WHICH PRICE INDEX FOR ESCALATING DEBTS?

by

Robert A. Jones

University of California,
Los Angeles

UCLA
Department of Economics
Working Paper #160
August 1979

WHICH PRICE INDEX FOR ESCALATING DEBTS?

Robert A. Jones

University of California, Los Angeles

The search for meaningful price indexes has been motivated by their usefulness both for macroeconomic description and for escalating contractual money payments. Existing theory of "economic price indexes" evolved for the purpose of description. This paper asks what form of index is appropriate for escalating deferred money payments. The theory of cost of living indexes is shown to be irrelevant for this problem. In circumstances where the optimal escalating index is unambiguous, it is a modified Paasche index, whether or not an invariant cost of living index of different form exists.

Wesley Mitchell, in his 1915 discussion of the making of a price index, states:

> The first step, forming a clear idea of the ultimate use of the results, is most important, since it affords the clue to guide the compiler through the labyrinth of subsequent choices. It is, however, the step most frequently omitted (1915, p.25).

One use for price indexes that motivated early researchers was their use for escalating deferred payments.[1] Linking money payments to a price index can reduce inequities and risks caused by unanticipated price changes. The theory of how to construct price indexes, particularly since Frisch's (1936) survey, has been motivated primarily by their use for macroeconomic description. An "economic price index" (Samuelson and Swamy, 1974; Diewert, 1979) measures changes in the cost of living (cost of production) by comparing the costs of achieving a given standard of living (level of output) at two different times. Used as price deflators, these indexes convert nominal into "real" quantities in a meaningful way. But how should one construct a price index for the purpose of escalating debts?

The purpose of this essay is to show that the index appropriate for escalating money payments is not a cost of living index. Even when the same cost of living index applies to everyone, individuals are better off with payments tied to a different function of prices.[2]

The reason for this result is not hard to find. Measuring changes in the cost of living does not, by itself, have any real effect on the economy; linking money payments to inflation does. The price index chosen for escalating deferred payments allocates price risks between payers and recipients. In any allocation problem, the question of what is desirable cannot be answered without reference to what is feasible.[3] But cost of living indexes reflect only preferences. They are invariant with respect to endowments, production possibilities and sources of relative price changes (without which there is no index number problem). Indexation arrangements must embody more than the cost of living for price risks to be allocated efficiently.

Section I of the paper examines optimal price-contingent payment arrangements between two individuals. Optimal arrangements are shown to depend on the consumption preferences, endowments and risk attitudes of the particular individuals involved. Section II points out that if all individuals engage in prior trade in competitive futures markets, then subsequent efficient deferred payment contracts can be expressed as multiples of a standardized contract, regardless of the contract size or pair of individuals involved -- a common "standard of deferred payment" can be used.[4] Section III represents this standard in terms of a price index and coefficient indicating the degree of escalation. The paper concludes by suggesting why the same good is often used for both medium of exchange and standard of deferred payment.

I. OPTIMAL PAYMENT ARRANGEMENTS

Consider two individuals arranging a contract that calls for one to make a money payment to the other, the level of which depends on future prices. Suppose there are n goods. Money is good 1 and is used as numeraire when representing prices. Future spot market prices are denoted by the vector $p = (1, p_2, \ldots, p_n)'$.[5] A payment arrangement can be described by a function $M^{hk}(p)$ specifying the amount to be paid by individual h to individual k when prices p prevail. A payment arrangement is optimal if there is no alternative arrangement that both individuals prefer.[6] The reason for the promised payment will not be explicitly considered in what follows. It could, for example, be in return for a quantity of current money, as in a loan contract, or in return for a quantity of labor, as in a wage contract. This section determines the form of optimal payment arrangements when future prices are unknown at the time the contract is made.[7]

