

An Examination of the Economic and  
Muthian Rationality of Price Level Forecasts

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## I. INTRODUCTION

Economic models frequently employ the hypothesis that individuals forecast inflation on the basis of past inflation rates. Static, adaptive, extrapolative, and regressive expectations adjustment schemes are examples of such forecasting rules. Although such naive forecasting rules ignore other information which is available, they may be warranted on grounds other than theoretical simplicity. As Feige and Pearce (1976) note, simple forecasting rules may be economically rational if the value of improved prediction from more sophisticated rules is outweighed by the cost of collecting and processing the additional information.<sup>1</sup> Moreover, as Mussa (1975) demonstrates, the rational and seemingly naive forecasting rules may be equivalent in some circumstances.

Using past inflation rates to forecast the rate of inflation ignores the fact that individuals observe prices not inflation rates. A change in observed prices can result from general price inflation, but it may also result from transitory shocks to the price level or observation error. A model of inflationary expectations should admit the possibility of these different sources of price changes. Such a model was employed by Jacobs and Jones (1979) in a study of consumer price expectations. The purpose of this paper is to extend that analysis to include wholesale price expectations and to examine the economic rationality and rationality in the sense of Muth of reported price forecasts.

The expectations model employed is a two level adaptive expectations algorithm in which individuals use the observed price level to revise their beliefs about the true underlying price level and its trend. The empirical results demonstrate that this rule accurately reproduces the Livingston survey data for the consumer and wholesale price indices. The adaptive algorithm implies forecasts which are a restricted distributed lag on past prices. The empirical results also indicate that this restricted lag is not significantly

different from an unrestricted distributed lag forecasting rule. We also examine the extent to which other information was employed in generating the reported forecasts and conclude that there is little evidence against the economic rationality of the adaptive forecasting rule. The final topic considered is the degree to which the reported forecasts differ from optimal forecasting rules for the CPI and WPI. The evidence indicates that forecasters of the CPI did not efficiently utilize information contained in the history of that series.

## II. EXPECTATIONS FORMATION

We assume that price expectations are formed through an adaptive learning process which relates future expectations to current and past observed prices. The maintained assumptions of our adaptive learning model are as follows:

(1) The value to individuals of basing forecasts of future prices on information beyond just the past history of prices is outweighed by the cost of collecting and processing the additional information. (2) Individuals behave as if the reduced form stochastic process generating prices belongs to the class of ARIMA processes described below. (3) Individuals incorporate new observations into their beliefs about future prices in an optimal Bayesian fashion.

Let  $P_t$  denote the observed price level and suppose that the process generating prices is

$$P_t = \bar{P}_t + u_t$$

$$(1) \bar{P}_t = \bar{P}_{t-1} + \bar{\pi}_t + v_t$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + w_t$$

where  $\bar{P}_t$  is interpreted as the true underlying price level,  $u_t$  as a transitory shock to prices (or observation error),  $\bar{\pi}_t$  as the true underlying inflation rate, and  $v_t$  and  $w_t$  as transitory and permanent shocks to the inflation rate.<sup>2</sup> The shocks  $u_t$ ,  $v_t$  and  $w_t$  are independent white noise processes with means zero and respective variances  $\mu$ ,  $\nu$  and  $\omega$ .<sup>3</sup>

Prior to observing  $P_t$ , the agent held prior expectations about the true price level to obtain at time  $t$  and the underlying inflation rate over the interval  $t-1$  to  $t$ .

We denote these prior beliefs about  $\bar{P}_t$  and  $\bar{\Pi}_t$  as  $P_t^e$  and  $\Pi_t^e$  respectively. After observing  $P_t$ , the agents posterior beliefs about  $\bar{P}_t$  and  $\bar{\Pi}_t$  will be denoted as  $P_t^{e'}$  and  $\Pi_t^{e'}$  respectively. It can be shown that the Bayesian revision rule for  $P_t^{e'}$  and  $\Pi_t^{e'}$  is given by<sup>4</sup>

$$(2) \quad \begin{aligned} P_t^{e'} &= P_t^e + \lambda_1 (P_t - P_t^e) \\ \Pi_t^{e'} &= \Pi_t^e + \lambda_2 (P_t - P_t^e) \end{aligned}$$

Forecasts of the price level expected to prevail at time  $t+1$  and the inflation rate over the interval  $t$  to  $t+1$  are then given by

$$(3) \quad \begin{aligned} P_{t+1}^e &= P_t^{e'} + \Pi_t^{e'} = P_t^e + (\lambda_1 + \lambda_2) (P_t - P_t^e) + \Pi_t^e \\ \Pi_{t+1}^e &= \Pi_t^{e'} = \Pi_t^e + \lambda_2 (P_t - P_t^e). \end{aligned}$$

