a factor $(1-\bar{s})$. The two estimates differ because in the logarithmic case a change in s shifts not only the demand curve for x_1 but also the demand curve for other goods y. When the average consumer surplus gains displayed in Figure 1 are computed for both x_1 and y, their sum is indeed V* of expression (21). This suggests that the average area calculation of expected gain in consumer surplus can be further generalized as an approximate measure of V*. This will be done in section 1.3.

But before extending this measure of the value of information to general utility functions, let us check the usefulness of the approximation by comparing the exact value of V* given by (20) with the approximation given by (21). Suppose s takes on two values $\bar{s} + \varepsilon$ and $\bar{s} - \varepsilon$ with equal probability. The variance of s is then ε^2. Defining $A \equiv -\ln(1-V^*/M)$, it follows that

$$(23) \qquad \frac{V^*}{M} = 1 - e^{-A}$$

where, from (20),

$$A = \frac{1}{2}\left[(\bar{s}+\varepsilon)\ln\left(1+\frac{\varepsilon}{\bar{s}}\right)+(\bar{s}-\varepsilon)\ln\left(1-\frac{\varepsilon}{\bar{s}}\right)+(1-\bar{s}-\varepsilon)\ln\left(1-\frac{\varepsilon}{1-\bar{s}}\right)+(1-\bar{s}-\varepsilon)\ln\left(1+\frac{\varepsilon}{1-\bar{s}}\right)\right].$$

From (21) the first order approximation to V* is given by

$$(24) \qquad \frac{V^*_a}{M} = \frac{\varepsilon^2}{2\bar{s}(1-\bar{s})} \quad .$$

The values of V*/M and V^*_a/M for selected values of $\bar{s}$ and ε are summarized in the following tables:

TABLE 1: The Value of Perfect Information as a Percentage of Income

ε \ $\bar{s}$	.01	.10	.30	.50
.01	.696	.056	.024	.020
.10		7.215	2.387	1.994
.30			23.994	17.532
.50				50.000

TABLE 2: Approximation of the value of Perfect Information as a Percentage of Income

ε \ $\bar{s}$	.01	.10	.30	.50
.01	.505	.056	.024	.020
.10		5.556	2.417	2.000
.30			21.750	18.000
.50				50.000

Since both V^* and V_a^* are symmetric in $\bar{s}$ around $\bar{s} = .50$, their values for $\bar{s} = .70$, .90 and .99 are the same as for $\bar{s} = .30$, .10 and .01 respectively.

Comparing the exact values of V^*/M in Table 1 with the corresponding values of the approximation V_a^*/M in Table 2 indicates that the approximation is quite good over the whole range of feasible values of $\bar{s}$ and ε, and extremely good when $\text{var}(s) = \varepsilon^2$ is small relative to $\bar{s}$. The mean difference between V_a^* and V^* expressed as a percentage of V^* for the ten computed values is less than 6.5%. This is reason for having some confidence in the measures developed in the next section as indicators of willingness to pay for information.

1.3 GENERAL UTILITY FUNCTIONS WITH IMPERFECT INFORMATION

The preceding sections were concerned with valuing information which eliminated all uncertainty about the effects of consuming particular goods for limited classes of utility functions. V^* represented a consumer's willingness to pay for perfect information about s. The results of this section extend the analysis in two directions: first, they apply as approximations for arbitrary twice-differentiable utility functions; second, the information whose value is to be determined may be less than perfect. The general validity of approximating techniques such as we employ is supported in other contexts by Samuelson (1970). The concept of information structure that we use is that developed by Blackwell (1953), Marschak and Radner(   ), and Marschak and Miyasawa (1968).

It is seldom feasible to eliminate all uncertainty about the characteristics of a good an individual might consume, an activity in which he might engage or an environment in which he might choose to live. Realistically, investigation can only narrow the range in which the true characteristics lie, decreasing but not eliminating the dispersion of a consumer's probability distribution over s. Valuing such less than perfect information is a necessary first step toward establishing a measure of the marginal value of information.

Let the outcome of research the consumer receives, or the particular information he ultimately obtains, be termed a *message* and denoted by $\alpha \in A$. A is the set of possible results. Depending on the context α may be the estimated value of some unknown parameter (e.g., reduction in life expectancy from moderate smoking, gasoline mileage of a particular automobile), the acceptance or rejection of a statistical hypothesis at a particular significance level (e.g., nitrates cause cancer) or the opinion of an informed individual (e.g., weather forecast, medical opinion of susceptibility to heart disease). Before the particular information is received α is a random variable in the mind of the consumer. Its relation to the uncertain state of the world s is embodied in a joint probability distribution function $F(\alpha,s)$ over $A \times S$; $F(s)$, $F(\alpha)$ and $F(s|\alpha)$ denote the associated marginal and conditional probability distributions. This pair $\{A, F(\alpha,s)\}$ is the *information structure* whose value we seek to determine.[3]

Suppose for the moment that the information is provided at zero cost and that only the ultimate state s rather than the message α itself affects the individual's ultimate welfare. Upon receiving α the consumer chooses a consumption vector x^α achieving a conditional expected utility of

$$U^\alpha = \underset{x}{\text{Max}} \{E_{s|\alpha} u(x,s) | p'x \leq M\} = E_{s|\alpha} u(x^\alpha, s). \tag{25}$$

The maximizer of the objective (25) defines the conditional demand functions $x^\alpha(p)$ for various price vectors. Prior to receiving α, x^α is a random variable; given α it is non-random even though the true s may still be uncertain. The individual's expected utility before knowing which message will arrive depends on income, prices and the information structure:

$$U^*(M,A) = E_{s,\alpha} u(x^\alpha, s) = E_\alpha E_{s|\alpha} u(x^\alpha, s).$$

In the absence of such information the individual would have chosen consumption vector x° to achieve expected utility

[3]The terminology is that of Marschak and Radner ( ). Marschak and Miyasawa (1968) present alternative representations of an information structure.

(26) $$U^\circ(M) = \underset{x}{\mathrm{Max}}\{E_s u(x,s) | p'x \leq M\} = E_s u(x^\circ,s).$$

The increase in expected utility resulting from having the information is thus

(27) $$U^*(M,A) - U^\circ(M) = E_\alpha E_{s|\alpha}[u(x^\alpha,s) - u(x^\circ,s)].$$

We shall also make use of the demand price vector p^α associated with bundle x° and message α. These are the prices which would induce the individual to consume x° after receiving α, defined by

(28) $$x^\alpha(p^\alpha) = x^\circ.$$

Expanding the innermost expectation on the righthand side of (27) in a Taylor series around x^α yields an approximation to the expected gain from receiving message α:[4]

(29) $$\begin{aligned} E_{s|\alpha}[u(x^\alpha,s) - u(x^\circ,s)] &= -E_{s|\alpha}[u(x^\circ,s) - u(x^\alpha,s)] \\ &= -E_{s|\alpha}[(x^\circ - x^\alpha)'u_x + \tfrac{1}{2}(x^\circ - x^\alpha)'u_{xx}(x^\circ - x^\alpha) + \ldots] \end{aligned}$$

[4]All vectors such as x, p, u_x, etc. are column vectors. The transpose of a vector x is denoted by x'. Partial derivatives of a function with respect to one of its arguments is indicated by a subscript: e.g., U^*_M. If the argument is a vector, such as in u_x, then the vector of partial derivatives $(\partial u/\partial x_i)$ is indicated. If the function and its argument are both vector valued, as in x^α_p, then the matrix of partial derivatives $[\partial x^\alpha_i/\partial p_j]$ is indicated.

The various partial derivatives of u are evaluated at x^{α}. Recall that x^{α} was the solution to maximization problem (29). Associate with that problem a Lagrangian expression

$$L^{\alpha}(x,\lambda) = E_{s|\alpha}u(x,s) + \lambda(M - p'x).$$

The first order condition for an interior maximum with respect to x is

$$L_x^{\alpha} = E_{s|\alpha}u_x(x^{\alpha},s) - \lambda^{\alpha}p = 0 \tag{30}$$

which, upon differentiation with respect to p yields the additional relation[5]

$$(E_{s|\alpha}u_{xx})x_p^{\alpha} = \lambda^{\alpha}I + p\lambda_p^{\alpha'}. \tag{31}$$

λ^{α} is interpreted as the expected marginal utility of income conditional on message α being received. Both relations will be used to simplify (29).

The individual's budget constraint requires that $p'x^{\circ} = p'x^{\alpha} = M$. Relation (30) then implies that

$$(x^{\circ} - x^{\alpha})'E_{s|\alpha}u_x = (x^{\circ}-x^{\alpha})'p\lambda^{\alpha} = 0,$$

which is the first term in our Taylor series expansion (29). This leaves us with

$$E_{s|\alpha}[u(x^{\alpha},s) - u(x^{\circ},s)] \approx -\frac{1}{2}(x^{\circ} - x^{\alpha})'(E_{s|\alpha}u_{xx})(x^{\circ} - x^{\alpha}). \tag{32}$$

Recalling the definition of demand price in (28) and linearly approximating

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[5] I signifies the identity matrix of order equal to the number of goods. $p\lambda_p^{\alpha'}$ is the outer product of column vectors p and λ_p^{α} -- the matrix $[p_i \partial \lambda^{\alpha}/\partial p_j]$.

the demand functions gives us

$$x^\circ - x^\alpha = x^\alpha(p^\alpha) - x^\alpha(p^\circ) \approx x_p^\alpha(p^\alpha - p^\circ) \tag{33}$$

in which x_p^α is the matrix of partial derivatives of the demand functions. Substituting (33) and (31) into (32) yields the alternative form

$$-\frac{1}{2}(x^\circ - x^\alpha)'(\lambda^\alpha I + p\lambda_p^{\alpha'})(p^\alpha - p^\circ).$$

The budget constraints imply that the $(x^\circ - x^\alpha)'p\lambda_p^{\alpha'}(p^\alpha - p^\circ)$ component of this expression vanishes, since $(x^\circ - x^\alpha)'p = 0$. Substituting (33) for the remaining $(x^\circ - x^\alpha)$ results in

$$E_{s|\alpha}[u(x^\alpha,s) - u(x^\circ,s)] = -\frac{1}{2}\lambda^\alpha(p^\alpha - p^\circ)'x_p^\alpha(p^\alpha - p^\circ)$$

as the first approximation to the expected utility gain conditional on message α being received. The expected utility increases from the information prior to knowing which message will be received is thus

$$U^*(M,A) - U^\circ(M) \approx -\frac{1}{2}E_\alpha\lambda^\alpha(p^\alpha - p^\circ)'x_p^\alpha(p^\alpha - p^\circ). \tag{34}$$

But what we seek is an indicator of how much the consumer would be willing to pay for the information structure $\{A,F(s,\alpha)\}$. His expected marginal utility of income prior to receipt of the information is $U_M^* = E_\alpha\lambda^\alpha$. Under the assumptions that the marginal utility of income, the demand prices p^α and the slopes of the demand functions x_p^α are statistically independent we may replace x_p^α in (34) by its mean value $x_p = E_\alpha x_p^\alpha$. The amount the consumer is just willing to pay for the information is V_A^* such that $U^*(M-V_A^*,A) = U^\circ(M)$. The first order approximation to V_A^* is thus

$$(35) \qquad V^*_A \approx \frac{U^*(M,A) - U^\circ(M)}{U^*_M} \approx -\frac{1}{2}E_\alpha(p^\alpha - p^\circ)'x_p(p^\alpha - p^\circ).$$

This result is summarised in the following proposition:

Proposition 1: A consumer with a twice differentiable utility function facing given prices p for goods with fixed money income considers the purchase of information structure $\{A,F(s,\alpha)\}$. Let x° be the vector of goods demanded if the information was not purchased, and p^α be the vector of prices at which x° would be demanded if message α was received. If x_p denotes the matrix of mean slopes of the consumer's demand functions and if the marginal utility of income is uncorrelated with the demand prices p^α, then the first order approximation to the maximum the consumer would pay for the information is $V^*_A = -\frac{1}{2}E_\alpha(p^\alpha - p)'x_p(p^\alpha - p)$.

This expression for V^*_A warrants some discussion. First it makes clear that research which could not alter a consumer's behaviour is valueless to him, even though it may significantly reduce uncertainties about s. If x° continues to be the optimal purchase no matter which message is received, then $p^\alpha = p$ for all α and $V^*_A = 0$.

Second, notice that the value of an information system is not dependent on the consumer's attitude toward risk -- at least in a first approximation. V^*_A is expressed solely in terms of the slopes and locations of theoretically observable demand functions and the prior probabilities of the various information outcomes α. Although this seems paradoxical, it may appear less so if we remember that the prospect of information does not alter the real uncertainties about s; it merely informs the consumer about what lies in store for him while there is still time to act. This is as valuable to a risk-seeker as to a risk-averter.

