UNCERTAINTY AND INFORMATION

IN ECONOMICS

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Traditional economic analysis passes over, in more or less
embarrassed silence, the problem of uncertainty. The central
elements of economic reasoning have been shaped into models of
ever-increasing precision, but models that presume a degree of
knowledge on the part of economic decision-makers that is pa-
tently unreasonable -- for example, that the firm knows its
demand function, not merely in the present but up to the eco-
nomic horizon. Such an unrealistic picture of the actual deci-
sion-making situation means that economic theory is of little
help to a business man facing an actual marketing choice. At
best the Law of Large Numbers operates to reduce the importance of uncertainty
at the market level, thereby allowing the fiction of the average or
"representative" individual.

Much more fundamentally, models postulating behavior-
al certainty are completely inconsistent with observable real-
world activities of the first importance -- among them insurance,
speculation, research, advertising, and even education.

In the past twenty years, however, an exciting new litera-
ture has addressed the problems of decision and market equili-
brium under uncertainty. This literature has two main founda-
tion-stones: (1) the Von Neumann-Morgenstern [1944] theory of
preference for uncertain contingencies and in particular the
"expected-utility theorem", and (2) Arrow's formulation of the
ultimate goods or objects of choice in an uncertain universe as

The modern literature on uncertainty and information divides into two rather distinct branches. The first deals with <u>market uncertainty</u>. Each individual is supposed to be fully certain about his own endowment and productive opportunities; what he is unsure about are the supply-demand offers of other economic agents. In consequence, search on the individual level, and disequilibrium and price dynamics at the market level, take the center stage -- replacing the traditional assumption of costless exchange at market-clearing prices [Stigler 1961, 1962; McCall 1965]. Explicit analysis of market uncertainty is leading toward a more realistic treatment of market "imperfections," with implications not only for microeconomics but for macroeconomics as well [Phelps <u>et. al.</u> 1970].

The second branch of literature has dealt with what is usually called <u>technological uncertainty</u>, though <u>event uncertainty</u> would be a better term. Depending upon the weather, for example, crops may be large or small. Here individuals are unsure about exogenous data such as resource endowments and/or productive opportunities. Analysts of event uncertainty generally have been content to employ, at least as an initial approach, the simpler traditional model of perfect markets in which all dealings (except possibly in the market for information itself) take place costlessly at equilibrium prices. However, as an important theme in the literature, while the markets that exist are supposed perfect the set of available markets may be assumed to be <u>incomplete</u> -- not every definable object of choice (commodity claim subscripted by state) may be separately exchangeable.

Part 1

THE ECONOMICS OF UNCERTAINTY

1.1 Decision Under Uncertainty

In decision-making under uncertainty the individual chooses among acts while Nature may be metaphorically said to "choose" among states. In principle both acts and states may be defined over a continuum, but for simplicity here a discrete representation will ordinarily be employed. Table 1 pictures an especially simple 2x2 situation. The individual's alternative acts a = (1,2) are shown along the left margin, and Nature's alternative states s = (1,2) across the top. The body of the Table shows the consequences c resulting from the interaction of each possible act and state. Expressed in fuller detail the individual's decision problem requires him to specify: (1) a set of acts a = {1,...,A}; (2) a probability function expressing his beliefs $\pi(s)$ as to Nature's choice of state s = {1,...,S}; (3) a consequence function c(a,s) showing outcomes under all combinations of acts and states; and, finally (4) a preference-scaling or utility function v(c) defined over consequences. Using these as elements, the "expected-utility rule" (Sec. 1.1.4 below) enables the individual to order the available acts in terms of preferences, i.e., to assign a utility function over acts u(a) so as to determine the one most highly preferred.

1.1.1 The Menu of Acts

We shall consider here two main classes of acts: terminal and non-terminal or informational.

Terminal actions represent making the best of one's existing combination of information and ignorance. For example, you might decide whether or not to take an umbrella on the basis of your past history of having been caught in the rain. In statistical theory, terminal action is exemplified by the balancing of Type I and Type II errors in coming to a decision (e.g., accepting or rejecting the null hypothesis) on the basis of the evidence or data now in hand. In contrast with the classical statistical problem, which may be likened to the decision situation of an isolated Robinson Crusoe, in the world of affairs studied by economics many interpersonal arrangements -- insurance contracts, futures markets, guarantees and collateral, the corporation and other forms of shared enterprise -- serve to widen the terminal-act options available to individuals. As we shall see, these market processes provide a variety of ways for sharing risks and returns among the decision-making agents in the economy.

Informational actions are non-terminal in that a final decision is deferred while awaiting or actively seeking new evidence that will, it is anticipated, reduce uncertainty. In statistics, informational actions involve decisions as to new data to be collected: choice of sampling technique, sample size, etc. Again, in the world of affairs interpersonal transactions open up ways of acquiring information apart from the sampling techniques studied in statistics: information may be purchased, or inferred by monitoring the behavior of others, or even stolen. To a degree, information acquisition and dis-

where there are opportunities for _informational_ action; as we shall see in Part 2, the value of acquiring information varies inversely with one's prior confidence.

1.1.3 The Consequence Function

By _consequence_ is meant a full definition of all relevant characteristics of the individual's environment resulting from the interaction of the specified act and state. A consequence can be regarded as a multi-commodity multi-date consumption basket; for simplicity, however, we will sometimes assume that it corresponds to the amount of a single summary variable like income.

In the case of a _terminal_ action, the consequences contingent upon each state might either be certain or probabilistic -- depending upon the definition of "states of the world" for the problem at hand. If the states are defined deterministically, as in "Coin shows Heads" versus "Coin shows Tails," and supposing the act is "Bet on Heads," the contingent consequences are the simple certainties "Win" in the one state and "Lose" in the other. But states of the world might sometimes represent alternative probabilistic processes. For example, for some terminal decision problem the two alternative states might be "Coin is fair (has 50% chance of coming up Heads)" versus "Coin is biased to come up Heads with 75% chance." In this situation the act "Bet on Heads" will have probabilistic consequences: "50% chance of winning" for the first state, and "75% chance" in the second.

For an _informational_ action, on the other hand, the consequences will in general be probabilistic even if the states of

the world are defined deterministically, since acquisition of information does not ordinarily eliminate all uncertainty. If the states are "Rain" versus "Shine," and the informational action is "Look at barometer," the consequences will only be improved likelihoods of behaving appropriately in each state -- since the barometer reading is not a perfect predictor of Rain or Shine.

1.1.4   The Utility Function and the Expected-Utility rule

In the theory of decision under uncertainty, utility as an index of preference attaches both to consequences c and to acts a. We shall sometimes find it convenient to distinguish the two by the notations $v(c)$ and $u(a)$, the problem being to derive the $u(a)$ for evaluating actions from the primitive utility valuations $v(c)$ for consequences.

To choose an act is to choose a row of the consequence matrix, as in Table 1. Given the assignment of probabilities to states, this is also choice of a probability distribution or "prospect." A convenient notation for the prospect associated with an act a, whose consequences $c_a = (c_{a1}, \ldots, c_{aS})$ are to be received with respective probabilities $\pi = (\pi_1, \ldots, \pi_S)$, is

$$a \equiv (c_{a1}, \ldots, c_{aS}; \pi_1, \ldots, \pi_S)$$

Or, more compactly:

$$a \equiv (c_a, \pi)$$

The connection between the preference scaling of acts and the preference scaling of consequences is provided by the Neumann-Morgenstern "expected-utility rule":

$$u(a) \equiv \pi_1 v(c_{a1}) + \ldots + \pi_S v(c_{aS}) \equiv \Sigma_{s=1}^S \pi_s v(c_{as}) \quad (1.1)$$

We can immediately verify that the expected-utility rule
(1.1) does give us the correct valuation at least of simple
reference-lottery prospects like that in (1.2) above:

$$u(\hat{\hat{c}},\hat{c};\pi^*,1-\pi^*) = \pi^* v(\hat{\hat{c}}) + (1-\pi^*)v(\hat{c}) = \pi^*(1) + (1-\pi^*)0 = \pi^*$$

More generally, this argument can be extended to any prospect
whatsoever -- for example, to the prospect $(c', c''; \pi, 1-\pi)$
where c' and c'' are any levels of income and π is any proba-
bility. This individual's $v(c)$ function tells us that the
contingent income c' is indifferent (equivalent in utility
terms) to some chance of success π' in the reference lottery --
$v(c') = \pi'$ -- and similarly c'' corresponds to some chance π'' --
$v(c'') = \pi''$. Then the prospect $(c',c''; \pi, 1-\pi)$ must be equiv-
alent in preference terms to having some computable <u>overall</u>
chance of success in the reference lottery. Specifically, the
prospect gives us the chance π of an income c' equivalent to
a probability of success π', and the chance $1-\pi$ of an income
c'' equivalent to a probability of success π''. Using the laws
of probability, the equivalent <u>overall</u> chance of success is
$\pi(\pi') + (1-\pi)(\pi'')$. But this is just the expected-utility rule:

$$u(c',c'';\pi,1-\pi) = \pi(\pi') + (1-\pi)(\pi'') = \pi v(c') + (1-\pi)v(c'')$$

The expected-utility rule, combined with the constructed
$v(c)$ function, works because the latter is <u>scaled as a proba-
bility</u>. The formula (1.1) for finding an overall $u(a)$ by
weighting the utilities of contingent consequences $v(c)$ is
exactly the formula for finding the overall probability as-
sociated with a set of contingent probabilities.

Marschak 1968], and involves technicalities that cannot be pursued here. Instead, what follows is an informal presentation (based mainly on Schlaifer [1959]) illustrating, by direct construction, the development of a personal cardinal preference-scaling function for use with the expected-utility rule (1.1).

For the purposes of this discussion, we will assume that the contingent consequences c are certainties, and also that c represents simply the quantity of generalized income. Let $\hat{c}$ represent the worst consequence (lowest level of income) contemplated by the individual, and $\hat{\hat{c}}$ the best consequence (highest level of income). As "cardinal" preference scales allow free choice of zero and unit interval, we can let $v(\hat{c}) = 0$ and $v(\hat{\hat{c}}) = 1$. Now consider the intermediate level of income c^*. We can suppose that the individual is indifferent between having c^* for certain and having some chance of success π^* in a prospect or "reference lottery" involving $\hat{c}$ and $\hat{\hat{c}}$. What numerical value can we attach to this common level of utility to allow use of the expected-utility rule? The answer is simply, the probability π^*. Thus

$$u(c^*) \equiv u(\hat{\hat{c}}, \hat{c}; \pi^*, 1-\pi^*) \equiv \pi^* \qquad (1.2)$$

Fig. 1 illustrates a situation in which $\hat{c} = 0$, $\hat{\hat{c}} = 1000$, $c^* = 250$, and $\pi^* = \frac{1}{2}$. That is, in the preferences of this individual a sure income of \$250 is indifferent to a 50% chance of winning in a lottery whose alternative outcomes are \$1000 or nothing. Hence, $v(250) = 1/2$. A similar introspective process generates the individual's entire $v(c)$ curve of Fig. 1, which is his preference-scaling function for consequences.

of certain income) to any probabilistic mixture of consequences
(lottery or prospect) having the same mathematical expectation.
In Fig. 1 we have seen that the reference lottery with equal
chances of $1000 or zero (and thus a mathematical expectation
of $500) is the preference equivalent of a sure income of only
$250. Thus, this person must prefer a sure income of $500 to
a risky lottery with a mathematical expectation of $500. It is
intuitively evident that this generalizes: any point P on a
concave v(c) curve will lie <u>above</u> the corresponding (vertically
aligned) point along the straight line connecting any pair of
positions on v(c) that bracket P. The point <u>on the curve</u> repre-
sents the utility of a given sure income; the ver-
tically aligned point <u>on the straight line</u> represents the uti-
lity of a lottery with a mathematical expectation equal to that
given amount. The generalization of this result, often referred
to as Jensen's inequality, can be expressed as:

$$v''(c) < 0 \rightarrow v(E(\tilde{c})) > Ev(\tilde{c})$$

Here E symbolizes the mathematical expectation operator, and the
tilde indicates that c is a non-degenerate random variable.

