IS IT A DEMAND CURVE, OR IS IT A SUPPLY CURVE?: PARTIAL IDENTIFICATION THROUGH INEQUALITY CONSTRAINTS

by

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Abstract

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Partial Identification through Inequality Constraints

In simultaneous equation systems with zero residual covariances, inequality constraints on some coefficients can be used to produce consistent bounds for otherwise underidentified parameters. In particular, in a supply-demand system, the interval between the direct and reverse regressions consistently bounds either the demand parameter or the supply parameter. Inequalities on coefficients of exogenous variables can be useful for selecting instruments.

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Is It a Demand Curve, or Is It a Supply Curve?: Partial Identification through Inequality Constraints*

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This article fully describes the sets of maximum likelihood estimates of parameters in two-equation under-identified simultaneous equation systems, and uses these characterizations to comment on the usefulness of inequality constraints on the parameters. It is shown in particular that in a demand-supply system with zero covariance between the residuals, it is proper to treat the regression of quantity on price as an attentuated estimate of the demand curve if the estimate is negative, and to treat it as an attenuated estimate of the supply curve if the estimate is positive. Heretofore, this estimation method has brought smiles to the faces of econometricians. In fact, the use of a method like this by Schultz (1928) can be said to have made him the reluctant mother of modern econometrics, the gang of fathers being Working [12], Leontief [6], and Frisch [2].

Section one of this paper is an analysis of the simple supply-demand system with uncorrelated residuals. It is shown that the interval between the least-squares estimate and the reverse least-squares estimate consistently bounds one slope or the other. Knowledge of the signs of the slopes then determines whether this interval applies to the demand curve or to the supply curve. In section two, the model is generalized to admit exogenous variables, and it is shown how inequalities on the coefficients of these variables can partially identify the supply and demand slopes. Finally, section three is an historical survey, which offers comments on why these simple results do not seem to be generally known.

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1. A Model without Exogenous Variables

We first consider estimates of the following simultaneous equations system

$$Q_t = \alpha + \beta P_t + \varepsilon_t \qquad t = 1, \ldots, T, \tag{1}$$

$$Q_t = \gamma + \theta P_t + u_t \tag{2}$$

where Q_t and P_t are observable, where α, β, γ and θ are fixed unobservable parameters, and where ε_t and u_t are serially and contemporaneously uncorrelated normal random variables with zero means and variances σ_ε^2 and σ_u^2. The reduced form of this model is

$$P_t = (\alpha + \varepsilon_t - \gamma - u_t)/(\theta - \beta)$$

$$Q_t = ([\alpha + \varepsilon_t]\theta - [\gamma + u_t]\beta)/(\theta - \beta).$$

Thus (P_t, Q_t) comes from a bivariate normal population with moments

$$E(P_t, Q_t) = (\alpha - \gamma, \alpha\theta - \gamma\beta)/(\theta - \beta) \tag{3}$$

$$V(P_t, Q_t) = \begin{bmatrix} \sigma_\varepsilon^2 + \sigma_u^2 & \theta\sigma_\varepsilon^2 + \beta\sigma_u^2 \\ \theta\sigma_\varepsilon^2 + \beta\sigma_u^2 & \theta^2\sigma_\varepsilon^2 + \beta^2\sigma_u^2 \end{bmatrix} (\theta - \beta)^{-2} \tag{4}$$

Maximum likelihood estimation requires that the sample moments be set equal to the population moments (3) and (4). If θ and β were known, the sample means could be used to solve uniquely for α and γ using Equations (3), provided that $\beta \neq \theta$. Since otherwise this places no restrictions on β and θ, it is the sample variance matrix which must be relied on to determine β and θ. Setting sample moments equal to population moments yields the three equations

$$(\hat{\theta} - \hat{\beta})^2 s_p^2 = \hat{\sigma}_\varepsilon^2 + \hat{\sigma}_u^2$$

$$(\hat{\theta} - \hat{\beta})^2 s_q^2 = \hat{\theta}^2\hat{\sigma}_\varepsilon^2 + \hat{\beta}^2\hat{\sigma}_u^2$$

$$(\hat{\theta} - \hat{\beta})^2 s_{pq} = \hat{\theta}\hat{\sigma}_\varepsilon^2 + \hat{\beta}\hat{\sigma}_u^2 .$$

where s_p^2 and s_q^2 are sample variances and s_{pq} is the sample covariance. The first and last of these equations can be written as

$$\begin{bmatrix} \hat{\sigma}_\varepsilon^2 \\ \hat{\sigma}_u \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \hat{\theta} & \hat{\beta} \end{bmatrix}^{-1} \begin{bmatrix} s_p^2 \\ s_{pq} \end{bmatrix} (\hat{\theta} - \hat{\beta})^2 = \begin{bmatrix} \hat{\beta}s_p^2 - s_{pq} \\ -\hat{\theta}s_p^2 + s_{pq} \end{bmatrix} (\hat{\beta} - \hat{\theta})^{-1}(\hat{\theta} - \hat{\beta})^2 \quad (5)$$