Third, (35) suggests strongly that V^*_A rises with income. Writing out the expression for V^*_A in summation form and rearranging into elasticities,

(36) $$V_A^* = -\frac{1}{2}E_\alpha(p^\alpha - p)'x_p(p^\alpha - p)$$

$$= -\frac{1}{2}E_\alpha\sum_{ij}\sum(p_i^\alpha - p_i)(p_j^\alpha - p_j)\partial x_i/\partial p_j$$

$$= -\frac{M}{2}E_\alpha\sum_{ij}\sum\left(\frac{p_i^\alpha - p_i}{p_i}\right)\left(\frac{p_j^\alpha - p_j}{p_j}\right)\eta_{ij}e_i$$

in which e_i is the i^{th} good's expenditure share and η_{ij} is the elasticity of demand for good i with respect to price j. To the extent that these parameters are independent of income level, the value of information rises with income.[6]

The expression for V_A^* is just the generalisation of the expected area expressions of sections 1.1 and 1.2. The horizontal shift in the demand curve for good i induced by the receipt of message α is $-\sum_j(p_j^\alpha - p_j)\partial x_i/\partial p_j$ while the vertical shift in the same curve is $(p_i^\alpha - p_i)$. (36) is then the expected value of the sum of triangular areas $-\frac{1}{2}\sum_{ij}\sum(p_i^\alpha - p_i)(p_j^\alpha - p_j)\partial x_i/\partial p_j$ representing the consumer surplus gain. The prospect of perfect information about s corresponds to an information structure with A = S and α = s. The message *is* the true state.

One final check on the plausibility of (35) as a measure of the value of an information structure is to verify that information cannot have negative value. This must be the case if the message itself, as opposed to the true state s, has no direct influence on utility. At worst, the consumer can ignore α and choose to always consume $x°$. The Slutsky relation of conventional demand theory requires that

[6]For homothetic utility functions, such as that in section 1.2, V_A^* is thus directly proportional to M.

$$\partial x_i/\partial p_j = \partial x_i^c/\partial p_j - x_j \partial x_i/\partial M$$

where $\partial x_i^c/\partial p_j$ is the slope of the income compensated demand curve for good i. In matrix form, $x_p = x_p^c - x_M x'$ in which $x \equiv x^\circ$ is the point at which the partial derivatives will be evaluated. Substituting this expression into (36) gives us

$$V_A^* = -\frac{1}{2}E_\alpha (p^\alpha - p)'(x_p^c - x_M x^{\circ\prime})(p^\alpha - p).$$

The budget constraints imply that $x^{\circ\prime}(p^\alpha - p) = M - M = 0$, whence

$$V_A^* = -\frac{1}{2}E_\alpha (p^\alpha - p)' x_p^c (p^\alpha - p). \tag{37}$$

The Slutsky matrix x_p^c is known to be symmetric and negative semidefinite, implying that the expectation is (37) is non-negative for all information structures.[7]

[7] The quadratic form in (37) is convex in the vector p^α. Hence higher "riskiness" of the post-information demand prices in the sense of Rothschild and Stiglitz (1970) implies greater value of the information structure.

1.4 THE MARGINAL VALUE OF INFORMATION

The analysis of sections 1.1 to 1.3 presumes that the alternative to purchasing information is to have no information beyond one's current beliefs. This final section addresses the question of how an individual would value an *increment* in the amount of information he will receive. What would he be willing to pay for higher quality of information about the unknown state of the world? The answer to this question together with knowledge of the incremental cost of obtaining better information equips us to discuss optimal levels of information acquisition.

The first difficulty encountered in discussing the marginal value of information is quantifying the amount of information received. It is tempting to suppose that information is measurable in some natural units like quantities of any other good. Arrow (1971, chap. 12), for example, measured quantity of information by the expected reduction in entropy it causes. But any scalar valued measure can be shown to lead to inconsistencies when used outside the context for which it was constructed. Information cannot generally be measured in the same manner as other goods.

An alternative approach to measuring quantity of information, based on notion that it cannot have negative value, is developed in the work of Marschak (1968), Blackwell (1953), and DeGroot (1962). Consider two information structures $\{A,F(\alpha,s)\}$ and $\{B,F(\beta,s)\}$. Information structure A is defined to be *more informative than* B if *every* decision maker would pay as least as much for A as for B, regardless of his preferences or decision problem faced.

Let α and β represent the messages from the two structures, $q = (q_\alpha)$ and $r = (r_\beta)$ the vectors of probabilities that the respective messages in A and B will be received, and $\pi^\alpha = (\pi_s^\alpha)$ and $\pi^\beta = (\pi_s^\beta)$ the posterior probabilities of states s occurring conditional on receipt of messages α and β. The matrices of these conditional probabilities are denoted by $\Pi_A = [\pi_s^\alpha]_{s,\alpha}$ and $\Pi_B = [\pi_s^\beta]_{s,\beta}$. The individual's prior beliefs about the probabilities of various s occurring is denoted by the vector $\pi^\circ = (\pi_s^\circ)$. Marschak and Miyasawa (1968, p. 154) demonstrate the following, which provides an operational method for determining whether information structure A is unambiguously better than B:

> Proposition (Marschak and Miyasawa): A is more informative than B if and only if there exists a non-negative matrix $M = [m_{\beta\alpha}]$ with $\sum_\beta m_{\beta\alpha} = 1$ such that $\Pi_A M = \Pi_B$ and $Mr = q$.

However the ordering induced on information structures by this definition is only partial. Some pairs of information structures may not be ordered in terms of informativeness. We thus conclude that any scalar measure of amount of information can only apply if we limit information structures to families which can be parameterized by a single parameter. After a brief examination of the value of an increment in information in the Marschak and Miyasawa sense, we will focus on examples of such parametrizable families.

Section 1.3 provided us with a first order approximation to the value of information structure A of

$$V_A^* = -\frac{1}{2}E_\alpha(p^\alpha - p)'x_p^c(p^\alpha - p)_s \tag{37}$$

in which p^α was the demand price associated with the uniformed consumption vector x° after receipt of message α. Define $p(\pi,x^\circ)$ to be the demand prices for vector x° associated with an arbitrary vector π of probabilities that various states s will occur. That is, it is the vector of prices such that

$$\text{(38)} \qquad \underset{x}{\text{Max}}\{\underset{s}{\Sigma}\pi_s u(x,s) \,|\, x'p(\pi,x^\circ) \leq M\} = \underset{s}{\Sigma}\pi_s u(x^\circ,s).$$

The argument x° of $p(\pi,x^\circ)$ is suppressed in what follows. Clearly, what was termed p^α equals $p(\pi^\alpha)$, and for the alternate information structure B, p^β equals $p(\pi^\beta)$.

The difference in value between two information structures A and B is, from (37),

$$\text{(39)} \qquad V_A^* - V_B^* = -\frac{1}{2}\{E_\alpha(p^\alpha - p)'x_p^c(p^\alpha - p) - E_\beta(p^\beta - p)'x_p^c(p^\beta - p)\}.$$

Denoting the consumer's prior beliefs over s by π°, and linearly approximating $p(\pi)$ by a Taylor series expansion around π° yields

$$\text{(40)} \qquad p(\pi) \approx p(\pi^\circ) + p_\pi(\pi - \pi^\circ) = p + p_\pi(\pi - \pi^\circ)$$

in which p_π indicates the matrix of partial derivatives $\partial p_i/\partial\pi_s$ of demand prices for goods i with respect to the probabilities of states s occurring. Since $p(\pi)$ gives the demand price for bundle x°, it follows that $p(\pi^\circ) = p$, the actual prices faced by the consumer. Thus $p^\alpha - p$ and $p^\beta - p$ in (39) can be expressed as

$$\text{(41)} \qquad p^\alpha - p = p_\pi(\pi^\alpha - \pi^\circ), \quad p^\beta - p = p_\pi(\pi^\beta - \pi^\circ).$$

Substituting (41) into (39) and remembering that q_α and r_β are the message receipt probabilities our final expression for the difference in value between A and B:

$$\text{(42)} \qquad \Delta V^* = V_A^* - V_B^* = -\frac{1}{2}[\underset{\alpha}{\Sigma}q_\alpha\pi^{\alpha'}G\pi^\alpha - \underset{\beta}{\Sigma}q_\beta\pi^{\beta'}G\pi^\beta].$$

The matrix $G = p_\pi' x_p^c p_\pi$ is symmetric and negative semi-definite since x_p^c is negative semi-definite. The $\pi°$ terms vanished in (42) since $\sum_\alpha q_\alpha \pi^\alpha = \sum_\beta r_\beta \pi^\beta = \pi°$ if the individual starts with the same prior beliefs for the two information structures.

ΔV^* in (42) is the closest we can get to valuing of an increment in information without imposing restrictions on the information structures involved. Its form suggests that ΔV^* corresponds to the increase in a generalized variance of posterior probabilities. ΔV^* tends to rise with the price sensitivity of demand for goods x_p^c and the sensitivity of demand prices to changes in beliefs about which state occurs p_π.

One can readily exhibit that ΔV^* is positive if A is more informative than B. Using the definition of more information introduced earlier, we rewrite (42) as

$$(43) \qquad -2\Delta V^* = [\sum_\alpha q_\alpha \pi^{\alpha'} G \pi^\alpha - \sum_\beta r_\beta \pi^{\beta'} G \pi^\beta]$$

$$= \sum_{\alpha\beta}\sum m_{\beta\alpha} r_\beta \pi^{\alpha'} G \pi^\alpha - \sum_\beta r_\beta (\sum_\alpha m_{\beta\alpha} \pi^\alpha)' G (\sum_\alpha m_{\beta\alpha} \pi^\alpha)$$

$$= \sum_\beta r_\beta [\sum_\alpha m_{\beta\alpha} (\pi^\alpha)' G (\pi^\alpha) - (\sum_\alpha m_{\beta\alpha} \pi^\alpha)' G (\sum_\alpha m_{\beta\alpha} \pi^\alpha)]$$

$$\leq 0$$

The non-positivity of the last bracketed expressions for each β follows from $\pi' G \pi$ being a concave function of π since G is negative semi-definite, and from Jensen's inequality since $\sum_\alpha m_{\beta\alpha} = 1$. Hence $\Delta V^* \geq 0$ as expected.

Suppose now that an individual is faced with an array of possible inforation structures from which to choose. Assume that the array is parametrized by a scalar ρ which unambiguously ranks the structures according to informativeness. Define his marginal value of information as

$$\text{MVI} = \lim_{\Delta\rho \to 0} \frac{V^*_{\rho+\Delta\rho} - V^*_{\rho}}{\Delta\rho}.$$

Suppose further that the cost of acquiring an information structure rises with ρ and is given by a cost function $c(\rho)$. The marginal cost of acquiring information is

$$\text{MCI} = \lim_{\Delta\rho \to 0} \frac{c(\rho+\Delta\rho) - c(\rho)}{\Delta\rho}.$$

When both such expressions are well defined, one can characterize the optimal choice of ρ, finding that level for which $V^*_p - c(\rho)$ is at a maximum, as finding the information structure for which MCI $=$ MVI. A solution could be depicted as in Figure 2. We next look briefly at the parametrizations possible in specific contexts.

EXAMPLE: Increase in precision of unknown parameter estimate.