It follows immediately that a risk-averse individual endowed
with a given sure income would never accept a <u>fair gamble</u>, a
lottery whose mathematical expectation of net return equals zero
(since it would shift him from a position on the v(c) curve to
a vertically aligned point below it). A gamble would have to
be somewhat better than fair, offer some positive mean return
(just how much depends upon his degree of risk-aversion) to be

that would be consistent with gambling over certain ranges of income and with avoiding gambles over other ranges [Friedman and Savage 1948, and see also Markowitz 1952]. These constructs run against the difficulty that individuals would never be at equilibrium in any risk-preferring range of their v(c) curves. To leave this range they would jump at the chance of accepting enormous riches-or-ruin gambles, provided only that these were available on a fair or nearly-fair basis. Such behavior is surely rare, and there is no indication of ranges of income that are thus depopulated. Except in more or less pathological cases, gambling at adverse odds is a recreational rather than income-status-determining activity for individuals. As evidence, we observe that actual gambling as in Las Vegas is mostly of a repetitive small-stakes nature, more or less guaranteed not to change one's overall income status in the long run [Hirshleifer 1966 p. 261].

That risk-aversion is the normal situation is indicated in a different way by Fig. 2. Here the familiar-looking indifference curves u^o, u', u'', ... represent the preference-scaling function v(c) of Fig. 1 in contingent-income or state-claim space. Specifically, assume for simplicity that there are only two states of the world s_1 and s_2, with corresponding fixed probabilities π_1 and $\pi_2 \equiv 1 - \pi_1$, and contingent consumption variables c_1 and c_2. Then (1.1) reduces to the special form (1.1'):

$$u(a) \equiv \pi_1 v(c_1) + \pi_2 v(c_2) \qquad (1.1')$$

If follows immediately, since du = 0 along any indifference curve,

that the indifference-curve slopes in Fig. 2 are related to the
marginal utilities $v'(c)$ via:

$$\frac{dc_2}{dc_1}\bigg|_{du=0} \equiv - \frac{\pi_1 v'(c_1)}{\pi_2 v'(c_2)} \qquad (1.3)$$

It is then elementary to show that the state claims preferred
to any given bundle C^* form a convex set if and only if $v''(c) < 0$ --
i.e., only if the preference-scaling function $v(c)$ is "concave."

Now let us suppose that the individual is a price taker in
a market where contingent claims c_1 and c_2 can be exchanged in
the ratio P_1/P_2. The price ratio, together with the individual's
endowment position (ω_1, ω_2), determines his budget line $L'L'$ in
Fig. 2. It is then geometrically evident that, given the standard
indifference-curve curvature that stems from risk aversion, C^* will
not normally be at the intersection of the line L' with one of the
axes -- i.e., the individual will want to "diversify" his holdings
of state claims. Following standard techniques, the optimum posi-
tion C^* along the budget line is the tangency determined by the
condition:

$$- \frac{dc_2}{dc_1}\bigg|_{du=0} \equiv \frac{\pi_1 v'(c_1)}{\pi_2 v'(c_2)} = \frac{P_1}{P_2} \qquad (1.4)$$

We can arrive at a much stronger result for the special
case where the price ratio P_1/P_2 equals the probability ratio
π_1/π_2. Since the condition for "fair" gambles can be expressed
as $\pi_1 \Delta c_1 + \pi_2 \Delta c_2 = 0$ -- the expected net return over conse-
quences is zero -- and since in market exchange
$\frac{\Delta c_2}{\Delta c_1} \equiv - \frac{P_1}{P_2}$, this equality of the price ratio and the probability

of the response of C^* to increments of <u>income</u> has led to a specification of interesting properties of the individual's utility function.

Specifically, an individual is said to have constant <u>relative risk-aversion</u> (RRA) if an increase in income, price ratios held constant, leads to a new risk-bearing optimum C^* which involves <u>proportionately</u> more holdings of claims for each and every state (i.e., the new C^* lies on a ray from the origin through the original C^*). Geometrically, if this held everywhere the individual's preference map in Fig. 2 must be homothetic. It can be shown that the condition for constant RRA is:

$$- \frac{c_s v''(c_s)}{v'(c_s)} = R \qquad (1.5)$$

where R is a constant. If R is an increasing function of income (<u>increasing</u> RRA), a rise in income leads to a new C^* proportionately closer to the 45° line than the original C^* -- and the reverse if R is a decreasing function of income (<u>decreasing</u> RRA).

An individual is said to have constant <u>absolute risk-aversion</u> (ARA) if a rise in income results in a final C^* position representing equal absolute increases in state-claim holdings in comparison with the original C^*, for each and every state. This corresponds geometrically to the C^* position shifting parallel to the 45° line as income increases. The analytic condition for constant ARA is given by

$$- \frac{v''(c_s)}{v'(c_s)} = A$$

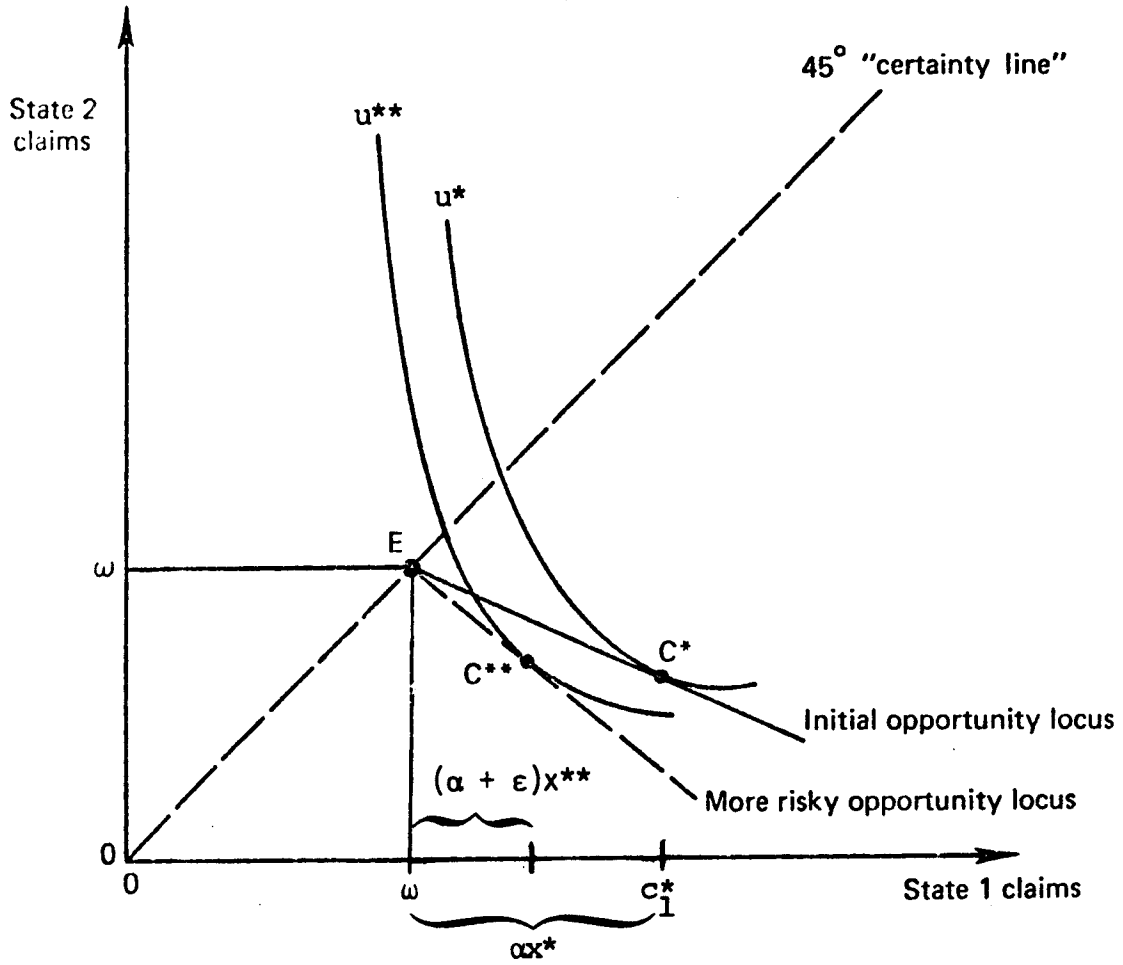where A is a constant. If A is a rising function of income

Fig. 3 — Investing in a risky asset

characteristic phenomena of search and of trading at non-clearing prices). Rather, we are dealing with <u>event uncertainty</u>. And we shall generally be assuming perfect but not necessarily complete markets: trading in consumption claims contingent upon alternative states of the world takes place at market-clearing prices, but not all definable claims may be separately tradable.

1.2.1. <u>Risk-Sharing</u>

If both parties in some transaction are risk-averse they will generally wish to enter into a contract in which the total risks and returns are shared. This can be illustrated by the Edgeworth box in Fig. 4 [Brainard and Dolbear 1971, Marshall 1976], which for concreteness may be thought of as illustrating a "share cropping" problem [Cheung 1969, Reid 1976]. The alternative states of the world are "good crop" or non-loss state N and "bad crop" or loss state L, with associated contingent claims c_N and c_L. Because of the difference in social totals of income in the two states, the box is vertically elongated. Given agreed-upon probabilities π_L, π_N ($=1-\pi_L$), the indifference curves for each agent have the same absolute slope π_L/π_N along their respective 45° certainty lines. It follows that the contract curve TT must lie between the two certainty lines.

Suppose that the Worker W has an alternative opportunity which would yield the certain bundle E along his 45° line (a fixed wage independent of the state). If the Landowner O offers E and thereby bears all the risk, it can be seen that there are **unexploited gains** from trade. Landowner and Worker have an incentive to negotiate an alternative risk-sharing contract somewhere on the contract curve TT. For the limiting case of a perfectly elastic supply of workers all the gains go to the Landowner and the equilibrium contract is the point F.

If the individuals were constrained to strict proportionate sharing of the income totals in the two states, the equilibrium would have to lie along the main diagonal of the Edgeworth box. (This would represent a kind of "incomplete market" for the trading of contingent claims.) Such a solution would not in general be Pareto-optimal, but it might be a rather close approximation of a point on the contract curve. Proportionate sharing in a world of unequal social totals of income would be strictly consistent with a Pareto optimal solution only if conjoined with side payments from one party to another. (With two states of the world a side payment in just one of the states would be required; with S states, a set of S-1 conditional side payments would be needed.)

