SHORT-RUN FLUCTUATIONS IN FOREIGN EXCHANGE RATES: AN EXPLORATION OF THE DATA

By

Daniel Friedman
Department of Economics
UCLA

and

Stoddard Vandersteel
Data Resources, Inc.
San Francisco

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I. Introduction

This paper examines a very interesting set of time series data: six recent years of daily Foreign Exchange spot rate movements for nine major currencies. For those involved in international business or economics, the data are of immediate practical significance. Econometricians and statisticians may also find the data fascinating for methodological reasons. Our analysis indicates that many of the standard statistical procedures based upon assumptions of stationarity and normality are inapplicable to these data. On the other hand, the time series are very long, containing over 1600 observations for each currency. As a result, the data present an unusual opportunity for using non-standard techniques to uncover the underlying patterns.

As we examine the data, we have in mind several questions. To what extent do the currency movements exhibit trends? How can one assess the volatility (i.e., departures from trend) of the various currencies? What underlying economic or institutional processes are consistent with the data? What sorts of behavior by various participants in the FX markets are appropriate in light of the observed fluctuations? This paper does not provide definitive answers to any of these questions, but we believe that it sheds considerable light on the first two, and will provide useful information for anyone seeking answers to the second two.

We have attempted to organize the paper in such a way as to make it accessible to a diverse audience. The reader who has completed a good course in basic statistics should be able to at least get the gist of the main part of the paper. Those whose interest in FX rate fluctuations is mostly practical will probably wish to skim the text and spend most of

II. The Data

Our raw data consists of daily Foreign Exchange (FX) spot rates, quoted in terms of the U.S. dollar, for the currencies listed in Table I, covering the period 1 June 1973 to 14 September 1979; these spot rates are graphed in Figures A1-A9. We are primarily concerned with the day-to-day fluctuations in the spot rates, defined by $r_t = \log (S_{t+1}/S_t)$, the continuously compounded rate of change of the spot rate on the t^{th} trading day, S_t, to that on the $(t+1)^{th}$ trading day, S_{t+1}. For technical reasons discussed in the Appendix, we prefer to work with r_t rather than the simple rate of change $(S_{t+1}-S_t)/S_t$; in any case the two ways of defining daily changes are ordinarily virtually indistinguishable.

Table I lists summary statistics for the r_t's of the various currencies. Although the mean rates of change are all quite small, ranging from about 0.04% per trading day appreciation for the Swiss Franc to 0.02% depreciation for the Italian Lira, very substantial daily changes do sometimes occur, e.g., a 6.7% appreciation of the French Franc on 3/4/73 and a 6.7% depreciation of the Deutsche Mark on 11/1/78. As a first indication of the magnitude of typical fluctuations, we list standard deviations. By this criterion most of our currencies appear to have about the same volatility, roughly 1/2% per trading day, the main exception being the Canadian dollar which seems more stable at 0.21%. In later sections, we will introduce more sophisticated measures of currency volatility.

The coefficients of skewness and kurtosis listed in the last two columns of the table pertain to the shape of the distribution of the FX rate fluctuations. A preliminary, more qualitative look at these distributions is provided in Figures B1-B9. These histograms indicate the probability densities of the observed fluctuations. The curve superimposed on each histogram indicates the Normal density of the same mean and standard deviation. All the currencies appear to give rise to essentially unimodal, approximately symmetric (i.e., "bell shaped") distributions, but with some anomalies to which we now turn.

The coefficient of skewness (defined as $SK = E(r_t - \mu)^3/\sigma^3$, the third moment around the mean divided by the cube of the standard deviation) detects asymmetries; a glance at Table I indicates moderately negative SK for all currencies with the exception of the essentially unskewed French Franc. Such negative skewness suggests that the lower tails of the distribution are longer than the upper, i.e., the largest downward fluctuations outweigh the largest upward fluctuations, a suggestion confirmed by comparing columns 2 and 3 of the Table and also by direct inspection of the histograms. Skewness will be discussed again in Section V; for now we will just comment that much of the observed skewness can be attributed to the events of a single trading day (11/1/78 for most currencies), so it is not unreasonable for most purposes to regard the distributions as symmetric.

are only .0000026% and $<10^{-20}$ respectively, confirming that the tails are indeed more massive than Normal. See the Appendix for a more precise description of this test.

For detecting abnormally long tails, Fama and Roll (1971) recommend highly the Studentized Range statistic, SR = (largest observation - smallest observation)/s, where s = sample standard deviation. Casual interpolation of the SR tables in David, Hartley and Pearson (1954) reveals for a sample of our size that any value of SR in excess of 8.5 indicates longer-than-Normal tails at the .005 confidence level. From Table I, one can readily compute that SR ranges from 13 for the Canadian Dollar to 21 for the French Franc, once again abundantly confirming leptokurtosis.

same. In the second - call it H_2 - we draw observations from a Normal distribution whose parameters are time-dependent.

The explanations H_0 - H_2 of leptokurtosis are not exhaustive, but appear to be the most attractive and the most commonly mentioned in the literature. The Appendix contains a more elaborate typology of models which produce leptokurtotic samples.

Several criteria have been suggested for chosing among the explanations of leptokurtosis. Perhaps the first to come to mind is to take increasing samples $R_k = \{r_1, r_2, \ldots, r_k\}$, $k \leq N(=\text{NOBS})$ and to plot the sample variances s_k^2 against k; if the s_k^2 appear to diverge, H_0 would be supported. The trouble with this procedure is that one can not really say if a series diverges by examining its initial segment. Also, some forms of H_2 are compatible with apparent divergence of s_k^2.

A more promising test is based on the observation that if independent observations are drawn from a stable distribution with scale parameter c_1 and exponent α, then the stability property implies that their sum has scale paremeter $c_k = c_1 k^{1/\alpha}$ (and exponent α). Therefore, α can be estimated by regressing $\ln c_k$ on $\ln k$ for various values of k; the slope parameter should be $1/\alpha$ and the intercept $\ln c_1$. See Fama and Roll (1971, pg. 334) for background and our Appendix for details of this procedure. Table II reports the estimates[3] of c_k and α, which strongly suggest $\alpha = 2$, thus supporting H_1 or H_2 at the expense of H_0.

The test most favored by Fama and Roll is similar in spirit, but based on order statistics. They first show that for $1 \leq \alpha \leq 2$ the scale parameter $c(=c_1)$ can be efficiently estimated by $\hat{c} = \frac{1}{2}(\hat{X}_{.72} - \hat{X}_{.28})/.827$, where $\hat{X}_f$ is the (N+1)f order statistic (i.e., the value at the 100f percentile in the observed sample). Then the statistic $\hat{Z}_{.97} = (\hat{X}_{.97} - \hat{X}_{.03})/2\hat{c} = .827$

$(\hat{X}_{.97} - \hat{X}_{.03})/(\hat{X}_{.72} - \hat{X}_{.28})$ is an estimator of the .97 fractile (97^{th} percentile) of the standardized ($d = 0$, $c = 1$) PSS distribution of exponent α. There is a monstonic (decreasing) relationship between $\hat{Z}_{.97}$ and α, tabulated in Fama-Roll (1968, p. 822). Thus one can form the statistic $\hat{Z}_{.97}$ and use the table to derive $\hat{\alpha}$ from any sample. If one forms a new sample of size $\simeq N/k$ by aggregating groups of k successive observations in the original sample, the resulting estimates $\hat{\alpha}_k$ should be essentially independent[4] of the degree of aggregation k if H_0 is correct. On the other hand, if H_1 is correct, $\hat{\alpha}_k$ should approach 2 as k increases, since the (aggregated) observations become more nearly "identically distributed" and the variance is finite. H_2 implies slightly different behavior: even for large k, the (aggregate) observations may not have nearly identical distributions, so $\hat{\alpha}_k$ may remain well below 2. However, if one aggregates randomly selected but non-overlapping groups of k individual observations, rather than successive observations, then H_2 becomes indistinguishable from H_1 and therefore implies convergence of α_k to 2.