Suppose an agent's utility depends on an unknown parameter s which can take on values 1, 2, ..., n. His prior beliefs about s are uniform: i.e., $\pi^\circ_s = 1/n$ for all s. The information structures available all have as possible messages an integer $\alpha = 1, 2, \ldots, n$ which gives the agent a maximum likelihood estimate of the true value of s. All messages are expected to occur with the same probability $q_\alpha = 1/n$. The agent's posterior beliefs about s upon receiving message α be described by

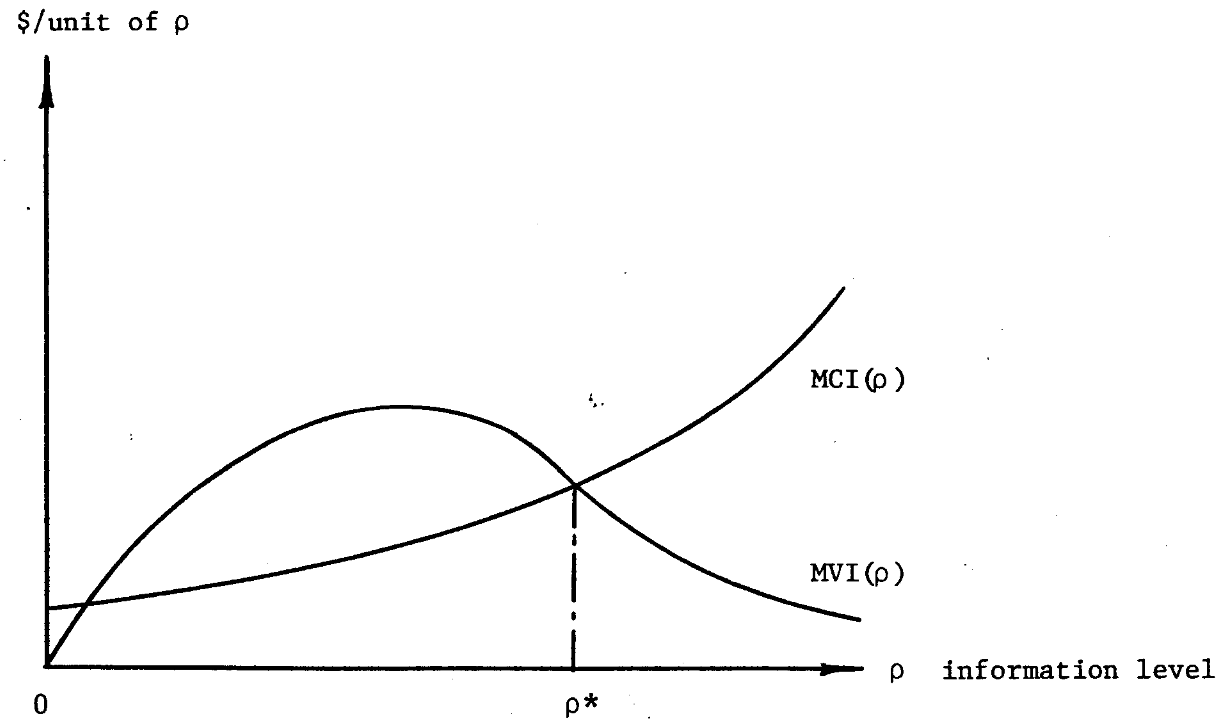

FIGURE 2: Choice of Optimal Level of Information

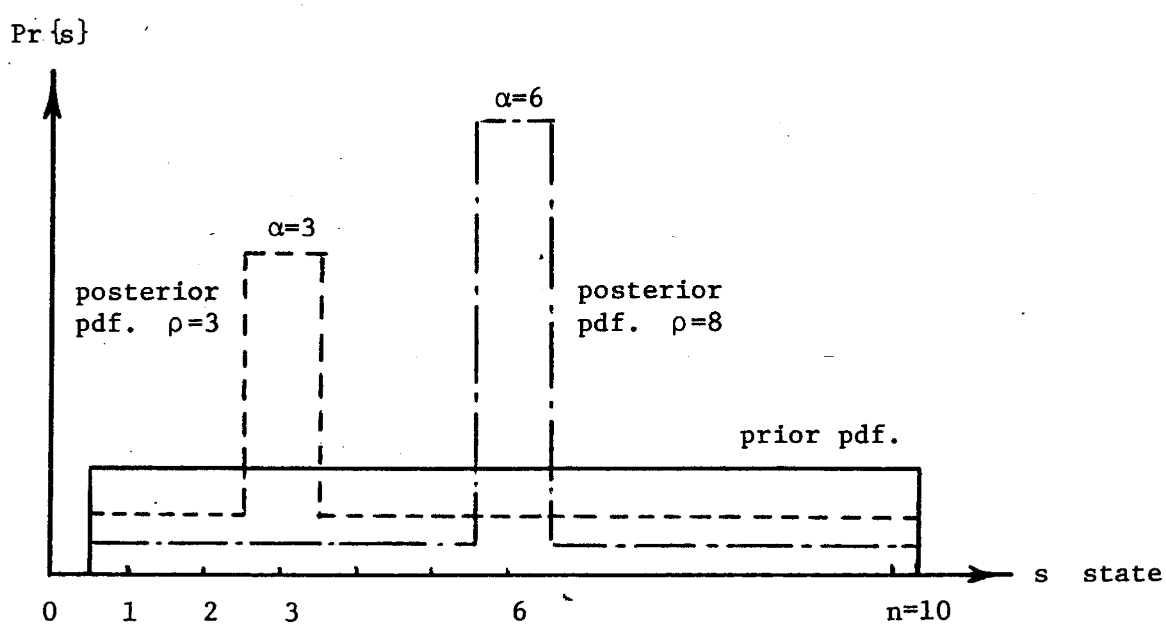

FIGURE 3: Prior and Posterior Probability Density Functions

$$\pi_s^\alpha = \begin{cases} \frac{1+\rho}{n+\rho} & \text{if } s = \alpha \\ \frac{1}{n+\rho} & \text{if } s \neq \alpha \end{cases}$$

where $\rho \geq 0$ is an informativeness parameter. The agent's probability density function and posterior density for selected values of α are depicted in Figure 3. When $p = 0$, the message provided is completely uninformative since $\pi^\alpha = \pi^\circ$; when $\rho \to \infty$ the message approaches perfectly informative giving certain knowledge of s. Higher ρ is associated with more peaked posterior density functions on s; the *precision* of α as an estimator of s is higher.

Consider two information structures A and A' in this family associated with information levels ρ and ρ' respectively. Substituting the expressions for π_s^α into (42) for the difference in information values results, with some manipulation, in

(44) $$V_A^* - V_{A'}^* = V_{A'}^* \cdot \{(\frac{\rho(u+\rho')}{\rho'(u+\rho)})^2 - 1\}.$$

For this example the *relative* values of two information structures depends only on the relative values of ρ, ρ' and n. Any change which increases the value of information structure A' increases the value of any increment in information by the same factor.

Let $\rho' - n$ and $V_{A'}^*$ be a benchmark level of information value. Solving (44) for V_A^* as a function of ρ gives

(45) $$V_A^* = 4V_{A'}^*(\frac{\rho}{n+\rho})^2.$$

The *marginal value* of additional information, as parametrized by ρ, is then

$$\text{(46)} \qquad \text{MVI} = \frac{dV_A^*}{d\rho} = \frac{8\rho n V_{A'}^*}{(n+\rho)^3} .$$

This curve has essentially the shape of MVI as graphed in Figure 2: MVI equals 0 at ρ=0, rises with ρ, then tapers off asymptotically toward 0.

EXAMPLE: Increase in the power of a statistical test.

Suppose an agent commissions research to statistically test a hypothesis, say, that smog is harmful to health. Let there be just two possible true states to the world: smog is harmless (null hypothesis, s = 1) or it is harmful (s=2). The outcome of the research may be either of two possible messages: accept the null hypothesis (α=1) or reject it (α=2). Assume the agent has given prior probabilities π_1°, $\pi_2^\circ = 1-\pi_1^\circ$, of smog being harmless or not, and must choose the quality of experiment to be performed.

For notational simplicity we confine our attention to experiments for which the prior probability of accepting the null hypothesis is the same as the prior probability that the null hypothesis is true, i.e., $q_1 = \pi_1^\circ$, $q_2 = \pi_2^\circ$. The available experiments are parameterized by their likelihood functions. Let $\lambda_{s\alpha}$ be the likelihood that outcome α is received given that s is the true state. Then λ_{22} is the *power* of the statistical test performed, or likelihood smog will be found harmful if it truly is so. λ_{12} is the *significance level* of the test performed, or likelihood that the null hypothesis is erroneously rejected when it is true. Our requirement of fixed prior probability of rejection together with the probability identity $q_2 = \pi_1^\circ\lambda_{12} + \pi_2^\circ\lambda_{22}$ implies that a fall in significance level accompanies a rise in power. In fact the prior probabilities of type I and type II errors in inference being committed are both equal to $\pi_2^\circ(1-\lambda_{22})$. Notice that a power of $\lambda_{22} = \pi_2^\circ$ can be achieved, consistent with our requirement that $q_2 = \pi_2^\circ$, simply by flipping a biased coin which has probability π_2° of coming up $\alpha = 2$. Thus we will rank the available experiments according to the amount by which λ_2 exceeds π_2°. A perfect experiment has $\lambda_{22} = 1$; there is no chance of erroneous inference. A useless experiment has $\lambda_{22} = \pi_2^\circ$; there is zero correlation between the state and the message. Take care to note that this is not the usual problem of trading off probabilities of type I and type II errors by choice of significance level for a given experiment. What is to be valued is a reduction in the probabilities of both errors. Furthermore any subsequent decision made on the basis of the experimental outcome will be made with full knowledge of the probabilities these errors were committed.

The conditional probabilities π_s^α can be computed from the given data as

$$\pi_1^1 = \frac{\pi_1^\circ-\pi_2^\circ(1-\lambda_{22})}{\pi_1^\circ} \qquad \pi_1^2 = 1-\lambda_{22}$$

(47)

$$\pi_2^1 = \frac{\pi_2^\circ(1-\lambda_{22})}{\pi_1^\circ} \qquad \pi_2^2 = \lambda_{22}.$$

These conditional probabilities can be substituted into $V_A^* = -\frac{1}{2}\sum_\alpha q_\alpha(\pi^\alpha - \pi^\circ)'G(\pi^\alpha - \pi^\circ)$ to obtain a rather complicated expression for the value of the experiment as a function of its power λ_{22}. Upon differentiation with respect to λ_{22}, we get the representation of the marginal value of information in this context:

(48) $$\frac{dV_A^*}{d\lambda_{22}} = -\frac{1}{2}\left(\frac{\pi_2^\circ}{1-\pi_2^\circ}\right)(\lambda_{22} - \pi_2^\circ)(g_{11}-g_{12}-g_{21}+g_{22}).$$

The terms g_{ij} are elements of the matrix $G = p'_\pi x_p^c p_\pi$. The last parenthesized expression can be shown to be non-positive since G is negative semi-definite. The term $\lambda_{22} - \pi_2^\circ$ is non-negative for reasons discussed earlier. Hence the marginal value of higher power is positive.

As in the previous example, the marginal value of information, as measured by $\rho \equiv \lambda_{22} - \pi_2^\circ$, is zero when the information level is zero. But with this parametrization of information structures MVI rises linearly with ρ until perfect information is obtained. This phenomenon of *rising* MVI at first appears paradoxical since it is contrary to economists' presumptions about the demand for more usual goods. An intuitive explanation for this is as follows: The greater the reliability of information already to be acquired (larger is λ_{22}), the greater will be the consumption response to that information, and more "costly" will be the receipt of a misleading message. Hence the value of further reducing the likelihood of such messages rises.

2. INFORMATION AND PRICE ADJUSTMENT

As analysed in chapter 1 of this report, information is valuable to the extent that consumption plans change with the message received. Loosely, the greater the optimal adjustment to the different messages the more an individual is willing to pay ex-ante for the provision of the information. Ignored, however, is the possibility that the receipt of information will have significant price effects.

Implicit in such a formulation is the assumption that prices are largely determined by cost conditions rather than the intersection of supply and demand curves. While this is a natural first approximation for a variety of applications it is particularly inappropriate for non-produced commodities of uncertain quality. One important case is the adjustment of land prices to reflect differences in air quality in an urban environment. It is this case that we shall focus on in the following sections.

We begin in section 2.1 by illustrating the implications of price adjustment on the value of information for a simple exchange economy. It is shown that *all* agents in an economy may be made worse off by the announcement that the true quality of a product will be made known prior to trading. Essentially the anticipation of information introduces an additional distributive risk which reduces each individual's expected utility. It is shown that each agent would prefer to engage in a round of trading prior to the revelation of product quality, thereby insuring himself against an undesirable outcome.

Then in section 2.2 a simple urban model is developed in which a fixed number of individuals must be located in two regions. The equilibrium allocation of individuals is first examined. Simple sufficient conditions for higher income groups to locate in the preferred environment are established

Surprisingly, it is shown that under not implausible alternative conditions *both* tails of the income distribution may locate in the preferred environment.

In an appendix at the end of the chapter the closely related question as to the optimal redistribution of income is considered. Starting with income equally distributed it is shown that optimization of a symmetric social welfare function in general requires an income transfer from those living in one zone to those in the other. Under the conditions which imply that in equilibrium the rich will locate in the better environment, it is optimal to transfer income to those in the better environment from the remainder of the population! The intuition behind this paradoxical conclusion is then developed.

Finally section 2.3 focusses on the implications of conducting research to resolve uncertainty about the nature of the environmental hazard. For the special case in which the marginal utility of income is constant (implying no aversion to income risk) it is shown that the value of such research can, in principle, be inferred from land value data.

2.1 INFORMATION ABOUT PRODUCT QUALITY WITH NEGATIVE SOCIAL VALUE

In the previous chapter information about product quality increases the expected utility of each consumer. However, when allowance is made for price adjustments this is no longer necessarily the case. Indeed, it is quite possible that if such information is made available prior to any trading, utility of every agent may be strictly lower!

