A PURE THEORY OF STRATEGIC BEHAVIOR AND SOCIAL INSTITUTIONS

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The dominant economic models of the interaction between maximizing individuals, the models of Cournot and Stackelberg, assume what economists have come to call "nonstrategic behavior." The individuals represented in such models cannot make and communicate prior commitments to reaction functions in order to influence the subsequent decisions of others. Yet the importance of strategic behavior in achieving realistic solutions to problems of individual interaction is becoming increasingly apparent. Professor Schelling in his 1960 classic, and several subsequent writers on bilateral bargaining, have given us numerous examples in which prior reaction commitments are required to produce realistic solutions to bilateral bargaining problems. Professor Buchanan has constructed examples in which prior reaction commitments are required to prevent charitable donations from worsening the conditions that the donors wish to improve. The present authors have recently shown (1979) that observed firm interaction in modern, concentrated industries can be understood only by assuming that some producers have precommitted themselves to certain reactions to the outputs of others.

More basic examples concern the institutions of exchange and property rights. Rational exchange cannot exist under Cournot or Stackelberg interaction in a finite horizon trading model. Under Stackelberg interaction, the last deliverer in an exchange or series of exchanges, having already obtained all he ever will from the others, has no incentive whatever to deliver; so any prior deliveries by others would also be irrational. Under Cournot interaction, where no party observes the actual deliveries of others (there are only estimates of the deliveries of others, estimates which are correct in equilibrium), no party has an incentive to deliver as one's receipts are independent of his deliveries. Rational exchange in a finite horizon model -- and a typical exchange in the real world -- requires a prior commitment that imposes greater costs for non-delivery than the goods are worth. Similarly, the existence of private property itself generally requires prior commitments to retaliate against potential aggressors such retaliation generally requiring expenditures by protectors in excess of the value of the property (Thompson, 1979).

The primary purpose of this paper is to develop an n-person model of strategic behavior for the "pure" case in which no individual suffers any direct costs of committing himself to or communicating any one of his possible reaction functions. Both the basic model, which also assumes a finite set of social alternatives, and the model's solution, are specified in Part I. Our central result, the Pareto optimality of the solution under strict preference relations, is demonstrated in Part II.

Part III is mainly pedogogical; it contrasts our solution to the standard non-cooperative solution to a prisoner's dilemma game in order to clarify possible ambiguities concerning the nature of our communication and commitment assumptions.

It is tempting to apply our central optimality result to the interactions between subgroups of communicating individuals in environments containing outside enforcers. In particular, it is tempting to infer that we are establishing the traditional conjecture that any subset of individuals, if they perfectly communicate with one another, will interact so as to achieve a joint optimum among themselves. But there is no reason to suppose that the outside enforcers would not affect the reaction functions of the insiders, thereby violating the condition that the individuals freely select from all possible reaction functions. Furthermore even if the outside enforcers induced no alterations in insider decisions, the fact that certain insiders, in general, devote overhead resources to establishing the priority of their individual commitments makes it possible for outsider intervention to benefit everyone by inducing reductions in such resource expenditures. If, however, these resource costs were somehow prevented, and if the only other imposition on the subgroup is a compensatory, common-law property rights system, then the so-constrained solution would be a Pareto optimum for the subgroup (Thompson-Faith, 1980). This amounts to a formalization of the Coase Theorem. The current paper, however, keeps the analysis at the level of the entire social group, wherein no outside imposition on feasible reaction functions can appear to threaten the optimality result and, because of the absence of outsiders and therefore an absence of anyone to whom we can appeal to gain an allocative correction, the resource losses in establishing a prior commitment become unavoidable deadweight costs. Correspondingly, our optimality result becomes a positive hypothesis rather than a normative statement, the conclusion being that, except for unavoidable overhead costs, equilibrium allocations, which always exist in our model, will be Pareto optimal. Alternatively, since the "institutions" that an individual faces are defined by the reactions of others to his own actions, our positivistic central conclusion states that equilibrium institutions always exist and are Pareto optimal.

The extreme assumptions regarding the commitment-making and communication abilities of our individuals mask the empirical relevance of the model. Part IV introduces plausible restrictions on the physical environment that allow us to reduce the information and commitment-making assumptions of the general model to plausible levels. The resulting special model amounts to a theory of social institutions with sufficient empirical power to at least suggest explanations for both certain, broad uniformities and certain, broad variations, historical and cross-sectional, in observed political and economic institutions.

Von Neumann and Morgenstern, in their pathbreaking work on the theory of games, also argued that the ultimate aim of their exercise was to determine institutions endogenously. Indeed, our basic model in Part I is merely a Von Neumann-Morgenstern "perfect information game" played over strategies rather than simple actions (or "plays of a game"). While Von Neumann and Morgenstern explicitly recognized (Sec. 11.3) that games could be constructed in which strategies are communicated in the same way as the actions in their perfect information games, they saw nothing novel about such games. For such games posed no new problem in the development of solution concepts or the existence of solutions. Perhaps, had they been more interested in evaluating the Pareto optimality of solution outcomes or in abstractly characterizing basic social institutions such as private property or contracts, they would have devoted some of their prodigious intellectual resources to characterizing games with perfect information concerning strategies as well as actions. But perhaps not too. For such games represent an uncomfortable hybrid within the corpus of game theory in that the games are, strictly speaking, neither "cooperative" nor "noncooperative." The games allow more information than non-cooperative games in that they explicitly allow preplay communication between the players. Yet the games are not "cooperative" either in that they contain no exogenous "characteristic functions" mysteriously assigning payoffs to "coalitions" of players and, correspondingly, no prior imposition of group rationality conditions. Our model is therefore distinct from other game-theoretic models in that it contains a theory of individually rational communication and, correspondingly, a theory of individually rational "cooperation."[1] Part V of this paper analyzes the basis of the difference between our theory and conventional cooperative game theory. In so doing, it exposes a basic defect in conventional cooperative game theory. It then applies the general argument to voting processes, showing that perfect strategic communication prevents the "voters' paradox" that underlies modern theoretical critiques of democracy.