which can be inserted into the second to produce

$$\hat{\theta}^2(\beta s_p^2 - s_{pq}) + \hat{\beta}^2(-\hat{\theta}s_p^2 + s_{pq}) = (\hat{\beta} - \hat{\theta})s_q^2 ,$$

which in turn can be rewritten as

$$(\hat{\theta} - b)\ (\hat{\beta} - b) = (r_{pq}^2 - 1)s_q^2/s_p^2 \quad (6)$$

where b is the ordinary least-squares estimate

$$b = s_{pq}/s_p^2 ,$$

and r_{pq}^2 is the squared sample correlation

$$r_{pq}^2 = s_{pq}^2/s_p^2 s_q^2 .$$

The set of maximum likelihood estimates of β and θ therefore is the hyperbola (6) intersected with the region which determines positive estimates of the variances (5), that is

$$(\hat{\beta} - b)/(\hat{\beta} - \hat{\theta}) \geq 0$$

$$(\hat{b} - \theta)/(\hat{\beta} - \hat{\theta}) \geq 0,$$

but these constraints are redundant since they are implied by (6) using the information that $r^2_{pq} \leq 1$.

The hyperbola of maximum likelihood estimates (6) is graphed in Figure 1 given the assumption that the least-squares estimate is positive. It may be noted that any estimate of β is possible, and any estimate of θ; but given one, there is a unique maximum likelihood estimate of the other. In particular, if one is known to be zero, then the estimate of the other is the reverse regression estimate

$$b_r = s^2_q/s_{pq} = b/r^2_{pq} \quad ;$$

or if one is known to be infinite, then the estimate of the other is least-squares, b.

The usefulness of inequality constraints is illustrated in Figure 2. If Equation (1) is taken to be the demand curve, $\beta < 0$, then the maximum likelihood estimates of the supply slope θ must lie between least-squares and reverse least-squares

$$0 < b < \hat{\theta} < b_r.$$

The knowledge that the supply slope is positive, $\theta > 0$, only restricts the estimate of β not to lie in this interval

$$\hat{\beta} < b \quad \text{or} \quad b_r < \hat{\beta}.$$

Together, the inequalities $\beta < 0$, $\theta > 0$ and a positive least-squares estimate, $b > 0$, imply the inequalities

$$\hat{\beta} < 0, \quad 0 < b < \hat{\theta} < b_r. \tag{8}$$

Alternatively, if the least-squares estimate is negative, these inequalities become

$$b_r < \hat{\beta} < b < 0, \quad 0 < \hat{\theta} \tag{9}$$

In other words, when the regression of quantity on price yields a positive estimate, we may assume that this is an attenuated estimate of the supply curve and that the data contain no useful information about the demand curve. If the estimate is negative, the number may be treated as an attenuated estimate of the demand slope, and we may assert that the data contain no useful information about the supply curve.

Under general conditions, maximum likelihood estimators are consistent, and there is nothing in this problem to suggest that (8) and (9) are not consistent bounds. It is easy to show this, using the fact that b and b_r are consistent estimates of the corresponding population moments

$$\text{plim } (b) = (\theta\sigma_\varepsilon^2 + \beta\sigma_u^2)/(\sigma_\varepsilon^2 + \sigma_u^2) \tag{10}$$

$$\text{plim } (b_r) = (\theta^2\sigma_\varepsilon^2 + \beta^2\sigma_u^2)/(\theta\sigma_\varepsilon^2 + \beta\sigma_u^2) \tag{11}$$

Equation (10) is a weighted average of θ and β. Therefore $\beta \leq \text{plim } b \leq \theta$. Also, if plim $(b) > 0$, then $\theta\sigma_\varepsilon^2 + \beta\sigma_u^2 > 0$, and (11) is a weighted average of θ and β with a negative weight on β and a positive weight on θ; thus $\theta < \text{plim } (b_r)$. Similarly, if plim $(b) < 0$, then plim $(b_r) < \beta$.