The adaptation coefficients are positive constants whose values depend on the relative magnitudes of  $\mu$ ,  $\nu$  and  $\omega$ . The variable  $\Pi_{t+1}^e$  is the expected underlying inflation rate over the period from  $t$  to  $t+1$ . This will differ from the expected observed inflation rate  $P_{t+1}^e - P_t$  over the same period by the estimated current transitory shock to prices,  $P_t - [P_t^e + \lambda_1 (P_t - P_t^e)]$ . Of course, if  $\mu = 0$  then  $\lambda_1 = 1$ , and (2) and (3) reduce to the usual adaptive rule on the observed inflation rate

Estimation of the expectations model requires solving difference equation system (3). If we define the column vectors

$$X_t = \begin{pmatrix} P_t^e \\ \Pi_t^e \end{pmatrix}_{t+1} \quad \text{and} \quad Z_t = \begin{pmatrix} \lambda_1 + \lambda_2 \\ \lambda_2 \end{pmatrix} P_t,$$

the model can be expressed in matrix form

$$(4) \quad X_t = [A] X_{t-1} + Z_t, \quad \text{where} \quad [A] = \begin{bmatrix} 1 - \lambda_1 - \lambda_2 & 1 \\ -\lambda_2 & 1 \end{bmatrix}.$$

For a vector  $X_0$  of initial values of  $P_0^e$  and  $\Pi_0^e$ , the solution of (4) becomes

$$(5) \quad X_t = [A]^t X_0 + \sum_{j=0}^{t-1} [A]^{t-j} Z_j.$$

Equation (5) provides the model's predicted values of the expectation variables at every point in time as a function of an observed price series and any given set of adaptation parameters and initial conditions. It is this predicted time history of expectations which we compare with reported expectations to assess the explanatory power of the model. During the estimation the unit of time will be taken as six months. The data, however, consist of price forecasts seven and thirteen months into the future. Since  $\Pi_t^{e'}/6$  is the best estimate of the current monthly inflation rate, the forecasts of the price level seven and thirteen months hence are  $(P_t^e)_7 = P_t^{e'} + \frac{7}{6} \Pi_t^{e'}$  and  $(P_t^e)_{13} = P_t^{e'} + \frac{13}{6} \Pi_t^{e'}$  respectively.

Equation (5) constrains the distributed lag of the one period ahead price forecast on current and past prices to a particular form. For example, the first few terms of the summation give

$$(6) \quad P_{t+1}^e = (\lambda_1 + \lambda_2) P_t + [(1-\lambda_1-\lambda_2)(\lambda_1+\lambda_2) + \lambda_2] P_{t-1} \\ + \{[(1-\lambda_1-\lambda_2)^2 - \lambda_2](\lambda_1 + \lambda_2) + \lambda_2(2-\lambda_1-\lambda_2)\} P_{t-2} + \dots$$

This constrained distributed lag expression for  $P_{t+1}^e$  is a direct consequence of the structure imposed on the process of price formation. We will test the validity of our expectations model against the alternative of an unrestricted distributed lag

$$(7) \quad P_{t+1}^e = \alpha_0 + \alpha_1 P_t + \alpha_2 P_{t-1} + \alpha_3 P_{t-2} + \dots$$

Equation (7) implies forecasts two and three periods into the future of<sup>5</sup>

$$P_{t+2}^e = \alpha_0 + \alpha_1 P_{t+1}^e + \alpha_2 P_t + \alpha_3 P_{t-1} + \dots$$

(8)

$$P_{t+3}^e = \alpha_0 + \alpha_1 P_{t+2}^e + \alpha_2 P_{t+1}^e + \alpha_3 P_t + \dots$$

The seven and thirteen month forecasts required for the estimation process will be obtained by quadratically interpolating between these three forecasts to obtain

$$(P_t^e)_7 = A_{11} P_{t+1}^e + A_{12} P_{t+2}^e + A_{13} P_{t+3}^e$$

(9)

$$(P_t^e)_{13} = A_{21} P_{t+1}^e + A_{22} P_{t+2}^e + A_{23} P_{t+3}^e$$

where the  $A_{ij}$  are elements of the matrix

$$A = \begin{bmatrix} 0.7639 & 0.3055 & -0.0694 \\ -0.0694 & 0.9722 & 0.0972 \end{bmatrix}.$$

The forecasting model of either (5) or (7) assumes that expectations were based solely on the information contained in current and past prices. This assumption will be tested by adding additional information Y to the process of expectations formation. We will assume that this information contributes  $\Delta P_{t+1}^e$  to the expected future price where  $\Delta P_{t+1}^e$  is the distributed lag

$$(10) \quad \Delta P_{t+1}^e = \beta_1 Y_t + \beta_2 Y_{t-1} + \beta_3 Y_{t-2} + \dots$$

