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ENDOGENOUS CONJECTURAL VARIATIONS IN DUOPOLY

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1. Introduction

A major difficulty -- perhaps the major difficulty -- of modeling oligopoly is the problem of specifying how each oligopolist anticipates his rivals will react if he varies his price or output. Depending on what anticipations each oligopolist is assumed to have, one obtains quite different solutions to the oligopoly problem.

It is widely recognized that the classical oligopoly theories have a serious common flaw: they arbitrarily and mechanically assume some particular anticipation on the part of the oligopolists which may not be confirmed by the oligopolists' experience. The Cournot solution, for example, simply assumes that each oligopolists anticipates no reaction at all by his rivals as he varies his own output. This zero "conjectural variation" would be belied by the oligopolist's experience. Alternatively, an asymmetrical but correct set of anticipations are postulated, as in the Bowley-Stackelberg solution.

This paper builds on earlier contributions by Leontief (1936) and Marschak and Selten (1978) in which correct and symmetrical anticipations are assumed. We call these anticipations endogenous conjectural variations (CVs), because one can view the oligopolists as learning from their experience of past reactions by their rivals. We distinguish two types of oligopolistic interactions: (a) passive, in which all actors adjust their own output levels so as to keep their profits at a maximum, disregarding the effect of the implied reactions on their rivals' outputs (but correctly anticipating the other actors' reactions), and (b) active, in which actors actively choose reactions that have anticipated effects on other actors' levels of output, and abandon the objective of maximizing their own profits when other actors' outputs are not at their equilibrium levels.

The model of active reactions goes beyond the work of Leontief, Marschak and Selten and is based on earlier work by Guttman (1978).[1] Active reactions are simply threats that if another ologopolist changes his output, one's own output will respond in a certain fashion. Unlike the cooperative game of Nash (1950), however, we explore the case -- which is considerably more relevant to oligopoly as well as to political behavior -- that these threats are not enforced, but rather are self-enforcing. We have not, however, developed a general theory of non-cooperative threat-making. In order to obtain determinate results, we have been forced to make strong simplifying assumptions. The oligopolists' "reaction coefficients" are assumed to be constants, i.e., each oligopolist's output is linearly related to other firms' outputs.[2] It is assumed, moreover, that the reaction coefficients are chosen before actual levels of output are chosen.

Neither of these assumptions departs from the (implied) assumptions of Leontief or those of the classical oligopoly theorists; this facilitates a comparison of the implications of active interactions to those of passive interactions. But our model of "matching behavior" cannot be regarded as more than a first exploration of this difficult problem. To appreciate the apparent necessity of limiting, in some way, the form of the reactions,

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[1]A note by Anderson (1977) suggests a similar concept of "matching behavior," but does not develop it in detail.

[2]One basis for such a linearity assumption would be an argument from the existence of imperfect information and, perhaps, bounded rationality. See, e.g., Simon (1959). (I am indebted to J. Hirshleifer for this point.) It may be argued that it takes time for each duopolist to learn its rival's reaction strategy, and that to impute higher-order terms in the rival's reaction strategy would be uneconomical, given the necessity of continuously revising one's CV in any case.

suppose each of two duopolists announces to his rival that his own output will be that which maximizes combined profits if the other duopolist does the same,[3] but at any other output of his rival, the first duopolist will choose so high an output that his rival's profits will be negative. Such a threat, when made and believed by both duopolists, would assure a result identical to a merger of the two firms. Such threats are seldom observed, however, perhaps because they would probably not be believed. If one firm were to call the other firm's bluff, would the other duopolist follow through with his threat? To do so would impose high costs on himself. Nevertheless, it must be admitted that all meaningful threats involve some costs to the threatening party if they are carried out: if each duopolist kept his output at its profit-maximizing level regardless of his rival's output, we would have the model of passive interactions suggested by Leontief. The feasible "degree of extremism" of threats depends on how often and for how long one's bluff is expected to be called, and this frequency of bluff-calling has yet to be modeled formally. We have simply chosen the simplest type of threat to consider, with the intention of illustrating the basic line of reasoning involved in active duopolistic interactions.