To illustrate this point, consider a two person economy in which aggregate endowments of two commodities, X and Y, are fixed and equal to unity. Both individuals have utility functions of the form

$$u(x_i,y_i;\theta) = (\theta x_i)^{1/2} + y_i^{1/2} \quad i = 1,2.$$

where θ is a parameter reflecting the 'quality' of the product. Prior to trading θ is unknown but both individuals believe that with equal probability θ takes on the values 0 and 1.

Then the expected utility of agent i is

$$(1) \qquad U^{\circ}(s_i,y_i) = \underset{\theta}{E}u(x_i,y_i;\theta) = \frac{1}{2}\, x_i^{\;1/2} + y_i^{\;1/2}$$

Without loss of generality we may set the price of y equal to unity. Then each agent chooses (x_i,y_i) to maximize U° subject to a budget constraint

$$px_i + y_i \leq p\bar{x}_i + \bar{y}_i$$

where $(\bar{x}_i,\bar{y}_i)$ is the agent's endowment.

Given our assumption on U°, the competitive equilibrium price of X is unique and equal to 1/2. Then if agent 1 has the entire endowment of X and agent 2 the entire endowment of Y, the final consumption vectors are given by [1]

$$(x_1,y_1) = (1/3,1/3) \text{ and } (x_2,y_2) = (2/3,2/3).$$

Substituting into (1) the expected utility levels of two agents are therefore

(2) $$U_1^\circ = \left(\frac{3}{4}\right)^{1/2} \quad \text{and } U_2^\circ = \left(\frac{3}{2}\right)^{1/2} .$$

We now contrast this with the final allocations when research is to be conducted which will provide consumers with full information prior to trade.

For our simple illustration there are only 2 possible outcomes, $\theta = 0$ or $\theta = 1$. If the former, the endowment of agent 1 is valueless hence there can be no trade *ex post*. The utility levels of the two agents are therefore

(3) $$u_1(=0) = 0 \quad \text{and} \quad u_2(=0) = 1$$

If $\theta=1$ each agent has an *ex post* utility function

$$u_i = x_i^{1/2} + y_i^{1/2}$$

Given the symmetry of endowments and preferences it follows directly that the equilibrium price of X is unity and that both agents consume half the aggregate endowment. Then

(4) $$u_1(\theta=1) = u_2(\theta=1) = \left(\frac{1}{2}\right)^{1/2} + \left(\frac{1}{2}\right)^{1/2} = 2^{1/2}$$

The question we wish to examine is whether either of the agents would place a positive value on completion of the research *before* knowing

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[1] For each agent the marginal rate of substitution of X for Y, $\frac{\partial U^\circ}{\partial x_i} \Big/ \frac{\partial U^\circ}{\partial y_i} = \frac{1}{2}\left(\frac{y_i}{x_i}\right)^{1/2}$. Setting this equal to the price p we have $y_i = 4p^2 x_i$. Hence $\sum_i y_i = 4p^2 \sum_i x_i$. Since the aggregate endowments of X and Y are both unity it follows that $p = 1/2$ and hence that $y_i = x_i$. Substituting into the budget constraint then yields the final consumption vectors.

the outcome. Since each places an equal probability on the two possible states, expected utility with the research is

$$U_i^* = \sum_\theta \pi_\theta u_i(\theta) \quad i = 1,2$$

Substituting from (3) and (4) we have

$$(5) \qquad U_1^* = \left(\frac{1}{2}\right)^{1/2} \quad \text{and} \quad U_2^* = \frac{1+2^{1/2}}{2}$$

Comparing (5) and (2) it follows immediately that the expected utility of both agents is lower with the provision of information.

Similar results have also been demonstrated by Hirshleifer (1971) and Fama and Laffer (1971) using a one commodity model with state contingent claims. As Hirshleifer points out, the prospect of information prior to trading creates uncertainty about the distribution of the aggregate endowment. This tends to reduce the expected utility of risk averse agents. Thus, in the absence of opportunities for insurance against such risk, suppression of the information can make every agent better off *ex ante*.

Alternatively, suppose agents can choose to conduct a preliminary round of trading prior to release of the results of the research. To analyse such trading it is necessary to make some assumption about the prices that consumers believe will prevail when the true state is known. Here it will be assumed that the future spot price of X is correctly predicted by each agent, that is, expectations are 'rational'. For our example the future spot price of X is dependent upon aggregate endowments but not on their distribution. It is therefore a straightforward matter to establish that the future spot price of X, $p(\theta)$, is given by

$$(6) \qquad p(\theta) = \theta, \; \theta = 0,1$$

If the spot price of X is $\hat{p}$, agents can select bundles (x_i, y_i) satisfying

$$(7) \qquad \hat{p}x_i + y_i = \hat{p}\bar{x}_i + \bar{y}_i$$

When the state is announced the agent then makes a second round of exchanges subject to the constraint

(8) $$p(\theta)x_i(\theta) + y_i(\theta) = p(\theta)x_i + y_i \qquad \theta = 1,2.$$

But if $\theta = 0$ the future spot price $p(\theta) = 0$. It follows that there will be no trading after the announcement, that is

$$(x_i(0),y_i(0)) = (x_i,y_i) \qquad i = 1,2$$

if $\theta = 1$ the future spot price, $p(\theta) = 1$. Given the symmetry of the indifference curves each agent will trade in such a way as to equalize his spending on two commodities.

$$\text{Then } (x_i(1),y_i(1)) = \left(\frac{x_i + y_i}{2}, \frac{x_i + y_i}{2}\right)^{1/2} \qquad i = 1,2$$

Expected utility of agent i is therefore

$$U(x_i,y_i) = \frac{1}{2}y_i^{1/2} + \left(\frac{x_i + y_i}{2}\right)^{1/2} \qquad i = 1,2$$

With a spot price of $\hat{p}$, agent i chooses x_i and y_i to maximize $U(x_i,y_i)$ subject to his budget constraint (7). The first order condition for expected utility maximization is therefore,

(9) $$\frac{\frac{\partial U}{\partial x_i}}{\frac{\partial U}{\partial y_i}} = \frac{\left(\frac{x_i}{y_i} + 1\right)^{-1/2}}{2^{-1/2} + \left(\frac{x_i}{y_i} + 1\right)^{-1/2}} = \hat{p}$$

It follows that $\frac{x_i}{y_i}$ is the same for both agents, hence equal to $\frac{\Sigma x_i}{\Sigma y_i} = 1$. Then from (9) $\hat{p} = 1/2$. Substituting into the budget constraint (7) it follows that

$$(x_1,y_1) = (1/3,1/3) \quad \text{and} \quad (x_2,y_2) = (2/3,2/3)$$

But this is exactly the consumption achieved by each agent in the absence of the information. Therefore the prior trading just eliminates the undesired distributive risk, and the expected value of the information is zero.[2]

A central feature of this and the earlier results is that agents correctly anticipate the price implications of the state revealing message. If consumers are unaware of these implications the analysis of section 1 applies. Each will therefore place a positive value on the information.

Of course it is a long leap from this simple example to a general proposition. However, it does seem reasonable that there will, in general, be a tendency for price adjustments to offset the anticipated gains associated with better information. Thus except in cases where there are solid grounds for arguing that prices are cost determined, the expressions for the value of information developed in section 1 seem likely to overstate true value.

[2] It should be noted that this result is not a general one. If individuals assign different probabilities to the two states or have different preferences, at least one individual will have a higher expected utility with trading before and after announcement of the true state. Moreover, by an appropriate redistribution of income both can be made better off in our ex-ante sense.

2.2 URBAN LOCATION AND LAND VALUES WITH ENVIRONMENTAL HAZARDS

One very important case in which price adjustment to a change in information is central, is that of urban location. In general, the price of land in any particular neighborhood will reflect the perceived quality of amenities and the surrounding environment. This insight has led to a considerable amount of empirical research attempting to place a dollar valuation on environmental difference by measuring difference in property values. For example Anderson and Crocker (1971), Lave (1972) and Wieand (1973) obtain estimates of the value of increasing air quality. More recently Freeman (1974, 1975) Polinsky and Shavell (1975) and Polinsky and Rubinfeld (1977) have further developed the theoretical underpinnings implicit in these empirical studies. One of the main points made by the latter group of authors is that, an improvement in the environment in one locality generates ~~in~~-migration which tends to lower land values elsewhere. Estimating local effects will therefore tend to overestimate the value of such improvements. However, if the locality is sufficiently small relative to the total urbanized area, changes in land values will indeed reflect willingness to pay for the environmental change.

Throughout our discussion of property values we shall adopt this simplifying assumption. In the pages following we set the stage by considering an urban area which is divided into two zones. In the first zone there is an uncertain environmental hazard. In the other zone (zone 0) which is assumed to be very large relative to zone 1, there is no such hazard. As a first step in the analysis we examine land holdings in the two zones under the assumption that preferences and incomes are identical. We then consider the location of individuals with different incomes.

It is shown that under not implausible assumptions it may be the higher income groups who locate in the hazardous zone! However, there is a slight presumption that it is more likely for the opposite to be the case.

This result is very relevant to the discussion in section 2.3 of the effects of resolving uncertainty about the environmental hazard. Suppose, for example, that research were to indicate that the environmental effects were less important than had been anticipated. This would make locating in the hazardous zone more attractive and result in a capital gain for those initially residing in the zone. Therefore, even if individuals begin with identical incomes and wealth, the information generates wealth differences and hence possible relocation.

Turning now to the basic model, let the utility of each individual be a concave function $U(x,y)$ of the area of his residence x and other consumption y. If there is any environmental hazard $\tilde{h}$ his utility drops to $U(x,y) - \tilde{h}$. Then, with uncertainty about the hazard, the expected utility of an agent locating in the hazardous zone 1 is

$$U(x_1, y_1) - \bar{h}$$

Suppose each individual purchases land from some outside landowner and all have identical incomes.[3] Let p_i be the price of a unit of land in zone i. For those locating in zone 0 the utility level achieved is

$$V(p_o, I) = \max_{x,y}\{U(x,y) \mid p_o x + y = I\} \tag{10}$$

Similarly for those locating in the first zone the (expected) utility level achieved is

$$V(p_1, I) - \bar{h} = \max_{x,y}\{U(x,y) \mid p_1 x + y = I\} - \bar{h} \tag{11}$$

[3]Alternatively the city owns the land and reimburses rents in excess of the agricultural value of the land in a lump sum manner.

In the absence of constraints on land purchases, the value of land in the hazardous zone must fall until utility is equated in the two zones. This is depicted in Figure 2.1.

At the level of an individual consumer, one measure of the cost of the smog is the extra income H that a person living in the second zone would have to be given in order to make him willing to move at *constant* prices. In formal terms this is the Hicksian compensation required to maintain the utility level of an individual in the hazardous zone at the higher land value p_o, that is

$$(12) \qquad V(p_o, I + H) = V(p_1, I) = V(p_o, I) + \bar{h}$$

This is also depicted in Figure 2.1.

With this background we can now ask which individuals live where, if incomes are not equally distributed. For expositional ease we shall restrict out attention to utility functions that are homothetic. Suppose that income is distributed continuously. Then for some income level $\hat{I}$ individuals will be indifferent between living in the two zones. We therefore have

$$V(p_o, \hat{I}) = V(p_1, \hat{I}) - \bar{h}.$$

An individual with income $I > \hat{I}$ locates in zone 0 if and only if

$$V(p_o, I) > V(p_1, I) - \bar{h}$$

Consider Figure 2.2. Those with incomes of $\hat{I}$ are indifferent between C_1 and C_o and hence between C_1' and C_o. Then

$$(13) \qquad V(p_o, \hat{I}) = V(p_o, \hat{I} + \hat{H}) - \bar{h}.$$

Moreover given our assumption that those with incomes of $I > \hat{I}$ locate in zone 0, they must prefer D_o to D_1, and hence prefer D_o to D_1'. Then