The two and three period ahead forecasts are then augmented by

$$\Delta P_{t+2}^e = \beta_1 \Delta P_{t+1}^e + \beta_2 Y_t + \beta_3 Y_{t-1} + \dots$$

(11)

$$\Delta P_{t+3}^e = \beta_1 \Delta P_{t+2}^e + \beta_2 \Delta P_{t+1}^e + \beta_3 Y_t + \dots$$

The contribution of this additional information to the seven and thirteen month forecasts is obtained by quadratic interpolation

$$(\Delta P_t^e)_7 = A_{11} \Delta P_{t+1}^e + A_{12} \Delta P_{t+2}^e + A_{13} \Delta P_{t+3}^e$$

(12)

$$(\Delta P_t^e)_{13} = A_{21} \Delta P_{t+1}^e + A_{22} \Delta P_{t+2}^e + A_{23} \Delta P_{t+3}^e$$

During the estimation we will substitute measures of the money supply and fiscal policy for  $Y_t$ .



### III. ESTIMATION OF THE EXPECTATIONS MODEL.

Every six months since 1947 Joseph A. Livingston conducted a survey of economists asking their forecasts of a number of key economic variables including the Consumer Price Index and the Wholesale Price Index. In a recent paper, John Carlson (1977) analyzed the original survey responses and generated a series for the expected CPI and WPI. Carlson emphasizes key timing aspects of the survey which have largely been ignored by other users of the data. Consider, for example, the forecasts published in June of the CPI or WPI for the following December and June. To meet the publication deadline, Livingston would mail the questionnaire in mid-May along with the latest available values of the price indices. Respondents would usually complete the questionnaire in late May. Consequently, we assume that survey participants know May prices and make seven and thirteen month forecasts for the following December and June.<sup>6</sup> For the December survey we assume that respondents know November prices and make seven and thirteen month forecasts for the following June and December.

Since the survey data are available every six months, that is taken as the unit of time for equation (5). Our timing assumptions imply that  $P_t$  is the price index for either May or November. At each point in time there are two observations:  $(\hat{P}_t^e)_7$ , the average survey response for the expected price seven months hence in either December or June and  $(\hat{P}_t^e)_{13}$ , the average survey response for the expected price thirteen months hence in either June or December. Carlson also reports standard deviations  $(\sigma_t)_7$  and  $(\sigma_t)_{13}$  which indicate the dispersion of individual forecasts. These standard deviations exhibit considerable variation over the data interval.

The ability of the model to reproduce the survey data is indicated by the weighted residuals  $\Delta s_{t,1} = [(\hat{P}_t^e)_7 - (P_t^e)_7 - (\Delta P_t^e)_7]/W_{t,1}$  and  $\Delta s_{t,2} = [(\hat{P}_t^e)_{13} - (P_t^e)_{13} - (\Delta P_t^e)_{13}]/W_{t,2}$  where the weights  $W_{t,1}$  and  $W_{t,2}$  are computed from the standard deviations of the price forecasts.<sup>7</sup> The estimation entails finding those values of the adaptation parameters and initial conditions which minimize the sum of squared transformed residuals

$$SSR = \sum_{t=1}^T \sum_{j=1}^2 (\Delta s_{t,j} - \rho \Delta s_{t-1,j})^2,$$

where  $\rho$  is the first order serial correlation coefficient. Since the parameter estimates are nonlinear in the initial conditions, adaptation coefficients and  $\rho$ , the minimization of SSR requires an iterative procedure. We use the technique of Donald Marquardt (1963) based on interpolation between the Taylor series and gradient methods.<sup>8</sup>

Estimation results for the adaptive expectations model of equation (5) are presented in rows one and seven of Table 1.<sup>9</sup> The initial conditions correspond to negative expected inflation rates of approximately five and seven percent/year respectively. Survey respondents expected both consumer and wholesale prices to fall in the immediate post-war period. When the negative expected inflation rates did not materialize  $\Pi_t^e$  turned positive as individuals adapted to the actual price behavior. Similar adaptive coefficients were obtained for both series which implies that survey respondents acted as if the stochastic processes generating the CPI and WPI had similar variances for  $u_t$ ,  $v_t$  and  $w_t$ .

Was the provision for beliefs in transitory shocks to the price level important? Rows two and eight contain estimates of the model with  $\lambda_1$  equal to unity so that individuals form their expectations on the basis of observed inflation rates. The F-tests reject this hypothesis at the 1% and 5% levels respectively against the alternative that  $\lambda_1$  exceeds unity.<sup>10</sup>

The results indicate that the distinction between the expected underlying inflation rate  $(P_{t+1}^e - P_t^{e'})$  and the expected observed inflation rate  $(P_{t+1}^e - P_t)$  is important in the process of expectations formation.<sup>11</sup> Many users of the survey data transform the price level forecasts into expected rates of inflation by subtracting  $P_t$  and converting to a growth rate. This transformation is suspect because it ignores the difference between  $P_t$  and  $P_t^{e'}$  when  $\lambda_1$  differs from unity.