Individual preferences over alternative payment arrangements are generated as follows. Assume that: (i) Each individual has a utility function $U^h(c^h;p)$ defined on his future consumption vector $c^h = (c_1^h, \ldots, c_n^h)'$. Prices enter the utility function directly because c_1^h represents nominal money balances held, while benefits from holding money are generally associated with real balances, somehow defined. Once future prices p are revealed and an individual's money wealth M^h is determined, he acquires a utility maximizing bundle of goods $c^h(p,M^h)$ and realizes a level of indirect utility $V^h(p,M^h) \equiv U^h(c^h(p,M^h);p) \equiv \mathrm{Max}_c\{U^h(c;p): p'c = M^h\}$. (ii) Each individual has a known endowment of future goods $w^h = (w_1^h, \ldots, w_n^h)'$. (iii) Individual beliefs about future prices can be represented by a joint probability distribution $\mu(p)$ over the possible prices. These beliefs are common to all individuals. For individual k participating in the payment arrangement, his prior expected future utility is $\int V^k(p,p'w^k + M^{hk}(p))d\mu(p)$. For individual h it is $\int V^h(p,p'w^h - M^{hk}(p))d\mu(p)$. We assume individuals rank alternative payment arrangements according to these levels of prior expected utility.

For the remainder of the paper, utility functions are confined to a class that allows explicit solutions to be obtained. Specifically, let utility functions have the form (up to a positive linear transformation)

$$(1) \qquad U(c;p) = \ln f(c - a;p)$$

where f is an arbitrary (positive) first-degree homogeneous function in c - a for each p. The vector a is a non-homotheticity parameter. Since utility becomes infinitely negative as c approaches a, we will refer to a as <u>necessities</u>. The function f and parameters a can differ across individuals. The consumption demand functions, obtained by maximizing U subject to a budget constraint p'c = M, have the form

$$(2) \qquad c(p,M) = a + (M - p'a)z(p).$$

The vector z depends only on p. The linearity of consumption in wealth, M, results from the first-degree homogeneity of f. This class of utility functions is reasonably flexible.[8] More importantly, it contains as a special case those consumption preferences (namely homothetic) for which invariant cost of living indexes exist, and hence suffices for our immediate purpose.

Some properties of the indirect utility function are used later. The indirect utility function, obtained by substituting (2) into (1), is

$$(3) \qquad V(p,M) = \ln f(c(p,M) - a;p) = \ln (M - p'a)f(z(p);p)$$

$$= \ln (M - p'a) + \ln f(z(p);p).$$

The second equality follows from the first-degree homogeneity of f for given p. The partial derivative of V with respect to nominal wealth and the Arrow-Pratt index of relative risk aversion are, respectively,

$$(4) \qquad V_M = 1/(M - p'a) \quad \text{and} \quad RRA \equiv -MV_{MM}/V_M = 1/(1 - (p'a/M)).$$

With positive necessities, $a > 0$, an individual becomes infinitely averse to income risk as wealth approaches the cost, $p'a$, of those necessities, but the risk aversion index declines toward 1 as M increases. For $a < 0$ the individual is neutral toward income risk at zero income, but the risk aversion index rises toward 1 as M increases. The individual's marginal utility of nominal wealth depends only on his nominal discretionary wealth $(M - p'a)$ left after purchasing necessities, even though his total utility depends also on prices and those aspects of tastes embodied in f.

A payment arrangement is optimal only if it is the one most preferred by individual h from among those to which k is indifferent. $M^{hk}(p)$ must maximize h's expected utility for a given expected utility of k. The Langrangian expression for this programming problem is

$$(5) \qquad L(M^{hk},\lambda) = \int V^h(p,p'w^h - M^{hk}(p))d\mu(p) + \lambda[\int V^k(p,p'w^k + M^{hk}(p))d\mu(p) - \bar{V}^k].$$

The first order condition (Euler equation) that must be satisfied for an interior maximum with respect to the function M^{hk} is

$$(6) \qquad V_M^h(p,p'w^h - M^{hk}(p))\, d\mu(p) = \lambda V_M^k(p,p'w^k + M^{hk}(p))\, d\mu(p),$$

where λ is a Lagrange multiplier independent of p (cf. Gelfand and Fomin, 1963, p. 42). Substituting for V_M from (4), rearranging terms, and letting β denote $\lambda/(1 + \lambda)$, gives us

$$(7) \qquad M^{hk}(p) = p'[(a^k - w^k) + \beta(w^h + w^k - a^h - a^k)] .$$

An optimal payment arrangement entails h contracting to pay k the difference between the market value of k's necessities and endowment (which may be negative), plus a fixed share of the difference between the value of their combined endowments and necessities. In effect, they share their discretionary wealth in fixed proportions, whatever prices occur. The "larger" is the payment (higher the required expected utility for k), the higher is β and larger is k's share.