We consider only output variations. Since we limit ourselves to duopolists producing an identical product, allowing price variations would seem to force us to accept Chamberlin's (1938) contention that if each oligopolist could foresee the reactions of his rivals, he would anticipate price-matching for all price cuts (though not, as Chamberlin implies, for all price rises), because such matching is necessary for the other firms to survive, leading us to accept the kinked-demand-curve model criticized on empirical grounds by Stigler (1947). This consideration suggests that output variations are

[3]Here we are assuming that there is a unique pair of joint-profit-maximizing outputs. This involves assuming rising marginal costs, if the firms are identical.

the more plausible case to analyze, at least in the framework of endogenous CVs developed here.

2. Passive Interactions

As indicated above, Stackelberg (1934) analyzed the case in which one firm forms correct expectations of how the other firm will react to changes in output. Stackelberg believed, however, that a "leader-leader" equilibrium was inherently unstable. In such an equilibrium, each duopolist would anticipate the reaction of his rival and take this reaction into account in determining his own output, always assuming the rival acts like a Cournot duopolist. The classic "Stackelberg equilibrium" is a "leader-follower" equilibrium, in which one duopolist (the follower) takes the output of his rival as given, as in the Cournot model, while the other duopolist takes the reaction function of his rival as given, and chooses an optimal output conditional on that function. Both duopolists' anticipations of their rivals' behavior are correct, i.e., the follower is viewed by the leader as choosing a reaction function, while the leader is (correctly) viewed by the follower as choosing an output. But these anticipations are asymmetrical, and this asymmetry leads to severe instability, at least as long as the firms' cost functions are not too different.

Provided the duopolists do not act simultaneously, this asymmetry is plausible. If the first duopolist to act chooses a reaction function, it is optimal for the other duopolist simply to choose an output. On the other hand, if the first duopolist knows the second will follow him in sequence, it may be optimal for him to choose a flat output and let the other choose a reaction function, i.e., react to his output.[4] Imposing

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[4] Cf. Thompson and Faith (1976).

such a time-sequence, however, requires either an enforcing agent (as in the Thompson-Faith model) or an arbitrary sequence of "whoever acts first," which, for non-identical actors, leads to indeterminacy.

Leontief (1936) showed how the "rational expectations" of Stackelberg duopolists can be made symmetrical.[5] Each duopolist knows the reaction of his rival if he were to change his own ouptut, and bases his choice of output on that expected reaction. Neither duopolist actively "chooses" a reaction function in the sense of an active matching or threat policy; rather, each passively reacts to the other firm's choice of output.

A formal model of such interactions was provided, in a more general context, by Marschak and Selten (1978). In their game, each firm chooses a level of output and a reaction strategy that shows how its output would change if the other firm were to depart from its current output. The equilibrium conjectural variations in Leontief's model are "stable" and "restabilizing" in the Marschak-Selten sense: i.e., no firm would change its output, knowing the reaction of its rival, and no firm would have an incentive not to follow through with its announced reaction if it is called upon to do so.

Leontief believed that his generalization of the Stackelberg model would only rarely yield an equilibrium.[6] This belief, however, appears to be incorrect. It is not difficult to show that under Leontief's assumptions of linear demand curves and upward-sloping, linear marginal cost curves, two identical duopolists will always find an equilibrium pair of CVs and

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[5]Negishi and Okugushi (1972) developed a similar model. Their model, however, postulates that each duopolist assumes that its rival is a "follower" in Stackelberg's sense. The Negishi-Okuguchi equilibrium simply makes the resulting CV's consistent with each other.

[6]Leontief described his equilibrium as "not very probable, but nevertheless theoretically significant." (1936, p. 556)

outputs. There are two equilibria, but only one is stable. As in the model of active interactions, we assume each duopolist's output reacts linearly to changes in his rival's output. In equilibrium, each firm's "reaction coefficient" (which is the CV anticipated by its rival) is its best reaction coefficient given the reaction coefficient of its rival.

Let the demand curve facing each of two identical duopolists be

$$p = K - \alpha(q_1 + q_2),$$

where

p = the price of their output

q_1, q_2 = their outputs, and

k, α = positive constants.