Stephen Turnovsky (1970), in a previous analysis of the Livingston data, concluded that the process of expectations formation was fundamentally different between the period after 1960 and the earlier period. Rows three and four and nine and ten of Table 1 contain separate estimates of the model for the 1947-1961 and 1961-1975 subperiods respectively. At first glance the estimation results appear to support Turnovsky. Quite different values of  $\lambda_1$  and  $\lambda_2$  are obtained for the two data intervals. However, these parameters have large standard errors for the shorter samples and the F-test indicates that the parameters do not differ significantly at the 5% level between the two periods. We find no compelling evidence of a change in the process of expectations formation for either series.

The remaining estimates of Table 1 test the consistency of the forecasts in the sense of Wold (1963). Consistency requires that forecasts for future periods be generated recursively using Wold's chain principle. This feature is embedded in the model because  $P_{t+2}^e = P_{t+1}^e + \Pi_t^{e'} = P_t^{e'} + 2\Pi_t^{e'}$ . The model contains the additional restriction that the expected rate of inflation is constant over the forecast horizon. In fact, survey respondents may have employed a time varying rate of inflation. If either consistency or the constant inflation rate assumption is violated by survey respondents, we would obtain different estimation results if the model were separately fit to seven and thirteen month forecast data. These estimates in table 1 give no indication that either restriction is violated.<sup>12</sup>

The parameter estimates in the first row of Table 1 imply the following distributed lag on past prices for the one period ahead forecast of the CPI

$$(13) \quad P_{t+1}^e = 1.32P_t - 0.33P_{t-1} + 0.08P_{t-2} - 0.02P_{t-3} + \dots$$

Estimating the unrestricted distributed lag of equation (7) gives the results presented in the first row of Table 2. Comparing the parameters with those of equation (13) illustrates that the unrestricted distributed lag is nearly identical to the constrained lag of the adaptive model. Row two indicates that the constant term is significantly different from zero at the 1% level of significance. Survey respondents expected prices to fall in the immediate post-war period and these expectations cannot be reproduced by the lagged prices. In the adaptive model these expectations were modeled by the negative value of  $\Pi_0^e$ . In the distributed lag model the negative constant performs this function. There is a slight increase in SSR with the distributed lag model because  $\Pi_0^e$  does a better job of reproducing this initial expectations data.

For the WPI data, the adaptive expectations model implies the following distributed lag on current and past prices

$$(14) \quad P_{t+1}^e = 1.21P_t - 0.13P_{t-1} + 0.003P_{t-2} - 0.01P_{t-3} + \dots$$

Estimation results for the unrestricted lag model of equation (8) are presented in row three of Table 2. In this instance, the lag coefficients differ somewhat from those implied by the adaptive expectations model; however, there is only a slight reduction in SSR. The constant term is significantly negative at the 1% level and helps reproduce the initial expected decline in wholesale prices.

The estimation results for the unrestricted distributed lag provide strong support for the two level adaptive expectations model developed in Section I. The adaptive model constrains the distributed lag to follow the particular pattern given by equation (6). If this constraint is relaxed there is little change in the goodness of fit or the pattern of coefficients.

Although the adaptive expectations model is capable of accurately reproducing both sets of survey data, the possibility exists that additional information was used to generate the survey responses. This hypothesis is investigated in Table 3. The first column contains the value of SSR from Table one for the adaptive expectations model under the assumption that only past prices were used to generate price expectations ( $\Delta P_{t+1}^e = 0$ ). The remaining columns include  $\Delta P_{t+1}^e$  in the expectations formation model with  $Y_t$  being either money supply definition M1 and M2 or the federal government budget deficit. The F-tests indicate that additional information did not have a significant impact on the formation of price expectations for the CPI or the WPI.<sup>13</sup> The results of table 3 are not inconsistent with the hypothesis that survey respondents formed their expectations on the basis of past prices only.

#### IV FORECASTING ERRORS

In this section we investigate the degree to which the reported expectations provided accurate forecasts of the price level seven and thirteen months hence. We will use two standards of comparison for the survey expectations. The first will be the adaptive expectations model with the parameters estimated to give the best one period ahead forecasts of the actual CPI and WPI series. The second will be a Box-Jenkins representation of these series.<sup>14</sup>

In the previous section the expectations model parameters were selected to give the best fit of the survey data. However, with these parameters the model may do a poor job of forecasting the actual price series unless the survey expectations were rational in the sense of Muth. To obtain the adaptive model which provides the best one period forecasts we seek those model parameters which minimize the sum square of the actual forecast errors. These parameters were obtained by estimating the model using the actual realized price level six months hence as data in place of the survey expectations. These estimation results are presented in rows one and three of table 4. The best forecasting rule for the actual CPI series involves significantly more rapid adjustment of the expected rate of inflation to forecasting errors than was obtained for the survey data. However, this is not the case for the actual WPI series compared to the survey results. In addition, the more rapid adjustment of expected inflation for the actual CPI compared to the actual WPI series indicates that the two series have different stochastic structures.<sup>15</sup> The similarity of adjustment coefficients for the survey data, however, indicates that survey respondents acted as if the two series had approximately the same structure.