I. THE BASIC MODEL AND ITS SOLUTIONS*

A. The Physical Environment.

An individual is denoted i, $i = 1,\ldots,n$. An action of individual i is denoted x_i, where $x_i \in X_i$, a finite set of feasible actions of individual i. A possible social choice, or allocation, is defined by an n-dimensional set of actions, and is denoted $x = (x_1, x_2, \ldots, x_n)$, so that $x \in \prod_{i=1}^{n} X_i$. To describe individual preferences, each individual, i, is given a complete, transitive, irrelexive, antisymmetric, binary relation, $\succ_i$, defined over $\prod_{i=1}^{n} X_i$. This description, in effect, assumes away indifference between any pair among the finite set of possible allocations. The motivation for this assumption and the effects of indifference on our central results will be discussed later. A Pareto optimum is an allocation, x', $x' \in \prod_{i=1}^{n} X_i$, for which there is no alternative allocation, x'', $x'' \in \prod_{i=1}^{n} X_i$, such that $x'' \succ_i x'$ for all i. Several Pareto optima may exist.

B. Institutional Possibilities

The institutions facing an individual can be completely described by the reactions of other individuals to his own actions. But institutions, or reactions, are not taken here as given; they are derived. This is done by allowing each individual to select, among all feasible reaction functions, a function which is maximal with respect to his preference relation. But we want individuals to know the institutions and thus the reaction functions of others. And for this to generally hold, the functions must be communicated in sequence. Thus, for the individuals to know the institutions, the first communicator, say individual 1, presents the reaction function,

$$x_1 = f_1(x_2, \ldots, x_n), \qquad (1)$$

to the other individuals; the second communicator, say individual 2, then presents

$$x_2 = f_2(x_3, \ldots, x_n) \qquad (2)$$

to individuals 3 through n; the third communicator then presents

$$x_3 = f_3(x_4,\ldots,x_n) \qquad (3)$$

to individuals 4 through n, and so on up to the $n - 1^{\text{st}}$ communicator, who presents

$$x_{n-1} = f_{n-1}(x_n) \qquad (4)$$

to the n^{th} individual, who has no need to communicate. Once the action of the n^{th} individual is taken, the action of the $n - 1^{\text{st}}$ individual is determined. Once this pair of actions is taken, the action of individual n - 3 is determined, and so on up until an allocation is determined as a chain reaction from the n^{th} individual's action. The set $(f_1, f_2, \ldots, f_{n-1})$ is thus a complete institutional description. The feasible choice set, or strategy set, of individual 1 is the set of <u>all</u> functions from $\prod_{i=2}^{n} X_i$ to X_1. This can be represented by the functional variable, F_1. Similarly, $F_2, \ldots, F_{n-1}$ can be used to represent the respective strategy sets of individuals 2 to n-1. The product space, $\prod_{i=1}^{n-1} F_i$, thus represent the world's institutional possibilities. The strategy set of individual n is X_n.

A question may arise as to why some individuals do not present reaction functions to other individuals who are higher up in the communication hierarchy. Consider individual n. Facing the prior strategies of the other n - 1 individuals, he sees that the eventual allocation must be consistent with the chosen reaction functions of each of the n - 1 prior selectors. Hence, if individual n responds to the prior selectors with a simple action, he will have a free choice over all allocations consistent with the prior reaction functions. But if n responds with a function of prior actions, thus giving further choices to the prior strategy selectors, he can only reduce his original choice out of the same set of possible allocations. He cannot expand the set of possible outcomes because any eventual outcome must be consistent with the given n-1 reaction functions.

Similarly, if the n-1st strategy selector presents a reaction function rather than an action to his prior strategy selectors for a given action of individual n, he is giving them the choice of actions consistent with the set of reaction functions he faces and thus can be no better off. This also applies, in like fashion, to individuals n-2 to 2, so that it is in no individual's interest to present a reaction function to a prior strategy selector.

The above world, which can now be viewed as a "game," differs from the standard, von Neumann-Morgenstern, "perfect information" (and majorant-minorant) games in that some individuals are allowed to communicate their strategies to others before the latter select their own strategies. Thus, in the von Neumann-Morgenstern world, a player will not adopt a special strategy in order to influence the subsequent strategies (and actions) of others simply because he cannot communicate it and therefore cannot use it to influence the subsequently chosen strategies. In contrast, in the above world, each of the first n-1 players communicates his strategy to all subsequent strategy selectors. And response strategies of the subsequent selectors are known a priori by the prior strategy selector because they are the rational responses to the given strategy of the prior selector.

Due to the additional information implied by having individuals communicate committed reaction functions to subsequent players before the latter select their strategies and before any actions are taken, the information implied by the above sequence of reaction functions is called "truly perfect." More specifically, under "truly perfect information," each player: (1) knows with certainty the reaction functions chosen prior to his strategy choice, (2) can freely choose (i.e. commit himself to) and communicate any reaction function consistent with $x \in \prod_{i=1}^{n} X_i$ and prior to reaction functions and (3) knows with certainty the rational responses to each of his possible reaction functions by all subsequent strategy selectors.

An alternative formulation of the above model, one which obviates the communication and commitment concepts used above, is to allow players in an n-person perfect information game with n moves an earlier, additional series of n-1 special moves in which each can perform a new kind of action, one which has the effect of reducing his subsequent feasible responses to the regular actions of others to a single, specified regular action. This formulation, a case of which is outlined in Schelling, produces the same result as above but adds a cumbersome, logically unnecessary step to the formal development.