1.2.2 Insurance

The Edgeworth box in Figure 4 can be given another interpretation: the risk sharing can be regarded as "mutual insurance." Indeed, all insurance is best thought of as mutual [Marshall 1974b]; insurance companies are only intermediaries in the risk—sharing process. Again L would correspond to a "loss" state of the world and N to a "non-loss" state of the world (recognizing that in general loss is a social phenomenon).

For the particular endowment point E all the initial risk is born by individual O since individual W is on his certainty line. Under complete contingent markets the two individuals trade contingent claims c_N and c_L at some price ratio P_L/P_N. If the price ratio were fair ($P_L/P_N = \pi_L/\pi_N$) we know from equation (1.4) above

tinct states of the world: loss suffered by (1) neither person (2) O only (3) W only, and (4) both persons. Let π_n be the probability that the number of losses is n. Continuing to assume equal initial incomes and constant loss amount, let us add the simplifying assumption of symmetry so that the probability of state 2 equals that of state 3 (and hence equals $\frac{1}{2}\pi_1$). The probabilities of each state can then be expressed in terms of the probability of loss by each person p ($\equiv\frac{1}{2}\pi_1 + \pi_2$), $q\equiv1-p$, and the correlation coefficient r between individual outcomes as:

$$\text{probability of state 1} \quad = \pi_0 = q(q+rp)$$
$$\text{probabilities of state 2 and 3} \quad = \frac{1}{2}\pi_1 = pq(1-r)$$
$$\text{probability of state 4} \quad = \pi_2 = p(p+rq) \quad (1.8)$$

The main lesson to be derived from this development is that, in general in a world of uncertainty, _full insurance_ (whereby each party attains his "certainty line") _is impossible_ [Hirshleifer 1953, Brainard and Dolbear 1971, Marshall 1974]. There is a "social risk" in that the social total of losses may be 0, 1, or 2; there is no way of arranging affairs so that _everyone_ can have the same personal income regardless of the social totals of income available. (Of course, _some_ individuals can achieve certainty positions -- but only if others' risks are correspondingly greater). Note also that, as a result of diminishing marginal utility, claims to income in state 4 will be the most valuable and claims to income in state 1 the least valuable (relative to the respective probabilities).

In conventional insurance arrangements, protection would ordinarily be offered to an individual like O without distinction between states 2 and 4 -- more generally, _without_ consideration

We see, therefore, that "social risk" is not exclusively due to small numbers; it persists even with large numbers if risks are on average correlated. In the language of portfolio theory, risks have a "diversifiable" element which can be eliminated by purchasing shares in many separate securities (equivalent to mutual insurance among a large number of individuals) and an "undiversifiable" element due to the average correlation between risks. It follows then that a particular asset will be more valuable the less is the correlation of its returns over states with the aggregate returns of all assets together -- the variability of which is the source of undiversifiable risk. As this concept is applied in modern investment theory, the correlation of returns on each particular security with the returns from the "market portfolio" consisting of all securities together is indicated by that security's "beta" parameter [Sharpe 1978, Ch. 6]. Securities with low or, even better, negative betas trade at relatively high prices (i.e., investors are satisfied with low expected rates of return on these assets) because they provide their holders with relatively large returns in just those states of the world where aggregate incomes are low (marginal utilities are high).

The "social risk" phenomenon therefore provides two reasons why insurance prices may not be fair or actuarial, so that purchase of coverage is ordinarily less than complete: (1) if the number of risks in the insurance pool is small, so that the Law of Large Numbers cannot fully work, or (2) even with large numbers, if risks are on average correlated.
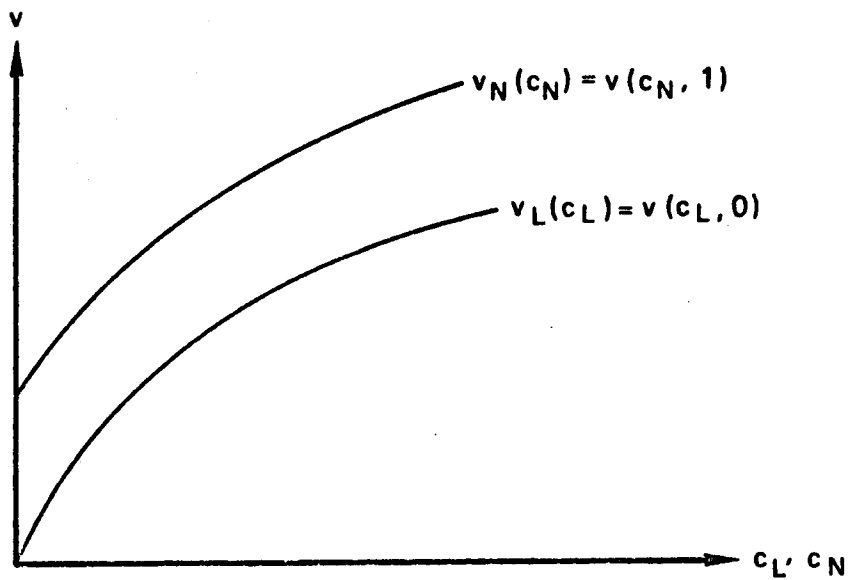
$$v_N(c_N) = v(c_N, 1)$$

$$v_L(c_L) = v(c_L, 0)$$

$$c_L, \; c_N$$

Fig. 5a — State-Dependent Utility

Income Certainty Line

45°

E

C*

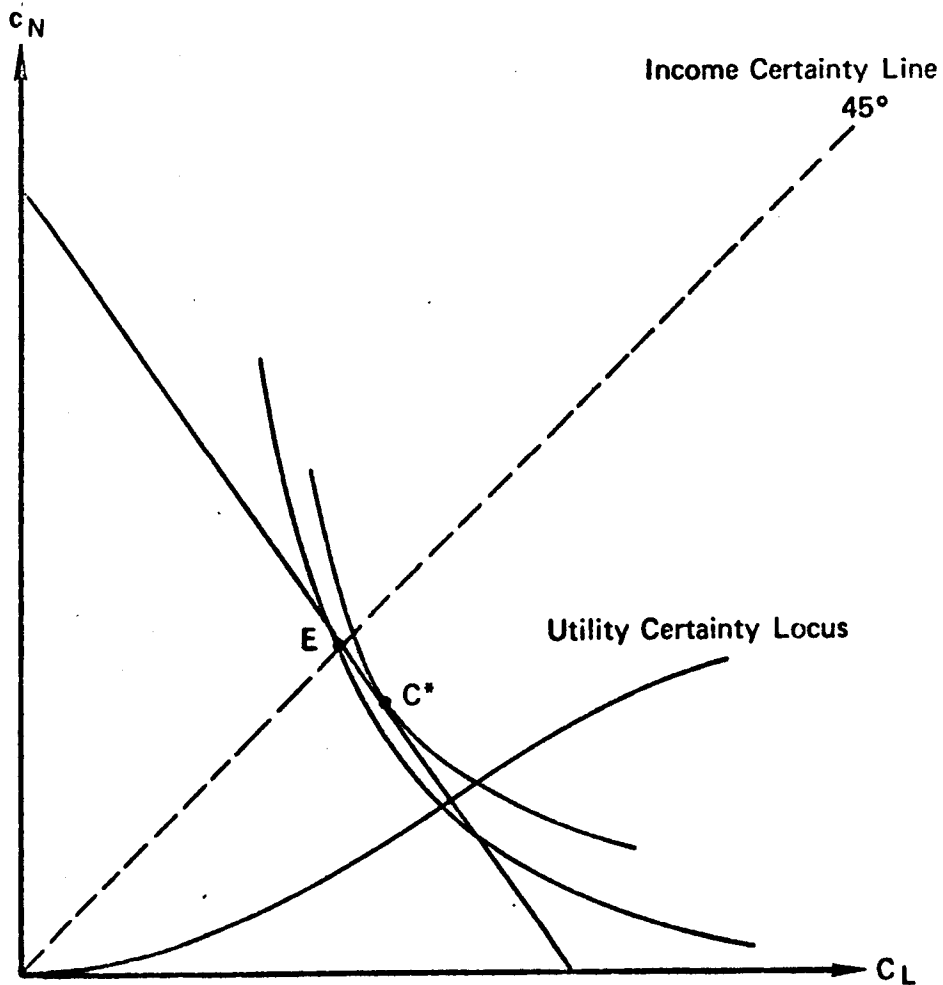Utility Certainty Locus

$$c_N$$

$$C_L$$

Fig. 5b — "Heirloom" Insurance

upon whether or not income c and the "heirloom" variable h

are Edgeworth substitutes -- i.e., whether the cross-derivative

of the cardinal utility function is negative. For an heirloom

such as an ancestral painting with negligible cash value it is

hard to establish an a priori case either way. We can thus ex-

pect to find that some people insure such objects while others,

similarly situated, do not.

However if h = 0 represents an injury leaving the indivi-

dual with major paralysis it seems reasonable that marginal

utility will be higher in the loss state (when paralyzed one

"needs" additional income to achieve a similar consumption bundle).

In such cases the optimum must therefore lie to the southeast

of the income certainty line -- but not necessarily southeast

of the utility certainty locus. That is, the individual will

buy insurance against loss, but not necessarily so much as to

be "fully insured" in the sense of not caring whether or not

the injury occurs.

The situation is very different if the variable h represents

the life of one's child. It then seems plausible that h and c

are complements; if your child dies (h=0), you have _less_ need

for income, since you planned to spend it mainly on him. In

such a case it is optimal to transfer income from the loss state

to the non-loss state. That is, such an individual would "reverse insure" --

would bet that the loss would not occur. (Contractually, instead

of insuring his child's life he might buy a life annuity for him.)