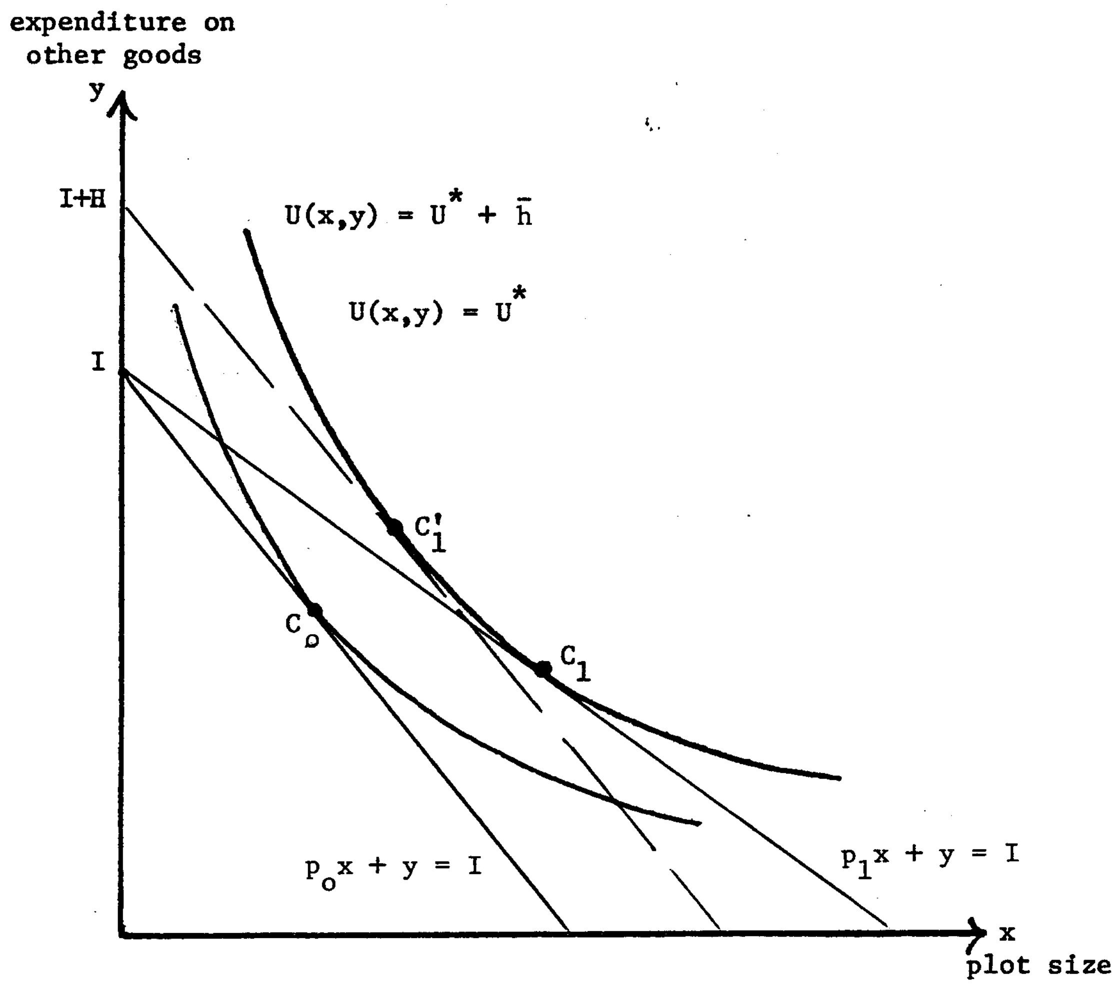
expenditure on
other goods
y
I+H
I
U(x,y) = U* + h̄
U(x,y) = U*
C'1
C1
Co
pox + y = I
p1x + y = I
x
plot size

Figure 2.1.

(14) $$V(p_o, I) > V(p_o, I + H) - \bar{h}$$

Combining (13) and (14) the higher income group prefer zone 0 if and only if

(15) $$V(p_o, I + H) - V(p_o, I) < V(p_o, \hat{I} + \hat{H}) - V(p_o, \hat{I})$$

For the special case of homothetic preferences depicted in Figure 2.2 we also have

$$\frac{OC_1'}{OC_1} = \frac{OD_1'}{OD_1}$$

Moreover, $$\frac{OC_1}{OD_1} = \frac{\hat{I}}{I} \quad \text{and} \quad \frac{OC_1'}{OD_1'} = \frac{\hat{I} + \hat{H}}{I + H}$$

It follows immediately that

$$\hat{H}/\hat{I} = H/I$$

We may therefore rewrite the necessary and sufficient condition (15) as

(15)' $$V(p_o, (\frac{\hat{I} + \hat{H}}{\hat{I}})I) - V(p_1, I) < V(p_o, \hat{I} + \hat{H}) - V(p_o, \hat{I})$$

Note that the left and right hand sides of (15)' are equal for $I = \hat{I}$. Then a sufficient condition for all those with higher incomes to prefer zone 0 is that the left hand side of (15)' be decreasing in I, that is

(16) $$\frac{1}{\hat{I}} [(\hat{I} + \hat{H}) V_I(p_o, \hat{I} + \hat{H}) - \hat{I} V_I(p_o, \hat{I}] < 0$$

In turn a sufficient condition for inequality (16) to hold for the required H_o is that it should hold for any $\hat{H}$. But this is the case if

$$\frac{\partial}{\partial I}[IV_I(p_o, I)] < 0$$

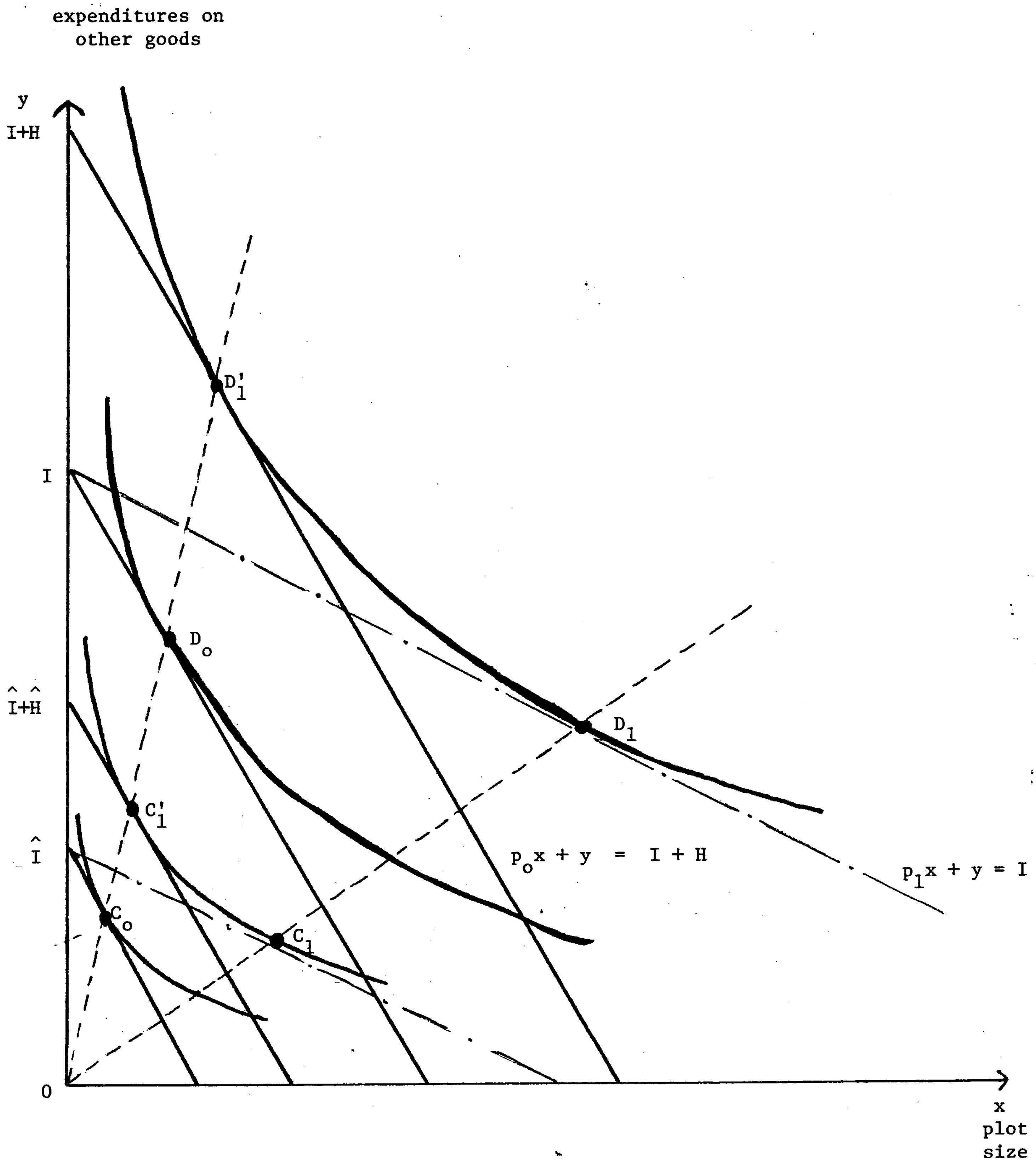
expenditures on
other goods
y
I+H
I
$\hat{I}+\hat{H}$
$\hat{I}$
0
D_1'
D_o
D_1
C_1'
C_o
C_1
$p_o x + y = I + H$
$p_1 x + y = I$
x
plot
size

Figure 2.2

that is

$$(17) \qquad \frac{-IV_{II}}{V_I} > 1$$

Thus with homothetic preferences a sufficient condition for the higher income groups to prefer the no-hazard zone is that the income elasticity of the marginal utility of income be greater than unity. Conversely, if each of the above inequalities is reversed, it follows that with homothetic preferences a sufficient condition for the higher income groups to prefer the hazardous zone is that the elasticity of marginal utility be less than unity.

We now note that this elasticity is also the coefficient of relative aversion to income uncertainty. Arrow (1971) has argued that the latter must be in the neighborhood of unity and increasing in income. Accepting this conclusion it follows that there is no clear presumption that income and environmental quality will be positively correlated. Indeed if relative risk aversion is less than unity for low incomes, and rises above unity as income increases it is possible for an equilibrium configuration with high and low income groups sharing the no hazard zone and middle income groups in the hazardous zone.

Of course this conclusion is very much dependent upon the underlying assumptions. Suppose that instead of entering additively, the environmental affects are multiplicative. That is, with the environment affected by an amount h expected utility is

$$u(x,y)\ \overline{w(h)}$$

where $w(o) = 1$ and $w'(h) < 0$.

Each consumer chooses x, y and his location to maximize expected utility or, equivalently, the logarithm of this utility, that is

$$\ln u(x,y) + \ln \overline{w(h)}$$

Setting $U(x,y) = \ln u(x,y)$ the problem becomes equivalent to the one already analysed. Therefore higher income groups will live in the less hazardous zone if the relative risk aversion of an individual with a utility function $\ln u(x,y)$ exceeds unity. Since $\ln(\cdot)$ is a strictly concave function, this individual's relative risk aversion exceeds that of an individual with a utility function $u(x,y)$. Therefore the sufficient condition is weakened and the presumption that higher income individuals will live in the less environmentally affected area is strengthened.

2.3 UNCERTAIN ENVIRONMENTAL QUALITY AND THE PROSPECT OF BETTER INFORMATION

In the previous section individuals made consumption and location decisions based on a given amount of information about an environmental hazard. We now ask how much society would be willing to pay for research which would resolve (or at least reduce) uncertainty about the hazard. For expositional ease we continue with the assumption that the hazardous zone is small relative to the rest of the urban area. Then, to a first approximation, land value and hence utility in the latter is unaffected by such information.

To capture the idea that information will arrive at a future date we extend the model to two periods. In the first period individuals lease their land holdings for two periods. At the end of this period the results of the research are announced. Individuals may then sublet land for the final period. While not essential to the analysis, it is assumed that there is no discounting and that the market interest rate is zero. It is also assumed that all individuals are paid the same income I per period. In the absence of the research, equilibrium is exactly as described in the previous section. The rental rate of land in zone 1, p_1, is just low enough to equate utility levels in each period and hence to equate lifetime utility levels. Formally, p_1 must satisfy

$$V(p_1,I) - \bar{h} = V(p_o,I) \tag{18}$$

Next suppose that individuals lease land knowing that information will be available at the end of the first period revealing the true state of the environmental hazard h_s. Let the per period rental rate in zone 1 be $\hat{p}$. Once h_s is known the land rental rate, p_s, will adjust to reflect the new information. Suppose that $\hat{p} = p_1$ and land holdings in zone 1, $\hat{x} = x_1$.

That is, the first period allocation and rental is exactly the same as with no information. This is depicted in Figure 2.3. For any realization h_s such that $p_s \neq p_1$ agents located in zone 1 can choose a consumption bundle (x_s, y_s) such that

$$U(x_s, y_s) > U(x_1, y_1)$$

Then the expected second period utility of an agent remaining in zone 1

$$\underset{s}{E}\ \{U(x_s, y_s) - h_s\}$$

$$> U(x_1, y_1) - \underset{s}{E}\ h_s$$

$$= U(x_o, y_o)$$

Therefore lifetime expected utility is slightly higher for an agent initially choosing zone 1. But this is impossible since every agent has a chance to purchase land in this zone. It follows that with the prospect of information, land rentals in zone 1 are bid up to some level $\hat{p}$ greater than p_1.

Formally we have

Proposition 1: The prospect of receiving information about the nature of the environmental hazard raises the pre-information land rental rate and hence the price of land in the hazardous zone.

Since all consumers have the same expected utility whether or not the information is released, the only group gaining from its provision are the original land owners. Thus the value of the research is precisely reflected in the difference between land values before and after the announcement that the research will be conducted. However, this by itself is not particularly useful since a decision as to whether the research costs are justified must be made *prior* to any such announcement. It is therefore necessary to consider more closely the locational equilibrium conditions.