The money payment in (7) can be viewed as tied to a price index of sorts. M^{hk} is the price of the fixed basket of goods that the bracketed expression in (7) represents. This is not a price index in the usual sense since the price of money (1 by definition) is accorded some weight. Moreover, the index depends not only on the specific characteristics and endowments of the contracting individuals (a^h, a^k, w^h, w^k), but also on the "size" (β) of the payment. Many aspects of consumption preferences are irrelevant to the form of the index -- namely those embodied in f^h and f^k. But the possibility of generally optimal price indexes for deferred payments appears even more remote than the existence of generally meaningful cost of living indexes.

II. A COMMON STANDARD OF DEFERRED PAYMENT

In addition to transferring future purchasing power, an optimal payment arrangement permits each individual to hedge against changes in the price of his necessities and to diversify his endowment against relative price risks. Since endowments and tastes vary, optimal contracts must be tailored to the specific individuals involved. But what if these basic hedging and diversification functions were performed by a conventional futures market, prior to negotiating deferred payment contracts? Would that impose more uniformity on the optimal payment arrangements? This section shows that it would.

A conventional futures contract is a commitment to purchase (or sell) in the future a fixed quantity of some good at a fixed money price. As such, it is an exchange of non-contingent claims to future goods (money claims for commodity claims); no current money or good is involved. Suppose that there is only one future period and that competitive futures markets exist for all goods. Let $p^0 = (1,p_2^0,\dots,p_n^0)'$ denote the equilibrium prices of claims to future goods in terms of claims to future money. An individual endowed with claims to future goods w can exchange them for any other vector of claims x satisfying the budget constraint $x'p^0 = w'p^0$. In the future these claims would be worth $p'x$ and permit him to achieve a utility level $V(p,p'x)$. Suppose, now, that individuals trade in the futures market with the objective of maximizing $E[V(p,p'x)]$, where the expectation is with respect to their beliefs about future prices. This requires that they ignore any yet to be negotiated deferred payment arrangements. From the Langrangian expression

$$(8) \qquad L(x,\lambda) = E[V(p,p'x)] + \lambda(w - x)'p^0$$

comes the first order conditions for an interior maximum:

$$(9) \qquad L_x = E[V_M p] - \lambda p^0 = 0$$

$$L_\lambda = (w - x)'p^0 = 0 \, .$$

Let the different individuals by indexed by h = 1,...,H and substitute $V_M^h = 1/(x^h - a^h)'p$ from (4) into (9). For the futures markets to be in equilibrium, the following conditions must hold simultaneously:

$$(10) \qquad E\left[\frac{p}{(x^h - a^h)'p}\right] = \lambda^h p^0 \qquad \text{for } h = 1,\ldots,H \text{ (first order conditions)}$$

$$(w^h - x^h)'p^0 = 0 \qquad \text{for } h = 1,\ldots,H \text{ (budget constraints)}$$

$$\Sigma_1^H x^h = \Sigma_1^H w^h \qquad \text{(market clearing condition).}$$

Letting $A \equiv \Sigma a^h$ denote aggregate necessities and $W \equiv \Sigma w^h$ aggregate endowments, a solution to this system of Hn + H + n equations is

$$(11) \qquad x^h = a^h + \left[\frac{(w^h - a^h)'p^0}{(W - A)'p^0}\right](W - A) \qquad \text{for } h = 1,\ldots,H$$

$$\lambda^h = 1/(w^h - a^h)'p^0 \qquad \text{for } h = 1,\ldots,H$$

$$\frac{p^0}{(W - A)'p^0} = E\left[\frac{p}{(W - A)'p}\right] \, .$$

That this is in fact a solution may be verified by substituting (11) into (10).

In equilibrium, each individual holds claims to his future necessities a^h, plus whatever proportionate share he can afford at prices p^0 of the remaining aggregate non-necessities (W - A). In this way the basic diversification and hedging objectives are fulfilled.[10]

Two aspects of this equilibrium warrant discussion. We have not mentioned the source of uncertainty about future prices. It cannot stem from uncertainty about the endowments of futures market participants, since w^h is assumed to be known. It could result from uncertain demand from outside the economy (i.e., from foreign trading partners), or from uncertain demand from within the economy by agents not participating in the futures market (e.g., from a government that will print money to finance as yet undetermined purchases). Or it could result simply from each individual's lack of knowledge about other individuals' future consumption preferences. If this is the case, we must ask whether a rational agent could infer enough from futures prices to eliminate this "strategic uncertainty" (Radner, 1968). But (11) shows that p^0 depends only on (W - A) and the common prior beliefs about p. Neither the f^h components of future preferences nor the distribution of wealth affect equilibrium futures market prices. Yet both of these affect equilibrium spot market prices. Since they cannot be inferred from p^0, uncertainty about p must remain.