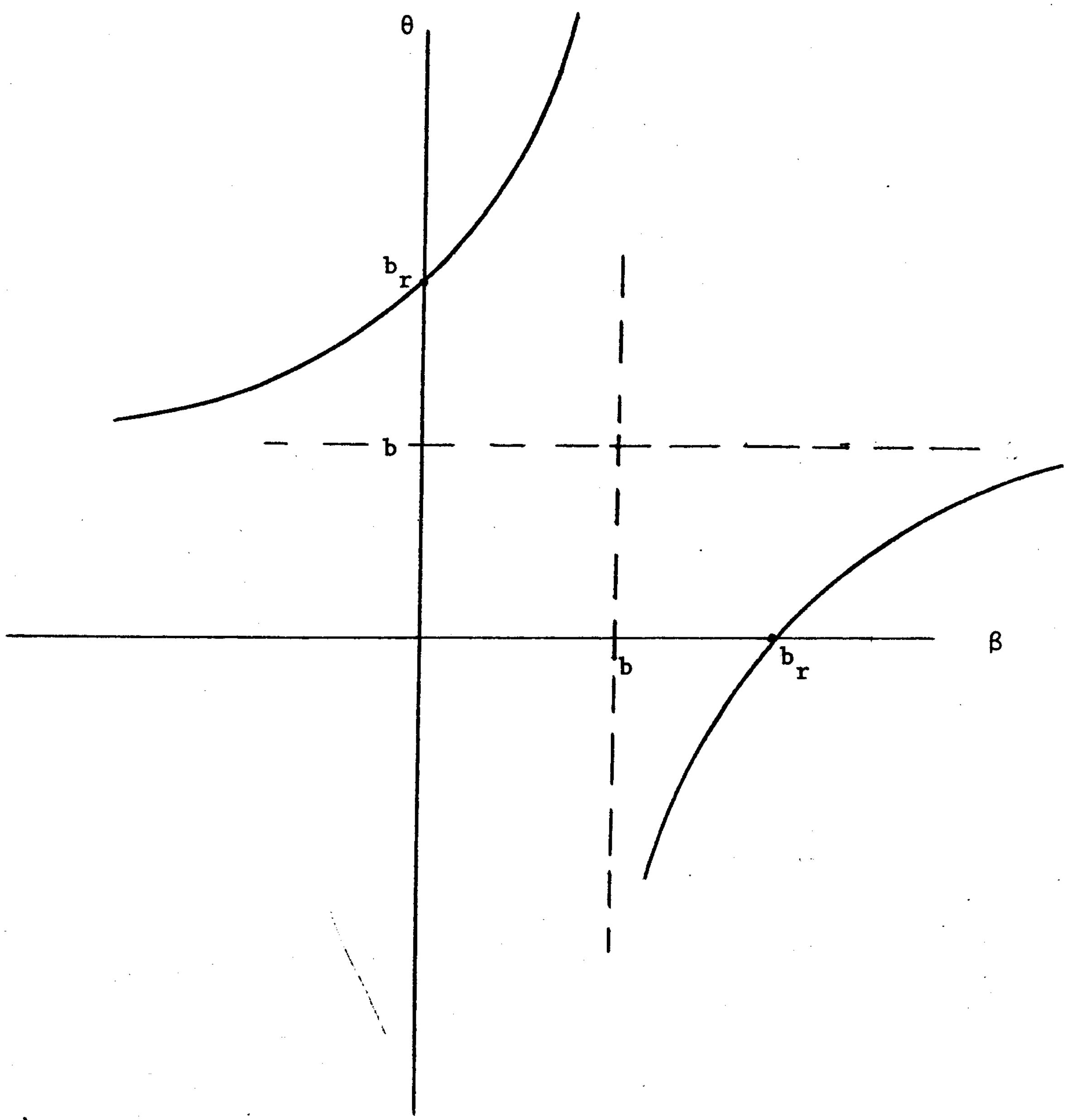


Figure 1. Maximum Likelihood Estimates of β and θ

$b = s_{pq}/s_p^2$, $b_r = s_q^2/s_{pq} = b/r_{pq}^2$

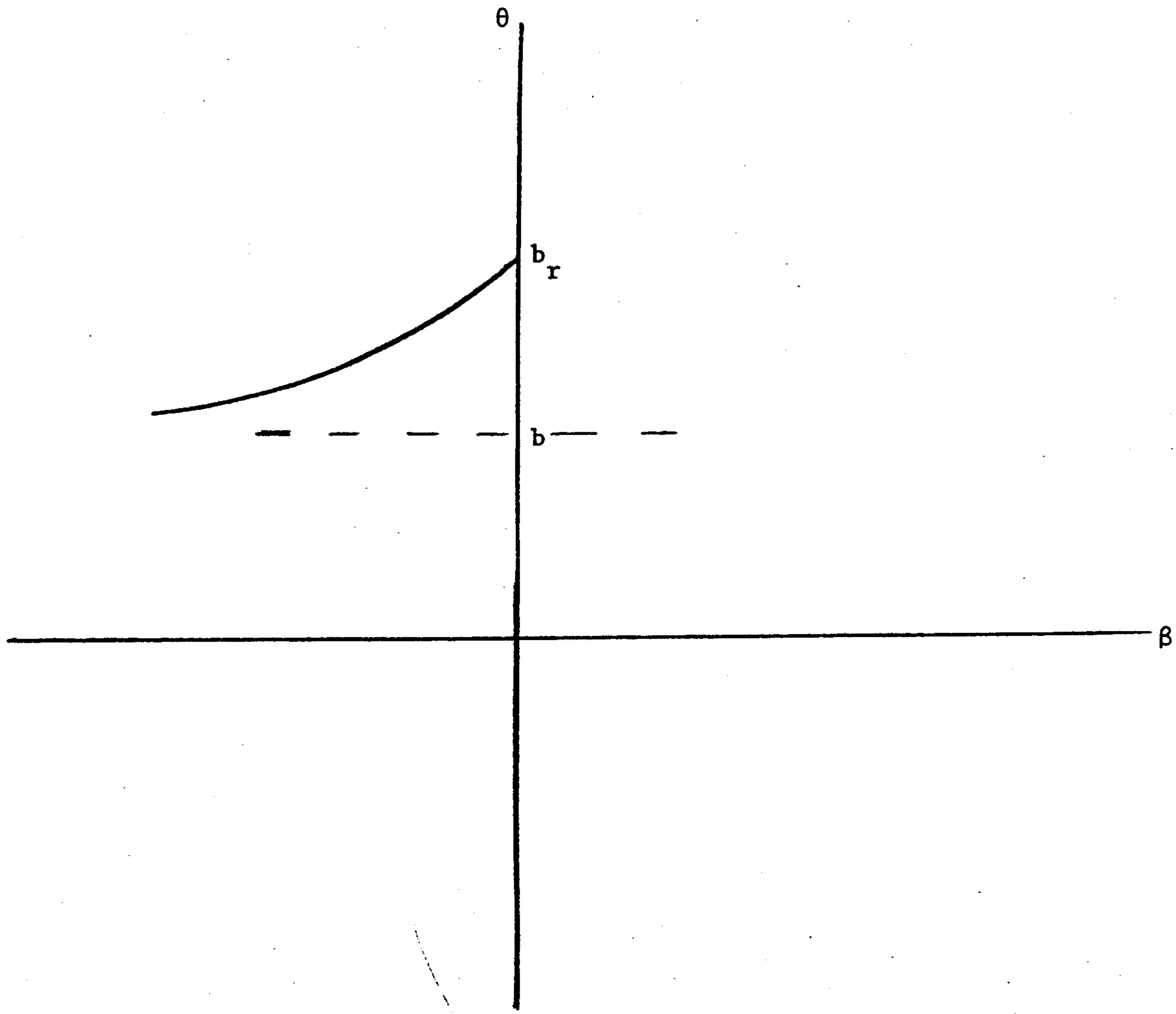


Figure 2. Maximum Likelihood Estimates of β and θ, given $\theta \geq 0$ and $\beta \leq 0$.

$b = s_{pq}/s_p^2$ $b_r = b/r_{pq}^2$

2. A Model with Exogenous Variables

We next consider a more general model in which an observable variable x_t affects both the quantity supplied and the quantity demanded. If there is no a priori information about the signs of the new coefficients, then the set of maximum likelihood estimates of β and θ is altered only in that the sample moments are computed after controlling for x_t. In other words, the direct and reverse regressions include the variable x_t. If there are equality or inequality constraints on the x-coefficients, then the set of estimates of β and θ may change more dramatically.

The model (1) and (2) is altered to allow α and γ to be functions of x:

$$\alpha = \alpha_0 + \alpha_1 x_t$$

$$\gamma = \gamma_0 + \gamma_1 x_t$$

This leaves the variance (4) unchanged but alters the means

$$E(P_t, Q_t | x_t) = (\alpha_0 - \gamma_0 + [\alpha_1 - \gamma_1]x_t, \alpha_0\theta - \gamma_0\beta + [\alpha_1\theta - \gamma_1\beta]x_t)/(\theta - \beta).$$