Moreover, let each duopolist's marginal cost curve be

$$MC_i = (K-\beta) + \lambda q_i \quad (1)$$

where β and λ are positive constants. Each firm's marginal revenue is

$$MR_i = K - \alpha\left(2 + \frac{d\hat{q}_j}{dq_i}\right) q_i - \alpha q_j, \quad j \neq i, \quad (2)$$

where the hat above dq_j/dq_i denotes "expected." Let the actual dq_j/dq_i be denoted as b_j. Then, in equilibrium, we have

$$b_j = \frac{d\hat{q}_j}{dq_i}, \quad (3)$$

and similarly for firm i. Equating MC_i and MR_i and using (3), we obtain, for optimal q_i,

$$q_i^* = \left[\frac{1}{\lambda + \alpha(2+b_j)}\right](\beta - \alpha q_j). \quad (4)$$

Differentiating with respect to q_j,

(5) $$\frac{dq_i}{dq_j} = b_i = \frac{-\alpha}{\lambda + \alpha(2+b_j)}$$

Writing out the corresponding expression for b_j and solving, we obtain

$$b_i = b_j = \frac{-(2\alpha + \lambda) \pm \sqrt{(2\alpha + \lambda)^2 - 4\alpha^2}}{2\alpha}$$

The expression under the square-root sign is positive when $\lambda > 0$, that is, when marginal costs are rising. In this case, we obtain two equilibria, but only one of these -- the one whose b's are smaller in absolute value -- is stable in the following sense: if one firm's b_i were to deviate from equilibrium, the ensuing interaction of the b's would lead back to equilibrium.

It should be noted that equilibrium in the "b-game" is a Nash equilibrium, i.e., each duopolist takes the other's b_j as given in determining his optimal b_i. This implicitly places a limit to the sophistication of the duopolists. One could imagine a more sophisticated model in which duopolists form expectations on how the rival's reaction (b_j) will change when one's b_i changes. Clearly, however, at some point this process of forming conjectures must stop. Our making the equilibrium (b_i, b_j) a Nash equilibrium is in the same spirit as our assumption that the duopolists treat their rivals' b_i's as constants.

To illustrate the passive reaction model with Leontief's example, suppose the duopolists' demand curves are