The second and related point is that, although each individual is risk averse, his claim holding is independent of all aspects of his future consumption preferences except a^h. This is somewhat surprising. One might have expected such individuals to acquire claims to goods they wish to consume. Yet, with $a^h = 0$, the person who consumes only bread holds the same portfolio as the person who consumes only wine. Futures prices are such that the higher expected return from holding the "market portfolio" just compensates for its added riskiness in terms of intended consumption goods.[11]

What implications does the futures market equilibrium carry for subsequently negotiated payment arrangements? Each individual departs the futures market with future claims $x^h = a^h + \gamma^h(W - A)$. Substituting x^h and x^k for w^h and w^k respectively in (7) implies that optimal payment arrangements have the form

$$(12) \qquad M^{hk}(p) = (\beta\gamma^h + \beta\gamma^k - \gamma^k)p'(W - A) \equiv \alpha^{hk}m(p) .$$

The function $m(p) \equiv p'(W - A)$ is independent of any characteristics of h and k. Since variations in the size (β) of the contract affect only α^{hk}, we can term m(p) a <u>standard of deferred payment</u> -- a standardized unit in terms of which future payments can be efficiently specified. Moreover, since m(p) does not depend on any characteristics of specific individuals, this standard is appropriate for all contractual payments, regardless of size or the consumption preferences and wealth of the individuals involved. Optimal payments still vary with prices, of course, but all debts can vary with prices in the same way. Unfortunately, such standardization is possible only if the basic hedging and diversification of endowment risk is accomplished first through futures markets or other institutions.

III. THE PRICE INDEX AND DEGREE OF ESCALATION

Most discussions of index-lined debt envisage the following sort of agreement: a fixed money payment is specified, which is to be increased in proportion to inflation between the time of contract and the time of payment. Inflation is to be measured by the change in some general price index; the percentage change in the money payment per one per cent change in the price level is the "coefficient of escalation" (Collier, 1969, p.51). Let us extract the inflation measure and coefficient of escalation implicit in the standard of deferred payment m(p).

Let current goods prices be denoted by $\bar{p} = (1,\bar{p}_2,\ldots,\bar{p}_n)'$. The payment arrangement (12) can be written $M^{hk}(p) = \bar{M}^{hk}[m(p)/m(\bar{p})]$, where $\bar{M}^{hk} \equiv \alpha^{hk}m(\bar{p})$ is the agreed upon payment if no prices change, and $m(p)/m(\bar{p})$ is the escalating factor if they do. We seek an expression for this factor of the form

$$(13)\quad m(p)/m(\bar{p}) = 1 + s[I(p,\bar{p}) - 1] .$$

$I(p,\bar{p})$ is a price index of the non-money goods 2,...,n with $\bar{p}$ as base period prices,[12] the bracketed expression is measured inflation, and s is the coefficient of escalation. From the fact that $m(p) \equiv \Sigma_1^n(W_i - A_i)p_i$, one can verify that (13) holds with

$$(14)\quad I(p,\bar{p}) = \Sigma_2^n(W_i - A_i)p_i/\Sigma_2^n(W_i - A_i)\bar{p}_i$$

$$s = v/(1 + v) \qquad \text{where } v \equiv \Sigma_2^n (W_i - A_i)\bar{p}_i/(W_1 - A_1).$$

If all individuals have homothetic preferences (all a^h and hence A equal zero), then the price index is the Paasche index, $I(p,\bar{p}) = \Sigma_2^n W_i p_i / \Sigma_2^n W_i \bar{p}_i$, since W represents quantities of goods exchanged in the future spot markets. Interestingly, Irving Fisher once recommended this index for escalating debts:

> To cut these Gordian knots, perhaps the best and most practical scheme is that which has been used in the explanation of the P in our equation of exchange, an index number in which every article and service is weighted according to the value of it exchanged at base year prices in the year whose level of prices it is desired to find .... What is repaid in contracts so measured is the same general purchasing power. (1922, p.217)[13]

If A is not zero, however, the index weights goods' prices according to the non-necessary part of aggregate future endowments. $I(p,\bar{p})$ is similar to the "marginal price index" suggested by Afriat (1974) for escalating nominal incomes to preserve a real income distribution. Compared with the Paasche index, price changes of relatively necessary goods (higher than average A_i/W_i) are weighted <u>less</u> heavily and price changes of the remaining "luxury" goods <u>more</u> heavily.