While Howard has produced a general class of games (called "jk-metagames") containing strategies contingent on the strategies of other strategy selectors, he does not assume truly perfect information. Correspondingly, he does not adopt a perfect information solution concept. Rather, he adopts, without substantive justification, the von Neumann-Morgenstern-Nash "no-regret" solution concept in which each strategy selector accepts as given the strategies of all other strategy selectors. This amounts, as Howard recognizes, to assuming uniformly zero information regarding the strategies of others at the time of strategy selection. For if the choice of strategy selector were perceived by subsequent strategy selectors, it would, in general, influence the latter's selections. Such games, besides being theoretically unsatisfying in that they typically generate a multiplicity of solution points, some of which are optimal and others nonoptimal (Howard, p. 58), are empirically unsatisfying in that observed commitments are, as pointed out in the Introduction, typically communicated to others in order to influence their strategy selections.

C. Equilibrium Institutions, or "Solutions", under Truly Perfect Information.

A solution, $(f_1^*, \ldots, f_{n-1}^*, x_n^*)$, is a set in which the ith variable is maximal with respect to $\succ_i$ for given values of $f_1, \ldots, f_{i-1}$. A solution can be constructed as follows: First, we find, for individual n, x_n^*, the point in X_n such that, for all $x_n \neq x_n^*$,

$$\{f_1(f_2,\ldots,f_{n-1},x_n^*),\ f_2(f_3,\ldots,f_{n-1},x_n^*),\ldots,x_n^*\} \succ_n$$
$$\{f_1(f_2,\ldots,f_{n-1},x_n),\ f_2(f_3,\ldots,f_{n-1},x_n),\ldots,x_n\}.$$

This solution determines a dependency of x_n^* on $f_1,f_2,\ldots,f_{n-1}$, which we write $x_n^*[f_1,\ldots,f_{n-1}]$. Then, for individual n-1, we find a reaction function, f_{n-1}^*, such that for all $f_{n-1} \in F_{n-1}$, $f_{n-1} \neq f_{n-1}^*$,

$$\{f_1(f_2,\ldots,f_{n-2},f_{n-1}^*,x_n^*[f_1,\ldots,f_{n-2},f_{n-1}^*]),\ldots,f_{n-2},f_{n-1}^*,x_n^*[f_1,\ldots,f_{n-2},f_{n-1}^*]\} \succ_{n-1}$$
$$\{f_1(f_2,\ldots,f_{n-2},f_{n-1},x_n^*[f_1,\ldots,f_{n-2},f_{n-1}]),\ldots,f_{n-2},f_{n-1},x_n^*[f_1,\ldots,f_{n-2},f_{n-1}]\}$$

This solution determines the dependency of f_{n-1}^* on $f_1,f_2,\ldots,$ and f_{n-2}, which we describe as $f_{n-1}^*[f_1,\ldots,f_{n-2}]$. Then, for individual n-2, we find a reaction function, f_{n-2}^*, such that, for all $f_{n-2} \in F_{n-2}$, $f_{n-2} \neq f_{n-2}^*$,

$$\{f_1(f_2,\ldots,f_{n-2}^*,f_{n-1}^*[f_1,\ldots,f_{n-2}^*],x_n^*[f_1,\ldots,f_{n-2}^*,f_{n-1}^*[f_1,\ldots,f_{n-2}^*]]),$$
$$\ldots,f_{n-2}^*,\ f_{n-1}^*[f_1,\ldots,f_{n-2}^*],x_n^*[f_1,\ldots,f_{n-2}^*,f_{n-1}^*[f_1,\ldots,f_{n-2}^*]]\} \succ_{n-2}$$
$$\{f_1(f_2,\ldots,f_{n-2},f_{n-1}^*[f_1,\ldots,f_{n-2}],x_n^*[f_1,\ldots,f_{n-2},f_{n-1}^*[f_1,\ldots,f_{n-2}]]),$$
$$\ldots,f_{n-2},\ f_{n-1}^*[f_1,\ldots,f_{n-2}],x_n^*[f_1,\ldots,f_{n-2},f_{n-1}^*[f_1,\ldots,f_{n-2}]]\}.$$

This solution thus determines the dependency of f_{n-2}^* on $f_1,f_2,\ldots,$ and f_{n-3}, which we write as $f_{n-2}^*[f_1,\ldots,f_{n-3}]$. The process continues until we have determined f_1^*. Since f_1^* does not depend on any prior functions, we can use it to determine the succeeding reaction functions by successively substituting starred values into $f_2^*[f_1]$, $f_3^*[f_1,f_2],\ldots,$ and $f_{n-1}^*[f_1,f_2,\ldots,f_{n-2}]$.

In this way, a solution, $(f_1^*,f_2^*,\ldots,x_n^*)$, which implies a solution allocation, $(x_1^*,x_2^*,\ldots,x_n^*)$, is determined.

The finite structure of the successive maximization problems, **along with the completeness and transitivity of** $\succ_i$, **assures us that a solution** **<u>always</u>** **exists.**

D. Determination of Priority and the Role of Commitments

The above game, with its predetermined priority, is not symmetric in that its solution will generally vary with the order of priority. While one may think of the priority in strategy making in the above model as being arbitrarily determined by an umpire of the game or some random device, it is much more realistic to determine the order of strategy selection in a higher-order game.