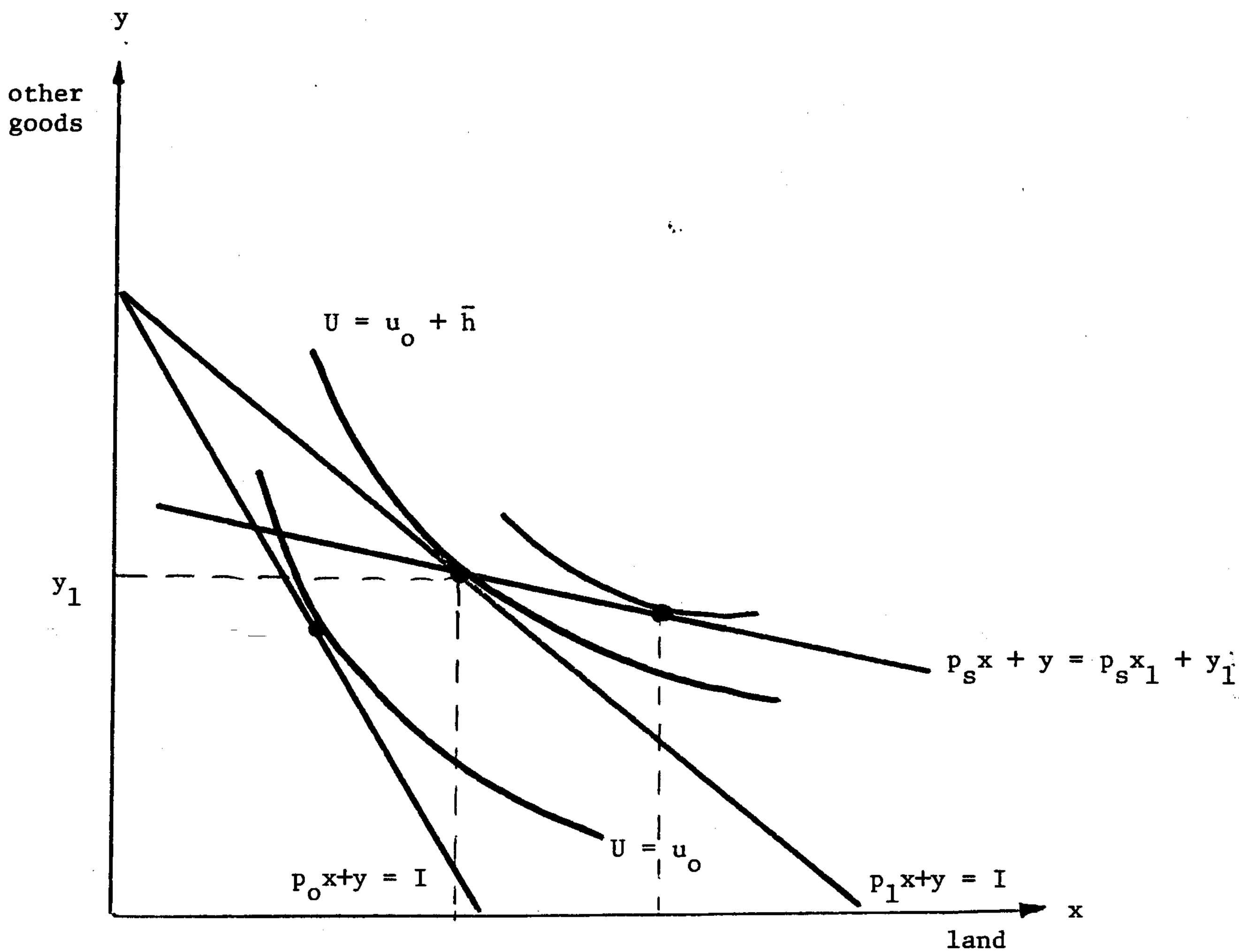

Figure 2.3: Learning About the True State

Suppose first that the research results have the effect of raising land rents in the hazardous zone. Then those taking a two period lease have a wealth gain of $(p_s-\hat{p})\hat{x}$. This increase in wealth creates a difference in hazard valuation between individuals located in the different zones. In the previous section it was shown that there is a mild presumption that higher income individuals will locate in the no-hazard zone. This being the case *all* those initially in zone 1 will relocate in zone 2. Those initially in zone 0 will enter zone 1 bidding up the price until the cost of land equates utility, that is

$$V(p_s,I) - h_s = V(p_o,I); \quad p_s > \hat{p} \tag{19}$$

Next suppose that the research results have the effect of reducing land rents in zone 1. This time those in zone 1 are poorer as a result of the announcement. Hence the more wealthy "outsiders" remain in zone 0. In addition enough of those in zone 1 leave to equalize second period utility, that is

$$V(p_s,I'' + (p_s-\hat{p})\hat{x}) - h_s = V(p_o,I'' + (p_s-\hat{p})\hat{x}) \tag{20}$$

where I'' is second period income. In principle expressions (19) and (20) can be used to solve for the second period rental p_s in each possible state s, as a function of initial land holdings, $\hat{x}$, rental rate $\hat{p}$ and second period income I''

$$p_s = p_s(\hat{x}, \hat{p}, I'') \tag{21}$$

An individual locating in zone 1 must choose at the beginning of period 1 not only x, but also how much income is to be spent in that period. Acting as a price taker this individual solves the following lifetime optimization problem

$$\underset{\hat{x},I'I''}{\text{Max}} \quad \{U_1^L \mid I' + I'' = 2I\}$$

where $U_1^L = U(\hat{x},I'-\hat{p}\hat{x}) - \bar{h} + \text{Prob}(p_s > \hat{p}) \underset{s}{E} \{V(p_s,I'') \mid p_s > \hat{p}\}$

$$+ \text{Prob}(p_s < \hat{p}) \underset{s}{E} \{V(p_s,I'' + (p_s-\hat{p})\hat{x}\} - \bar{h}$$

From the first order conditions it is in principle possible to solve for $\hat{x}$ and I" as a function of p_s and $\hat{p}$ combining these results with (21) yields expressions for all other unknowns in terms of the first period rental $\hat{p}$. The final step is to choose $\hat{p}$ to equate expected utility in the two zones.

Such a procedure will in general require non-analytical methods. However, by making the assumption that the marginal utility of income is constant, the problem is greatly simplified. For analytical convenience we shall consider the special case in which the utility function U(x,y) is given by

(22) $$U(x,y) = \alpha \ln x + y$$

The indirect utility function is then

(23) $$V(p,I) = -\alpha \ln p - \alpha + I$$

In the absence of the research the land rental rate in the hazardous zone, p_1, is given by (18). We therefore have

(24) $$p_1 = p_o e^{-\bar{h}/\alpha}$$

Moreover, from (19) and (20) the second period land rental rate must satisfy

(25) $$p_s = p_o e^{-h_s/\alpha}$$

Since e^{-z} is a convex function of z we have from Jensen's Inequality

Proposition 2: If $U(x,y) = \alpha \ln x + y$. The prospect of information revealing the true state at the end of the first period raises the expected second period land rental in the hazardous zone, above that which would prevail in the absence of such information.

Conditions (19) and (20) require that those at the appropriate income level are indifferent between the two locations. However for the special case the indirect utility function is additively separable. Therefore *all* individuals must be indifferent between locating in zone 0 and zone 1. We may therefore contrast the utility of an agent choosing to locate and remain in zone 1 with that of an agent spending his lifetime in zone 0. Expected utility of the former is

(26) $$U_1^L = \underbrace{\alpha\ln\hat{x}+I'-\hat{p}\hat{x}-\bar{h}}_{\substack{\text{first period}\\ \text{expected utility}}} + \underbrace{\underset{s}{E}\{-\alpha\ln p_s-\alpha+I''+(p_s-\hat{p})\hat{x}\}-h_s}_{\substack{\text{second period}\\ \text{expected utility}}}$$

Since U_1^L is linear in income the only choice variable is the size of the period 1 land holdings. Differentiating U_1^L with respect to $\hat{x}$ yields

(27) $$\frac{\alpha}{\hat{x}} = 2\hat{p}-\bar{p} \qquad \text{where} \qquad \bar{p} = \underset{s}{E}p_s$$

Substituting this back into (26) and making use of (25) we have

(28) $$U_1^L = -2\alpha\ln\alpha - 2\alpha - \bar{h} + 2I - \alpha\ln(2\hat{p}-\bar{p}) - \alpha\ln p_o$$

The utility of an individual living always in zone 0 is

$$U_1^L = 2(-\alpha\ln\alpha - \alpha + I - \alpha\ln p_o)$$

Therefore for locational equilibrium in period 1 it is required that

$$-\bar{h} - \alpha\ln(2\hat{p}-\bar{p}) - \alpha\ln p_o = -2\alpha\ln p_o$$

Rearranging we have finally,

$$\begin{aligned} 2\hat{p} &= \bar{p} + p_o e^{-\bar{h}/\alpha} \\ &= \bar{p} + p_1 \qquad \text{from (24)} \end{aligned}$$

Hence

(29) $$\hat{p} = \frac{1}{2}(\bar{p}+p_1) > p_1 \qquad \text{from Proposition 2}$$

Formally, we have

Proposition 3: If $U(x,y) = \alpha \ln x + y$ the prospect of information revealing the true state at the end of the first period raises the land rental rate to the mean of the rental rate with no information and the expected second period rate.

With a zero interest rate the value of land in period 1 is simply the sum of the land rental rates. Using capital letters to denote land values we then have, from (29),

$$\hat{P} - P_1 = \bar{p} - p_1 \tag{30}$$

The value of the research is therefore equal to the difference between the expected future rental rate with the information and the rental rate in the absence of this information. The latter is the prevailing rental rate prior to consideration of the research proposal. The former can, in principle, be inferred from environmental and land rental data in other localities.

More generally, with an infinite time horizon, a market interest rate and private discount rate of r, and research which will reveal the true state after n periods we have

$$\hat{P} - P_1 = \frac{1+r}{r}\left[p_1 + \frac{1}{1+r(1+r)^{n-1}}(\bar{p}-p_1)\right] \tag{31}$$

It is a straightforward matter to establish that $\hat{P} - P_1$ is a decreasing convex function of n. Thus the smaller the lag between the decision to conduct the research and the announcement of the results, the larger the marginal value of a further reduction in this lag.

Appendix: OPTIMAL URBAN LOCATION

In section 2.2 the positive implications of urban environmental differences were examined. It turns out that there are also rather puzzling normative implications, at least if one adopts the usual approach of maximizing a symmetric social welfare function. Suppose that initially all individuals have the same income. Some locate in a hazardous 'smoggy' zone (hereafter $h_s=s$). A naive view might be that those living in the smog should be compensated by an income transfer from those in the smog-free zone. Not so, an economist would almost certainly respond. If individuals are free to move from one zone to the other, land values will adjust to equalize utilities.

While the response is correct as far as it goes, it does not necessarily follow that the sum of all the utilities, or indeed any symmetric function of each utility, is maximized as a result. For expositional ease we shall consider only the Benthamite welfare function. Let a_i be the area of zone i, n_i the number assigned to this zone, $\bar{n}$ the total population and $\bar{y}$ the total income. We seek to maximize the utility sum

$$W = \sum_{i=1}^{2} n_i[U(\frac{a_i}{n_i}, y_i) - s_i]$$

subject to the constraints

$$n_1 + n_2 = \bar{n}; \; n_1y_1 + n_2y_2 = \bar{y}$$

To solve we form a Lagrangian

$$L = W + \lambda(\bar{n} - n_1 - n_2) + \mu(\bar{y} - n_1y_1 - n_2y_2)$$

Necessary conditions for a maximum are therefore,

(A1) $$\frac{\partial L}{\partial y_i} = n_i(U_{y_i} - \mu) = 0$$

and

(A2) $$\frac{\partial L}{\partial n_i} = U(x_i,y_i) - s_i - x_i U_{x_i} - \lambda - \mu y_i = 0$$

where $x_i = a_i/n_i$.

Suppose that the optimal distribution of land and individuals is $(x(s_i),y(s_i))$ $\quad i = 1,2$

Differentiating the two first order conditions with respect to s we have

(A3) $$\frac{d}{ds}(U_y) = 0$$

and

$$U_x x'(s) + U_y y'(s) - 1 - x'(s)U_x - x\frac{d}{ds}(U_x) - \mu\frac{dy}{ds} = 0$$

Substituting for μ from (A1) this reduces to

(A4) $$\frac{d}{ds}(U_x) = -\frac{1}{x}$$

Writing out the derivatives in (A3) and (A4) we therefore have,

$$\begin{bmatrix} U_{xx} & U_{xy} \\ U_{yx} & U_{yy} \end{bmatrix} \cdot \begin{bmatrix} x'(s) \\ y'(s) \end{bmatrix} = -\frac{1}{x}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Applying Cramer's rule yields

(A5) $$x'(s) = -\frac{1}{x}\frac{U_{yy}}{|H_u|} \quad \text{and} \quad y'(s) = \frac{1}{x}\frac{U_{xy}}{|H_u|}$$

where H_u is the Hessian matrix of the function $U(x,y)$. Given the concavity of U the principal minors of H_u must alternate in sign thus $x'(s) > 0$. It follows that the optimal plot size is larger for those located in the smoggy zone.