We see that once allowance is made for state-dependent utility,

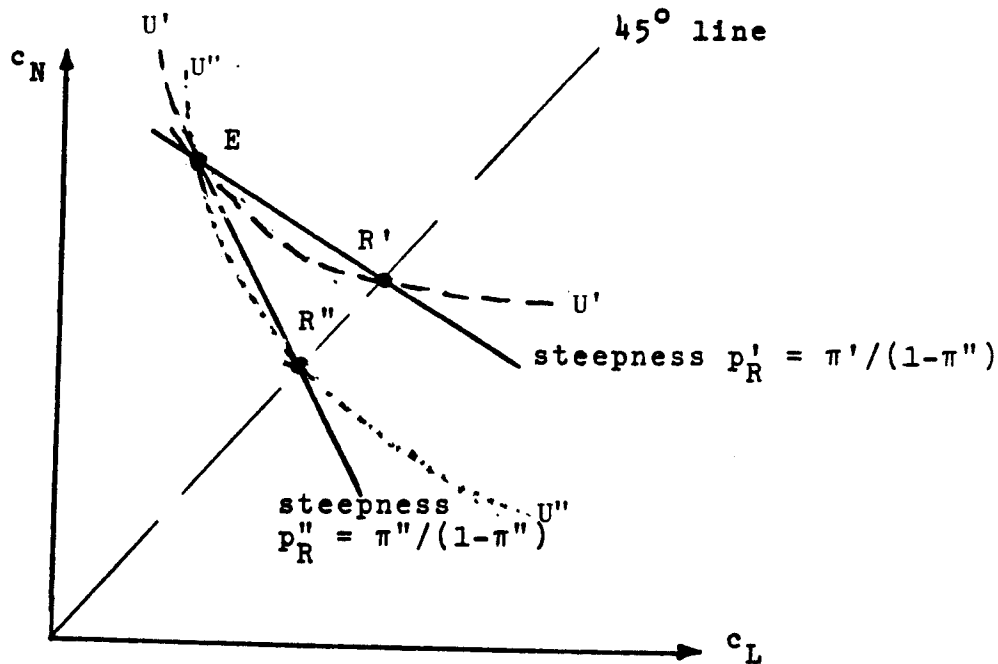it can no longer be presumed that individuals offered actuarial in-

Fig. 6a: "Reservation price ratios" for two risk classes

$$p_R' = \pi'/(1-\pi'')$$
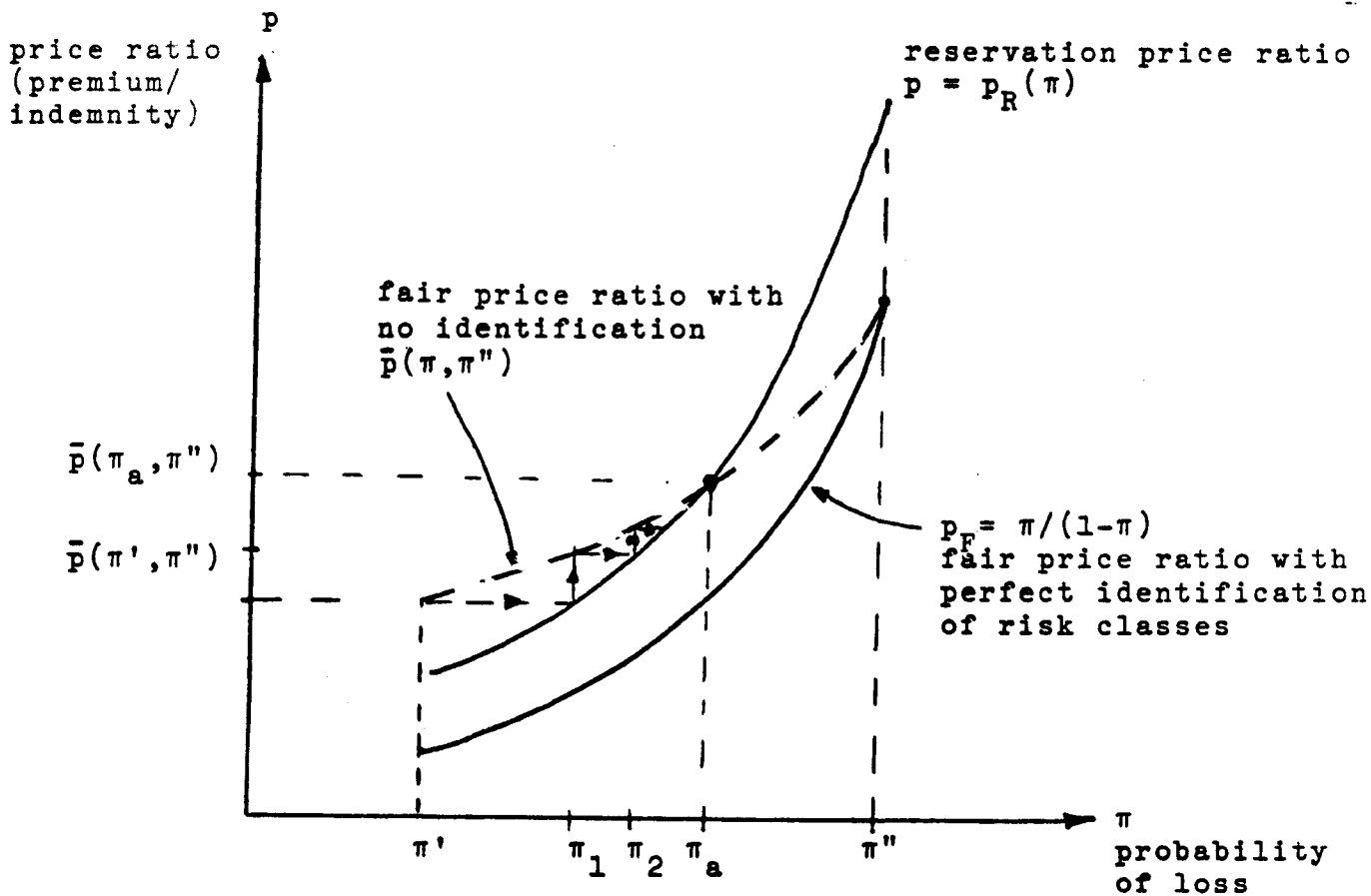
$$p_R'' = \pi''/(1-\pi'')$$

Fig. 6b: Adverse Selection

But now suppose that the insurers (other members of the mutual insurance pool) have no way of distinguishing individuals belonging to different risk classes. Instead they offer insurance at the price ratio $\bar{p}(\pi',\pi'')$ based on the average probability of loss, $\bar{\pi}$, across all risk classes $(\bar{p} = \bar{\pi}/(1-\bar{\pi}))$. If the difference between risk classes is sufficiently great there will be some probability of loss π_1 such that $p_R(\pi) < \bar{p}(\pi',\pi'')$ for all risk classes $\pi < \pi_1$. That is, as depicted in Figure 6b, the lowest risk classes have a reservation price ratio which is lower than the fair price ratio when all risk classes are pooled. It follows that only those with loss probability greater than π_1 will purchase full-coverage insurance. To cover expected claims insurance companies must, therefore, raise the price ratio to $\bar{p}(\pi_1,\pi'')$ which reflects the average probability of loss for all risk classes for whom $\pi \geq \pi_1$. But this in turn results in further exit by risk classes in the interval (π_1, π_2) and again insurance companies are forced to raise the premium/indemnity ratio $\bar{p}$. Only when all those risk classes with a loss probability π less than π_a have withdrawn is an equilibrium reached.

This is the problem of <u>adverse selection</u>. While we have described it in the insurance context, it is a much more general phenomenon. Wherever buyers are only able to observe average quality, there is a tendency for sellers not fully rewarded for high quality to withdraw from the market. In one extreme model of

$$\frac{P_{N \cdot x}}{P_{L \cdot x}} = \frac{\pi_{N \cdot x}}{\pi_{L \cdot x}}$$

That is, the insurance rate will reflect the level of care x. For any fixed level of x, fair insurance results in full coverage. Therefore, if the individual faces the prospect of a loss of ℓ and initial income is ω, expected utility after insurance is:

$$u(x) = v(x, \omega - \ell\pi_{L \cdot x}), \tag{1.11}$$

where $\ell\pi_{L \cdot x}$ is the premium so that $\omega - \ell\pi_{L \cdot x}$ is a constant income received in either state.

The individual then chooses that policy for which the marginal disutility of additional care is just high enough to offset the marginal utility of the lower premium resulting from the extra care.

More realistically, however, monitoring is at best imperfect. This leads to the problem known as <u>moral hazard</u>. In the extreme case, if insurance is offered at some fixed price ratio independent of any loss-prevention activity, it is evident that an individual will be motivated to entirely eliminate all such activity.

Insurers have two main ways of coping with the problem [Arrow 1963, Pauly 1968]. The first is to require the insured party to bear some portion of the risk, for example by a "deductible" provision (indemnity will be less than the loss by a fixed amount) or by "coinsurance" (indemnity will only be a proper fraction of the loss). Then insurance will continue to be provided [Shavell 1977], but moral hazard persists in the sense that insureds are motivated to engage in less preventive activity than would be efficient with costless information.

independent (i.e., that none of them could be expressed as a linear combination of the others). We shall call this a regime of Complete Contingent Markets. From (1.12) each agent has the same marginal rate of substitution between every pair of state claims, hence in equilibrium the trades t^i result in a Pareto-efficient allocation.

·Consider instead a "stock market economy". This will be defined as a situation where there are F distinct types of tradable assets, each consisting of some total vector of state claims $\omega^f = (\omega_1^f, \ldots, \omega_S^f)$. Individual i has an untradable endowment $\omega^i = (\omega_1^i, \ldots, \omega_S^i)$ plus endowed amounts of tradable <u>shares</u> $(\bar{\alpha}_1^i, \ldots, \bar{\alpha}_F^i)$ of the F "firms" in the economy.

If each firm's holding has market value V_f, the individual's decision problem is to choose a portfolio $(\alpha_1^i, \ldots, \alpha_F^i)$ subject to his marketable wealth constraint:

$$\Sigma_f \alpha_f^i V_f = \Sigma \bar{\alpha}_f^i V_f \qquad (1.13)$$

His final consumption is $c^i = \omega^i + t^i$ where:

$$t^i = \Sigma_f (\alpha_f^i - \bar{\alpha}_f^i) \omega^f \qquad (1.14)$$

The individual then chooses a portfolio to maximize :

$$u(\omega^i, \alpha_1^i, \ldots, \alpha_F^i) = \Sigma_s \pi_s v(c_s^i) \qquad (1.15)$$

subject to (1.13) and (1.14). To achieve this he expands or contracts his holdings in the different firms until the expected marginal utility of a dollar invested in each asset is equated to his expected marginal utility of wealth, λ^i, that is:

$$\frac{\Sigma_s \Pi_s v'(c_s^i) \omega_s^f}{V_f} = \lambda^i \qquad \text{for all f and all i,} \qquad (1.16)$$

The stock market model is particularly interesting when we pass from the realm of pure exchange to consider aggregate endowments and attendant uncertainty as being generated endogenously by production decisions. The vector ω^f for firm f now becomes the result of a production decision on the part of owners of the firm's shares, subject of course to constraints in the form of the production possibilities available. For expositional ease we shall characterize the firm's decision as a scalar x, generating a final output $\omega^f(x)$.

A question that has received considerable attention is whether maximization of a firm's market value V_f is in the interests of all its shareholders, and therefore would be unanimously chosen by them. In general the answer is in the negative. However, suppose that when firm f announces a new plan $x + \Delta x$, there is only a negligible effect on any shareholder's marginal utility of income in the different states. Then the expected marginal utility of wealth is unchanged and, from (1.16) the new market value of the firm must satisfy:

$$\frac{\sum_s \pi_s v_i'(c_s^i) \, \omega_s^f(x + \Delta x)}{V_f + \Delta V_f} = \lambda^i \tag{1.18}$$

Multiplying both sides of (1.16) and (1.18) by the value of firm f and then subtracting we have:

$$\sum_s \pi_s v_i'(c_s^i) \{\omega_s^f(x + \Delta x) - \omega_s^f(x)\} = \lambda^i \Delta V_f, \text{ for all i.} \tag{1.19}$$

The left hand side of this expression is individual i's expected marginal utility of the change in the firm's production plan. Since $\lambda^i > 0$ this is positive for all i if and only if ΔV_f is positive. That is, any plan will have the unanimous support of the stockholders if and only if it raises the value of the stock.

shrinks toward zero as the number of consumer-shareholders becomes very large [Hart 1978]. Then shareholders again would unanimously support value-maximization as the goal of the firm.