Such higher-order games come in two forms. One form corresponds to a world containing a higher-authority, an outside player who assigns hierarchical positions according to competitive bids for the positions. In such a world, the outside player may also serve the function of an enforcer who assigns hierarchal position and punishes any player who does not carry out his announced strategy. This could eliminate insider resources devoted to establishing prior commitments and also assure the unrestricted commitment ability which characterizes the basic model, thereby preventing the inefficiency possibilities raised in the Introduction above. The game of contracting with outside parties to guarantee commitments, the higher-order game which determines the order of strategy selection, and the game described in the above sections may all be combined into a single game in which players interact to determine the method of enforcement, the order, and the specific form of all strategy commitments. (See Thompson-Faith, 1979) for a specification of such a game and an existence proof for the game.) Since a player in such a game can lower the bids against him for a given position in the hierarchy by choosing a strategy that yields more benefits to his competing subordinates, the solutions to such a game differ somewhat from those described above (See Thompson-Faith). So our optimality theorem does not generally apply to these games. Whether outside authorities establish Pareto optimal rules cannot be theoretically determined when there is an outside authority.

The second form of higher order game is a war-like affair with no higher-authority. Pareto inefficient, Nash-VNM, non-cooperative games apply in determining the order of strategy selection. But the solution characteristics of our own, lower-order game are unaffected by the higher-order game. War losses are strictly sunk costs once a hierarchy is established and our game is ready to be played. Hence, we shall concentrate our applications of the model on raw states of the world, where outside authorities do not exist (Part IV).

E. Lack of Realism

It is grossly unrealistic to arrange all of the individuals in any observed society into a hierarchy of strategy selectors in which each must receive the strategies of the previous selectors before transmitting his own strategy to others. Furthermore, it is similarly unrealistic to give all (but one) individuals the ability to adopt strategies which commit them to carrying out actions which, at the time of their undertaking, may be irrational.

However, as we shall see in Part IV, under certain, rather realistic, specializing assumptions regarding the physical environment, the model does not require either of these extreme characteristics.

II. PARETO OPTIMALITY

Besides unqualified existence, the solution has the important property of Pareto optimality. That is, institutions formed under truly perfect information always imply Pareto optimal allocations.

To prove this, suppose the solution allocation, $(x_1^*,..,x_{n-1}^*,x_n^*) = x^*$, is not Pareto optimal. Then there is a point, $x^o = (x_1^o,\ldots,x_n^o) \in \prod_{i=1}^{n} X_i$ such that $x^o \succ_i x^*$ for all i. A set of reaction functions generating x^o as an allocation is given by $(f_1^o,\ldots,f_{n-1}^o)$. Of course, $(f_1^*,\ldots,f_{n-1}^*,x_n^o) \neq (f_1^o,\ldots,f_{n-1}^o,x_n^o)$; otherwise x^o would be the solution. Now let individual 1 consider:

$$\text{(A)}\quad f_1(f_2,..,f_{n-1},x_n) = \begin{cases} f_1^o \text{ if } (f_2,..,f_{n-1},x_n) = (f_2^o,..,f_{n-1}^o,x_n^o) \\ f_1^* \text{ otherwise.} \end{cases}$$

This may induce each subsequent strategy selector to reorder his strategy in $f_2^o,\ldots,f_{n-1}^o,x_n^o$ relative to $f_2^*,\ldots,f_{n-1}^*,x_n^*$. However, as it does not alter the allocations resulting from non-solution strategies <u>other than</u> $f_2^o,\ldots,f_{n-1}^o,x_n^o$, it does not alter anyone's ordering of these other strategies relative to $f_2^*,\ldots,f_{n-1}^*,x_n^*$. Therefore, because $x^o \succ_1 x^*$, individual 1 is no worse off under (A) than under his original strategy.

We next let individual 2 consider, in view of (A),

$$\text{(B)}\quad f_2(f_3,\ldots,f_{n-1},x_n) = \begin{cases} f_2^o \text{ if } (f_3,\ldots,x_n) = (f_3^o,\ldots,x_n^o) \\ f_2^* \text{ otherwise.} \end{cases}$$

This similarly cannot hurt individual 2. We continue on to individual n, who now faces (A), (B),..., Thus, $(f_1^o,\ldots,f_{n-1}^o,x_n^o) \Rightarrow x^o$ will result if he picks $x_n = x_n^o$; and $(f_1^*,\ldots,f_{n-1}^*,x_n^*)$ if he picks his solution action. Since $x^o \succ_n x^*$, he picks the former. The supposition that there is a Pareto nonoptimal solution is thus immediately contradicted: For the supposition implies that the players individually prefer a non-solution set of strategies $(f_1^o,\ldots,f_{n-1}^o,x_n^o)$ to the solution set, $(f_1^*,\ldots,f_{n-1}^*,x_n^*)$.

III. CONTRAST TO PRISONER'S DILEMMA GAME

The above game contrasts sharply with the familiar Prisoner's Dilemma Game. A Prisoner's Dilemma payoff matrix is illustrated in Figure 1. In a Prisoner's Dilemma Game with conventional "perfect information," the standard, VNM, perfect information solution applies. Following VNM, the player who has the "second move," say the row player, R, has his strategy set expanded beyond the set of simple actions to include that player's possible reactions to the various actions of his opponent. The column player, C, then has the "first move." The normal form of the game is shown in Figure 2, where, for example $x_R'|x_C''$ means that R adopts his first action if C adopts his second. Solving the game, C peruses each column to determine which action R will select (i.e., which action maximizes R's payoff) for each of the given actions of C and then selects the action which maximizes his own payoff given the resulting action of R. C's optimal strategy, in light of R's rational response, is to play x_C''. This leads R to play x_R'' [or $(x_R'|x_C', x_R''|x_C'')$], resulting in the jointly inefficient outcome (x_R'',x_C''). (Since x_R'' is a "dominant strategy" for R, (x_R'',x_C'') also represents a Nash-VNM "no-regret" solution to the normal form, the pair of strategies such that no player can increase his payoff by changing his strategy, given the strategies of the other players.) In contrast, under our assumption of strategic communication, R, while moving second, is able to commit himself to a strategy and communicate it to C before C moves. R rationally commits himself to $(x_R'|x_C',x_R''|x_C'')$ in view of C's rational responses to R's various possible strategies. R's strategy thus becomes a committed reaction function rather than a narrowly rational response function. C then rationally chooses x_C' so the solution is the jointly efficient solution (x_R',x_C'). Hence, as long as the second mover is able to communicate his strategy to the first before the first makes his move, the solution is the jointly efficient outcome.