The other aspect of the contract is the degree of escalation. As long as aggregate endowments exceed aggregate necessities, $W >> A$, v is positive and the coefficient of escalation is less than one -- debts should be less than fully escalated.[14] Efficient risk allocation requires that everyone share the real risks associated with money endowments, not just the holders and recipients of money.

A special case can provide some notion of how much escalation is appropriate. Suppose that any new money, when created, is not distributed via transfer payments. Since money held today, through storage, provides its owner with an endowment of money tomorrow, net private claims to future money, W_1, must equal the current money stock. Further suppose that $A = 0$, that money is the only durable good, and that aggregate endowments of non-money goods $W_2,\ldots,W_n$ are the same in every period. Then v equals the current period's income velocity of money, $\Sigma_2^n W_i\bar{p}_i/W_1$. The higher is velocity, the closer is the coefficient of escalation to 1. When A is not zero, then the more is money a luxury relative to other goods (lower is A_i/W_i), the smaller is the coefficient of escalation.

The purpose of this paper is to show that cost of living indexes are not appropriate for escalating debts. This can be shown by an example. Suppose all individuals have the same Cobb-Douglas utility function $U^h = \ln c_2^{\alpha}c_3^{1-\alpha}$. The well-known invariant cost of living index for these preferences is $I(p,\bar{p}) = (p_2/\bar{p}_2)^{\alpha}(p_3/\bar{p}_3)^{1-\alpha}$. But since these preferences are homothetic, the Paasche index must be used in (13) to efficiently allocate price risks. For any collection of payment arrangements linked to the cost of living index, there is another linked to the Paasche index that all individuals prefer.[15] When an invariant cost of living index does exist, the efficient contract raises money payments by a smaller proportion than the rise in the cost of living, both because the coefficient of escalation is less than 1 and because the Paasche index understates the rise in cost of living. When the cost of living falls, these two effects tend to offset each other. The Paasche index overstates the fall in cost of living, but money payments are reduced by only some fraction of that amount.

IV. RELATED ISSUES

One heartening aspect of the analysis is that is made sense to talk of an optimal escalating price index in situations where there are no invariant cost of living indexes -- namely when individual tastes differ and are not homothetic (as long as they belong to the assumed class). With standardized contracts, a central authority could collect all price data and announce the escalation factor $m(p)/m(\bar{p})$ applicable to all contractual money payments. But how could an authority determine the value of W - A, on which m(p) depends?

Suppose the authority could estimate the distribution of future prices (e.g., on the basis of past price experience) and that its estimate corresponds with individual beliefs $\mu(p)$. Upon observing futures contract prices, p^0, the third equation in (11) could be solved for W - A up to a scale factor. That, together with current prices, $\bar{p}$, and future spot prices, p^0, suffices to determine $m(p)/m(\bar{p})$. In this idealized context, the appropriate escalation factor can in principle be inferred from observed prices.

Doe the analysis shed light on why indexed debts are seldom observed? Even in "non-inflationary" times there is considerable uncertainty about the future prices of specific goods. Private contracts should arrange for these risks to be shared efficiently. Let us add a little more structure. Suppose that, whatever uncertainty attends the demand for other goods, the aggregate demand function for money is stable and of the form $M^d(\Sigma_2^n W_i p_i)$. That is, prices affect the demand for cash balances only to the extent that they affect the aggregate money value of non-money goods exchanged. Any rationale for such a relation would lie with the medium of exchange function that money performs. Further suppose that tastes are homothetic and that the future money stock, M, is known with certainty, though it need not be identical to the current stock. The knowledge that prices will adjust until M equals M^d restricts rational beliefs about future prices. Although p may be uncertain, $\Sigma_2^n W_i p_i$ is not. With A = 0, this implies that m(p) takes on the same value for all future p consistent with equilibrium. Money alone is the optimal standard of deferred payment. This does not mean that individuals are indifferent to the standard used; on the contrary, money is preferred over any other good or basket of goods. When the inflation rate to be measured by the Paasche index is known, money represents fixed shares of total future endowments; its use as standard of payment efficiently allocates relative price risks caused by demand uncertainties. Admittedly, this is a very special case. However, it suggests that the observed use of the same good as both medium of exchange and standard of deferred payment might be more than mere coincidence or a reflection of costs of negotiating other arrangements.