Furthermore, substituting from (A5) we also have

$$\frac{dU}{ds} = x'(s)U_x + y'(s)U_y$$

(A6)
$$= -\frac{1}{x}\frac{(U_xU_{yy} - U_yU_{xy})}{|H_u|}$$

$$= -\frac{1}{x}\frac{\begin{vmatrix} U_x & U_{xy} \\ U_y & U_{yy} \end{vmatrix}}{|H_u|}$$

Consider an individual located in zone i facing a land price of p_i, and having an income of I. Given that he is to remain in this zone, he chooses a consumption (x_i,y_i) yielding the solution of

$$\mathrm{Max}\{U(x_i,y_i)\,|\,p_ix_i + y_i = I\}$$

Introducing the Lagrangian λ (equal to the marginal utility of income) the following first order conditions must be satisfied

$$U_x = \lambda p_i$$

$$U_y = \lambda$$

Suppose income I were increased. Differentiating the first order conditions we have

$$\begin{bmatrix} U_{xx} & U_{xy} \\ U_{yx} & U_{yy} \end{bmatrix}\begin{bmatrix} x_i'(I) \\ y_i'(I) \end{bmatrix} = \lambda'(I)\begin{bmatrix} p_i \\ 1 \end{bmatrix} = \frac{\lambda'(I)}{\lambda(I)}\begin{bmatrix} U_x \\ U_y \end{bmatrix}$$

Then applying Cramer's rule

(A7)
$$\frac{dx_i}{dI} = \frac{1}{\lambda}\frac{d\lambda}{dI}\frac{\begin{vmatrix} U_x & U_{xy} \\ U_y & U_{yy} \end{vmatrix}}{|H_u|}$$

Combining (A6) and (A7) we have

$$\frac{dU}{ds} = \frac{-\frac{1}{x}\frac{dx_i}{dI}}{\frac{1}{\lambda}\frac{d\lambda}{dI}} = \frac{E(x_i,I)}{E(\lambda,I)}$$

The expected utility of an individual residing in zone i is $U(x_i,y_i) - s_i$. Therefore the change in expected utility as the smog level s increases is

$$\text{(A8)} \qquad \frac{dU}{ds} - 1 = \frac{E(x_i,I)}{E(\lambda,I)} - 1$$

Therefore if the right hand side is positive for any price p_i and income level I, it is optimal for those in the smoggy zone to have a higher utility. Conversely, if the right hand side is always negative it is optimal to transfer income to those in the less smoggy zone!

for the special case of homothetic preferences examined in the previous section $E(x_i,I)=1$. Therefore in such cases it is optimal to transfer income to those in the less smoggy zone if and only if the income elasticity of marginal utility exceeds unity. Thus the condition obtained in section 2.2 ensuring that the higher income groups will locate in the less smoggy zone also ensures that for a population with equal incomes, the utility sum is maximized with a transfer of income to those in the less smoggy zone!

Such paradoxical results have already been noted in the urban literature by Mirrlees (1972) Riley (1974) and others, although the usual emphasis has been on the implications of differential transportation costs. Recently Arnott and Riley (1977) have attempted to explain the origin of these results as a production asymmetry. While their analysis does not carry over directly, to this more complicated case the basic issues are the same. Suppose we begin with incomes equally distributed, as in Figure 2.1. Since land is cheaper in the smoggy zone plot sizes are larger, unless land is a Giffen good. That is, C_1 lies to the right of C_o. Moreover, if land is a normal good C_1' is above and to the right of C_o. Arnott and Riley note that for a normal good the marginal utility of income rises with a Hicks compensated fall in the price of the good. That is, the marginal utility of income rises around the curve from C_1' to C_1. With diminishing marginal utility of income marginal utility falls in moving from C_o to C_1'. If the latter effect outweighs the former (and this will be the case with a sufficiently high income elasticity of marginal utility) marginal utility is lower at C_1 than at C_o. Maximization of any differentiable symmetric social welfare function therefore requires a transfer of income from those in the low marginal utility, smoggy zone to those in the less smoggy zone.

3. EFFECTS OF THE PROSPECT OF INFORMATION ON CONSUMPTION

The first chapter of this report examined the value to an individual of receiving information about goods' characteristics prior to making any consumption decisions. Consumption decisions were binding once made and could not be altered if information about s subsequently arrived. But it generally takes time to produce information through experimentation and research after a decision to acquire it had been made. In the meantime current consumption choices must still be made, although future plans may be appropriately revised upon receipt of the experimental outcomes.

This chapter examines the impact of the *prospect* of future information on decisions made prior to the receipt of that information. Analysis of this problem reinforces Hart's (1942) original claim that the anticipation of learning alters individual behaviour even before the learning takes place; an agent's optimal current behaviour cannot be determined solely from his current beliefs about s, but must take into account the timing of future information. Our basic result states that if the harmful effects of consuming a particular good depend on whether accumulated lifetime consumption exceeds some unknown maximum safe level, then the prospect of receiving information about the safe level reduces current consumption of the good. Furthermore, the sooner is the information expected the larger is the reduction in current consumption.

Sections 3.1 and 3.2 characterize an agent's response to the prospect of learning in a two period context: the agent must rely on prior beliefs when choosing first period consumption but may receive additional information before choosing second period consumption. Section 3.3 analyses the agent's response to variations in the expected time of arrival of the information in a continuous time framework. The relationship of the results to the literature on "irreversibilities" is discussed.

3.1 LEARNING PROSPECTS DO NOT AFFECT CURRENT CONSUMPTION

We first exhibit circumstances in which the prospect of future information about a good has *no effect* on current consumption decisions. Let x_1 and x_2 denote an individual's consumption in two successive periods of the good with uncertain characteristics s. Expenditures on all other goods in the two periods will be denoted by y_1 and y_2. Ignoring rate of time preference considerations and adopting the constant marginal utility of income assumption of section 1.1, let us assume the agent's "lifetime" utility has the form

$$U(x_1,x_2,y_1,y_2;s) \equiv u(x_1,x_2;s) + y_1 + y_2.$$

Without loss of generality we may choose the units of x so that its price is 1 and, ignoring interest rate considerations, write the agent's budget constraint as $x_1 + x_2 + y_1 + y_2 \leq M$.

This section examines the case in which lifetime utility is additively separable in the two periods' consumptions. The consumer's objective is thus to maximize the expected value of

$$U = u_1(x_1;s) + u_2(x_2;s) + y_1 + y_2 \tag{1}$$

subject to $x_1 + x_2 + y_1 + y_2 \leq M$.

If *no* further information is forthcoming the agent relies on his prior probabilistic beliefs about s to choose x_1 and x_2 maximizing

$$E_sU \equiv E_su_1(x_1;s) + E_su_2(x_2;s) + M - x_1 - x_2 \tag{2}$$

Assuming concavity and differentiability of u_1 and u_2, the necessary first order conditions for a maximum are

$$E_s\frac{\partial u_1}{\partial x_1} = 1, \qquad E_s\frac{\partial u_2}{\partial x_2} = 1. \tag{3}$$

The maximizing levels of consumption are denoted x_1^o and x_2^o, and the ex ante level of expected utility is

$$U^o(M) = \max_{x_1, x_2} E_s[u_1(x_1;s) + u_2(x_2;s) + M - x_1 - x_2]. \tag{4}$$

If *perfect* information about s is forthcoming after x_1 is chosen but before x_2 is chosen, then x_2 can be adjusted according to s revealed. The level of expected utility attainable becomes

$$U^*(M) = \max_{x_1, x_2(s)} E_s[u_1(x_1;s) + u_2(x_2(s);s) + M - x_1 - x_2(s)].$$

The maximum principle of dynamic programming permits this to be rewritten as

$$\begin{aligned} U^*(M) &= \max_{x_1} E_s \max_{x_2}[u_1(x_1;s) + u_2(x_2;s) + M - x_1 - x_2] \\ &= \max_{x_1} E_s\{u_1(x_1;s) + M - x_1 + \max_{x_2}[u_2(x_2;s) - x_2]\} \\ &= \max_{x_1} E_s[u_1(x_1;s) + M - x_1] + E_s \max_{x_2}[u_2(x_2;s) - x_2]. \end{aligned} \tag{5}$$

The last two equalities follow from additive structure of the utility function. The first order conditions for an interior maximum are

$$E_s \frac{\partial u_1}{\partial x_1} = 1, \qquad \frac{\partial u_2}{\partial x_2} = 1 \text{ for all s.} \tag{6}$$

Denoting the optimum consumption levels by x_1^* and $x_2^*(s)$, it follows from a comparison of (3) and (6) that $x_1^o = x_1^*$. The prospect of learning the true value of s before x_2 is chosen leaves unaltered the optimal *current* level of consumption of the risky good x_1. Any effect of this prospect on x_1^* must therefore hinge on some relation between the level of x_1 and the marginal utility of future consumption.

3.2 UTILITY AFFECTED BY ACCUMULATED CONSUMPTION

Of frequent concern in the study of consumption hazards is the effect of consuming particular goods, ingesting contaminants or continuing polluting activities over long periods of time. Current activities may have a negative effect on future welfare. One way to capture such concerns is to suppose that lifetime utility depends in part on accumulated consumption of x over time. Let us assume a utility of the form

$$U = u_1(x_1) + u_2(x_2) + y_1 + y_2 + v(x_1 + x_2;s). \tag{7}$$

To focus on cumulative consumption effect and yet maintain notational simplicity we further assume that the immediate effects of consuming x are known: i.e., u_1 and u_2 are independent of s.

How might this cumulative consumption term $v(x_1 + x_2;s)$ be interpreted and what properties might it have? In environmental problems the concern is often that continuing an activity at high levels over long periods may ultimately have a large negative impact on welfare, although lower levels of activity may be tolerated without ill effects. Examples include the ingestion of cumulative toxins such as heavy metals, exposure to carcinogenic substances, and continued pollution of water bodies leading to eutrophication.[1] The accumulated level of contamination which may be tolerated without harmful effects, however, is generally not known for certain. The essential structure of these situations is captured by a $v(x_1 + x_2;s)$ of the form

$$v(x_1 + x_2;s) = \begin{cases} 0 \text{ if } x_1 + x_2 \leq s \\ -\alpha \text{ if } x_1 + x_2 > s. \end{cases} \tag{8}$$

[1]Further examples and discussion may be found in Kneese (1977) and Mills (1978).

The potential loss α is assumed very large, but finite. It is interpreted as the cost of clean-up, cure or compensation if cumulative consumption exceeds the initially uncertain "safe level" s.[2] The agent's prior beliefs about s are represented by a probability density function f(s).

The following analysis shows that the prospect of receiving perfect information about s before x_2 is chosen, compared with no information, reduces the optimal current consumption x_1 of the risky good.

First, suppose no further information is forthcoming. Neither x_1 nor x_2 may be chosen contingent on s. The maximum level of expected lifetime utility attainable is

$$(9) \qquad U^o(M) \equiv \max_{\substack{x_1,x_2 \\ y_1,y_2}} E_s[u_1(x_1) + u_2(x_2) + y_1 + y_2 + v(x_1 + x_2;s)]$$
$$\text{subject to } x_1 + x_2 + y_1 + y_2 \leq M.$$

Substituting the budget constraint and form of v from (8) into (9) yields

$$(10) \qquad U^o(M) = \max[u_1(x_1) + u_2(x_2) + M - x_1 - x_2 - \alpha \int_0^{x_1+x_2} f(s)ds].$$

Denoting by x_1^o and x_2^o the maximizing values of x_1 and x_2, the first order conditions for an interior maximum are

$$(11) \qquad u_1'(x_1^o) = 1 + \alpha f(x_1^o + x_2^o)$$
$$u_2'(x_2^o) = 1 + \alpha f(x_1^o + x_2^o).$$

Next, suppose that s is revealed to the agent prior to x_2 being chosen. In contrast with (9), the maximum expected utility attainable is

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[2]Such discontinuous jumps in welfare associated with critical pollution levels have been pointed out by Dorfman (1977, p. 36) and Cropper (1976).

$$U^*(M) \equiv \max_{\substack{x_1, x_2^s \\ y_1, y_2^s}} E_s[u_1(x_1) + u_2(x_2^s) + y_1 + y_2^s + v(x_1 + x_2^s; s)] \tag{12}$$

subject to $x_1 + x_2^s + y_1 + y_2^s \leq M$ for all s.