$$p = K - (q_1 + q_2)$$

and that their marginal cost curves are

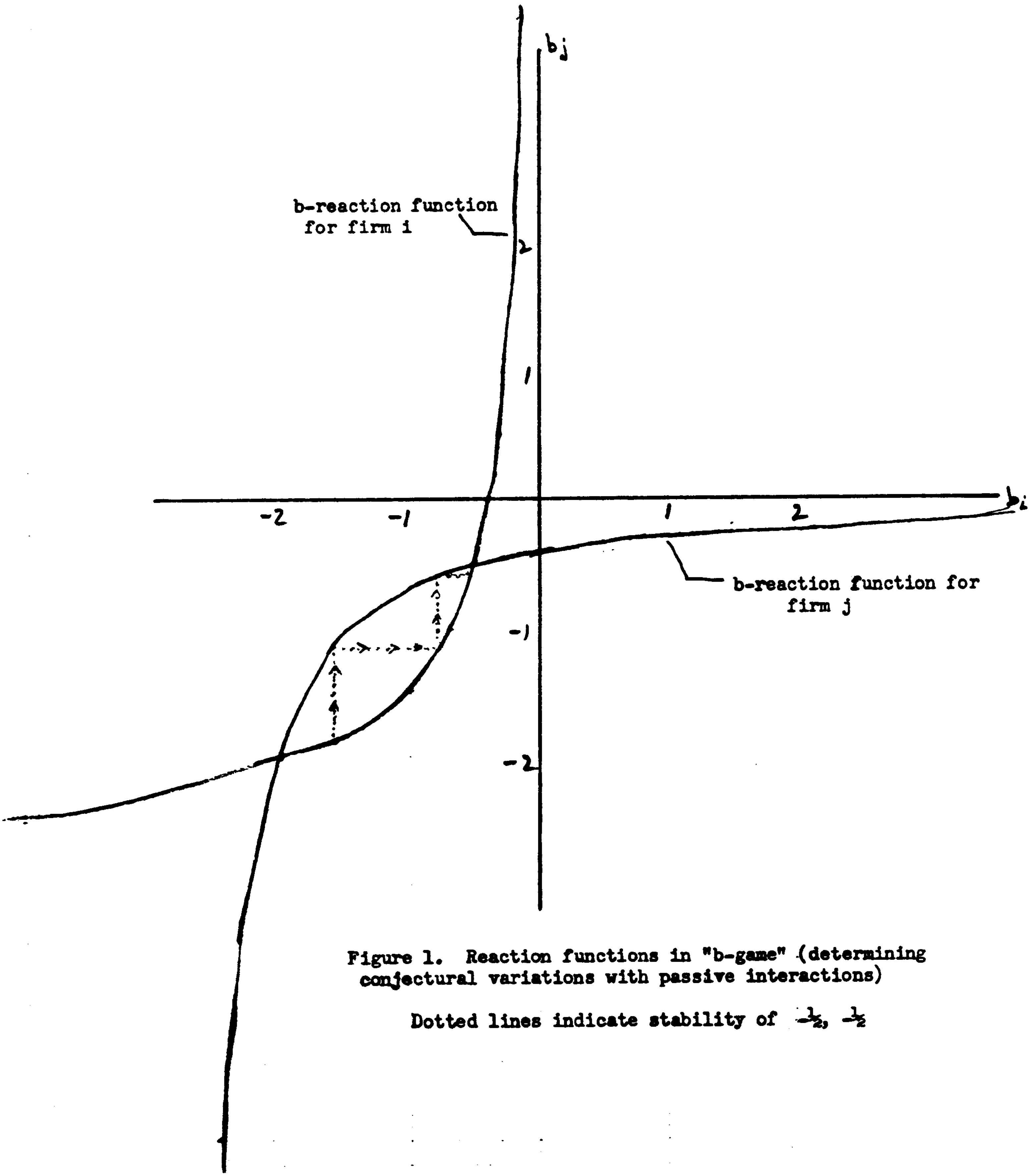

Figure 1. Reaction functions in "b-game" (determining conjectural variations with passive interactions)

Dotted lines indicate stability of $-\frac{1}{2}$, $-\frac{1}{2}$

$$MC = \frac{1}{2} q_i, \quad i = 1, 2.$$

Then the equilibrium b_1, b_2 are

$$\frac{1}{2} \left(- \frac{5}{2} \pm \frac{3}{2}\right) .$$

Thus the two equilibrium CVs are $(- \frac{1}{2}, - \frac{1}{2})$ and (-2, -2), but only the former is stable. The "reaction curves" in the b-game, given by equation (5), are depicted in Figure 1. By starting at a point other than $(- \frac{1}{2}, - \frac{1}{2})$, it can be seen, following through the interactions, that one is lead to that point. Such a movement towards equilibrium is shown by the dotted lines.

Once the equilibrium (b_1, b_2) are determined, it is a simple matter to find the equilibrium outputs. In this case, they would be determined by the reaction functions given in (4) as

$$q_1 = \frac{1}{2} (K - q_2)$$

or

$$q_i = K/3.$$

This may be compared with the Cournot duopolist's output of .286K, and with the output of each duopolist if the two firms merged and maximized joint profits, which is .22K. Thus, the model of passive reactions predicts a larger output than either of these two alternative models. This is to be expected, because each duopolist expects the other to cut back when he increases his output.

3. Active Reactions

The model presented in the previous section assumed that each duopolist simply adapts his output to the output and expected reaction of the other duopolist. It was noted that, in equilibrium, duopolist i expects the other to reduce his output if duopolist i increases his output; this conjectural variation is not postulated a priori, but is derived as a consequence of equilibrium. Each duopolist, however, would eventually realize that if he were to adopt a policy of positively matching his rival's output, his rival would be induced to choose a lower output, resulting in a higher price for their product. The model of active reactions is an attempt to capture this idea of matching behavior formally.

In order to make the model comparable to the Leontief-Stackelberg analysis, as well as to keep the analysis simple, we continue to assume that the reaction coefficients of both duopolists (i.e., their rivals' CVs) are constants, and that they are determined before the actual levels of output are determined.

To visualize how such a process could occur without any coordination between the duopolists, consider a sequence of outputs by the duopolists continuing over time. Occasionally, one firm "tests" the other by varying its output from its profit-maximizing level and watching the response of the other firm. With this information on the response of its rival, the firm adjusts its output so that, taking its rival's reaction into account, it is maximizing its profits. Each firm determines its reaction so that its profits are maximized, taking into account how its choice of reaction coefficient affects its rival's output and thus affects its own profits.

This maximization of profits by varying one's reaction coefficient is accomplished by predicting the equilibrium levels of outputs of both firms given any pair of reaction coefficients, b_1 and b_2. The firm's profits are not maximized at any other pairs of outputs. When either firm is "tested," its rival has moved away from this equilibrium, and thus the firm being tested has an incentive to fail to react in its earlier-revealed manner. We assume, however, that each firm views such costs as being transitory and therefore negligible in any long-run calculation such as the one considered here. The benefits of reacting as previously revealed or "announced," in contrast, are permanent -- they are maintaining one's credibility and thus maintaining an equilibrium pair of outputs that maximizes the firm's long-run profits.