The two x-coefficients in this reduced form are estimated by regressing P_t on x_t and Q_t on x_t respectively. These will be indicated by $\hat{d}_p$ and $\hat{d}_q$

$$\begin{bmatrix} s_{px}/s_x^2 \\ s_{qx}/s_x^2 \end{bmatrix} \equiv \begin{bmatrix} d_p \\ d_q \end{bmatrix} = \begin{bmatrix} \hat{\alpha}_1 - \hat{\gamma}_1 \\ \hat{\alpha}_1\hat{\theta} - \hat{\gamma}_1\hat{\beta} \end{bmatrix} (\hat{\theta} - \hat{\beta})^{-1} ,$$

which can be solved for $(\hat{\alpha}_1, \hat{\gamma}_1)$ in terms of $(\hat{\theta} , \hat{\beta})$

$$\begin{bmatrix} \hat{\alpha}_1 \\ \hat{\gamma}_1 \end{bmatrix} = \begin{bmatrix} d_q - \hat{\beta} d_p \\ d_q - \hat{\theta} d_p \end{bmatrix} \tag{12}$$

The logic that leads to estimates of β and θ is the same as before except that the sample moments control for x. The least-squares estimate of b in (6) is therefore replaced by least-squares controlling for x which for ease of notation will still be denoted by b:

$$b = b_{pq\cdot x} = s_{pq\cdot x}/s^2_{p\cdot x} \quad .$$

The other moments are similarly altered. Thus the couple $(\hat{\beta}, \hat{\theta})$ lies on an hyperbola centered at least-squares and the x-coefficients are computed using (12).