Table III provides strong support for H_2 over both H_0 and H_1. The column k=1 lists full-sample estimates of the exponent α, given that H_0 is correct; the extent to which an entry is less than 2 (two) may be taken as an index of leptokurtosis for the observed distributions. The stability property of PSS distributions implies that the entries of the other columns also are estimates of the same α if H_0 is correct, although the error will increase with k since the sample size [N/k] decreases. However, for each currency the entry for k=20 exceeds the k=1 entry, casting severe doubt on H_0. For comparative purposes we generated 1640 independent Normally distributed random numbers with the mean and variance of the Deutsch Mark series and

Table III: Estimates of the Exponent α

A. Direct Aggregation of degree k:

	k =	1	2	5	10	20
1.	German Mark	1.45	1.47	1.50	1.46	1.63
2.	Swiss Franc	1.39	1.46	1.36	1.60	1.63
3.	British Pound	1.29	1.39	1.39	1.54	1.62
4.	Japanese Yen	1.11	1.18	1.30	1.37	1.33
5.	Dutch Guilder	1.48	1.47	1.47	1.57	1.49
6.	French Franc	1.36	1.38	1.37	1.54	1.74
7.	Canadian Dollar	1.55	1.58	1.67	1.48	1.74
8.	Belgian Franc	1.39	1.39	1.45	1.54	1.52
9.	Italina Liva	1.12	1.23	1.14	1.18	1.24
10.	Random DM	2.12	1.96	2.12	2.44	2.00

B. Scrambled Aggregation of degree k
(Average of five random permutations)

	k =	1	2	5	10	20
1.	German Mark	1.45	1.58	1.72	1.74	1.92
2.	Swiss Franc	1.39	1.48	1.65	1.77	1.72
3.	British Pound	1.29	1.47	1.64	1.75	1.76
4.	Japanese Yen	1.11	1.39	1.65	1.69	1.75
5.	Dutch Guilder	1.48	1.58	1.79	1.76	1.79
6.	French Franc	1.36	1.48	1.71	1.86	1.72
7.	Canadian Dollar	1.55	1.70	1.84	1.88	2.06
8.	Belgian Franc	1.39	1.5.	1.63	1.93	2.15
9.	Italian Liva	1.12	1.37	1.47	1.54	1.67
10.	Random DM	2.12	1.94	1.97	1.98	1.78

f = .97 NOBS = [1640/k]

of the observations R_t^{65}. That is, one throws out largest 25% and the smallest 25% of the observations (thus gaining "resistance" to outliers and speeding adjustment to shifts of μ_t) and then takes the mean of what remains (thus maintaining reasonable efficiency). One similarly defines the Moving Upper (Lower) Semi-Mid-Means UMM_t(LMM_t), essentially by taking the MMM of the upper (lower) 50% of R_t^{65} -- see the Appendix for details. UMM_t (LMM_t) is a robust, resistant and reasonably efficient estimator of essentially the 75^{th}(25^{th}) percentile point.

Figures C1-9 plot UMM_t, MMM_t, and LMM_t, t = 65, 70, 75, . . . , 1640, for the nine currencies. The middle line (MMM_t) indicates the trend, while the outer lines (UMM_t, LMM_t) enclose a 50% confidence interval around the trend. For the Deutsche Mark, for instance, one observes a trend which reverses itself several times during 1973-5 before stabilizing near 0 during 1976-7. Meanwhile, the DM became increasingly less volatile, indicated by the narrowing gap between UMM_t and LMM_t from 1973-77. In late 1977, the DM began a new upward trend, associated with much higher volatility, which (apart from a short lull in early 1978) persisted until almost the end of 1978; from then until the end of our sample period (Sept. 1979) the DM was relatively trendless and less volatile.

The patterns displayed in the Mid-Mean graphs for the other currencies are quite varied but equally striking. Are they perhaps just artifacts of our statistical techniques and of no economic significance? Their significance can be confirmed in two different ways. First, one can in effect use an "experimental control" by applying the mid-mean treatment to random noise.[7] We randomly drew 1640 observations from a normal distribution of mean .00024 and standard deviation .0058, the values for the German Mark;

suggests that the usual autocorrelation coefficients provide useful information even in the presence of leptokurtosis. Accordingly, we computed the autocorrelation coefficients $\rho_s = \text{corr}(r_t, r_{t-s})$ for various lags s and all currencies; the results are summarized in Table 4. Most coefficients are insignificant, but there are more "significant" t-statistics than one would expect from uncorrelated Normal data. Bearing in mind that the data are leptokurtotic, not Normal, these t-statistics must be taken with a grain of salt. In order to check the robustness and stability of some of these "significant" coefficients, we examine more closely in Table 5 the four largest: the 1st and 16th order autocorrelations of the Italian Lira, 10th UK Pound and the 2nd Swiss Franc. The first of these appears to be the largest and most significant, but evidently the observed autocorrelation arises entirely from events in the first half of the sample period, since the coefficient changes sign and becomes insignificant in the second half of the sample. The second line shows that this coefficient again changes sign and becomes insignificant when the sample is trimmed; evidently this full sample estimate of .123 arises from a few large observations, probably during the precipitous depreciation of the Lira in early 1976.

A similar pattern appears for the other large autocorrelation coefficients: estimates are no longer significantly different from zero in the 10% trimmed sample, and differ between the two sample periods. We conclude that the FX rate fluctuations are probably not autocorrelated to any significant degree, that the larger observed autocorrelation coefficients are generally not stable, and probably arise mostly from "random" placement of the larger fluctuations.

TABLE V: Stability of Selected Autocorrelation Coefficients

	Currency:	Italian Lira		Swiss Franc	UK Pound
	Lag:	1	16	2	10
1) Full Sample		.123 (4.98)	.092 (3.72)	-.106 (-4.28)	0.92 (3.71)
2) 10% Trimmed Sample		-.045 (-1.64)	.014 (0.51)	-.028 (-1.00)	.003 (.093)
3) First Half of Sample (6/2/73-7/23/76)		.176 (5.04)	.094 (2.70)	-.137 (-3.92)	.076 (2.165)
4) Second Half of Sample (7/24/76-9/14/76)		-.031 (-.88)	.052 (1.482)	-.073 (-2.09)	.094 (2.70)

<u>Notes</u>: t-stats in parentheses.
10% trimmed sample involves estimates based on the central 80% of the ordered data.

is of great interest, and robust methods are certainly called for. Data is available for forward as well as spot FX markets; its analysis should also be of great theoretic and practical value. This work will have to wait for another occasion.