Eliminating the budget constraint through substitution and utilizing the maximum principle yields

$$U^*(M) = \max_{x_1} E_s \max_{x_2}[u_1(x_1) + u_2(x_2) + M - x_1 - x_2 + v(x_1 + x_2; s)]. \tag{13}$$

The inner maximum with respect to x_2 takes both s and x_1 as given; hence its maximizer $x_2^*(x_1, s)$ is a function of both variables.

This second period reaction function $x_2^*(x_1, s)$ must be determined before the optimal initial consumption level x_1^* can be characterized. First, let us define $\bar{x}_2$ to be the level of x_2 that would be consumed if the good had no harmful consumption effects (i.e., $v = 0$ for all x_1, x_2). If such were the case then consumption would rise to where $u_2'(\bar{x}_2) = 1$, and the consumer surplus realized from second period consumption of x would be $u_2(\bar{x}_2) - \bar{x}_2$. Second, let us make more precise the meaning of α being "large." By large we mean that α exceeds $u_2(\bar{x}_2) - \bar{x}_2 - u_2(0)$, implying that once s is revealed the agent would reduce x_2 to 0, if necessary, to avoid the penalty of exceeding the safe cumulative consumption level. Of course if it turns out that previous consumption x_1 already exceeded s, and if negative consumption is ruled out, then nothing more can be lost and $x_2 = \bar{x}_2$ will be chosen. The structure of v thus leads to the second period reaction function

$$x_2^*(x_1, s) = \begin{cases} \bar{x}_2 & \text{if } s < x_1 \\ s - x_1 & \text{if } x_1 \leq s \leq x_1 + \bar{x}_2 \\ \bar{x}_2 & \text{if } x_1 + \bar{x}_2 < s. \end{cases} \tag{14}$$

Turning our attention to the outer maximum of (13), x_1 is chosen to attain expected utility

$$(15) \qquad U^*(M) = \underset{x_1}{\text{Max}}\ E_s[u_1(x_1) + u_2(x_2^*) + M - x_1 - x_2^* + v(x_1 + x_2^*;s)]$$

$$= \underset{x_1}{\text{Max}}\{u_1(x_1) + M - x_1 + E_s[u_s(x_2^*) - x_2^*] - \alpha\int_0^{x_1} f(s)ds\}.$$

Note that the penalty α is incurred only if s turns out to be less than x_1. The first order condition for an interior maximum is

$$(16) \qquad u_1'(x_1^*) = 1 - E_s[u_2'(x_2^*) - 1]\frac{\partial x_2^*}{\partial x_1} + \alpha f(x_1^*).$$

From (14) $\partial x_2^*/\partial x_1$ is 0, except when $x_1 \leq s \leq x_1 + \bar{x}_2$ in which case it is -1. Substituting x_2^* from (14) into (16) gives

$$(17) \qquad u_1'(x_1^*) = 1 + \alpha f(x_1^*) + \int_{x_1^*}^{x_1^*+\bar{x}_2} [u_2'(s-x_1^*) - 1]f(s)ds.$$

The last term is the expected increase in second period consumer surplus obtainable if one unit less x had been consumed in the first period, and is non-negative.

Finally we may compare first period consumptions with and without the prospect of information. Solving both (11) and (17) for 1 and equating provides

$$(18) \qquad u_1'(x_1^o) - \alpha f(x_1^o + x_2^o) = u_1'(x_1^*) - \alpha f(x_1^*) - \int_0^{\bar{x}_2} [u_2'(\delta)-1]f(x_1^*+\delta)d\delta.$$

The integral in (17) has been rewritten using the change of variable $\delta = s - x_1^*$. One further assumption is needed to obtain unambiguous results: assume the density function f(s) is *non-increasing*. The exponential and uniform distributions would have this property, for example.

Our objective is to demonstrate that $x_1^o \geq x_1^*$. If $x_1^o = x_1^*$ then the right hand side of (18) would generally be smaller than the left hand side both because $f(x_1^o) \geq f(x_1^o + x_2^o)$ and because the right hand side integral is non-negative. But the derivative of the right hand side of (18) with respect to x_1 is negative at x_1^* — that is the necessary second order condition for the maximum in (15). Hence x_1^* must be *less* than x_1^o for (18) to hold. The prospect of learning the true value of s thus reduces the optimal consumption level of the risky good prior to the information's arrival.

This phenomenon has some connection with recent literature on "irreversibilities." The works of Arrow and Fisher (1974), Henry (1974) and Fisher, Krutilla and Cicchetti (1972) suggest that when further information relevant to valuing an irreversible action is anticipated, then a lower value should be placed on that action relative to others. Put another way, the value of the *option* to change your future action rises if you expect to learn more. In the current context, exceeding the safe consumption level is the irreversible action. Reducing current consumption increases the probability of having the option to choose future consumption so as not to exceed s. The individual pays in the form of foregone current consumer surplus for this enhanced option.

3.3 ARRIVAL TIME OF ANTICIPATED INFORMATION

The previous section demonstrated that if consuming a good is risky because the safe level of cumulative consumption might be exceeded (resulting in large loss), then the prospect of learning the safe level part way through one's life reduces initial consumption of that good. But the comparison between receiving such information and not is, in essence, a comparison between receiving information before and after second period consumption is chosen. It was the prospect of receiving information *sooner* which curbed initial consumption until the safe level was revealed. This section casts the problem in a continuous time framework, in which information may arrive at any point during the agent's life, to show that the sooner is perfect information forthcoming the lower is the optimal pre-information consumption level.

Let the units in which time is measured be such that the agent lives one period, λ denote the time at which s is revealed, and x_t, y_t represent the rate of consumption of x and other goods at time $0 \leq t \leq 1$. Assuming the agent's instantaneous utility function for consuming x is identical at all points in time, the lifetime utility function analogous to (7) is

$$(19) \qquad U = \int_0^1 u(x_t)dt + \int_0^1 y_t dt + v(\int_0^1 x_t dt; s).$$

The agent's budget constraint is

$$(20) \qquad \int_0^1 (x_t + y_t)dt \leq M.$$

The maximum level of expected utility attainable with wealth M if s is revealed at time λ is

$$(21) \qquad U^*(M,\lambda) = \max_{x_t^s, y_t^s} E_s[U] \quad \text{subject to } \int_0^1 (x_t^s + y_t^s)dt \leq M$$

$$\text{and } \left.\begin{aligned} x_t^s &= x_t^{s'} \\ y_t^s &= y_t^{s'} \end{aligned}\right\} \text{for } t < \lambda \text{ for all } s, s'.$$

The second constraint expresses the fact that if s is unknown before time λ then consumption prior to that time cannot be contingent on s. From the form of the utility function and absence of positive interest it can be readily shown that x_t is constant before and after time λ, although the levels in these two intervals may differ. Denoting by x_1 and x_2^s the constant consumption levels before and after time λ, eliminating the budget constraint through substitution and integrating over these intervals $[0,\lambda]$, $[\lambda,1]$ yields the more tractable expression

$$U^*(M,\lambda) = \max_{x_1, x_2^s} E_s[\lambda u(x_1) + (1-\lambda)u(x_2^s) + M - \lambda x_1 - (1-\lambda)x_2^s + v(\lambda x_1 + (1-\lambda)x_2^s;s)]. \tag{22}$$

The loss $v(\cdot,s)$ from cumulative consumption $\lambda x_1 + (1-\lambda)x_2^s$ exceeding s is α as in section 3.2. The prior probability density function on s is f(s).

The optimal initial level of consumption x_1^* may now be characterized using the same analysis as employed in the previous section. Let $\bar{x}$ denote the optimal rate of consumption of x if the good had no harmful cumulative consumption effects: i.e., $u'(\bar{x}) = 1$. If α is sufficiently large then the optimal level of x_2 as a function of x_1, s and λ is given by the reaction function

$$x_2^*(x_1,s,\lambda) = \begin{cases} \bar{x} \text{ if } s < \lambda x_1 \\ \dfrac{s - \lambda x_1}{1 - \lambda} \text{ if } \lambda x_1 \le s \le \lambda x_1 + (1-\lambda)\bar{x} \\ \bar{x} \text{ if } \lambda x_1 + (1-\lambda)\bar{x} < s. \end{cases} \tag{23}$$

Applying the maximum principle, x_1 is chosen to attain expected utility

$$(24) \qquad U^*(M,\lambda) = \underset{x_1}{\text{Max}}\ E_s[\lambda u(x_1) + (1-\lambda)u(x_2^*) + M - \lambda x_1 - (1-\lambda)x_2^* + v(\lambda x_1 + (1-\lambda)x_2^*;s)]$$

$$= \underset{x_1}{\text{Max}}\{\lambda u(x_1) + M - \lambda x_1 + (1-\lambda)E_s[u(x_2^*) - x_2^*] - \alpha \int_0^{\lambda x_1} f(s)ds\}.$$

Again, the penalty α is incurred only if cumulative consumption λx_1 prior to s being revealed exceeds s.

The first order condition for an interior maximum with respect to x_1 in (24) is

$$(25) \qquad \lambda[u'(x_1^*) - 1] + (1-\lambda)E_s[u'(x_2^*) - 1]\frac{\partial x_2^*}{\partial x_1} - \alpha\lambda f(\lambda x_1^*) = 0$$

From (23) it is clear that $\partial x_2^*/\partial x_1 = 0$ except when $\lambda x_1 \le s \le \lambda x_1 + (1-\lambda)\bar{x}$, in which case $\partial x_2^*/\partial x_1 = -\lambda/(1-\lambda)$. Making these substitutions into (25) gives us

$$(26) \qquad u'(x_1^*) - 1 - \int_{\lambda x_1^*}^{\lambda x_1^* + (1-\lambda)\bar{x}} [u'(\frac{s - \lambda x_1^*}{1 - \lambda}) - 1]f(s)ds - \alpha f(\lambda x_1^*) =$$

$$u'(x_1^*) - 1 - \int_0^{(1-\lambda)\bar{x}} [u'(\frac{\delta}{1-\lambda}) - 1]f(\delta+\lambda x_1^*)d\delta - \alpha f(\lambda x_1^*) = 0$$

The change of variable $\delta = (s-\lambda x_1^*)$ was used to rewrite the integral in the latter expression. The second order condition for (26) to indicate a maximum (used later) is

$$(27) \qquad u''(x_1^*) - \lambda \int_0^{(1-\lambda)\bar{x}} [u'(\frac{\delta}{1-\lambda}) - 1]f(\delta+\lambda x_1^*)d\delta - \alpha\lambda f'(\lambda x_1^*) < 0$$

We are now in a position to answer how x_1^* varies with λ. If $\lambda = 1$, so that no information about s arrives before all consumption decisions have been made, then (26) reduces to

(28) $$u'(x_1^*) = 1 + \alpha f(x_1^*).$$

That is x is consumed at a rate where its immediate marginal utility of consumption just equals the marginal utility of the other goods foregone *plus* the marginal expected utility loss from increasing the likelihood that s is exceeded. Totally differentiating (26) with respect to λ yields the relation between x_1^* and λ

(29) $$\left[u''(x_1^*) - \lambda \int_0^{(1-\lambda)\bar{x}} [u'(\tfrac{\delta}{1-\lambda}) - 1] f(\delta+\lambda x_1^*) d\delta - \alpha\lambda f'(\lambda x_1^*)\right]\frac{dx_1^*}{d\lambda}$$

$$= - \bar{x}[u'(\bar{x}) - 1] f(\cdot) + \int_0^{(1-\lambda)\bar{x}} u''(\tfrac{\delta}{1-\lambda})\frac{\delta}{(1-\lambda)^2} f(\cdot)d\delta$$

$$+ \int_0^{(1-\lambda)\bar{x}} [u'(\tfrac{\delta}{1-\lambda}) - 1] x_1^* f'(\cdot) d\delta + \alpha x_1^* f'(\lambda x_1^*).$$

The expression multiplying $dx_1^*/d\lambda$ on the left side of (29) is negative from the second order condition (27). The first term on the right side is zero from the definition of $\bar{x}$, the second negative since u is assumed concave, the third and fourth negative under the assumption that f(s) is non-increasing introduced in section 3.2. Hence it follows that $dx_1^*/d\lambda \geq 0$: The sooner knowledge of s is anticipated (smaller is λ), the smaller is the optimal initial consumption of the risky good.

The fact that the appropriate current consumption depends on the prospect and timing of future information has qualitative policy implications. To obtain the full benefit of information produced about consumption hazards, it must be announced that the research is in progress together with an estimate of when the results will be available! Alternatively, if consumption of the good is regulated, then the sooner is determination of the safe level expected, the more stringent should be current restrictions on its use.

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