		don't confess x'_C	confess x''_C
don't confess	x'_R	4,4	0,10
confess	x''_R	10,0	1,1

Figure 1. Payoff Matrix

R	C	don't confess x'_C	confess x''_C
don't confess	x'_R	4,4	0,10
confess	x''_R	10,0	1,1
$x'_R \mid x'_C$, $x''_R \mid x''_C$		4,4	1,1
$x''_R \mid x'_C$, $x'_R \mid x''_C$		10,0	0,10

Figure 2. Normal form of prisoners' dilemma

IV. SPECIALIZATIONS AND EMPIRICAL APPLICATIONS

A. The Nature of the Solution under an Additional Assumption

To give some empirical power to the above model, we shall now assume that for any individual below the first strategy selector, there is some individual higher-up in the decision hierarchy who "can punish" him. To formally define this, first let x^1 be the allocation which individual 1 prefers to all other x in $\prod_{i=1}^{n} X_i$. Individual k is said to "punish" individual j if k selects an $x_k = x_k^{P(j)}$ such that $x^1 \succ_j (x_1, \ldots, x_k^{P(j)}, \ldots, x_n)$ for any $(x_1, \ldots, x_{k-1}, x_{k+1}, \ldots, x_n)$ in $\prod_{i \neq k} X_i$. Individual k "can punish" individual j iff $x_k^{P(j)} \varepsilon X_k$. Our assumption, then, is that for any j, j=2, ..., n, there is a k, k < j, such that $x_k^{P(j)} \varepsilon X_k$. In other words, for any j, there is someone higher-up in the hierarchy that can select an action which makes him worse off than he would be under x^1. The assumption does not appear to be unrealistic once we recognize that, in the real world, almost any healthy adult can inflict damages on almost any, single, other individual to the extent that the victim would prefer serving as a slave to suffering the damages.

The assumption implies that individual 1 can punish individual 2, that individuals 1 or 2 can punish 3, that 1, 2 or 3 can punish 4, etc. It follows that individual 1 can, by adopting the appropriate strategy, effect the punishment of any other single individual if that individual chooses an action which is not an element of x^1. To see this, let j now represent the individual of lowest hierarchial rank that chooses an action which is not an element of x^1 and let k_r be the first individual, going up the hierarchy, who can punish j. If $k_r = 1$, individual 1 can effect the punishment of j directly. If $k_r > 1$, let k_{r-1} be the first individual, again going up in the hierarchy, who can punish k_r. If $k_{r-1} = 1$, individual 1 can adopt a strategy in which he punishes k_r if $x_j \neq x_j^1$ and chooses x^1 if $(x_2,\ldots,x_n) = (x_2^1,\ldots,x_n^1)$. If 1 adopts such a strategy, k_r's optimal strategy is one in which he punishes j if $x_j \neq x_j^1$ and chooses $x_{k_r}^1$ if $(x_{k_r+1},\ldots,x_n) = (x_{k_r+1}^1,\ldots,x_n^1)$. For, given 1's strategy, any strategy of k_r which does not have this characteristic will, by inducing j to pick an $x_j \neq x_j^1$, generate a punishment action by 1 and thus an allocation which is worse for him than the alternative, x^1. If $k_{r-1} > 1$, but k_{r-2}, the first person up in the hierarchy who can punish k_{r-1}, equals 1, then 1 can adopt a strategy in which he punishes k_{r-1} if $x_j \neq x_j^1$ and chooses x^1 if $(x_2,\ldots,x_n) = (x_2^1,\ldots,x_n^1)$. For any such strategy of individual 1, k_{r-1}'s optimal strategy is one in which he punishes k_r if $x_j \neq x_j^1$ and adopts $x_{k_{r-1}}^1$ if $(x_{k_{r-1}+1},..,x_n) = (x_{k_{r-1}+1}^1,..,x_n^1)$ in view of prior reaction functions which assure him that x^1 will result if $(x_{k_{r-1}},\ldots,x_n) = (x_{k_{r-1}}^1,\ldots,x_n^1)$. If $k_{r-2} > 1$, the same argument applies so that k_{r-2} will be induced by prior strategies to punish k_{r-1}, who will then be induced to punish k_r, if $x_j \neq x_j^1$. Eventually, k_{r-i} must equal 1 for some i. At that point, where i = r, individual 1 effects the punishment of j as above and the argument stops. This applies to any j, where j is again the first player who chooses an action which is not an element of x^1. Of course, the set, $(k_1,\ldots,k_r)$ depends on j and may be empty, meaning that 1 can punish j.

Hence, under the additional assumption on the physical environment, individual 1's optimal reaction function is given by

$$x_1 \begin{cases} = x_1^1 \text{ if } (x_2,\ldots,x_n) = (x_2^1,\ldots,x_n^1) \\ = x_1^{P(k^1(j))} \text{ otherwise, where } j = \max i \text{ for which } x_i \neq x_i^1 \text{ and} \end{cases}$$

$$k^1(j) \begin{cases} = j \text{ if 1 can punish } j \\ = k_1(j) \text{ otherwise.} \end{cases}$$

Given this, regardless of the reaction functions in between 1's and $k_1(j)$'s, the optimal reaction functions from $k_1(j)$ to $k_r(j)$ become:

$$x_{k_1} = \begin{cases} x^1_{k_1} & \text{if } (x_{k_1+1},\ldots,x_n) = (x^1_{k_1+1},\ldots,x^1_n) \\ x^{P(k^2(j))}_{k_1} & \text{otherwise, where} \end{cases}$$

$$k^2(j) = \begin{cases} j & \text{if } k_1(j) \text{ can punish } j \\ k_2(j) & \text{otherwise,} \end{cases}$$