Finally, what is the significance of our analysis to a participant in the FX markets? If he is interested in trends and volatility over only the next few weeks, our estimators MMM and $\hat{\sigma}$ could prove helpful. The expected trend over the next k trading days would be $kMMM_T$ and the expected volatility[8] would be $\sqrt{k}\,\hat{\sigma}_T$, where T = today, and $k \leq 30$. If one wishes to look further out into the future, some sort of econometric or judgemental forecast would seem necessary. Of course, even if one has estimates of future trends and volatility (and even perhaps covariances and forward market behavior), the analysis of appropriate behavior of various participants in the FX markets is far from trivial. Such analysis will also have to wait for another occasion.

normal. No such restrictions apply to r_t or its distribution.

(c) If the spot rate is quoted in foreign instead of U.S. terms, the analysis should not be affected in any significant way. This is the case with r_t; if $F_t = 1/S_t$ is the spot rate in foreign terms, then the rate of change is $r_t' = \log (F_{t+1}/F_t) = \log (S_t/S_{t+1}) = -\log (S_{t+1}/S_t) = -r_t$, as expected. However,

$$P_t' = (F_{t+1} - F_t)/F_t = S_t/S_{t+1} - 1 = (S_t - S_{t+1})/S_{t+1} = -P_t \ (S_t/S_{t+1}).$$

The extra factor (S_t/S_{t+1}) ensures that S_t and F_t can't both be martingales, a severe theoretical difficulty.[9]

However, r_t and P_t do agree up to first order; by Taylor's expansion for the log function, we have

$$r_t = \log (S_{t+1}/S_t) = \log (1 + P_t) = P_t - P_t^2/2 + P_t^3/3 - \ldots$$

3. Notes on the Kurtosis Coefficient

By Jensen's inequality, $(\int x^2 \, dF)^2 \leq \int x^4 \, dF$; therefore KURT $\geq$ 1. If the mass of the distribution F is concentrated equally on two points (*e.g.*, arises from a coin-flip experiment), then the lower bound is attained. A direct computation shows that KURT = 9/5 for a uniform distribution on an interval, and manipulations of the characteristic function show that KURT = 3 for a normal distribution. For symmetric stable distributions with $1 \leq \alpha < 2$, KURT is not defined; finite sample kurtosis from such distributions will $\to\infty$ as the sample size $\to\infty$.

4. The "fat tail" test

In sampling from a normal distribution with mean μ and standard deviation σ, the probability p of drawing an observation x such that $|x - \mu| \geq 3\sigma$ is $2\,(1 - Z(3)) \simeq 2\,(1 - .9987) = .0026$. The probability of drawing k or more

TABLE 6: The 3σ-test

Currency	NOBS	k	p(k)
German Mark	1640	28	$<10^{-16}$
Swiss Franc	1640	36	$<10^{-20}$
British Pound	1640	29	$<10^{-16}$
Japanese Yen	1640	26	$<10^{-12}$
Dutch Guilder	1640	20	2.61×10^{-8}
French Franc	1640	33	$<10^{-20}$
Canadian Dollar	1640	24	$<10^{-12}$
Belgian Franc	1640	33	$<10^{-20}$
Italian Lira	1640	34	$<10^{-20}$

k = number of observations at least three sample standard deviations from the sample mean.

p(k) = probability that k or more out of NOBS observations drawn from a normal distribution will lie at least three standard deviations from the mean.

long as there isn't some μ_0 such that $P[\mu(\lambda) = \mu_0] = 1$). An easy computation shows that skewness is zero for the r_t's because it is zero for $f|\lambda$. For kurtosis, first note that since $f|\lambda$ is normal, $E_{f|\lambda}(x - \mu(\lambda))^4 = 3\sigma^4(\lambda)$, and by symmetry, $E_{f|\lambda}(x - \mu(\lambda)) = E_{f|\lambda}(x - \mu(\lambda))^3 = 0$. By definition, $\sigma^4 \cdot \text{KURT} = E_\lambda E_{f|\lambda}(x - \mu)^4 = E_\lambda E_{f|\lambda}[(x - \mu(\lambda)) + (\mu(\lambda) - \mu)]^4$; after expanding and simplifying, we obtain:

$$\text{(3)} \quad \text{KURT} = \sigma^{-4} E_\lambda \left(3\sigma^4(\lambda) + 6\sigma^2(\lambda)(\mu(\lambda) - \mu)^2 + (\mu(\lambda) - \mu)^4\right).$$

Two special cases suggest themselves:

(a) $\mu(\lambda) = 0$, all $\lambda \varepsilon \Lambda$. Then $\text{KURT} = 3 E_\lambda \sigma^4(\lambda)/\sigma^4$ and $\sigma^2 = E_\lambda \sigma^2(\lambda)$; if $\sigma^2(\lambda)$ is not essentially constant, we have (by Jensen's inequality): $\sigma^4 = (E_\lambda \sigma^2(\lambda))^2 < E_\lambda \sigma^4(\lambda)$, so in this case the distribution is leptokurtotic.[10]

(b) $\sigma^2(\lambda) = \sigma_0^2$, all $\lambda \varepsilon \Lambda$. Then $\sigma^2 = \sigma_0^2 + \sigma_\mu^2$, where σ_μ^2 is the variance of $\mu(\lambda)$, by equation (2). In view of the fact that $\sigma_\mu^2 = E_\lambda(\mu(\lambda) - \mu)^2$ and $E_\lambda(\mu(\lambda) - \mu)^4 = \sigma_\mu^4 \text{KURT}_\mu$, equation (3) becomes

$$\text{KURT} = 3 \frac{\sigma_0^4 + 2\sigma_0^2\sigma_\mu^2 + \sigma_\mu^4 \text{KURT}_\mu/3}{(\sigma_0^2 + \sigma_\mu^2)^2}$$

Evidently, the kurtosis of r_t in this case depends entirely on the kurtosis of $\mu(\lambda)$; if $\mu(\lambda)$ is lepto(platy)kurtotic, then so will be r_t, although to a lesser extent.

One can generate many distributions by mixing normals; <u>e.g.</u> Student's t arises from assuming $\mu(\lambda) = \mu_0$ and $\sigma^2(\lambda)$ is distributed χ_k^2. Even Paretian stable symmetric distributions of exponent $\alpha < 2$ can be so obtained -- one assumes that the variance $\sigma^2(\lambda)$ has a (Levy) positive stable distribution of exponent $\alpha/2$ (cf. Mandelbrot (1973)).