As an alternative standard of comparison, both price series were represented by the ARIMA (1,1,1) process with a constant term illustrated in Table 5.

The two models of the CPI and WPI give approximately the same distributed lag coefficients on past prices for each series. The adaptive and ARIMA models for the CPI imply that

$$P_{t+1}^e = 1.65P_t - 0.81P_{t-1} + 0.35P_{t-2} - \dots \text{ (Adaptive)}$$

and

$$P_{t+1}^e = 1.75P_t - 0.88P_{t-1} + 0.16P_{t-2} - \dots \text{ (ARIMA)}$$

For the WPI these models imply

$$P_{t+1}^e = 1.44P_t - 0.48P_{t-1} + 0.14P_{t-2} - \dots \text{ (Adaptive)}$$

and

$$P_{t+1}^e = 1.54P_t - 0.56P_{t-1} + 0.02P_{t-2} - \dots \text{ (ARIMA)}$$

In addition, the adaptive model and ARIMA process give an almost identical pattern of forecasting errors for both series. Since the two alternative forecasting rules are equivalent, we will confine our attention to the adaptive model.

Given the adaptive models for the actual CPI and WPI we are interested in comparing their forecasting errors with those of the survey data. This comparison is made in table 6. The adaptive model has significantly smaller forecast errors for both price series. The survey forecasts appear to be irrational in the sense that a model which reproduces the survey data can provide significantly better forecasts by using a different set of parameters.

This comparison, however, may be biased against the survey data because of the large forecast errors at the start of the data set. The expected fall in prices which did not materialize in the immediate post war period leads to significant forecast errors for the first few years of the survey. If we allow time for survey respondents to adapt to the structure of prices after the war the survey results are significantly better. This fact is illustrated in table 7

which compares forecast errors for the period after 1950 when there was a significant decline in the survey forecast errors. The WPI survey errors are almost equal while the CPI survey errors are significantly greater than the errors of the best forecasting model. The WPI survey forecasts appear to have made efficient use of the information contained in past prices, however, this is not true for the CPI survey forecasts. Additional evidence on this point is provided by the estimate in rows two and four of table 4. For these estimates, the adaptation coefficients were constrained to equal the values which give the best fit of the survey data. The initial conditions were then estimated in order to correctly forecast the immediate post war price data. For the WPI series there is no significant difference between the adaptation coefficients which reproduce the survey data and those which best forecast the actual WPI data. For the CPI series, however, there is a significant difference between these two sets of adaptation coefficients.<sup>16</sup>

The results of this section indicate that the survey forecasts of the CPI were irrational in the sense that the same model can generate the survey and actual forecasts but that different parameters are required for these tasks. This implies that the survey respondents could have made significantly more accurate forecasts of the future price level. It could be argued that this is an unfair comparison because the forecasting model for the actual series was fit to the entire data set. As a result, its forecasts are based on more information about the structure of the price series than was available to survey respondents at the time their forecasts were made. This argument is, however, blunted by some additional evidence on the irrationality of the CPI survey forecasts.



Fama (1975) has argued that the rate of interest on Treasury Bills embodies a forecast of the rate of CPI inflation for the time interval to maturity. Agents in the market can be viewed as efficiently processing all information and making rational forecasts of the expected rate of inflation  $\Pi^e$ . The rate of interest  $i$  is then determined so the T-Bill yields a constant expected real return  $r$  given by  $r = i - \Pi^e$ . Fama tests this hypothesis on bills having maturities which range from one to six months and finds strong empirical support for this efficient market hypothesis.

To evaluate the survey data we want to compare errors in the rate of inflation forecasts implied by the six month Treasury Bill rate with errors in the rate of inflation forecasts implied by the seven month survey price forecasts. The comparison is limited to the period 1959 to 1975 because of the lack of data on six month bills prior to 1959. Using Fama's value for the real rate of return on six month bills,  $r = 1.76\%/annum$ , we compute forecasts of the rate of inflation for each May and November survey data. When these forecasts are compared to the subsequently realized rate of inflation we obtain a RMS forecasting error of  $1.77\%/annum$ . The survey forecasts of the CPI for the same interval imply an error in the rate of inflation of  $2.38\%/annum$ . For this same interval the adaptive model of Table 4 gives a forecast error of  $1.76\%/annum$ .<sup>17</sup> The close agreement between the forecasts of the adaptive model fit to the actual CPI series and the forecasts implied in the T-Bill rate provides further support for the conclusion that survey respondents did not efficiently utilize available information when responding to the survey.