For the sake of interpretation, it may be shown that if $\hat{\beta}$ is least-squares, then $\hat{\alpha}_1$ is least-squares, and if $\hat{\beta}$ is reverse least-squares then $\hat{\alpha}_1$ is reverse least-squares. The least-squares estimate of α_1 given β is formed by regressing $Q_t - \beta P_t$ on x_t

$$\hat{\alpha}_1 = s_x^{-2}[s_{xq} - \beta s_{xp}] = d_q - \beta d_p,$$

which is just Equation (12). The reverse regression estimate of $\hat{\alpha}_1$ is computed by first estimating the reverse equation $P_t = (Q_t - \alpha_0 - \alpha_1 x_t - \varepsilon_t)/\beta$ and then letting $\hat{\alpha}_1$ be the coefficient on x divided by the coefficient on Q. Given β, this estimate is formed by regressing $P_t - \beta^{-1}Q_t$ on x_t:

$$\hat{\alpha}_1 = -\beta\, s_x^{-2}\, [s_{xp} - \beta^{-1} s_{xq}] = d_q - \beta d_p,$$

which is just Equation (12).

Now we consider the usefulness of information about α_1 and γ_1 for estimating β and θ. The traditional identifying restriction is that the coefficient of the exogeneous variable is zero in one equation. If α_1 is known to be zero, then (12) determines the estimate of β

$$\hat{\beta} = d_q/d_p \equiv b_{2SLS}$$

which is the two-stage least-squares estimate. The other equation is also identified since given $\hat{\beta}$ we can solve for $\hat{\theta}$ using hyperbola (6)

$$\hat{\theta} = b_H = b_{pq\cdot x} + (r^2_{pq\cdot x}-1)s^2_{q\cdot x}/s^2_{p\cdot x}(b_{2SLS}-b_{pq\cdot x})$$

which will henceforth be called the "hyperbolic estimate."

Less precise knowledge of α_1 may also be useful. The exact algorithm for finding maximum likelihood estimates given inequality constraints on α_1 and γ_1 as well as β and θ is tedious, but not difficult. Given equations (12), the sign pattern for α_1 and γ_1 selects a quadrant located at (b_{2SLS}, b_{2SLS}) within which ($\hat{\beta}$, $\hat{\theta}$) must lie. This may or may not further restrict $\hat{\beta}$ and $\hat{\theta}$ given that they are already restricted to lie in the quadrant $\hat{\beta} < 0$, $\hat{\theta} > 0$, as in Figure 2. Suppose for example that it is known that $\alpha_1 > 0$ and $\gamma_1 < 0$. Then from (12) $d_q - \hat{\beta}d_p > 0$ and $d_q - \hat{\theta}d_p < 0$. If the least-squares estimate of β is positive, so that Figure 2 applies, then these further restrictions are redundant if and only if they are satisfied at the extreme points of the feasible set in Figure 2, that is, if $d_q > 0$, $d_p > 0$, $d_q - b_r d_p < 0$ and $d_q - bd_p < 0$. The last two inequalities together with $b > 0$ require that the direct and reverse regressions have coefficients conforming in sign to the supply equation, namely price has a positive effect and x a negative effect on quantity. In addition it is necessary that the simple correlation between price and x, and between quantity and x both be positive ($d_q > 0$, $d_p > 0$). The intuitive logic for this second restriction escapes me.

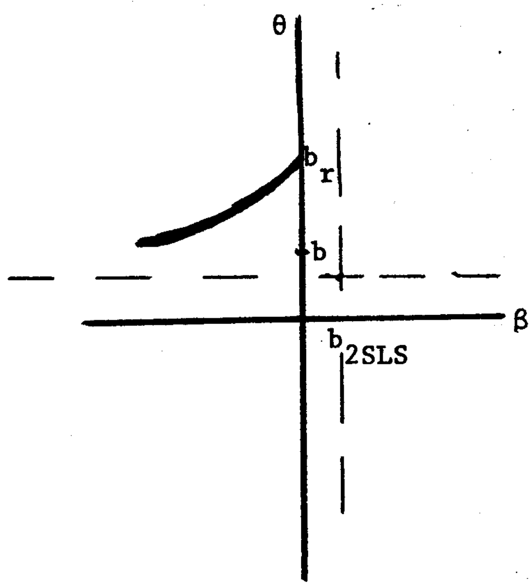
When the new constraints reduce the set of feasible points, they necessarily divide the set in two, leaving one part no longer feasible. Two examples are illustrated in Figure 3. By inspection, there are five possibilities, given $b > 0$:

Size of Estimates	Constraints on $\hat{\beta}$ and $\hat{\theta}$
$0 \leq d_q$, $0 < d_p$, $0 \leq b_{2SLS} \leq b$	$\hat{\beta} < 0$, $b < \hat{\theta} < b_r$
$0 \leq d_q$, $0 < d_p$, $0 < b \leq b_{2SLS} < b_r$	$b_H < \hat{\beta} < 0$, $b_{2SLS} < \hat{\theta} < b_r$
$0 \leq d_q$, $0 < d_p$, $0 < b_r \leq b_{2SLS}$	no solution
$d_q < 0$, $0 < d_p$, $0 < b$	$\hat{\beta} < b_{2SLS} < 0$, $b_H < \hat{\theta} < b_r$
$d_p < 0$, $0 < b$	no solution

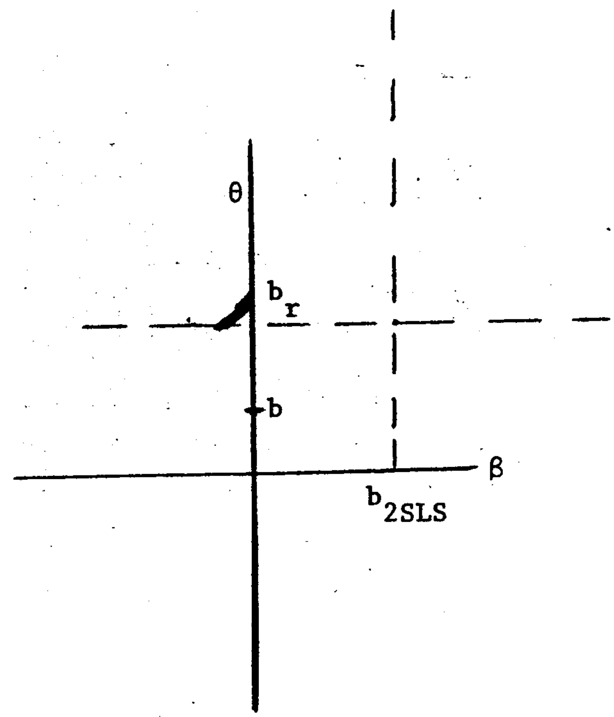
where $b = s_{pq\cdot x}/s^2_{p\cdot x}$, $b_r = b/r^2_{pq\cdot x}$.

When "no solution" is indicated, the sample moments violate the constraints of the process. If this is treated as a small sample aberration, then maximum likelihood estimation can be done given the list of constraints. In some cases, this will lead to a unique maximum likelihood estimate.

Other cases are straightforward generalizations of the preceding. Each exogenous variable with a coefficient of known sign in one equation restricts the slope of the same equation, and therefore restricts the couple (β, θ) to a half-plane. If these restrictions are not mutually exclusive, then there are at most two binding constraints. The variables corresponding to these constraints are then used individually as instruments for two-stage least-squares estimation, which in turn are used to form the hyperbolic estimate of the other coefficients, thereby delineating the region in which the couple $(\hat{\beta}, \hat{\theta})$ must lie.



Case 1 Redundant constraints



Case 2 Binding Constraint

Figure 3. Sets of Maximum Likelihood Estimates $b > 0$, $d_2 > 0$, $d_p > 0$, $\beta < 0$, $\theta > 0$, $\alpha_1 > 0$, $\gamma_1 < 0$

$b_{2SLS} = d_q/d_p$

3. Historical Notes

The supply-demand system was the mother's milk of the econometric profession and it is curious that the simple results in this paper do not seem to be known. To put it more strongly, there is defect in an intellectual tradition which studies so carefully so simple a problem, yet which fails to produce these simple results. I believe that the problem lies in excessive preoccupation with issues of statistical estimation, and in particular, with identification, to the exclusion of issues of inference. The econometric pioneers Pigou (1910), Moore (1914), and Schultz (1928), generally did ask the inference question: "what can be learned from a given data set," but modern econometricians beginning with Koopmans (1949) were satisfied with establishing that a consistent estimator does not exist. The bridge from the one tradition to the other was a sequence of articles by Elmer Working (1927), Leontief (1929), Frisch (1933) and finally Schultz (1938), who surrendered his early intuitive judgements in the face of the imposing mathematical attack.

Although Schultz (1928) recommended the use of lagged prices to identify the supply equation, with considerable qualification he did take a positive estimate to be a supply equation and a negative estimate to be a demand equation (p. 131):

> In reaching a decision on these questions some help may be derived from economic theory if we recall that there is little or no theoretical justification for the assumption of the existence of negatively inclined supply curves in a regime of free competition. This means that we shall not go wrong often if we reject all negative correlations between prices and production on the ground that they can not generally lead to a true supply curve, and direct our attention primarily to the positive correlations between prices and production.