Although we have described the process of duopolistic interaction as being a continuous process extending over time, it is simplest to model this process as consisting of two single-period games. In the first of these games the reaction coefficients of the duopolists are determined. (In reality, these coefficients would be revealed by the duopolists over time in response to testing by their rivals.) In the second game, "flat" levels of output are determined, the duopolists having "understood" that these flat outputs would be matched by their rivals at the rates revealed by the reaction coefficients. These flat levels of output are chosen so that the full output of the firm (including its matching of the other firm's flat output) is at its profit-maximizing level, given the other firm's flat output and its reaction coefficient.

By modeling duopolistic interaction in this way, we are introducing an important difference between the model of active reactions and the Leontief model of passive reactions. In the Leontief model, the reaction coefficients are only hypothetical in nature: if firm 2 changes its output, firm 1 would respond in a certain way. These reaction coefficients, moreover, are not consciously "chosen," but are passively "determined" by the equilibrium conditions, i.e., (a) that each firm maximizes its profits given the expected reaction of its rival (CV) and (b) that each firm's CV is the actual reaction of its rival. In the model of active reactions, the reaction coefficients are not hypothetical: an increased reaction coefficient means an undertaking to match one's rivals' flat output (not simply changes in his output) at a higher rate. The reaction coefficient, moreover, is now actively chosen with an eye on the resulting equilibrium of flat contributions.

The model's specification that actual flat outputs (and not merely changes in outputs) be matched is a device for making large threats more costly than small ones. If only changes in outputs were threatened with matching behavior, infinitely large matching rates (reaction coefficients) would be chosen, as long as no changes in the rival's output (i.e., "tests") were expected.[7] One would then need to develop a model predicting the frequency of testing of threats. But if, as in our model, flat outputs are matched and not merely changes in outputs, the threats are continuously tested and large threats become costly relative to small threats.[8]

To describe the model formally, let a_i be the "flat" output of firm i, and b_i be its reaction coefficient. Then

$$q_i = a_i + b_i a_j$$

[7]Alternatively, one would begin at, say, a Cournot equilibrium, and make the matching rates applicable only to changes in rivals' outputs from that point. Here a one-time change in output takes place, which can be predicted and modelled.

[8]One might ask why not have total outputs matched, and not only "flat" outputs, i.e., a model in which $q_i = a_i + b_i q_j$? One answer is that it can easily lead to no equilibrium -- if $b_i b_j > 1$. A second answer is that there is no "point" in matching the component of one's rival's output which itself is matching one's own output, at least not at the same rate as one matches the "autonomous" ("flat," in our terminology) component of the rival's output.

is the output of firm i, where a_j is the flat output of firm j, its rival. We continue to assume linear demand curves with slopes of -1:

$$p = K - (q_i + q_j)$$

The restriction of the slope to -1 could be dropped without affecting our results. The cost curves of the equation are quadratic and marginal costs increase with output:

$$C_i = \lambda_i q_i + \mu_i q_i^2, \qquad \lambda_i, \mu_i > 0.$$

When the reaction coefficients (b_i) are determined, the flat outputs (a_i) are determined in a Nash non-cooperative game. The reaction functions, which give the profit-maximizing a_i^* for given a_j, are

$$(6) \qquad a_i^* = \frac{K - \lambda_i}{2(1+\mu_i) + 2b_j} - a_j \left[\frac{1 + b_i(2+2\mu_i + b_j)}{2(1 + \mu_i) + 2b_j}\right] .$$

Equation (6) indicates that a_i^* is a linear function of a_j, and that da_i^*/da_j depends on the matching rates b_i and b_j. Depending on the magnitudes of b_i and b_j the equilibrium of (a_i, a_j) may be unique and stable in the Cournot sense, or non-unique and unstable. The possibilities are illustrated in Figure 2. In Case (a), there is a single, stable equilibrium. In Cases (b) and (c), one reaction line encloses the other, again leading to a single equilibrium -- at a corner, where one flat output is zero. In Case (d), the two reaction lines intersect, but the interior equilibrium point is unstable in the sense that any movement from that point will not be lead