The idealizing assumptions used in the analysis limit its application. That utility functions belong to a particular class might be tolerated since they approximate a much broader class. Uncertainty about future endowments, either from natural sources or from adapting future production to prices, is not considered. Any effect that indexing arrangements themselves have on the distribution of future prices is similarly ignored. However the assumption that futures markets are in equilibrium prior to negotiating payment arrangements is essential if there is to be a single best standard of payment. Obviously, complete futures markets do not exist in practice; and if they did, one could argue that indexation is redundant. Since the efficient standard of payment is linear in prices, debts could be fixed in money terms and efficient risk-sharing restored by a second trip to the futures markets. In the absence of these markets, the question is whether existing institutional arrangements -- asset markets, bankruptcy laws, labor contracts, explicit forward purchases, carrying of inventories, implicit agreements between suppliers and regular customers, insurance contracts, together with the knowledge that certain prices move together -- provide an efficient risk allocation from which additional contracts can be negotiated. If so, and if the additional contracts are appropriately indexed, then the cost of renegotiating prior arrangements is saved.

At the least, Mitchell's advice must be taken to heart. There are as many price indexes based on economic theory in a given situation as there are distinct uses to which they might be put.[16]

FOOTNOTES

* An earlier version of this paper was presented at the Miami University Conference on Inflation, May 15-16, 1975. I am indebted to Robert Clower, Herschel Grossman, Keizo Nagatani, Joseph Ostroy and John Riley for helpful comments and suggestions.

1. The use of "tabular standards" for deferred payments was suggested as early as 1807 by John Wheatley, investigated by the British Association for the Advancement of Science between 1887 and 1889, and supported by a long line of distinguished economists in both this century and the last. Robert Giffen (1888, p.181) ranked the use of indexes for deferred payments ahead descriptive uses when listing the needs measures of price change could fulfill. Irving Fisher stated that "Perhaps the most important purpose of index numbers is to serve as a basis for loan contracts" (1922, p.208).

2. There is some evidence that economists presume the two price index problems to be closely related, if not the same. Samuelson and Swamy cite the importance of price indexes for "sliding scale wage and other contracts" (1974, p.566) in addition to macroeconomic description when introducing their survey of cost of living indexes; Fischer, when considering relevant mutual funds of indexed bonds, suggests one indexed on 'the appropriate "ideal" [cost of living] price level' (1975, p.522); and Niehans (1978, chap. 7) devotes space to discussing the bias in various indexes as measures of the cost of living in a chapter on indexed debts.

The issue here is not simply the degree of escalation of money payments that is desirable; the issue is what measure of inflation should be used as the basis for indexing arrangements. One price index is a better <u>basis</u> for deferred payments than another if, for any set of contracts with money payments specified as <u>functions</u> of the latter, there is another set of contracts with payments specified as functions of the former that all individuals prefer.

Throughout this paper, cost of living index refers to a theoretical index giving the ratio of minimum expenditures needed to obtain a given utility at two different sets of prices, not to any published consumer price index based on the price of a fixed basket of goods.

3. That linking payments to a price index could not generally eliminate all price risk for both the payer and recipient was convincingly demonstrated by Harry Brown (1909).

4. The term "standard of deferred payment" appears to have originated with Francis A. Walker, who vigorously objected to Jevon's use of the ambiguous term "standard of value" (1877, pp.10-13). I am indebted to Robert Clower for pointing this out.

5. All vectors are column vectors; p' denotes the transpose of p; p'x is the inner product of p and x. $G_x(x,y)$ denotes the partial derivative of function G with respect to x.

6. The term optimal payment arrangement and criterion of efficient risk allocation are those of Shavell (1976).

7. If future prices p are known in advance, then the form of the payment arrangement is indeterminate. The manner in which payment M^{hk} would have varied if p had been different is irrelevant to both individuals.