FIGURE E. Models with Leptokurtotic Distributions: A Family Tree

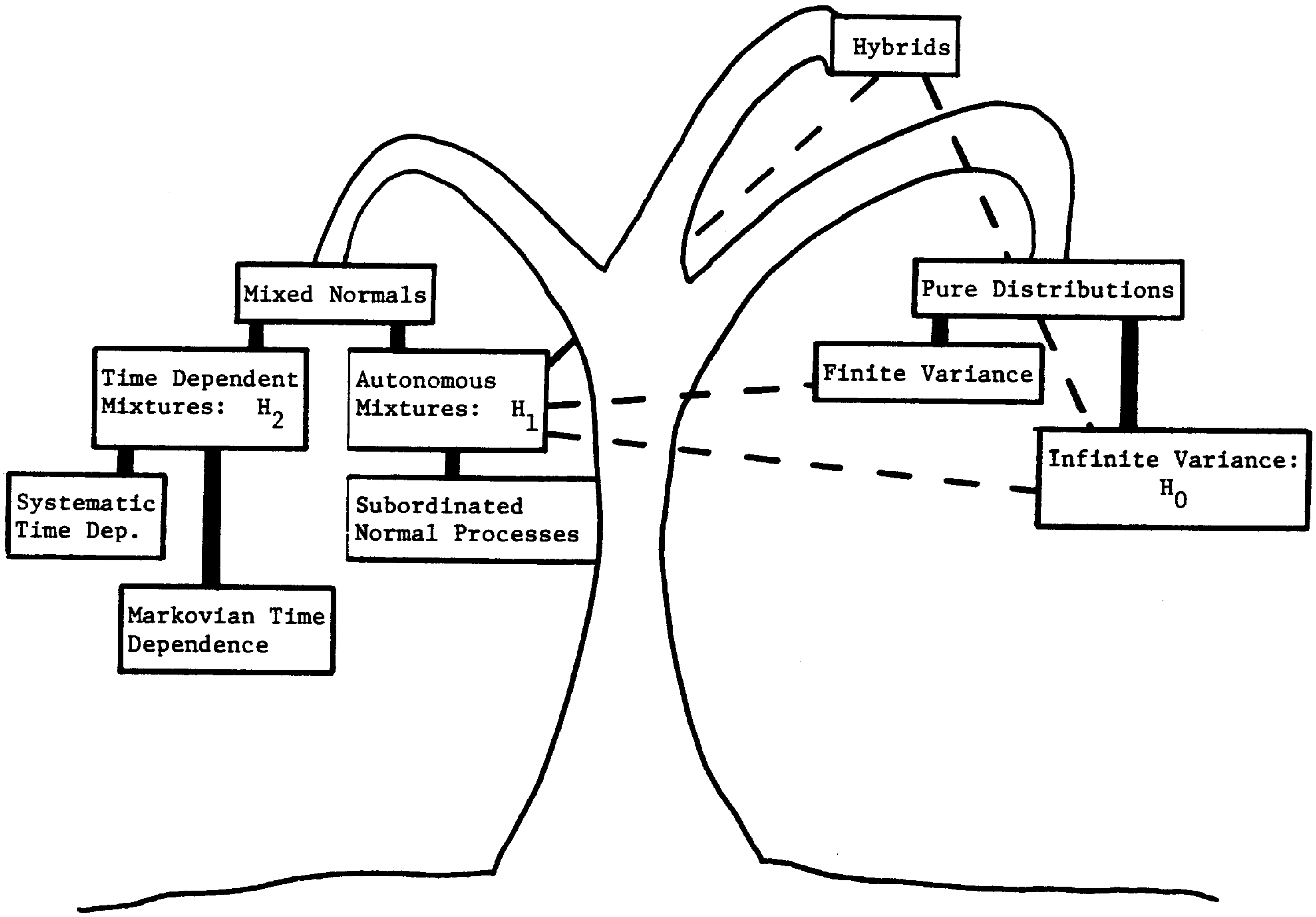


FIGURE A9

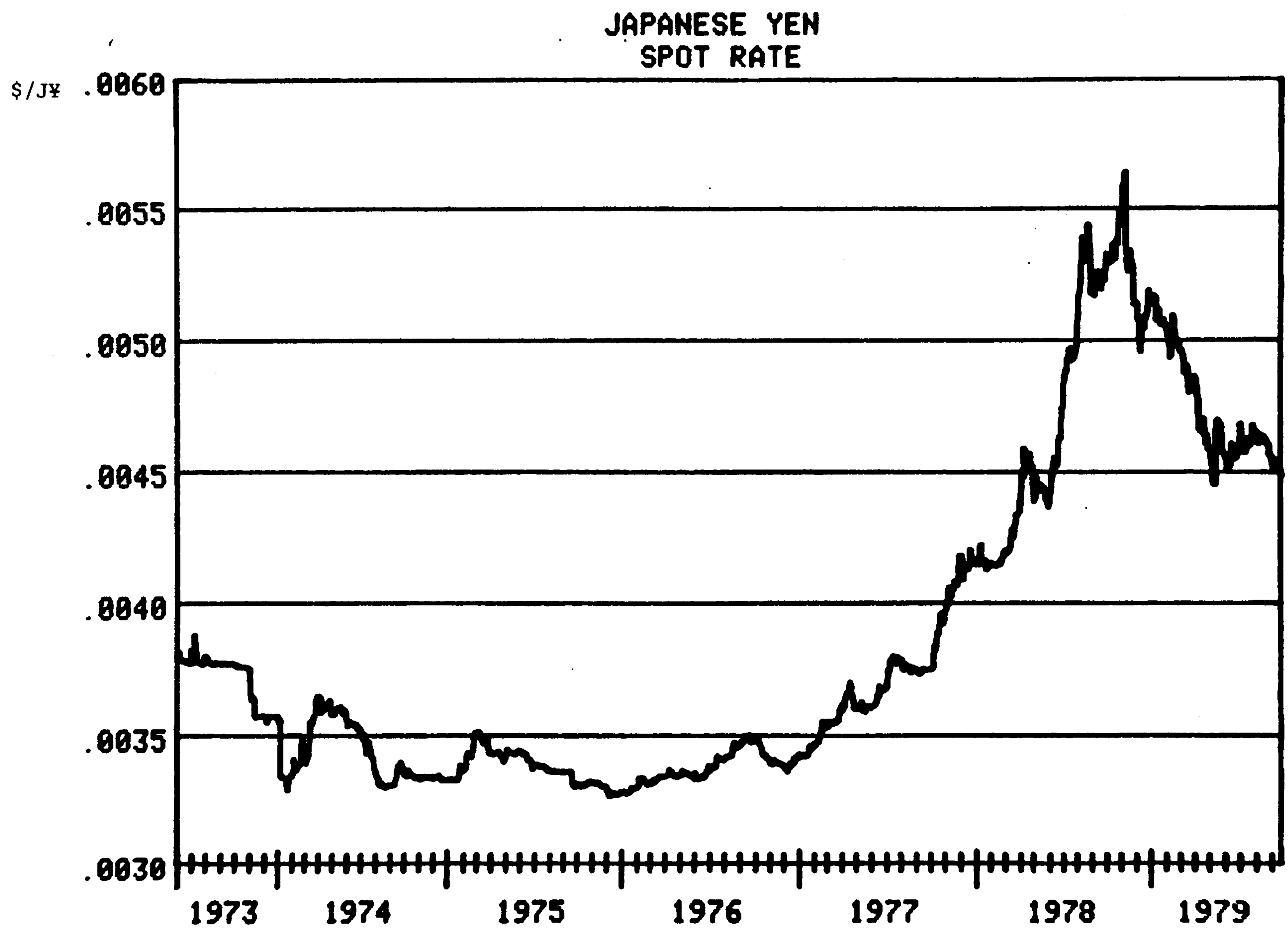

JAPANESE YEN
SPOT RATE
$/J¥
.0060
.0055
.0050
.0045
.0040
.0035
.0030
1973
1974
1975
1976
1977
1978
1979

FIGURE B2: SWISS FRANC
Histogram of Percentage Changes in Daily FX Rates
1 June 1973 -- 14 September 1979