1.2.4 Other Applications

In this Part 1 we have provided a relatively extensive treatment of insurance; under that heading we have been able to expound and illustrate, in rather simple format, most of the basic ideas of modern uncertainty theory. (Of course, we have scarcely been able to hint at the many exciting developments of a more advanced nature.) We have also referred, briefly, to two other applications of uncertainty theory: (1) investments and portfolios, and (2) share cropping. A number of other significant applications can only be mentioned here: (3) optimal contracts between agent and principal, for example to elicit ideal performance on the part of corporate managers [Marschak and Radner 1972, Harris and Raviv 1978, Shavell 1978, Cheung 1969, Groves 1973, Alchian and Demsetz 1972, Jensen and Meckling 1976, Zorn 1978]; (4) corporate finance, and in particular the balance between debt and equity funding [Modigliani and Miller 1958, Lintner 1962, Hirshleifer 1966, Fama and Miller 1972, Ch. 4]; (5) optimal behavior and equilibrium with respect to accidents [Vickrey 1968, Calabresi 1970, Baumol 1972, Diamond 1974]; the "value of life" appropriate for risk-taking decisions [Mishan 1971, Thaler and Rosen 1975, Conley 1976, Schelling 1968, Jones-Lee 1974, Bergstrom 1974]; and (6) choice of discount rate for public investment [Hirshleifer 1966, Arrow and Lind 1970, Sandmo 1972, Bailey and Jensen 1972, Mayshar 1977].

That is, the revised or _posterior_ probability $\pi_{s.m}$ assignable to state

s after receiving message m equals the ratio of the _joint_ probability π_{sm}

of state s and message m both occurring, divided by the _prior_ probability

q_m of message m. Furthermore, using standard laws of probability, the numerator and

the denominator on the right hand side of (2.1) can be expressed in terms of the prior

probabilities π_s of the different states and the conditional probabilities

or "likelihoods" $q_{m.s}$ of any message m given state s:

$$\pi_{sm} = \pi_s q_{m.s} \qquad (2.2)$$

$$q_m = \sum_s \pi_{sm} = \sum_s \pi_s q_{m.s} \qquad (2.3)$$

EXAMPLE: Suppose in a coin-tossing situation that an individual

initially assigns equal prior probabilities of 1/3 to three states of the

world: (1) coin is 2-headed, (2) coin is 2-tailed, and (3) coin is fair.

The possible messages, on a single toss of the coin, are Heads (H) and Tails

(T). Suppose Tails comes up. Then $\pi_{1.T}$, the posterior probability of state 1,

must obviously be zero (since $q_{T.1}$, the likelihood of the message Tails given

state 1, is zero). Using equations (2.2) and (2.3), the posterior probability

of state 2 is the fraction with numerator $\frac{1}{3}(1)$ and denominator $0 + \frac{1}{3}(1) + \frac{1}{3}(\frac{1}{2}) = \frac{1}{2}$,

whose value is 2/3. Similarly, the probability of state 3 can be found to be

1/3.

Fig. 7 is a suggestive illustration of Bayesian recalculation of

probabilities on the basis of a given message m, where the possible states

of the world are a continuum of values of s from zero to some upper limit S.

The prior distribution shows that the bulk of the initial probability weight

happens to lie toward the high end. But, the likelihood function indicates,

the message (evidence) received is much more likely if s has a small rather

than a large value. The posterior distribution is a compromise or average

of the other two curves, derived by multiplying (for each s) the prior
probabilities and likelihoods as in equation (2.2), and then re-scaling so
that the integrated probability weight comes out to unity. We can concept-
ually picture the process as:

| Prior beliefs π_s | Message m and Likelihoods $q_{m.s}$ |

Posterior beliefs $\pi_{s.m}$
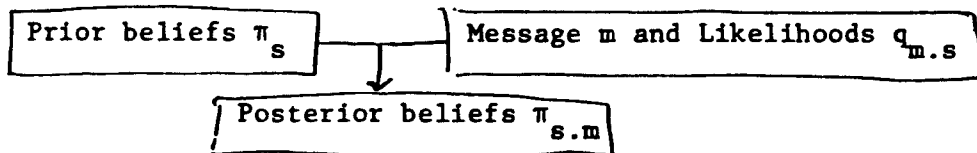
The individual's <u>confidence</u> in his initial beliefs is indicated by the
"tightness" of his prior probability distribution -- the degree to which he
approaches assigning 100% prior probability to some single possible value
for s. Evidently, the higher the prior confidence the more the posterior
probability distribution will resemble the prior, for any given weight of
evidence as summarized in the likelihood function. It follows quite directly,
as we shall see again below, that greater confidence implies attaching
lesser value to acquiring evidence.

While the prior beliefs will ordinarily be "personal" or "subjective"
probabilities, in at least some cases the likelihoods might be "objective"
in the sense of being calculable via the laws of probability. For example,
if two tosses of a coin yield the message "Heads both times," then <u>given</u>
the state of the world that the coin is fair probability theory says that
the likelihood is $q_{m.s}=1/4$. A main theorem of Bayesian statistical theory is that as
the sample size increases, the weight of the evidence tends to rise relative
to the importance of the prior probabilities -- so that, in the limit,
objective evidence tends to swamp out divergences in personal prior beliefs.
(In some cases, however, there may be an irreducible subjective element in
the likelihoods as well as in the initial probabilities.) One other point
worth noting is that, other things equal, "more surprising" evidence (low

$$\Delta_m = u(a_m, \pi_{s.m}) - u(a_0, \pi_{s.m}) \qquad (2.5)$$

Note that Δ_m, which is necessarily non-negative, is an _ex post_ valuation. It represents the expected gain from revision of best action, estimated in terms of the revised probabilities.

However, the decision to seek information must necessarily be made _ex ante_. One is never in the position of choosing whether or not to receive the particular message m; the essence of the problem is that the information-seeker does not know in advance which of the set of possible messages m = 1,...,M he will obtain. What the agent can actually purchase is not a particular message but an _information service_ μ -- generating a probability distribution of messages m.

An information service is best thought of as characterized by its matrix of likelihoods $Q = [q_{m.s}]$. In the example with three possible states of the world for a coin (2-headed, 2-tailed, and fair), the information services associated with sample sizes of one and two are represented by the matrices Q_1 and Q_2 below:

		(Message) # of Heads					(Message) # of Heads		
		1	0				2	1	0
	(State)					(State)			
$Q_1 =$	1	1	0		$Q_2 =$	1	1	0	0
	2	0	1			2	0	0	1
	3	.5	.5			3	.25	.5	.25

Note that a purveyor of an information service μ could not "objectively" characterize it by its probabilities q_m of generating the different possible messages, since -- as equations (2.1) to (2.3) indicate -- not only the likelihoods $q_{m.s}$ but also the "subjective" prior probabilities π_s are involved in determining the message probabilities q_m. However, given his prior
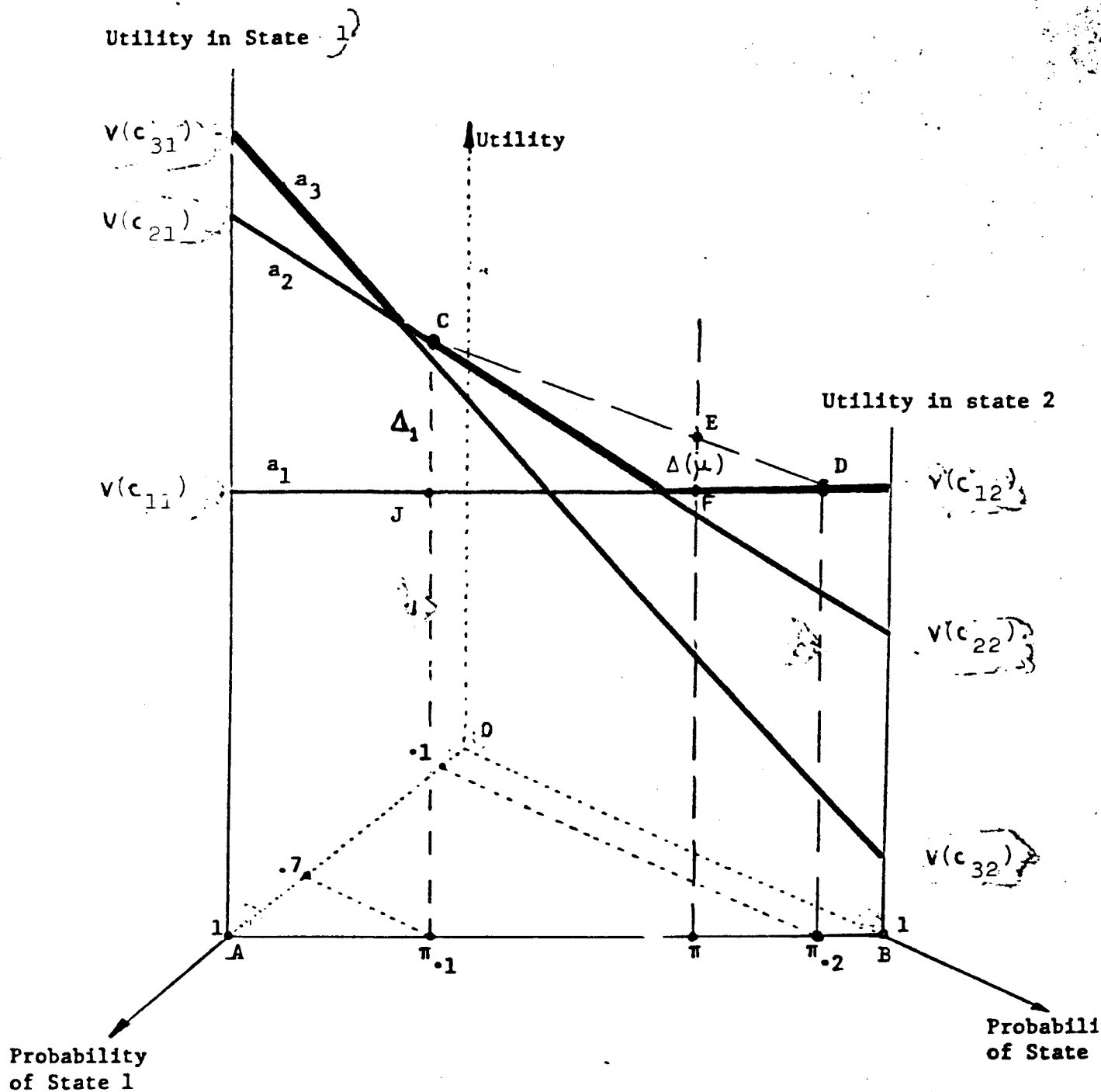
Figure 8: The Value of Information

a more informative information service would be preferred (cost being equal) independently of the decision-maker's particular situation. There was some initial hope of using Shannon's "entropy" measure of communication theory for this purpose [Shannon 1948]. "Entropy" in the informational sense is the expected number of binary messages needed to communicate the output of an information service. This is indeed a measure of the amount of information, somewhat analogous to ton-miles as a measure of amount of transportation. However, Kenneth J. Arrow (1971) and Marschak (1973) have shown that only in a special logarithmic case does the entropy measure effectively scale the <u>value</u> of information.