## V. CONCLUSIONS

We found that a two level adaptive expectations model, in which individuals use the current price to revise their beliefs about the true price level and its trend, was able to accurately reproduce the Livingston survey data on the expected future CPI and WPI. The model implies that the expected future price is a restricted distributed lag function on current and past prices. When the forecasting model is estimated with unrestricted lag coefficients, we obtain approximately the same lag pattern implied by the adaptive model and the same fit of the survey data. Adding additional information, such as the money supply or government deficit, to the process of expectations formation did not significantly enhance our ability to explain the survey data. It thus appears that survey respondents formed their expectations solely on the basis of information contained in past prices.

The final question addressed was the rationality of the survey responses in the sense of efficiently utilizing available information. This hypothesis was tested in a weak form which requires the forecasts to efficiently utilize information contained in past prices. The WPI forecasts satisfied this test of rationality but the same was not true for the CPI forecasts. For the CPI, a forecasting rule which involved more rapid adjustment of the rate of inflation to forecasting errors would have significantly improved the survey forecasts. This conclusion was supported by evidence which indicates that the inflation forecasts embodied in nominal interest rates provided significantly better predictions of the future than did the survey respondents. The survey respondents used approximately the same rate of adjustment of  $\Pi_t^e$  to forecast errors in forecasting both the CPI and WPI. This forecasting rule happens to work for one series but not for the other.

TABLE 1  
Adaptive Expectations Model

Estimate	$P_o^e$	$\pi_o^e$	$\lambda_1$	$\lambda_2$	$\rho$	D-W	SSR	F-test
Data 1947-1975	4.163	-0.023	1.225	0.094	0.487	1.952	0.00415	-
Data 1947-1975 $\lambda_1 = 1.0, (P_o^e = P_o)$	-	-0.025	-	0.090	0.538	1.922	0.00534	9.319**
Data 1947-1961	4.161	-0.016	1.317	0.072	0.442	1.685	0.00289	1.209
Data 1961-1975	4.499	0.006	1.08	0.111	0.378	2.043	0.00088	
47-75, ( $\hat{P}_t^e$ ) <sub>7</sub> only	4.170	-0.023	1.239	0.083	0.264	1.962	0.00166	
47-75, ( $\hat{P}_t^e$ ) <sub>13</sub> only	4.156	-0.023	1.238	0.100	0.562	1.804	0.00231	0.544
Data 1947-1975	4.265	-0.033	1.098	0.112	0.245	2.014	0.01488	-
Data 1947-1975 $\lambda_1 = 1.0, (P_o^e = P_o)$	-	-0.035	-	0.118	0.275	2.031	0.01673	4.041*
Data 1947-1961	4.292	-0.050	1.192	0.170	0.170	1.904	0.00950	1.385
Data 1961-1975	4.548	-0.006	1.048	0.089	0.250	2.050	0.00384	
47-75, ( $\hat{P}_t^e$ ) <sub>7</sub> only	4.264	-0.030	1.096	0.100	0.173	1.994	0.00717	0.130
47-75, ( $\hat{P}_t^e$ ) <sub>13</sub> only	4.264	-0.035	1.108	0.119	0.302	2.030	0.00755	

Note: \* indicates significant at the 5% level while \*\* indicates significant at the 1% level.

TABLE 2

## Distributed Lag Model

Data 1947-1975	$\alpha_0$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	$\alpha_6$	$\rho$	D-W	SSR	F-test
Expected CPI	-0.148	1.323	-0.360	0.104	-0.106	0.045	0.028	0.368	2.060	0.00421	-
Expected CPI ( $\alpha_0 = 0$ )	-	1.336	-0.378	0.149	-0.109	0.037	-0.035	0.874	2.482	0.00550	18.998
Expected WPI	-0.237	1.199	-0.228	0.128	-0.021	-0.186	0.161	0.077	2.153	0.01417	-
Expected WPI ( $\alpha_0 = 0$ )	-	1.109	-0.187	0.148	-0.055	-0.136	0.061	0.638	2.751	0.01983	24.764

TABLE 3  
 Impart of Additional Information on Expectations Formation

Data (1947-1975)	SSR, Adaptive Expectations on Past Prices	SSR/(F-test)		
		$Y_t = M1$	$Y_t = M2$	$Y_t = \text{Budget Deficit}$
Expected CPI	0.00415	0.00379 (0.934)	0.00390 (0.630)	0.00412 (0.071)
Expected WPI	0.01488	0.01234 (2.024)	0.01344 (1.053)	0.01426 (0.428)

Note: The 5% and 1% significance levels for these tests are 2.76 and 4.13 respectively.