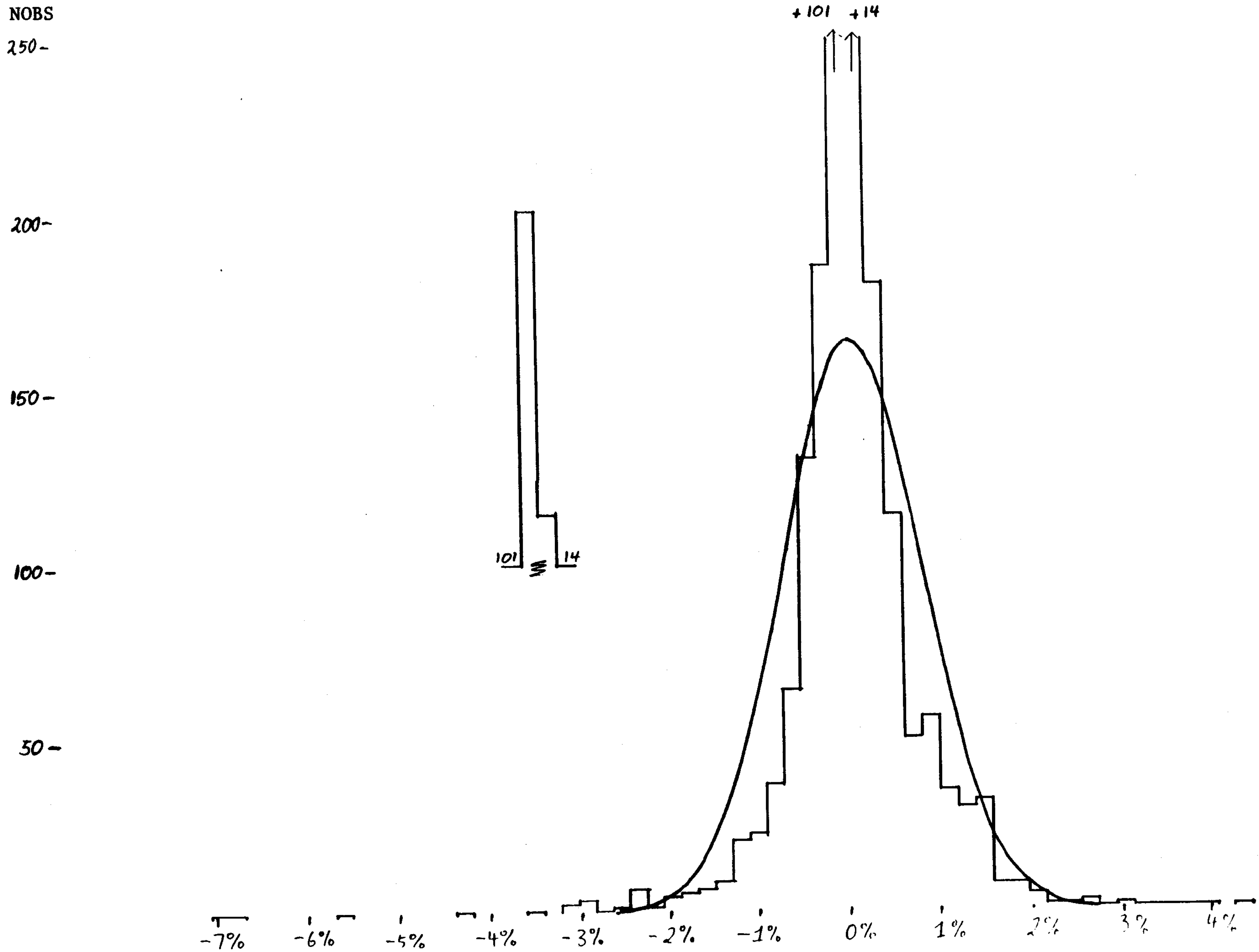


FIGURE B4: JAPANESE YEN
Histogram of Percentage Changes in Daily FX Rates
1 June 1973 -- 14 September 1979

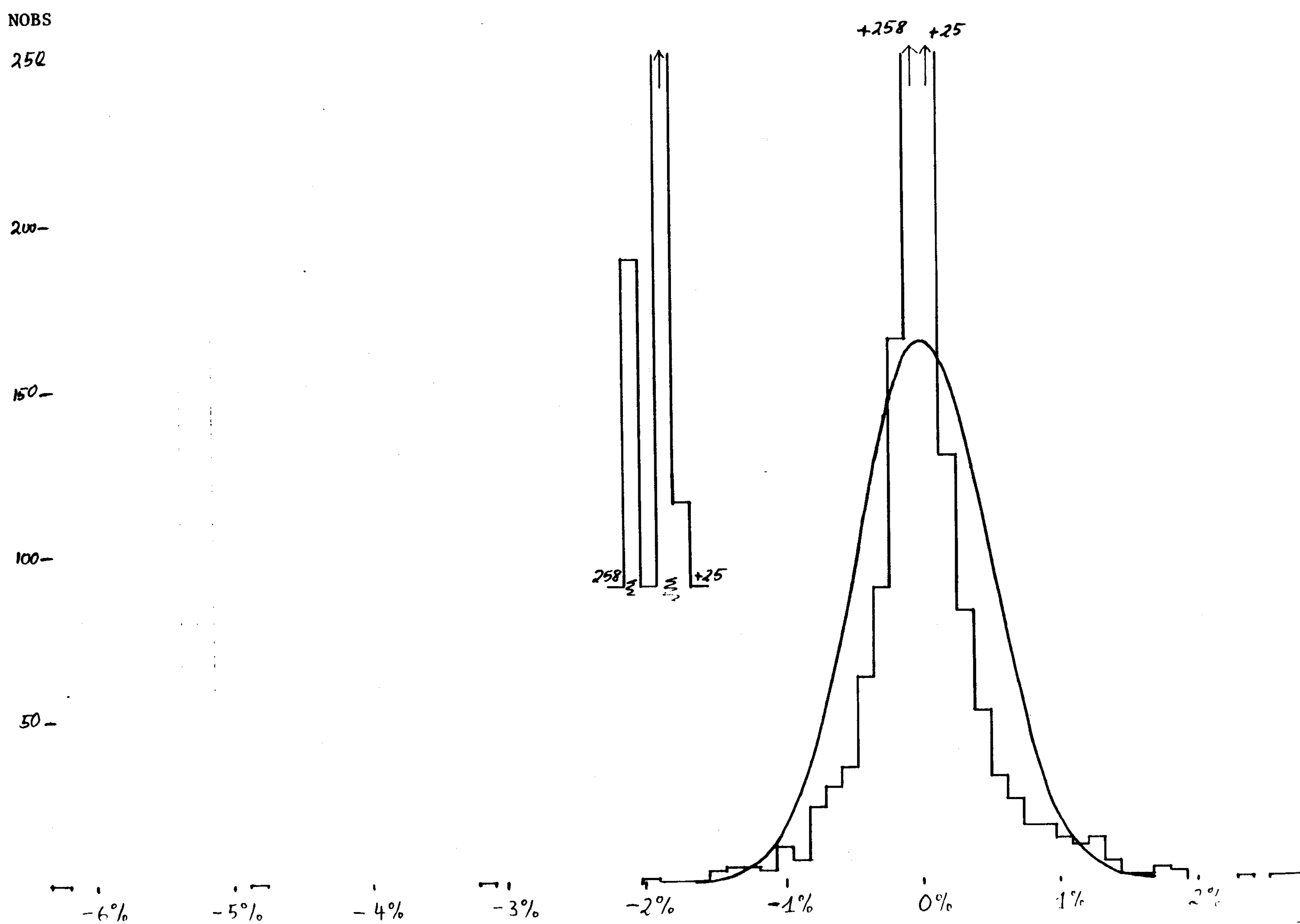


FIGURE B6: FRENCH FRANC
Histogram of Percentage Changes in Daily FX Rates
1 June 1973 -- 14 September 1979