Between some information services an informativeness ordering is clearly possible; a random sample of 2, we know, must be more informative than a sample of 1. But in general informativeness can only be partially ordered. The condition for $(\hat{\Pi},\hat{q})$ to be more informative than (Π,q) is that the posterior probability vector, associated with each message under the latter, is a "reduced version" in the sense of a convex combination of the posterior probabilities under the more informative service. That is, if:

$$\pi_{.m} = \sum_m \theta_m \hat{\pi}_{.m} \qquad , \text{ where } \theta_m \geq 0 \text{ and } \sum_m \theta_m = 1 \qquad (2.7)$$

This condition is easily visualized in terms of Fig. 8 , which pictures a particular information service (Π,q) leading to posterior probability vectors $\pi_{.1}$ and $\pi_{.2}$. Suppose an alternative information service $(\hat{\Pi},\hat{q})$ also had two possible messages 1 and 2, but $\pi_{.1}$ were to lie to the left of $\pi_{.1}$ and $\hat{\Pi}_{.2}$ to the right of $\pi_{.2}$. The alternative service must lead to higher utility

$\pi^*_{.1}$ and $\pi^*_{.2}$. An important special case of garbling is where distinctions are obliterated: two or more distinct messages of a more informative service are reduced to a single message of a less informative service [Radner 1968, Hart 1975]. This may lead to a situation where different agents have different partitionings of the states of the world, with consequent difficulty for the negotiation of contingent contracts.

Fig. 8 also indicates an important "non-concavity" (condition of increasing marginal returns) in the valuation of information services [Radner and Stiglitz 1975]. Starting with the null information service with posterior probabilities equal to the priors π, suppose a slightly informative $\hat{\mu}$ comes along with posterior probabilities $\hat{\pi}_{.1}$ just barely to the left of π and $\pi_{.2}$ barely to the right. If the probability changes are small, neither message changes the associated best action, and hence there can be no utility gain. So the marginal return of improved information will be zero over a certain range, before becoming positive at the point where the improvement begins to affect action taken after at least one of the possible messages.

Thus far we have interpreted messages essentially as sample evidence, for which likelihood functions that follow from the laws of probability can be calculated for use in Bayes Theorem. More generally, however, a message can take a form for which the likelihood function is not so easily assessed. One important example is "expert opinion." In general, it would seem that a decision-maker should take account of the opinions (if available) of all other parties who have some information not accessible to himself [Shavell 1976]. But such "expert" opinion would generally be a posterior probability vector that would in part depend upon objective evidence, but also in part

members of the group may agree <u>not</u> to seek information where an impartial

observer would advise them to do so.

If there are conflicts of interest as well as lack of consensus, the

<u>redistributive effect</u> of information becomes important. The group tends

to agree to acquire information, even at a collective loss, once each member

has staked out a position whereby he expects to benefit thereby at the

expense of the others -- as by a wager. Note that even if there were no

initial conflicts of interest, wagering would convert mere differences of

opinion into conflicts of interest.

2.1.2 Other informational activities

So far, under the heading of informational decision-making we have

only considered the <u>acquisition</u> of evidence -- as by generation of sample-

data (the production of socially "new" information) or the receipt of expert

advice (the interpersonal transfer of "old" information). But other types

of informational activities can also be very important. The possibility of

acquiring information from others, as discussed in connection with "expert

opinion" above, immediately suggests the reverse activity -- the <u>dissemination</u>

of information to other economic agents. This might be done for a price,

as when one is hired as an expert, but (as we shall see below) sometimes

it may pay to disseminate gratuitously, or even to incur cost to "push"

information to others [Hirshleifer 1973]. Advertising is an obvious example.

There is also a choice as to disseminating publicly ("publishing"), or else

privately to a select audience. As a question of authenticity might arise

in all such cases, the receiver of information may devote effort to the

process of <u>evaluation</u>, possibly assisted by <u>authentication</u> activities (or hampered

2.2 Emergent Information

In the section preceding we thought of information as being newly

generated by an informational action like a sampling experiment or, al-

ternatively, as acquired from others via

a transaction like the purchase of expert opinion. But in some

cases information may autonomously emerge simply with the passage of time,

without requiring any direct action by recipients. Tomorrow's weather is

uncertain today, but the uncertainty will be reduced as more

meterological data flows in and will in due course be conclusively

resolved when tomorrow arrives. Direct informational actions might still

be useful, by providing knowledge earlier then it would autonomously arrive.

But under conditions of emergent information a kind of "indirect" informational

action becomes available -- simply waiting before taking terminal action.

2.2.1 The value of flexibility

Suppose a choice must be made now between immediate terminal action

and awaiting emergent information. This choice can only be interesting,

of course, where there is a trade-off between two costs: (1) a cost of

waiting, versus (2) an "irreversible" element in the possible loss suffered

from mistaken early commitment. Exactly these elements have been involved

in analyzing the benefit of actions that irreversibly transform the

environment [Arrow and Fisher 1974, Henry 1974] and in discussions of the

value of "liquidity" [J. Marschak 1949, Hirshleifer 1972] or of "flexibility"

[T. Marschak and Nelson 1967, Jones and Ostroy 1978].

The essential idea is pictured in Fig. 9 (which has the same structural

framework as Fig. 8 but suppresses the 3-dimensional background). The

individual, if he decides upon immediate terminal action, has a choice

among a_1, a_2, or a_3. As shown here, he would choose either a_2 or a_3 de-

pending upon his beliefs π (in the diagram, he prefers a_3 yielding utility F).

As the diagram is drawn, he would never choose a_1 as a terminal action.

But suppose that a_1 has a "flexibility" property. To wit, <u>after</u> receiving

emergent information the individual can shift from a_1 to a_2, achieving

the intermediate overall utility indicated by the line a_{12}

or, should the information point the other way, he can shift from

a_1 to a_3 with overall utility payoff indicated by line a_{13}. In the situation

as drawn, had he initially chosen the "flexible" action a_1, then if message

1 is received (leading to the posterior probability vector $\pi_{.1}$) the individual

would shift to a_2, thus attaining overall utility indicated by point C

on line a_{12}. Similarly, message 2 would allow him to attain point D on

line a_{13}. His expected utility is then E, superior by the amount EF to

the result of the best immediate terminal action a_3.

The element of "irreversibility" appears here in the fact that line

a_{12} lies below a_2 in the range where both of these are preferred to a_1,

and similarly a_{13} lies below a_3 in the corresponding range. One has to

pay a price to retain flexibility; the price is that you cannot do as well

as if you had made the best choice among "irreversible" actions in the

first place. Indeed, if the incoming information had somewhat lesser

weight, so that the posterior belief vectors $\pi_{.1}$ and $\pi_{.2}$ were not so different

from the original π, the price of flexibility can become too great; point E

would then be somewhat lower in the diagram, and might well fall below the

point F representing the utility of the best immediate terminal action (a_3).

state s — at price P_{gs}. After state s* obtains, posterior trading

becomes possible among the G remaining valid claims c_{gs*}. But suppose for

the moment that individuals ignore the possibility of posterior trading in

their prior-round dealings. Then the optimality conditions include ratios

of the following form, where g' and g" are any two goods:

$$\frac{\pi_s \frac{\partial v}{\partial c_{g's}}}{\pi_s \frac{\partial v}{\partial c_{g"s}}} = \frac{P_{g's}}{P_{g"s}} \qquad (2.9)$$

After the conclusive information arrives, and state s* is known to obtain,

in the only condition of (2.9) that remains relevant π_{s*} now equals unity.

But this makes no difference; in fact, π_s cancels out whatever its value.

Therefore the price ratio on the RHS of (2.9) continues to sustain the solution

attained in the prior round.

Thus we see that even though posterior trading is <u>possible</u>

(there remain G tradable claims c_{gs*}), with CCM in the prior round no-one

will find such trading <u>advantageous</u>. In effect, markets for GS prior claims

plus G posterior claims are "more than are needed." Qualification: If

prior-round traders failed to correctly ·forecast that the conditional price

ratio on the RHS of (2.9) would remain unchanged in the posterior round,

they would be led to make "erroneous" prior-round transactions, in turn

requiring "corrective" posterior-round transactions,

So Complete Contingent Markets in the prior round suffice for Pareto-efficiency

only subject to a proviso of "correct conditional price forecasting."

Consider now another special case. We return to the assumption of ·a

Interesting questions arise, however, when we analyze market regimes that are <u>incomplete</u> (as, of course, they must actually be in the world). There are many different possible patterns of incompleteness. We will consider here only the particular case of emergent <u>conclusive</u> information; then the set of M messages collapses into the set of S states so that individuals are only concerned with c_{gs} claims, GS in number. Equation (2.9) represented the first-order optimality conditions holding under CCM for this case. Two types of incomplete-market regimes, Numeraire Contingent Markets (NCM) and Futures Markets (FM) will now be discussed.

Arrow $[1953]$ has shown that the same allocation as indicated by conditions (2.9) under CCM (trading in GS claims) is achievable with prior contingent trading in only a <u>single</u> commodity. Let us think of this commodity as the numeraire good, g=1. Then under NCM only S claims of form c_{1s} would be tradable in the prior round. We can think of these as side-bets in numeraire units as to which state of the world is going to obtain. These bets determine the individual's posterior wealth under each possible state of the world, which he can then use to purchase a preferred consumption basket in the posterior round.