8. A function of form (1), through choice of f and a, can provide a second order differential approximation locally to any twice differentiable concave utility function. It does impose important global restrictions on attitudes toward risk and on consumption preferences (e.g., income expansion paths are straight lines through some fixed point a), of course. In approximating a utility function with a member of this class, n - 1 degrees of freedom in the vector a are utilized to match the directions of the income

expansion path, and the remaining degree of freedom is utilized to match the curvatures (RRA) of the utility functions along that path. The function f is chosen to match the shapes of the indifference contour through the point at which one is approximating, and thus matches the compensated demand curves.

9. Obviously, we are abstracting from real-world considerations such as margin requirements or collateral to ensure performance.

10. Our maintained assumption of identical beliefs precludes speculative behaviour of the type emphasized by Hirshleifer (1975).

11. There is no requirement that all claims are to goods which directly enter some utility function -- some may be interpreted as equities or durables. It is required, however, that all goods which do enter direct utility functions are included in the list (or that some combination of the existing "assets" has a price perfectly correlated with that of the omitted good).

12. That is, a function with the property that $I(p,\bar{p}) = 1$ when $p = \bar{p}$ and that is first-degree homogeneous in $p_2,\ldots,p_n$.

13. To see that it is a Paasche index referred to, note that $\Sigma p_i W_i / \bar{p}_i W_i = \Sigma (p_i/\bar{p}_i)(\bar{p}_i W_i / \Sigma \bar{p}_j W_j)$. The Paasche index is an average of $p_i/\bar{p}_i$ weighted in the manner Fisher describes.

14. A similar point is made in a context with one consumption good (or fixed relative prices) by Liviatan and Levhari (1975).

15. It is possible, or course, that the cost of living and efficient escalating price indexes coincide. The Paasche price index is an exact cost of living index for the Leontief utility function $U = \ln \text{Min}\{c_i/\alpha_i\}$.

16. ... which leads to the last question: What sort of price index is appropriate for deflating nominal balances when one talks of the demand for "real" money balances?

REFERENCES

Afriat, S. N., "The Marginal Price Index Method (I and II)," Research Paper No. 21, University of Ottawa, December 1974.

Brown, H. G., "A Problem in Deferred Payments and the Tabular Standard," Quarterly Journal of Economics, August 1909, 23, 714-18.

Collier, R. P., Purchasing Power Bonds and Other Escalated Contracts, Buffalo Book Co., Taipei, 1969.

Diewert, W. E., "The Economic Theory of Index Numbers: A Survey," Discussion Paper No. 79-09, University of British Columbia, March 1979.

Fischer, S., "The Demand for Index Bonds," Journal of Political Economy, June 1975, 83, 509-34.

Fisher, I., The Purchasing Power of Money, 2nd ed. (1922), A. M. Kelley, New York, 1963.

Frisch, R., "Annual Survey of General Economic Theory: The Problem of Index Numbers," Econometrica, January 1936, 4, 1-38.

Gelfand, I. M., and Fomin, S. V., Calculus of Variations, trans., R. A. Silverman, Prentice-Hall, Englewood Cliffs, 1963.

Giffen, R., "Second Report of the Committee appointed for the purpose of investigating the best method of ascertaining and measuring Variations in the Value of the Monetary Standard," Report of the British Association for the Advancement of Science, 1888, 181-88.

Hirshleifer, J., "Speculation and Equilibrium: Information, Risk and Markets," Quarterly Journal of Economics, November 1975, 89, 519-42.

Liviatan, N., and Levhari, D., "Risk and the Theory of Indexed Bonds," Research Report No. 67, Hebrew University of Jerusalem, March 1975.

Mitchell, W., "The Making and Use of Index Numbers," Bulletin of the Bureau of Labor Statistics, July 1915, 173, 5-114.

Niehans, J., The Theory of Money, Johns Hopkins University Press, Baltimore and London, 1978.

Radner, R., "Competitive Equilibrium Under Uncertainty," Econometrica, January 1968, 36, 31-58.

Samuelson, P. A., and Swamy, S., "Invariant Economic Index Numbers and Canonical Duality: Survey and Synthesis," American Economic Review, September 1974, 64, 566-93.

Shavell, S., "Sharing Risks of Deferred Payment," Journal of Political Economy, February 1976, 84, 161-68.

Walker, F. A., Money, (1877), MacMillan, London, 1891.

Wheatley, J., An Essay on the Theory of Money and Principles of Commerce, Cadell and Davies, London, 1807.