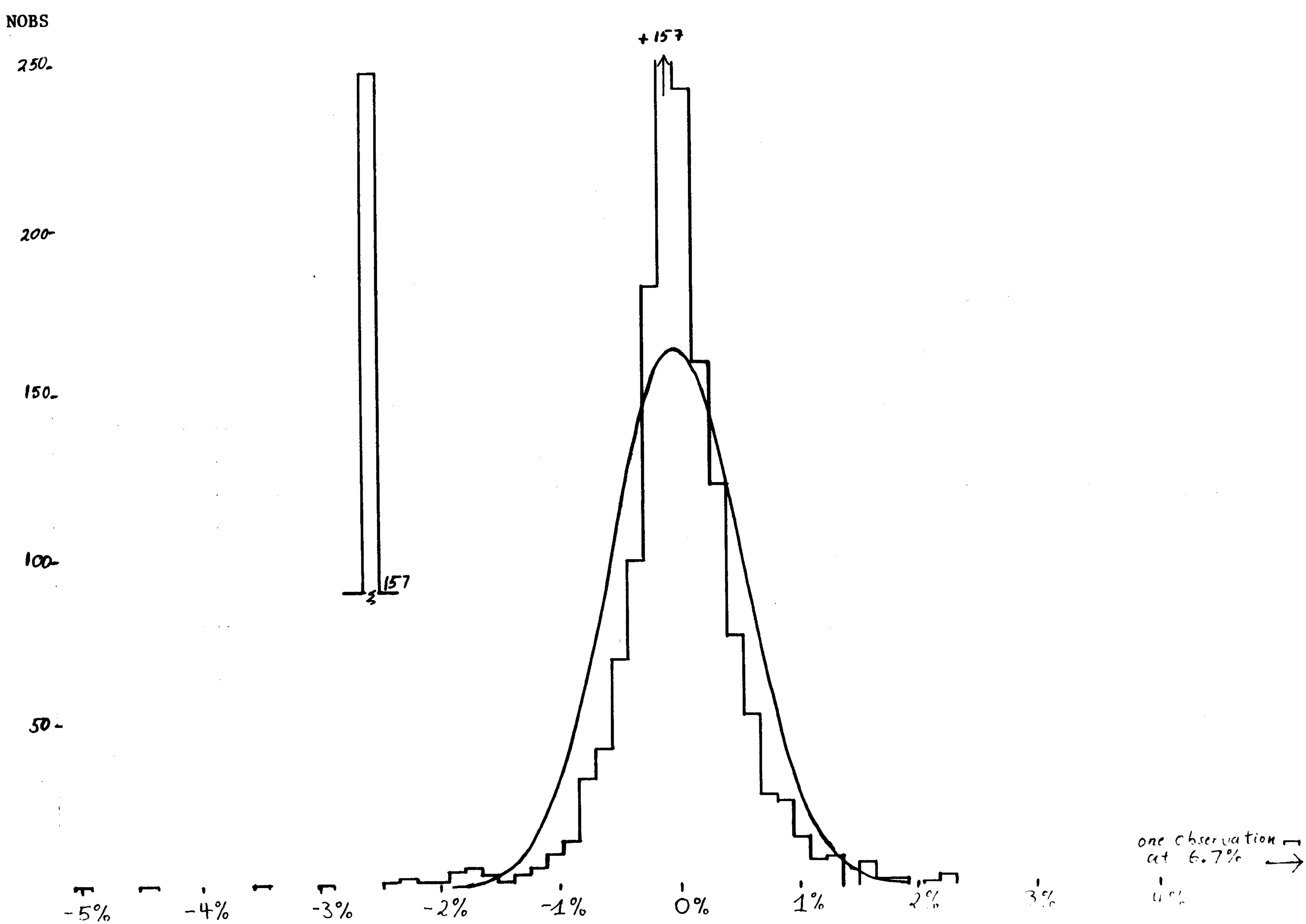


FIGURE B8: BELGIAN FRANC
Histogram of Percentage Changes in Daily FX Rates
1 June 1973 -- 14 September 1979