Under either regime of markets, the individuals is seeking to maximize expected utility $U = \sum_s \pi_s v(c_{1s}, \ldots, c_{Gs})$. Under the CCM regime, the constraint is:

$$\sum_s \sum_g P_{gs} c_{gs} = \sum_s \sum_g P_{gs} \omega_{gs} \qquad (2.11)$$

where ω_{gs} indicates an endowment quantity. For the NCM regime, prices will be symbolized as $\emptyset_{1s}$ for the prior markets and $\emptyset_{g.s}$ for the posterior market (after it is known that state s obtains). In the prior round, the individual will transact at prices $\emptyset_{1s}$ to arrive at an intermediate "trading position"

Table 2

COMPLETE CONTINGENT MARKETS (CCM) VERSUS NUMERAIRE CONTINGENT MARKETS (NCM)

Equilibrium for Representative Trader-Pair J, K

DATA		J		K	
Preference-scaling function v		$\ln c_1 + \ln c_2$		$\ln c_1 + \ln c_2$	
Beliefs:	(π_1, π_2)	(.7, .3)		(.5, .5)	
Endowment:	$\begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix}$	$\begin{bmatrix} 200 & 200 \\ 0 & 0 \end{bmatrix}$		$\begin{bmatrix} 0 & 0 \\ 400 & 160 \end{bmatrix}$	

COMPLETE CONTINGENT MARKETS (CCM)

		J		K	
Equilibrium prices: $P_{gs} =$	$\begin{bmatrix} .6 & .4 \\ .3 & .5 \end{bmatrix}$				
Consumption:	$\begin{bmatrix} c_{11} & c_{12} \\ c_{22} & c_{22} \end{bmatrix}$	$\begin{bmatrix} 116\frac{2}{3} & 75 \\ 233\frac{1}{3} & 60 \end{bmatrix}$		$\begin{bmatrix} 83\frac{1}{3} & 125 \\ 166\frac{2}{3} & 100 \end{bmatrix}$	

NUMERAIRE CONTINGENT MARKETS (NCM)

Equilibrium prices:

		J		K	
Prior:	$\phi_{1s} = (.6, .4)$				
Posterior:	$\phi_{g.s} = \begin{bmatrix} 1 & 1 \\ .5 & 1.25 \end{bmatrix}$				
Trading position:	$\begin{bmatrix} c_{11}^t & c_{12}^t \\ c_{21}^t & c_{22}^t \end{bmatrix}$	$\begin{bmatrix} 233\frac{1}{3} & 150 \\ 0 & 0 \end{bmatrix}$		$\begin{bmatrix} -33\frac{1}{3} & 50 \\ 400 & 160 \end{bmatrix}$	
Consumption	$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$	$\begin{bmatrix} 116\frac{2}{3} & 75 \\ 233\frac{1}{3} & 60 \end{bmatrix}$		$\begin{bmatrix} 83\frac{1}{3} & 125 \\ 166\frac{2}{3} & 100 \end{bmatrix}$	

Under NCM the S tradable claims in the prior round just sufficed,with re-trading among the G claims in the posterior round, to reproduce the same result. (The proviso as to "correct conditional price forecasting" being assumed to hold in each case.) It will be evident that the G+G tradable claims in two rounds under FM cannot do as well as the S+G under NCM, if G < S. However, this negative conclusion tends to be overcome once we allow for the fact that emergent information in the world is only rarely conclusive. If improved but not yet conclusive information emerges repeatedly, the multiple opportunities for trading among the G goods that are re-created after each informational input increase the effectiveness of FM relative to NCM and CCM. (Again, the proviso as to correct conditional price forecasting is required under FM as well.)

We shall not provide here a formal statement of the optimality or equilibrium conditions for FM (see Hirshleifer [1977], Townsend [1978]). The crucial point is that, in the prior round, the difference between the elements c_{gs}^t of the trading position for any good g and the corresponding elements ω_{gs} of the endowment position is a constant g^t:

$$c_{gs}^t - \omega_{gs}^t = g^t \quad , \ s=1,\ldots,S \qquad (2.15)$$

Here g^t is simply the unconditional purchase (or sale, if negative) of good g in the prior round. Table 3 illustrates that, if G=S (=2 in this example), then with conclusive emergent information the same final result for the representative trader-pair J,K can be attained under FM as shown in Table 2 for CCM and NCM. Table 3 also illustrates an alternative

representative trader-pair L,M -- differing from J,K only in that L,M share identical beliefs. The L,M world generates the same market equilibrium prices as the J,K world though the individual outcomes are different. Note that L and M choose not to trade in the prior round, a point that will take on significance in the discussion of speculation that follows.

2.2.3 Speculation

The term "speculation" has caused a good deal of confusion. Some authors loosely apply the word to arbitrage between markets, or to storage of goods over time or carriage over space -- activities which do not involve uncertainty in any essential way. For our purposes, speculation is purchase with the intention of re-sale, or sale with the intent of re-purchase, where the uncertainty of the future spot price is the source of both risk and gain. The probabilistic variability of price is in turn due to anticipated emergence of information. Each possible message (in the conclusive-information case that we shall be assuming here, this is equivalent to the advent of a single possible state) leads to an associated equilibrium posterior price vector, benefiting agents who adopted trading positions generating relatively high conditional wealths for that state.

From the discussion in the preceding section showing that re-trading possibilities are not needed in a regime of Complete Conditional Markets (CCM), we see that speculation is a response to incompleteness of prior-round markets /Feiger 1976/. (However, if the proviso as to correct conditional price forecasting fails to hold, "corrective" re-trading would be needed even under CCM.) Also involved as determinants of speculative activity are initial risk-exposure in the form of unbalanced state-endowments, degree of risk-aversion, and probability beliefs as to future states of the world.

$\phi_2 = .8$ -- the mathematical expectation of the conditional posterior prices $\phi_{2.1} = .5$ (with probability .6) and $\phi_{2.2} = 1.25$ (with probability .4) -- so that neither need pay anything, on average, for divesting himself of price risk via prior-round trading. Nevertheless, in the situation of Table 3 any such trading would make it impossible for the individuals to attain a Pareto-optimal position.

Strong results that are fully general are hard to come by [Salant 1976, Feiger 1976, Hirshleifer 1976, 1977]. But under reasonable simplifying assumptions Working's contention tends to be borne out: differences of belief, and not differences in risk-tolerance, are the sine qua non of speculative activity. On the other hand, given any difference in belief the degree of risk-aversion affects the **extent** of traders' speculative commitments. And, our above discussion suggests, unbalanced state-endowments may also play a role under incomplete market regimes. Even without differences of belief or of risk-aversion, the restricted trading opportunities under a FM regime may require **both** prior and posterior trading to attain a Pareto-efficient allocation.

A patentee might instead maximize returns by granting _exclusive_ licenses
(in which case the social value of the excluded uses is of course lost)
or by imposing fee structures that distort the marginal production
decisions of licensees. On the other side of the picture, because of
the elusiveness of property in ideas there is a good deal of uncertainty
and unreliability in the legal protection of patents and copyrights, and
of course relatively little protection for trade secrets not made subject
to patent or copyright. The result is that a good deal of unlicensed uses
escape control. Short of ideal conditions there will be losses from _both_
underutilization and underinvestment, and in practice something of a
trade-off: provision of greater legal protection to inventors tends to
ameliorate the underinvestment problem, but to worsen the underutilization
problem.

More recent investigations have indicated, however, that not all the
important elements of the picture have been captured by this analysis. These
newer results turn upon the possibility of _overinvestment_ in the production
of ideas ("a rush to invent").

The first such factor is the _fugitive resource_ (or _common-property resource_)
nature of undiscovered ideas [Barzel 1968]. For concreteness, we can use
as metaphor the "over-fishing" model of Gordon [1954]. Suppose there are
perfect property rights in fish caught, but complete free entry into fishing
(i.e., there are no property rights that exclude others from engaging in fishing as
an activity). Then in competitive equilibrium there will be over-fishing;
private marginal cost will equal price, but the true social marginal cost in fishing
exceeds the private marginal cost. The reason is that a certain fraction of

defective rights _in_ ideas reduce what otherwise might be an excessive "rush to invent."

Recapitulating at this point, we have seen two distinct possible justifications for limiting property rights in ideas -- for example, by granting patents only for a term of years. The first is that some protection to inventors is traded off against protection to users of invention; e.g., after the patent expires use of the idea is unconstrained. The second is that the "rush to invent" tendency is moderated by reducing the capturable value of the invention itself.

There is still another motivation that may lead to excessive devotion of resources to invention. Ideas of course vary enormously in their significance, and some among them will have far-reaching consequences. This opens up a new channel of reward for inventors. Instead of, or possibly in addition to selling the information via patent license or otherwise, an inventor might be able to speculate by taking long or short positions in assets whose values will be affected by the invention [Hirshleifer 1971]. The cotton gin, for example, had speculative implications for the prices of cotton and cotton land, the business prospects of firms engaged in cotton warehousing and shipping, the site values of key points in the cotton transportation network -- in addition to more indirect implications for competitor industries like wool and complementary ones like textile machinery. Nor does an idea have to be as momentous as the cotton gin to have profitable speculative implications. An oil firm that has developed a new method of deep recovery might, for example, reap a speculative payoff by buying up options on tracts whose petroleum now lies too deep to be recovered. One important implication of the speculative reward for invention is that it motivates the

does not warrant going to the other extreme. It would not be in order to conclude that patent protection is _not_ justified, but only that the arguments pro and con are more complex than had previously been realized.

For the labor market Spence [1973], Stiglitz [1975], and Riley [1976, 1979] have argued that educational credentials may constitute signals with regard to jobs in which productivity is difficult to determine. As long as there is a negative correlation between productivity and the (money and time) costs involved in achieving any education level, the marginal cost of education is lower for the higher-quality workers. The latter are then able to signal by attaining higher educational credentials. On the other side of the market, the complementary process to signalling is called screening; employers are able to use education signals to screen for quality differentials.

Rothschild and Stiglitz [1976] and Wilson [1977] make parallel arguments for the insurance market. Section 1.2.3 above illustrated how, in the absence of ability to distinguish between better and poorer risks, insurance premiums reflect the average risk quality. Hence adverse selection occurs, with lower-quality risk classes tending to insure more than others. However, ceteris paribus, the higher the probability of loss the higher is the marginal loss in utility associated with accepting less than full coverage. Thus, the marginal cost of accepting a high deductible is greater for those with low-quality risks (high loss probabilities). Insurance companies can then screen for differences in risk by offering a menu of policies. Some policies can be offered at the low premiums appropriate for high-quality risks but with high deductibles to deter acceptance on the part of poor risks. Other policies, intended to appeal to poorer risks, offer smaller deductibles but steeper premiums.

In contrast to the situations examined in section 2.2, in signalling models the flow of information from seller to buyer is generated endogenously. This has important consequences for the stability of informational equilibria.
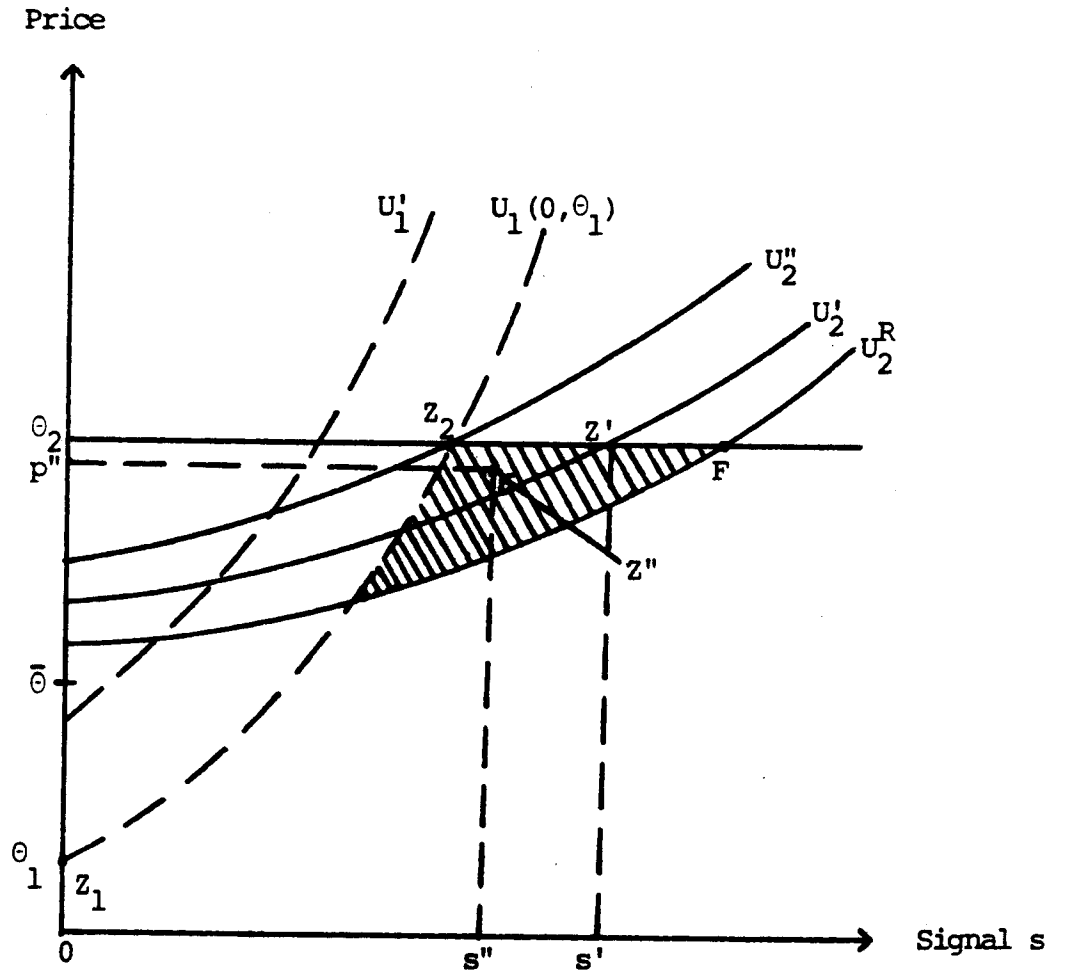
Fig. 10: Cournot-Nash Equilibrium