$\vdots$

$$x_{k_r}(j) = \begin{cases} x^1_{k_r} & \text{if } (x_{k_r+1},\ldots,x_n) = (x^1_{k_r+1},\ldots,x^1_n) \\ x^{P(j)}_{k_r} & \text{otherwise.} \end{cases}$$

If $x_{k_r}(j)$ did not select this strategy so that j failed to adopt x^1_j, $k_{r-1}(j)$ would, according to the prior functions in the above sequence, punish him. Given this chain of reaction functions, the $j^{\underline{th}}$ individual, $j < n$, is irrational if he chooses a reaction function other than one in which

$$x_j = x^1_j \text{ if } (x_{j+1},\ldots,x_n) = (x^1_{j+1},\ldots,x^1_n).$$

(Since by definition, players j+1,...,n choose $(x^1_{j+1},\ldots,x^1_n)$, j's responses to other actions are irrelevant.) And, of course, if j=n, j is irrational to pick $x_n \neq x^1_n$. This is because the allocation resulting from such deviations will result in an allocation which is less preferred by individual j. Hence, assuming rationality, as defined in our solution concept, first n will not diverge from x^1_n in order to save being punished, so $j \neq$ n-1; then n-1 will not diverge for the same reason, so $j \neq$ n-1; and so on. Since no j exists under the definition of a solution, the solution must be x^1.

The result of our rather plausible, apparently innocent, assumption is thus that any solution to social interaction under truly perfect information is dictatorial in the sense of Arrow. The result thus takes the bite out of Arrow's possibility theorem, since dictatorial social welfare functions are always possible under Arrow's reasonability conditions. A social welfare function may be constructed from the preferences of individual 1, who is the dictator, and Arrow's problem vanishes.

The punishability condition -- when complemented by another rather realistic specialization described below -- will also allow us to substantially reduce the informational requirements of our model.

B. Reducing the Informational Requirements of the Model

As mentioned earlier (Part I.E.), it is highly unrealistic to give all individuals (except one) the ability to (1) communicate reaction functions to all subsequent strategy selectors and (2) select reaction functions which require a prior commitment to narrowly irrational behavior. This lack of realism can now be removed by adding the assumption that the set of individuals $(k_1,\ldots,k_r)$ is independent of j for any $j > m$, a relatively small number compared to n. The rest of the individuals, the bulk of the population, seeing that these players enforce x^1, will simply choose their element of x^1 without having to observe the strategies selected by the others in the group. I.e., they simply set $x_i = x_i^1$ and do not exhibit any punishment actions, thus producing a set of degenerate, constant reaction functions, x_i^1, $i > m$. This greatly reduces the information requirement of our solution in that the large majority of the population need neither communicate their reaction functions to the others nor commit themselves to narrowly irrational reactions.

As mentioned in Part I, when there is no predetermined outside-authority to assign a hierarchy and enforce commitments to the announced reaction functions, a question arises as to how the hierarchy and commitment abilities are formed. The problem is greatly simplified under the punishment assumption introduced above. For, under our assumption of the existence of punishment actions, the only hierarchical position worth having is the first one. Competition for the top spot would plausibly occur in a war-like game to first establish a commitment to punish deviant players. As this battle for dictatorship preceeds our basic game, and has no influence on its solution given the remaining resources and the identity of the victor, we need not model it here. It remains true, however, that the various solutions to our basic game -- corresponding to the various possible dictators -- do generally affect the identity of the victor in the dictatorship battle. For example, more benevolent players meet with less resistence than their less benevolent competitors and are therefore more likely to wind up as dictator. Also, to reduce the resources devoted to subsequent battles in a life-cycle environment, a dictator is likely to train and appoint the next generation's dictator.

C. A Possible Application to Explaining Broad Features of Observed Institutions

We can use our special model with its optimality feature to offer an explanation, albeit speculative, of the broad features of observed political-economic institutions without imposing a standard, paradoxical, who-guards-the-guards, enforcement mechanism and without assuming some sort of inexplicit cooperation among individuals (i.e., "social contracts.")[2]

To see this let us view the set of individuals $(k_1, . . . ,k_r)$ as a hierarchy representing the individuals in a military chain of command subject to a dictator, individual 1. The military hierarchy rationally establishes reaction functions which ensure the dictator's benchmark set of actions x^1 are carried out. This appears to work well for families and small, tribal societies, which are decidedly hierarchal, stable, command societies (M. Nash). Although many tribal societies admit some form of private property and exchange in final, consumer goods, this is apparently because of the lack of information on the part of the leader concerning others' preferences combined with the fact that such exchanges do not harm the leader. Free exchange in inputs is a different story. Here the leader's income depends substantially on how inputs are used and here the tribal leader maintains substantial direct control (M. Nash).

For larger societies, it becomes implausible to assume that the first strategy selector knows what is to him the relevant set of feasible individual actions or the relevant particular actions that are actually undertaken. That is, it is implausible to assume that he knows the capacities and actual performances of his subjects. To get around these information problems and still achieve about the same, efficient allocation, the first strategy selector could appoint local leaders who could fairly easily discover the potentials and monitor the actions of the peasants in his territory. When the first strategy selector cannot, in our sense, punish the local leaders, and the latter thereby become about as wealthy as the former, the local leaders can afford the risk of ownership and -- because of the obvious incentive value of their being residual claimants given that first strategy cannot easily observe their behavior -- they would rationally become landlords over their territories, paying mainly lump sum rents to the overlords. This is, in essence, feudalism. Since a local lord, or vassal, would not differ much with the overlord regarding the welfare of the lord's subjects, little conflict would arise on this issue. Therefore -- assuming no technical interdependence between the regions -- the vassals would make about the same, efficient, decisions as the super-informed overlord appearing in our formal model.