A year previous to the publication of Schultz' Statistical Laws of Supply and Demand, Working (1927) wrote his famous paper "What do Statistical 'Demand Curves' Show?", which is the by-now-familiar graphical analysis of the lack of identifiability of the supply-demand system. This in turn seems to have stimulated Leontief (1929) to seek a solution to the identification conundrum.

If a modern econometrician is defined to be one who manipulates the mathematical relationships implied by a model between sample moments and population parameters, then Leontief (1929) offers the first modern econometric treatment of the supply-demand system.[1] His article includes the hyperbola (6) although expressed in the form $\hat{\beta} = (\hat{\theta}s_{pq} - s_q^2)/(\hat{\theta}s_p^2 - s_{pq})$. Actually he uses a pair of hyperbolas formed by splitting the data set into two parts, and is able to solve for a pair of estimates which jointly satisfy both hyperbolic equalities as illustrated in Figure 4. This procedure brought down upon him the wrath of Frisch's (1933) Pitfalls, which is devoted almost completely to debunking the method. Frisch's treatment is both modern and complete. It uses alternately both the assumptions that $\text{cov}(u,\varepsilon) \neq 0$ and the assumption that $\text{cov}(u, \varepsilon) = 0$, in the latter case showing that the least squares procedure estimates a weighted average of the slopes $s_{qp}/s_{pp}^2 = (\theta\sigma_\varepsilon^2 + \beta\sigma_u^2)/(\sigma_\varepsilon^2 + \sigma_u^2)$. However, possibly ignoring the fact that on the previous page he assumed knowledge of the signs of the elasticities, Frisch on page 17 asserts (using my notation)

> Even if the shifts are uncorrelated, the absolute value of the slope of the (q,p) regression does therefore not tell us anything about the elasticities θ and β. Trying to evaluate the elasticities θ and β from the slope of the observed (q,p) regression and the observed (q,p) correlation, without making assumptions about the relative violence $\sigma_u^2/\sigma_\varepsilon^2$ would be the same as trying