high-quality workers accept Z_2; as for the low-quality workers, being indifferent between Z_2 and Z_1 they have no motive to shift from the latter.

Unfortunately, this result is not general. Suppose instead the indifference curves of the high-quality sellers differ less from those of the low-quality sellers, as in Figure 11. Starting from the $\{Z_1, Z_2\}$ solution, consider the alternative offer $Z^* = <s^*, p^*>$ in the shaded region. It is strictly preferred by sellers of both quality levels; moreover, since $p^* < \bar{\theta}$, it yields expected profits to the buyer. Therefore the set of offers $<Z_1, Z_2>$ is no longer a Cournot-Nash equilibrium. However, no "pooling" offer like Z^* can be a Cournot-Nash equilibrium either since it is always possible to skim the high-quality sellers off the top of the pool. In Fig. 11, if all workers were receiving Z^*, a firm could enter and offer Z^{**} to high-quality sellers only. Since $p^{**} < \theta_2$ such an offer is profitable.

How then would such a market behave? Plausibly, in the absence of collusion, each buyer would eventually expect some <u>reaction</u> by other agents to changes in his own list of offers. Suppose that a new offer would be profitable in the absence of any reaction, but yields losses once another buyer reacts with a strictly profitable counter-offer. Suppose furthermore that the latter's response is riskless, in the sense that <u>further</u> response by any other buyers would not impose losses on the first reactor. Then it seems reasonable that the potential initial "defector" would eventually recognize that his new offer would bring on such a reaction, and hence would be deterred from making it. This suggests the following strategic equilibrium concept [Riley 1977], which builds on the development by Wilson [1977].

REACTIVE EQUILIBRIUM: <u>A set of offers is a reactive equilibrium if, for any additional offer which yields an expected gain to the agent making the offer</u>

there is another which yields a gain to a second agent and losses to the first. Moreover, no further addition to or withdrawal from the set of offers generates losses to the second agent.

The general derivation of the existence and uniqueness of the reactive equilibrium is somewhat delicate. However, it is relatively easy to check that in Fig. 11 $\{Z_1, Z_2\}$ is a reactive equilibrium. The initial "defector" must make an offer like Z^* to generate an expected profit. But then another buyer can counter with Z^{**}, thereby attracting away some high-quality products. As this process continues, $\bar{\theta}$ will fall until Z^* generates losses, while Z^{**} remains strictly profitable since $p^{**} < \theta_2$.

To conclude, the endogenous revelation of information via markets is, after all, explainable as a non-cooperative equilibrium phenomenon. While in general there is no Cournot-Nash equilibrium, recognition of reasonable reactions by other agents always result in a stable equilibrium.

2.4.2 Informational Inferences From Market Prices

We now consider the problem of information leakage. In Section 2.2.3 the process of speculation was interpreted as largely due to differences of information and belief. Nevertheless, the problem of leakage did not arise there, because no trader regarded any other individual's knowledge or beliefs as intrinsically superior to his own. Here, we will suppose, everyone recognizes that some traders do and others do not possess an informational advantage. (Though, it may or may not be the case that traders with an informational advantage are publicly identified as such.) In Section 2.4.1 above, better-informed individuals were seeking to overcome the informational disparity by signalling to potential trading partners. In this section, in contrast, the better-informed individuals are trying to capitalize on the disparity, by adopting a speculative position before their informational advantage disappears.
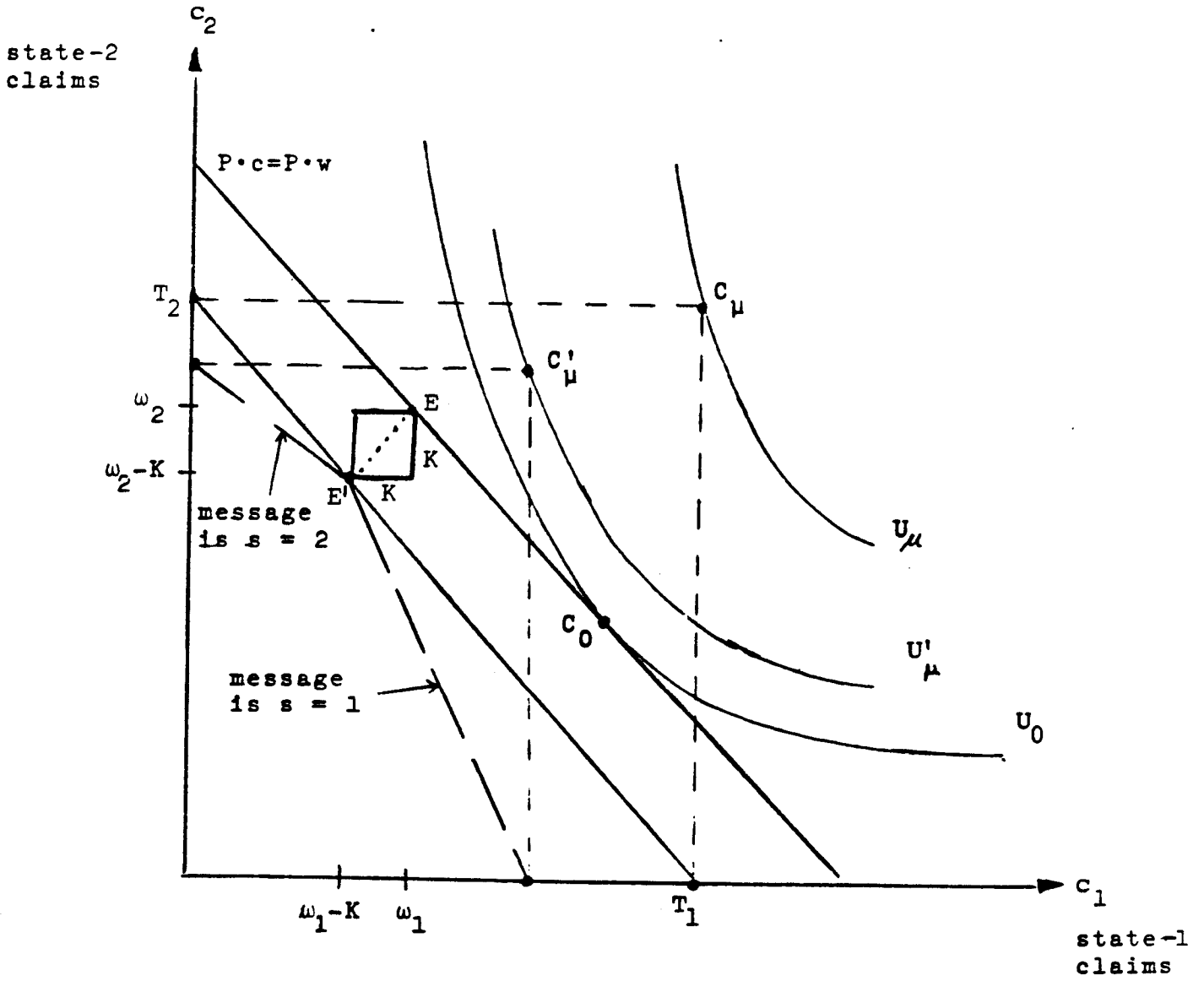
Figure 12: Trading by informed agents

With a finite number of information states Γ, it is almost certainly the case that even in very incomplete markets, the function $p(\Gamma)$ is invertible [Radner 1977]. Thus there is almost certainly a "fulfilled-expectations" equilibrium in which each agent correctly infers aggregate information ℓ from the price vector p.

As in the case of signalling models there is a market externality → tending to break down any equilibrium in which information is obtained only at a cost: if none are informed there is potential profit in becoming informed, yet if anyone invests in information and trades accordingly he loses relative to those not having invested. The analog of the "reactive equilibrium" concept here would evidently be the corner solution with no informational investments. However, in contrast with the signalling case, an interior solution can be obtained by introducing noise or lags. If only imperfect information about the state of nature can be inferred by observing prices (as will generally be the case with a continuum of information states [Grossman 1977b, Jordan 1976]), or if the informed individual can make his commitments before the uninformed can fully react, there will tend to be an equilibrium _fraction_ of traders who choose to become informed.

2.4.3 _Informational Efficiency_

In the Arrow-Debreu world of complete information about traded commodities and competitive markets for all contingent claims, market equilibrium is efficient in the sense that an omniscient observer would not be able to reallocate resources to the advantage of all agents. However, if information is costly, and as a result incomplete, it is hardly surprising that markets no longer have this same effi-

cient market hypothesis" to mean that prices fully _convey_ information. In terms of our illustration, efficiency then requires that the uninformed are able to infer (from the price changes) the content of the evidence processed by the informed group. As we have seen in the previous section, under these conditions the market for private information is not viable.

The underlying problem here, in our opinion, is confusion between "efficiency" as used to characterize _equilibrium configurations_ and as applied to _disequilibrium processes_. Consider an analogous problem with regard to "profit." In an efficient competitive equilibrium, there are no competitive profit opportunities -- and yet, this outcome is the end-result of a process in which profit provides the motivating force. Whether competitive profit-seeking takes place at the ideal _rate_, and is therefore dynamically efficient, is quite another question, to answer which we as yet lack adequate tools. In its recent usage, "informational efficiency" means essentially that there is no way of making money from informational activities. Since information-involved activities (information seeking, processing, disseminating, etc.) by their very success tend, like profit-seeking activities, to eliminate their own _raison d'etre_, it is certainly true that _in equilibrium_ there will be no way of making money via such activities. But this says nothing whatsoever about the dynamically optimal level of informational activities -- which are, of their very nature, disequilibrium processes.

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