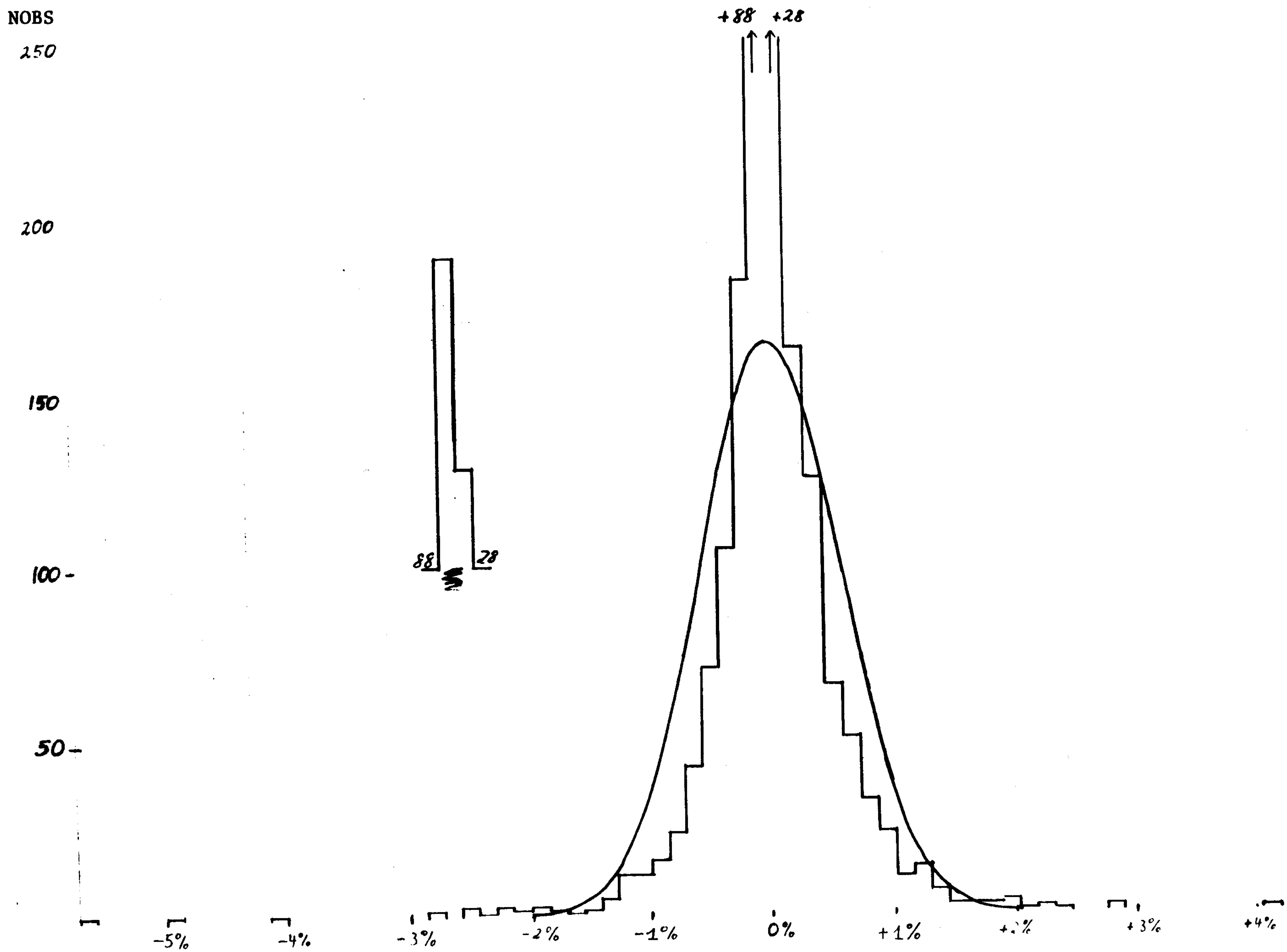
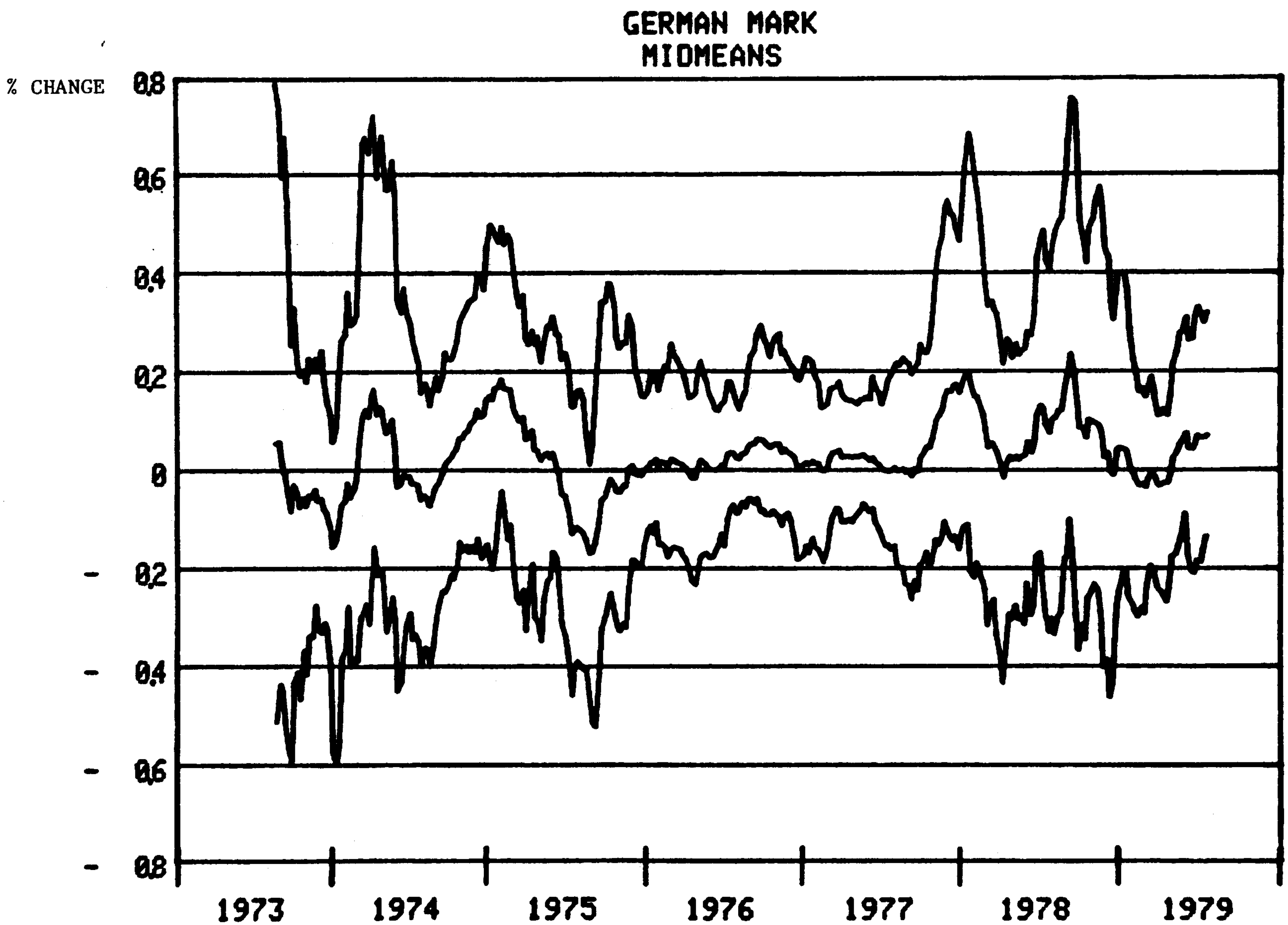
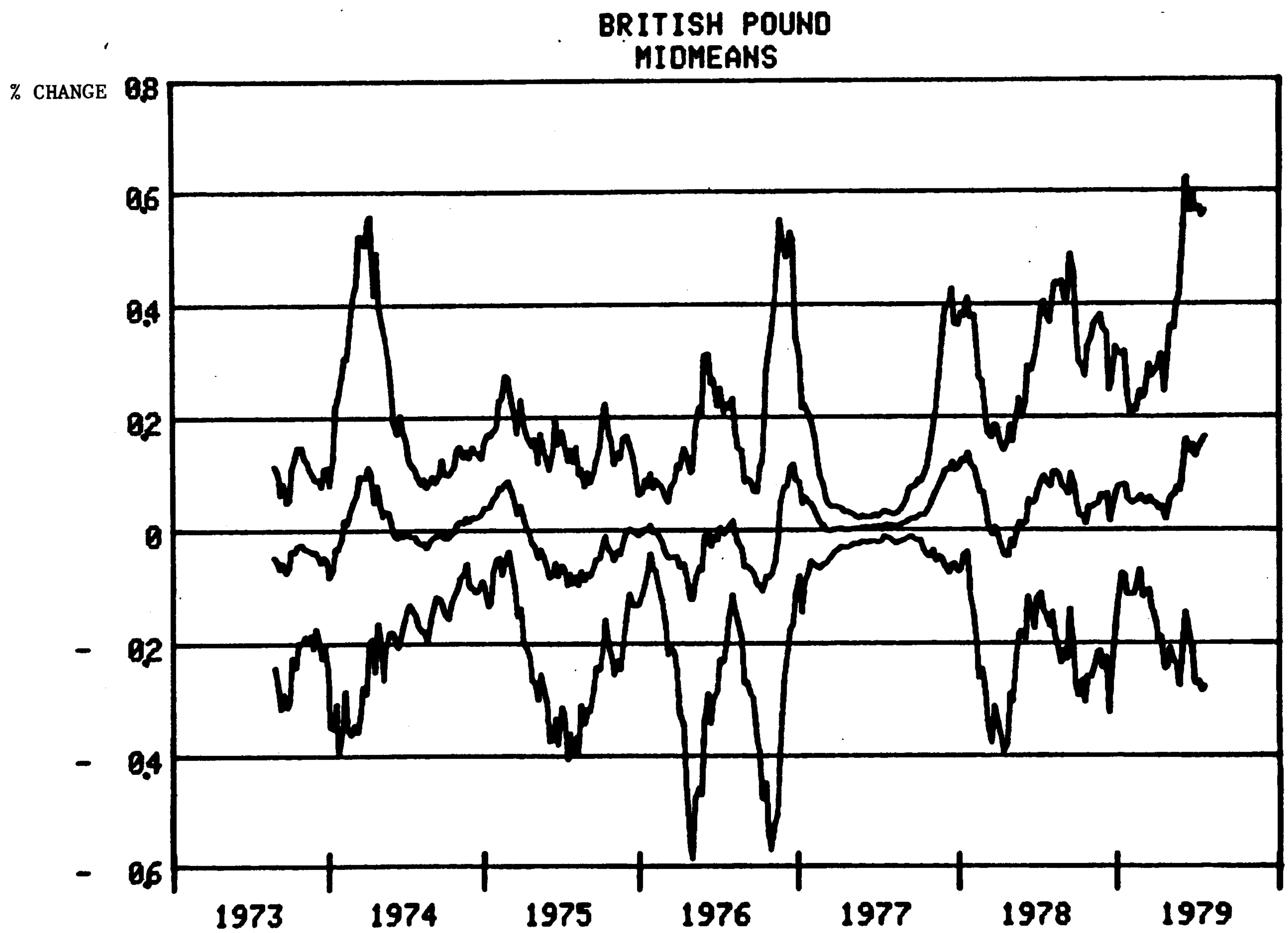