TABLE 4  
Models Which Minimize the One Period Forecast Error

Model	$P_0^e$	$\pi_0^e$	$\lambda_1$	$\lambda_2$	SSR	F-test
Actual Realized CPI						
Adaptive Expectations	4.217	0.014	1.386	0.266	0.00737	-
Adaptive Expectations: $\lambda_1$ and $\lambda_2$ from row one of Table 1	4.222	0.012	(1.225)	(0.094)	0.00881	5.275
Actual Realized WPI						
Adaptive Expectations	4.367	0.009	1.284	0.154	0.03750	-
Adaptive Expectations $\lambda_1$ and $\lambda_2$ from row seven of Table 1	4.370	0.008	(1.098)	(0.112)	0.04012	1.886

Note: The 5% and 1% significance levels for the F-tests are approximately 3.18 and 5.06 respectively

TABLE 5

Box-Jenkins Representations of  
May/November CPI and WPI Data

<u>DATA</u>	<u>ARIMA MODEL</u>	<u>SSR</u>	<u>Q(Degrees Of Freedom)</u>
CPI	$(1 - 0.559B)(1 - B)P_t = 0.0068 + (1 + 0.186B)a_t$	0.00796	30.4(34)
WPI	$(1 - 0.498B)(1 - B)P_t = 0.0087 + (1 + 0.04B)a_t$	0.03895	14.7(34)

TABLE 6

Forecast Errors 1947-1975

RMS Price Forecast Error

Converted to %/annum Rate of Inflation Error

Series	Model	7 Month Forecast	13 Month Forecast
CPI	Survey Data	3.54	3.36
	Adaptive Model Fit to Actual CPI	2.26	2.27
WPI	Survey Data	6.75	5.73
	Adaptive Model Fit to Actual WPI	5.05	4.53



TABLE 7

Forecast Errors 1951-1975

RMS Price Forecast Error

Converted to %/annum Rate of Inflation Error

Series	Model	7 Month Forecast	13 Month Forecast
CPI	Survey Data	2.15	2.01
	Adaptive Model Fit to Actual CPI	1.56	1.69
WPI	Survey Data	4.16	3.78
	Adaptive Model Fit to Actual WPI	3.95	3.57

FOOTNOTES

<sup>1</sup> Feige and Pearce use Granger (1969) causality as a criteria to measure the predictive benefits of using information other than the past values of the series to be forecast. Darby (1976) also analyzes expectations formation under conditions of costly information by comparing the cost and value of reducing the forecast variance through the use of additional information.

<sup>2</sup>  $\bar{\pi}_t$  is actually the rate of change in the price level  $\bar{P}_t$ . For estimation purposes, however,  $\bar{P}_t$  will be the logarithm of the price so that  $\bar{\pi}_t$  represents the rate of inflation.

<sup>3</sup> With  $\nu$  equal to zero, the process generating prices is identical to that used by Nerlove and Wage (1964) to demonstrate the optimality of adaptive forecasting. With  $\bar{\pi}_0$  and  $\omega$  equal to zero, the process is identical to that examined by Muth (1960).

<sup>4</sup> Jacobs and Jones (1977) derive these equations as the asymptotic form of a Bayesian revision rule. See Winters (1960) for an early application of this model to the problem of forecasting sales.

<sup>5</sup> To obtain forecasts for future periods, we employ Wold's chain principle of forecasting.

<sup>6</sup> Carlson assumes that survey respondents know April prices and make eight and fourteen month forecasts to the following December and June. Although the April indices would be the only published data available, survey participants would have observed the behavior of prices during May. Jacobs and Jones (1979) demonstrate that the assumption that participants know May prices provides a significantly better fit of the survey data.

<sup>7</sup> Since the model is estimated in logarithmic form, the standard deviations reported by Carlson were divided by the price level to obtain approximate standard deviations  $(\sigma_t)_7$  and  $(\sigma_t)_{13}$  for the logarithm of the forecast price level. These standard deviations exhibit considerable variation which creates a potential problem of heteroscedasticity if the poorer quality observations have larger residuals and dominate the estimation process. As a remedy for this problem, we adopt the following weighting scheme. We first compute the average standard deviation  $\bar{\sigma}$  and the normalized values  $(\sigma_t^*)_7 = (\sigma_t)_7/\bar{\sigma}$  and  $(\sigma_t^*)_{13} = (\sigma_t)_{13}/\bar{\sigma}$ . The weights are then computed as  $W_{t,1} = 1 - \alpha + \alpha (\sigma_t^*)_7$  and  $W_{t,2} = 1 - \alpha + \alpha (\sigma_t^*)_{13}$  where  $\alpha$  is a constant between zero and one. If  $\alpha = 1$  the weights equal the standard deviations normalized to unity while if  $\alpha = 0$  all observations receive unit weight. A value of  $\alpha = 0.75$  was employed since this value minimized the weighted residual variance.