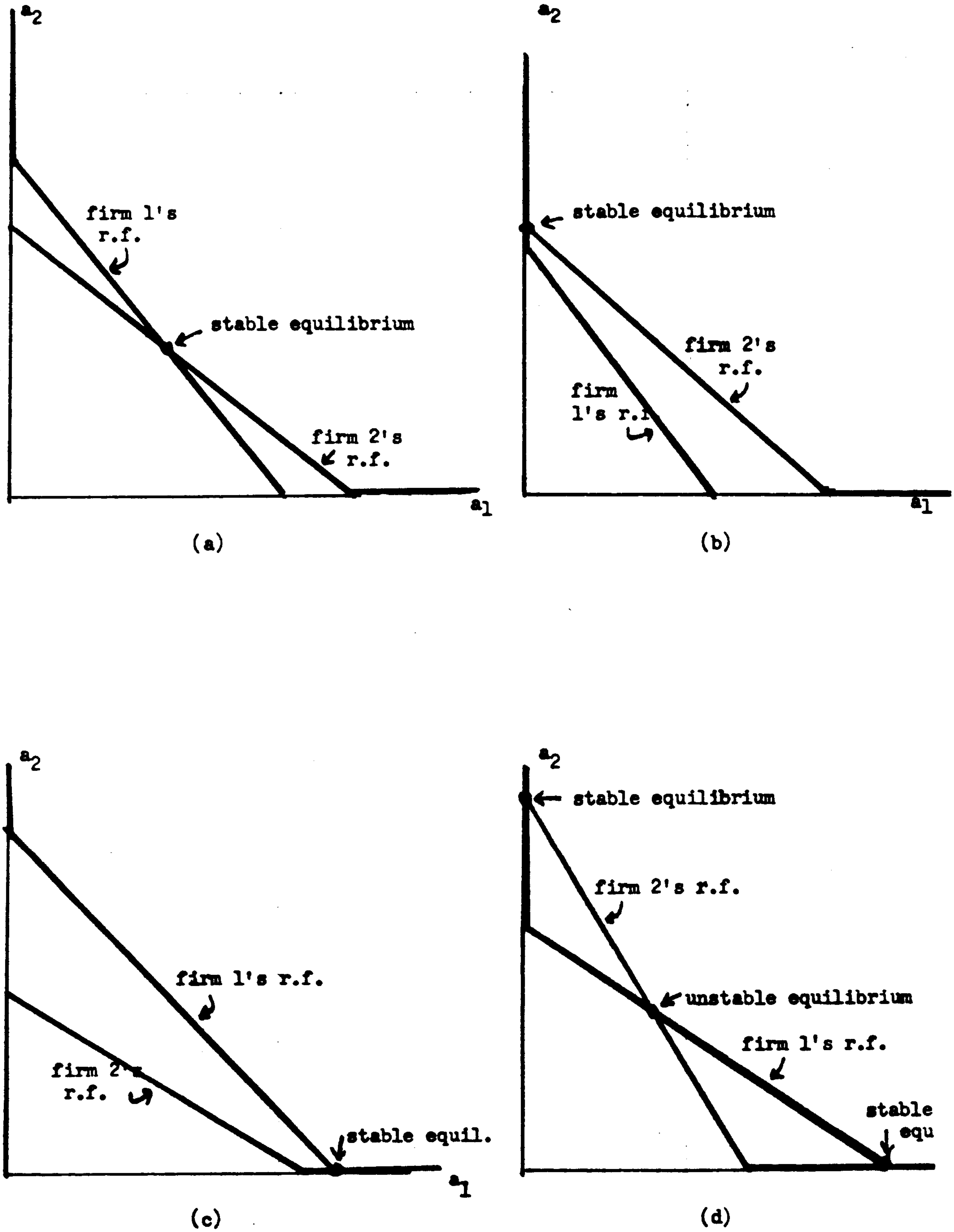

Figure 2. Possible configurations of reaction functions in "a-game", model of active interactions

back to the equilibrium if the actors act sequentially.[9]

As each firm contemplates alternative choices of its reaction coefficient, b_i, it must predict the equilibrium (or equilibria) resulting from its choice, given the reaction coefficient of its rival, b_j. To do this analytically poses serious difficulties, because as b_i changes one moves from one case to another in Figure 2, resulting in possible discontinuities in the payoff function. Therefore, it appears to be necessary to simulate the model numerically. An additional difficulty is posed by the fact that multiple equilibria may result in the game determining the flat outputs. Our approach to solving this problem is, first, to examine only "stable" equilibria, and, second, to let the firm's expected profit be the mathematical expectation of its profits in those equilibria -- with the two equilibria considered equally probable. An alternative solution would require dominance of all equilibria given (b_i, b_j) over (b_i', b_j), for b_i to be chosen over b_i'. Adopting this alternative does not seem to alter our results.