Once the overlord acquires punishment power over the local leaders, as occurred for example over the past few centuries as nation states have come to replace the old feudal states (Batchelder-Freudenberger), the wealth of the local leaders is scaled way down as our dictatorial solution takes over. This simple distributional change means that the now relatively middle-class local leaders can no longer afford either the risk of territorial ownership or the same kind of benevolence toward the masses that the over-lord possesses. As a result, local leaders become special, fixed wage, individuals, often granted the power to govern only after the local populations have also approved of them by popular vote. This is essentially a modern nation state and also describes most of the "great empires" of the past, those with strong central military control. At the same time, to provide additional protection of the masses from exploitation by their new local leaders, either private property would be extended to inputs for the masses (see Thompson-Faith, 1980) or extremely rigid, centrally directed, input controls would be imposed.

The industrial revolution introduced technologies of easy duplication rather than revolutionizing the quality of the goods which a dictator could receive. Hence, the new dictatorial solution gave many more goods to the subjects, assuming some benevolence on the part of the dictator. Since observed voting mechanisms have provided alterations in the private property system which are far superior for the adult voters to those provided by a benevolent dictator armed with the best economic theories (e.g., Thompson, 1974, 1979), their main drawback being that they also induce a serious exploitation of children when the typical voter is very poor (Thompson-Ruhter) the much greater wealths of the subjects of certain countries induced by the industrial revolution made it attractive for efficient dictators in these countries to adopt voting systems to improve upon the basic private property system.

The apparent control that voters have over the benefits of their dictators (military leaders) in democratic countries is, we submit, an illusion. Military leaders, as a group, have both tenure and an absence of significant institutional constraints on the goods and services they can command. The frequency of military takeovers of democratic governments that generate unsatisfactory results from the standpoint of the military in medium-poor countries is evidence for the dominance of the military. It is also further evidence, given the additional fact that wealthier countries typically have non-military authorities while poorer countries typically have direct military rule, for the above argument on the rationale for popular democracies. Additional evidence in support of this illusion, together with an argument explaining how the illusion is in the joint interest of the members of the society, appears in Thompson (1979, Section 1E).

While most modern analyses of democracy recognize its underlying internal efficiency tendencies in that an allocation cannot be an equilibrium under democracy if there is an alternative allocation in which all voters would be better-off, the analyses are also critical of democracy in that it is -- within the standard model -- unable to achieve a determinate solution. Cyclical majorities, or "voters' paradoxes," arise. They mean either a never-ending series of generally undesirable redistributions and a corresponding drain on societies resources or such severe constraints on the agenda that it is likely that many efficiency-enhancing bills will never be voted upon. But the arguments for voters' "paradoxes" rest on a Nash interaction. The "paradoxes" do not arise under truly perfect information! (See Section V.) In fact, very few legislative ballots are secret. It is relatively easy to observe votes and communicate voting reaction functions in legislatures. So the "paradoxes" appear to exist only within highly inappropriate theoretical models.

We have used the strong efficiency and distributional implications of our model to help predict the occurrence of: (a) feudalism or modern, nation statehood and (b) democracy or authoritarian government. We can also use these implications to predict whether a nation will adopt socialism or capitalism. We have been assuming that the various territories of a country are technically independent. Communication between leaders of the various areas is of no importance under this condition. But suppose that each area has a definite comparative advantage at producing a particular durable input and that these inputs are complementary. If strategic interaction between the leaders of these areas is sufficiently costly that the leaders will not initially (say within an effective legislature) strategically communicate -- but not so costly that it would fail to emerge even after certain investments were made -- then none of these areas will produce their particular complement and the decentralized nation would be underdeveloped (Thompson, 1980). In this case, with democratic legislatures ineffective, authoritarian control of investment is required for the nation to reach a developed state. This may explain why spread-out, resource-rich nations, like China and Russia, nations with notoriously uncohesive regional leaders (Eisenstadt), have employed centralized control of investment in emerging from feudalism to modern nation statehood.

D. Rationalization for Hierarchies in Nature

The observed universality of hierarchal social organization among social primates (Farb), while supportive of our basic assumption, is not directly explainable by the social efficiency of such organization. While would-be dictators generally favor this form, would-be subjects may well be better off without the physical ability to receive or understand the reaction functions of others. Our explanation for the predominance of hierarchal organization is that since most primate evolution has taken place in isolated families or small, family-like clans, wherein the members live or die together on the basis of the joint efficiencies of their separate groups, biological evolution has selected against families whose individuals could not receive or communicate reaction functions.

V. CONTRAST TO OTHER THEORIES OF CONFLICT RESOLUTION

A. Cooperative Game Theory

Cooperative game theory is founded on the assumption that any subset of n can form a "blocking coalition," a group of players which can, presumably by a given set of actions, achieve a given payoff for themselves and thereby prevent certain outcomes. The excluded outcomes are the "non-solution" outcomes to the game. The assumption guarantees each player a payoff at least equal to the minimum of what he can achieve in a one-man coalition. On the additional assumption that any Pareto optimum can be achieved by a coalition of all n players, the theory guarantees that any solution must be Pareto optimal. For any Pareto nonoptimal solution would be blocked by an n-person coalition. The theory is then devoted to the search for a solution out of the resulting set of "imputations," i.e., Pareto optimal points which give each player at least the minimum of what he would receive in a one-man coalition. The standard solution set indicated above, the core, is the set of unblocked outcomes. A chronic problem with this theory is that its solution sets are often empty. Other solution sets, such as VNM's "stable set" and the "bargaining set" are less frequently empty but have the chronic problem of admitting a superabundance of outcomes in their solution sets (see, for example, Owen).