<sup>8</sup> If the actual error term has first order serial correlation then the estimation procedure is equivalent to obtaining maximum likelihood estimates. The only difference is the term  $\log(1 - \rho^2)$  in the likelihood function which would have a negligible impact on our estimates. The model was iterated to convergence starting from widely different initial conditions in order to avoid obtaining a local maximum.

<sup>9</sup> There are no values of  $R^2$  listed in Table 1 because the large changes in  $\ln(P_t)$  over the data interval insure that it will be close to unity. For example, the first row estimate of Table 1 would give a value of  $R^2$  equal to 0.99998. A value of  $R^2$  computed from first differences would be approximately 0.89 and is a more realistic measure of the goodness of fit.

<sup>10</sup> Residuals for the seven and thirteen month forecasts at each point in time are highly correlated. While this residual correlation will not lead to inconsistent parameter estimates, it could create problems with F-tests because the number of data points overstates the number of degrees of freedom. Based on the residual correlation, we have reduced the number of effective data points in the F-test to seventy as opposed to actual one hundred and sixteen data points used in the estimation.

<sup>11</sup> Estimating the model in terms of the price level rather than its logarithm would not alter the conclusions reached in this paper or the outcome of the various hypothesis tests. To obtain the same goodness of fit, however, the model would have to be extended to include a time varying drift in the variable  $\bar{\pi}_t$  (see Jacobs and Jones (1979) for this extension of the model). The drift term is necessary because a constant rate of inflation forecast implies a time varying rate of change of prices.

<sup>12</sup> In an earlier study of the Livingston data, Pesando (1975) finds that the CPI survey data violate this consistency requirement. Carlson notes that Pesando was using a data set which was judgementally altered by Livingston and which ignores the survey timing. Carlson repeats Pesando's consistency test for his new CPI and WPI series and finds that the CPI fails the consistency test. In a recent article, Mullineaux (1978) notes that if the seven and thirteen month forecasts do not have the same error structure the F-test will be biased against finding consistency. He finds this to be the case for the Pesando and Carlson regressions. Our findings differ from those of Carlson for two reasons. First, the weighting scheme yields a similar error structure for the seven and thirteen month regressions and, therefore, eliminates the bias noted by Mullineaux. Second,

our tests are performed using the actual price forecasts whereas Carlson transforms the data into equivalent rates of inflation. As previously noted, this transformation is permissible only if the transitory disturbance to prices is zero.

<sup>13</sup> The current and five lagged terms of the money supply or government deficit were used in these regressions. Changing the number of lagged values would not significantly alter the results of Table 3.

<sup>14</sup> This type of comparison is usually interpreted as a test of the survey expectations rationality in the sense of Muth. Muthian rationality requires that expectations be generated by the same reduced form equation in the exogenous variables which generate the actual variable which is being forecast. A weaker definition suggested by Rutledge (1974) is usually employed in empirical tests. According to this weak form definition of rationality, expectations are rational if they fully incorporate all information contained in the current and past values of the variable being forecast. This same weak form definition was previously employed by Nelson (1970) and Sargent (1972) in studies of the term structure of interest rates. In another study of the term structure of interest rates, Modigliani and Shiller (1973) expand the information set used in the rationality test to include other variables. A serious problem exists with these tests for finite samples of the forecast variable. The forecasts may be rational "ex-ante" but there could exist an "ex-post" model of the particular realization which provides better forecasts. We will ignore this problem, however, and presume that the tests are a test of Muthian rationality.

<sup>15</sup> Adding distributed lags in the money supply or budget deficit does not significantly improve the forecasting accuracy of the model fit to the actual price series. This result has been previously discussed by Feige and Pearce.

<sup>16</sup> The finding that the CPI forecasts are irrational in the sense of not efficiently using information contained in current and past prices is easy to explain if one examines the pattern of forecast errors. For the period 1951 to 1975 the seven month forecast underpredicts the realized price 44 out of 52 times while the thirteen month forecast underpredicts the price level 46 times. By contrast, the best forecasting model underpredicts the realized price 28 and 25 times respectively. The systematic bias present in the survey forecasts is eliminated in the best forecasting model because of the more rapid adjustment of the rate of inflation to errors in forecasting the price level. Our findings on rationality of the CPI differ from those of Mullineaux for three reasons. First, we have used the actual price forecasts in our test rather than the rate of inflation implied by the survey forecasts. Second, Mullineaux concentrates on the period 1959-1969 when the persistent forecasting errors were smaller. Finally, he includes a constant in his tests which negates the effect of the persistent forecasting bias.

<sup>17</sup> A single adaptive expectations model on the rate of inflation,  $\Pi_t^e = \Pi_{t-1}^e + \lambda (\Pi_t - \Pi_t^e)$ , estimated just to the 1959-1975 data interval would give a six month RMS forecasting error of 1.54%/annum.

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