In addition, we restrict the a_i and b_i to be non-negative. The restriction on the a_i appears required for intuitive reasons: it is difficult to conceive of a negative output, "flat" or otherwise. The restriction on the b_i is made partly to avoid negative outputs. Alternatively, we would

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[9] We beg the question as to whether sequential action "makes sense" in a Nash equilibrium context, and thus whether this notion of stability is meaningful. An interpretation of a Nash equilibrium in which no naivete is assumed requires that we eschew a sequential-action interpretation, in favor of the following: Each duopolist somehow correctly predicts his rival's action and chooses his best response to that action. The basis for such a prediction, however, is unclear without one's rival having prior information on one's own choice. To postulate that firm i's rival j predicts that i's output will be i's best response to firm j's output also involves prior information, this time on the part of firm i. Ultimately, there appears to be no justification for simultaneous-action interpretations of Nash equilibria other than the internal consistency of such equilibria.

require complicated constraints on the combinations (a_i, b_i) which would lead to a hopelessly complex analysis. A second reason for restricting the b_i is that there is little point in revealing negative b_i: such b_i would only encourage larger outputs by one's rival and reduce one's own profits. If a small output is desired, a zero reaction coefficient is preferable.

Table 1 presents the results of some simulations of the workings of the model, and compares the resulting equilibria Q* with the Cournot equilibrium (Q_c) and the equilibrium resulting from letting the firms merge and maximize point profits (Q_m). In all cases, the firms' marginal cost curves slope upwards; otherwise, there is no equilibrium. Two patterns emerge: First, firms with smaller marginal costs choose larger reaction coefficients. This is an implication which is testable, in principle, e.g. with experimental data. Second, an interesting relationship between Q*, Q_c, and Q_m emerges. The predicted equilibrium output (which, with non-identical firms, involves a slight indeterminacy) is approximately one-third of the way from the Cournot Q_c to the joint-profit maximizing equilibrium Q_m. The questions of whether this holds in all cases or whether similar regularities appear with more than two actors remain to be investigated. Nevertheless, it is significant that the present model goes <u>part</u> of the way to explaining cooperation by duopolists without invoking the usual requirement of cooperation -- enforceable agreements.

Table 1

Simulations of Model of Active Interactions

Cost functions of duopolists	Reaction coefficients	Q^*	Q_c	Q_m
(a) $MC_1 = MC_2 = K-1+2q_i$, $i = 1,2$	$b_1 = b_2 = 2.0$	.375	.40	.333
(b) $MC_1 = K-1+2q_1$ $MC_2 = K-1+3q_2$	$b_1 = 2.6$ $b_2 = 1.2$	.344- .353	.368	.3125
(c) $MC_1 = (K-1.1)+2q_1$ $MC_2 = (K-1)+2q_2$	$b_1 = 2.6$ $b_2 = 1.6$	.391- .397	.42	.35
(d) $MC_1 = MC_2 = K-1+q_i$, $i = 1,2$	$b_1 = b_2 = 6.0$	.467	.50	.40
(e) $MC_1 = (K-1.0)+2q_1$ $MC_2 = (K-1.2)+2q_2$	$b_1 = 1.3$ $b_2 = 3.5$	.409- .418	.44	.367

4. Concluding Remarks

We have described two alternative models of endogenous conjectural variations: a model of "passive interactions" originally suggested by Leontief, and a model of "active interactions" based on the notion of matching behavior. On the theoretical front, a number of improvements and generalizations could be made. The analysis could be generalized to the case of more than 2 firms; non-linear demand curves and variable reaction coefficients could be investigated. But perhaps a more promising line of future inquiry would be empirical. While reaction functions are difficult to estimate statistically, in an experimental situation they can be identified more easily. Most experimental work on oligopoly has focussed on output decisions and not on interactions among oligopolists.[10] In a future draft of this paper, we hope to pursue this line of investigation.

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[10]But see the work of Hoggatt (1967), who finds evidence of matching behavior.

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