We object to cooperative game theory because of its inexplicit communication process and related absence of committed strategies. These weaknesses result in insufficient constraints on the set of blocking coalitions. This point requires some elaboration.

Blocking coalitions exist in a general form as a by-product of interaction under truly perfect information. For any subset of reaction functions effectively blocks all outcomes which do not simultaneously satisfy these functions; and the players in the subset may be thought of as a blocking coalition. However, in our model, the players may be worse-off under their blocking behavior but still engage in it because they recognize the effect of their strategies on the strategies of others. The commitment of the players to these strategies simultaneously prevents them from forming blocking coalitions with subsequent strategy selectors merely because they would be better off in these coalitions under the narrowly rational, uncommitted, reactions characteristic of standard game theory.

Consider, for example, a "majority game" three person, zero sum, game in which, say, a dime and a nickel are to be shared by the three players. If players 1 and 2 each select certain actions implying that they "get together," 2 gets a dime and 1 gets a nickel. If 1 and 3 each select certain actions, where the action is different for 1 than in the former case, then 1 gets a dime and 3 gets a nickel. If 2 and 3 each select new actions implying that they "get together," then 3 gets a dime and 2 gets a nickel. Cooperative game theory offers no meaningful solution to this game because, for any distribution of coins, there is a blocking coalition.[3] Under truly perfect information, where the order of strategy selection is, say, 1, 2, 3, player 1 will adopt the following strategy: "I will get together with 2 if he gets together with me; otherwise, I will perform my part of getting together with 3." Player 2 then selects: "I will perform my part of getting together with 1 regardless of the action of player 3. Player 3 gets nothing no matter what he does. It is easy to verify that there is no other solution. In sharp contrast, under cooperative game theory, 3 would offer to get together with 1, who -- being unable to commit himself to a fixed strategy -- would be unable to refuse the offer. And we would be off on the never-ending cycle of coalition formation characteristic of existing cooperative game theory.

B. Voting Theory

A specification of these acts of "getting together" enables us to see that voters' paradoxes cannot arise under truly perfect information. Instead of the above as one world with three abstract "meetings," think of the world as one that uses majority rule voting over three possible bills. Correspondingly, let the abstract, "getting together" of players 1 and 2 represent their both supporting the bill that shuts out the player 3, the "getting together" of players 1 and 3 represent 1 and 3's supporting the bill that shuts out 2, and the "getting together" of 2 and 3 represent their supporting the bill that shuts out 1. It is easy to see that whatever bill becomes law, one of the other bills will defeat it <u>if</u> voting follows only the narrow self-interest of the voters. This is the "voters' paradox." But the paradox does not arise under the communication of commitments to narrowly irrational but, of course, broadly rational voting strategies. Following our example, player 1 rationally adopts; I will vote to shut out player 3 if player 2 does, otherwise, I'll vote to shut out player 2. Player 2 then votes to shut out player 3 and player 3 is shut out. While player 3 tempts player 1 with a payoff of 10 if he will vote instead to shut out player 2, player 1 is committed not to so vote. If he were not previously committed to some strategy prior to 3's strategy choice, player 1 would himself be shut out by players 2 and 3.

C. Supergame Theory

Supergame theory, like cooperative game theory, is a result of the inability of standard non-cooperative game theory to allow for sufficient communication to generally achieve Pareto optimal outcomes. The advantage of supergames relative to cooperative games is the absence of imposed, collective rationality conditions. Supergames are the result of the temporal replication of ordinary two-person games in which strategy sets are expanded to include actions contingent on actions in prior games. The standard supergame strategy, due to Luce and Raiffa, is to play a Pareto optimal action if the other player has played his corresponding Pareto action in the preceeding period; and otherwise play a Nash action for the remainder of the supergame. One type of supergame is the Luce-Raiffa, finite-horizon supergame; the other is the Aumann-Friedman, infinite-horizon supergame.

In the finite-horizon supergame, a problem arises in that it generally pays each player to play a Nash action in the last period. This shortens the supergame by one period; but the same argument applies to the shortened supergame and continues applying up through the first period. Thus, the only solution is the Nash solution. Luce and Raiffa argue that the Nash strategy is dominated by their supergame strategy, where they leave to be determined the point at which it pays a player to switch to a Nash action. The problem is that a determination of this point, playing the supergame game as a Nash game reveals that it always pays to switch just before the other player switches. So the players "switch" to a Nash solution in the first period. It never pays to play their Luce-Raiffa strategy in a finite supergame (Rapoport, Selten). An alteration of the game, in which one player can communicate a fixed, committed strategy to the other before the latter chooses his strategy, will change this result; but it also will make the supergame an unnecessary construct, as we have seen.

An infinite supergame does not have this problem, for there is never a last period in which it pays to switch to a standard Nash solution. However, the standard Nash solution is always a possible solution to the supergame. If one player plays an ordinary Nash strategy, so does the other. So, while the Luce-Raiffa strategies -- extended to infinite replication -- are a possible solution;

so are the ordinary Nash strategies. Furthermore, as Friedman shows, sufficiently high discount rates will assure a Nash solution. Still further is the weakness that the theory does not determine which of the generally many outcomes which are Pareto superior to the Nash Solution will be actually chosen.

But the most obvious weakness of supergame theory is the requirement of an infinite horizon. While it is plausible to assume that individuals behave as if the world may last forever, it is implausible to assume that individuals behave as if they, as continually acting individuals capable of continually exhibiting strategies, may last forever.

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FOOTNOTES

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[1] This is not to say that conventional cooperative game theory cannot be reformulated to produce a game-theoretic model similar to our own generalization of Schelling's bargaining model. Indeed, such a reformulation has been recently achieved by Rosenthal.

[2] The weakness of conventional cooperative game theory in describing cooperative behavior is discussed in Part V.

[3] While the core and VNM's stable set are empty, the bargaining set contains all possible allocations.