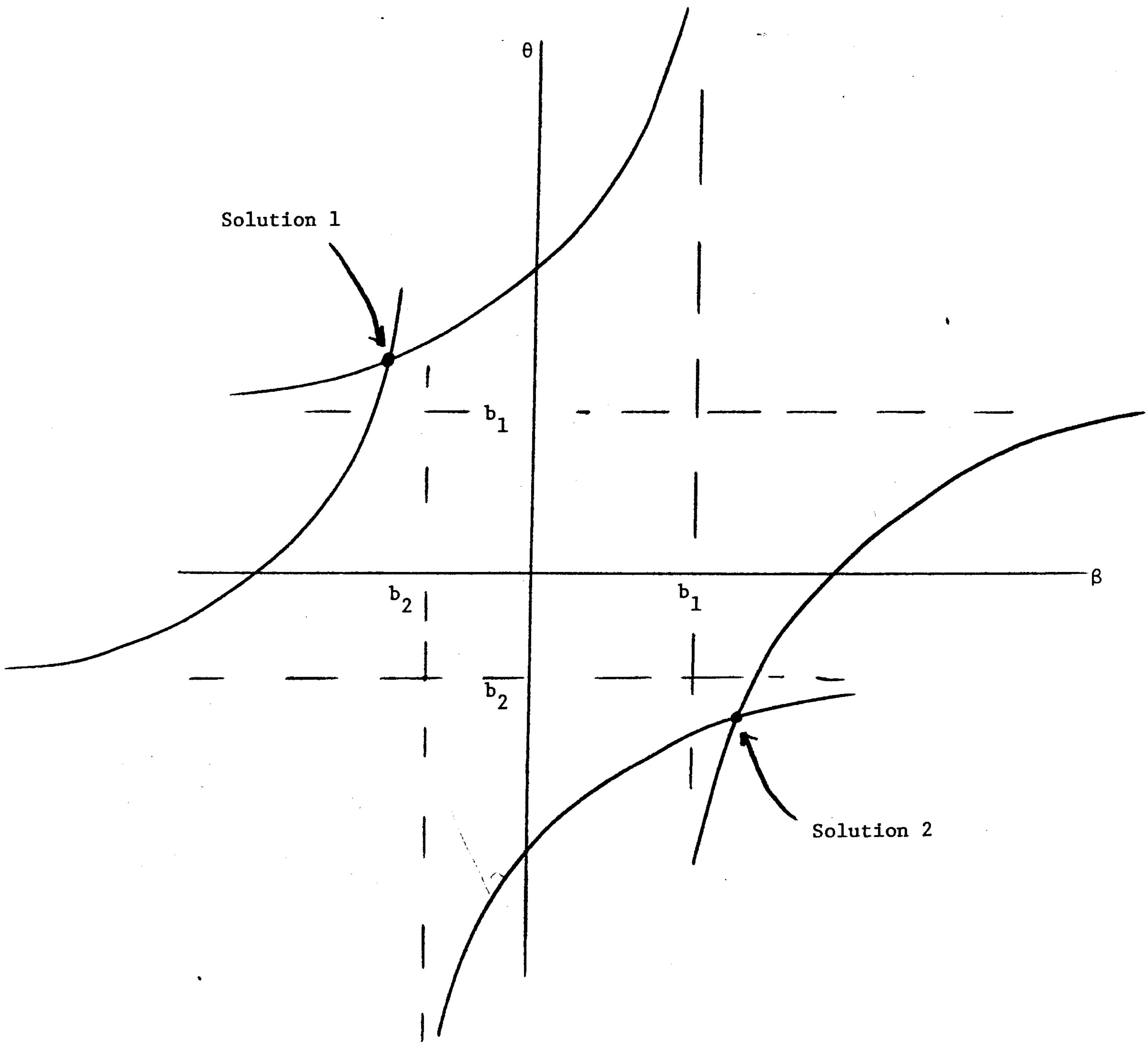


Figure 4. Leontief's Two Estimates of the Supply-Demand System

b_1 = least squares estimate with first half of data

b_2 = least squares estimate with second half of the data to evaluate the two parts of a cake by only knowing the size of the whole cake.

Perhaps thinking of Frisch, Schultz (1938, p. 95) offers the rather backhanded complement of Leontief:

> Students of the subject will, however, always be grateful to Leontief for his bold and painstaking attempt to deduce the true statical, Cournot-Marshall demand and supply curves from statistics. His efforts will not have been wasted if they serve to convince economists and statisticians of the futility of trying to obtain these curves without first examining, by means of 'mental experiments,' whether and to what extent the Cournot-Marshall law of supply and demand has meaning in terms of operations.

This advice was ignored by Schultz himself in the sense that he surrendered his earlier intuition that the supply and demand curves could be separated. Moreover, Leontief's contribution seems to have shrunk away in ignominy. It is not referred to by Marschak (1942) nor by any of the papers in the Hood and Koopman's (1953) volume, nor by any modern econometrics text.

There is of course a problem with the Leontief procedure. If the variances σ_u^2 and σ_ε^2 are the same in both halves of the sample, then asymptotically the two least-squares estimates will necessarily coincide, as will the two hyperbolas. Moreover, the two estimates of the variance ratio necessarily differ in any finite sample. The method thus rests on the unlikely assumption that the slopes β and θ are constant over time but the variances are not. Still Leontief did have the hyperbola properly defined, which is only one short step from the results in this paper.

It is interesting to note that Schultz (1938, p. 89) complains about the case when all four of Leontief's estimates are negative: "Which of these two negatively sloping curves is the demand curve, and which are the supply curves, ...?" Yet Schultz does not realize that knowledge of signs can be used partially to identify the system. And by 1938 (p. 75), he seems to have given up: "When both curves have shifted, it is, therefore, impossible to deduce their forms statistically from the data of consumption and prices. We must have more information."

The Working-Leontief-Frisch sequence culminated in the work at Cowles on identification reported in the Hood and Koopmans (1953) volume. The supply-demand system is extensively discussed in Koopmans (1949) and in Bronfenbrenner (1953). Koopmans mentions the possibility of using the inequality constraints $\beta < 0$ and $\theta > 0$. Both authors consider models with cov $(u,\varepsilon) = 0$. Doubtlessly many others since then have fiddled with the algebra of these systems. Why aren't these results known?

I believe there is a simple answer, and I believe there is a lesson to be learned here. The econometric literature is concerned almost exclusively with issues of estimation rather than issues of inference. Bronfenbrenner (1953), for example, titles his article correctly: "Sources and Size of Least-Squares Bias in a Two-Equation Model." He considers exclusively properties of least-squares estimation, and never asks the more direct question "What can be learned from a given data set?" By this I do not mean to question the usefulness of the estimation results; I am only pointing out that there are other questions that can be considered. The literature on identification as in Koopmans (1949) is a prime example of this. When a parameter is found to be unidentified, discussion tends to terminate because the parameter cannot be consistently estimated. This is clearly giving up too soon since the parameter may nonetheless be consistently bounded as in the errors-in-variables problem and the supply-demand systems in this paper. And even if that can't be done, there is almost certainly information about the unidentified parameter generated by a priori subjective probabilistic constraints on the other parameters, Leamer (1978, 187-193).

As I see it, the econometrics profession ought to refocus its attention away from issues of estimation and towards issues of inference, in particular, the study of likelihood functions.

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