FIGURE C1



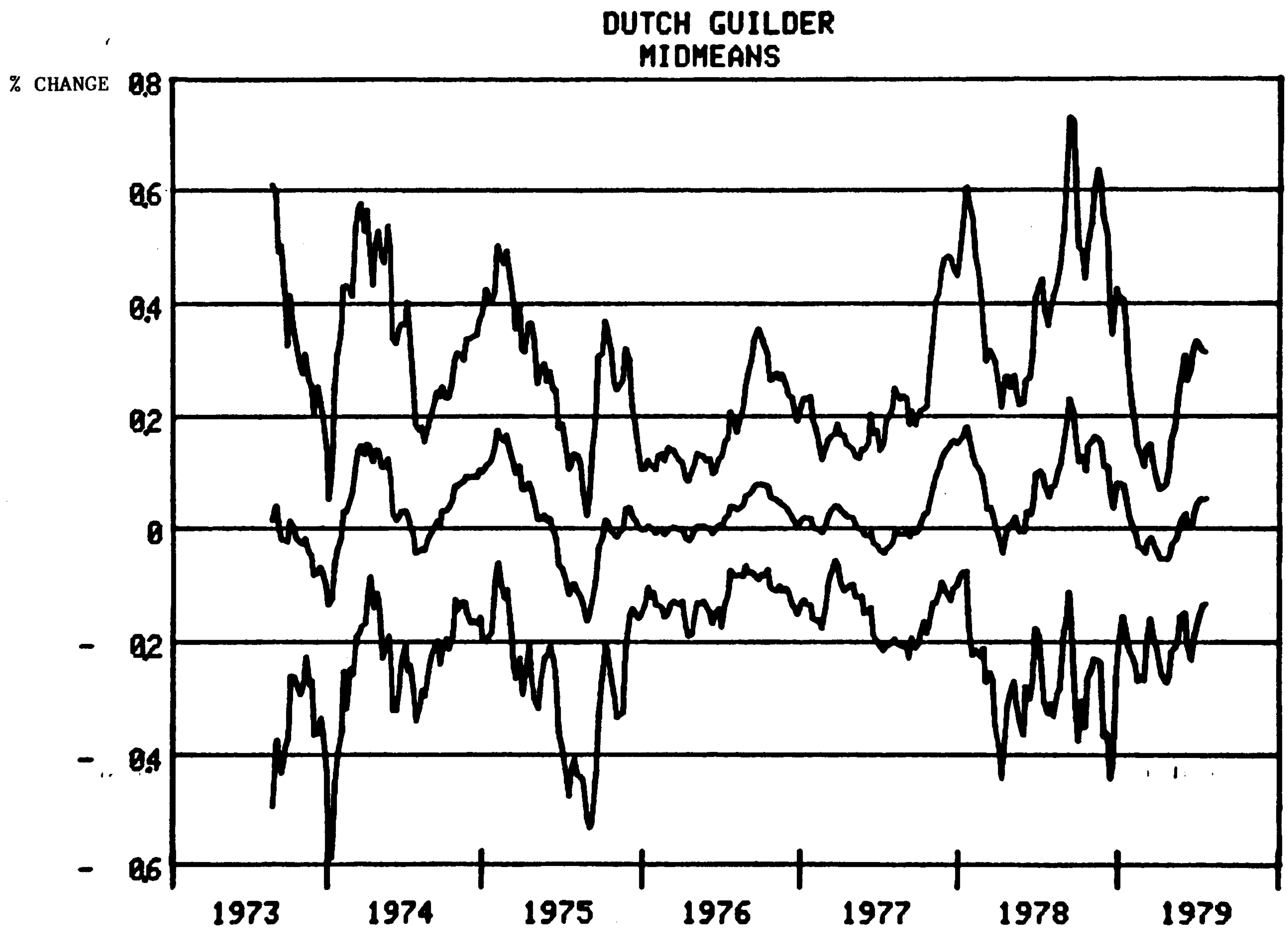
65-day window

FIGURE C3



65-day window

FIGURE C5



65-day window

AGGR LEVEL: 20	NOBS: 82	1.85	1.70	1.92	1.84	1.81	1.82

***** N0R *****

F LEVEL: .95	RUN:	I	II	III	IV	V	AVG
AGGR LEVEL: 1	NOBS: 1646	2.18	2.18	2.18	2.18	2.18	2.18
AGGR LEVEL: 2	NOBS: 823	2.08	1.92	1.90	2.18	1.76	1.97
AGGR LEVEL: 5	NOBS: 329	2.22	1.85	1.76	1.90	1.78	1.90
AGGR LEVEL: 10	NOBS: 164	2.04	1.86	1.76	1.70	2.24	1.92
AGGR LEVEL: 20	NOBS: 82	1.95	1.61	1.58	1.88	2.04	1.81

F LEVEL: .97	RUN:	I	II	III	IV	V	AVG
AGGR LEVEL: 1	NOBS: 1646	2.12	2.12	2.12	2.12	2.12	2.12
AGGR LEVEL: 2	NOBS: 823	2.00	1.95	1.95	2.01	1.82	1.94
AGGR LEVEL: 5	NOBS: 329	2.21	1.85	1.87	2.06	1.86	1.97
AGGR LEVEL: 10	NOBS: 164	2.21	2.04	1.82	1.75	2.11	1.98
AGGR LEVEL: 20	NOBS: 82	1.81	1.65	1.69	2.00	1.74	1.78

F LEVEL: .99	RUN:	I	II	III	IV	V	AVG
AGGR LEVEL: 1	NOBS: 1646	2.02	2.02	2.02	2.02	2.02	2.02
AGGR LEVEL: 2	NOBS: 823	2.01	1.97	1.94	2.03	1.88	1.96
AGGR LEVEL: 5	NOBS: 329	2.13	1.91	1.92	1.97	1.94	1.97
AGGR LEVEL: 10	NOBS: 164	2.14	2.07	1.93	1.92	2.05	2.02
AGGR LEVEL: 20	NOBS: 82	2.03	1.95	1.85	2.11	1.99	1.99

***** N1R *****

F LEVEL: .95	RUN:	I	II	III	IV	V	AVG
AGGR LEVEL: 1	NOBS: 1646	1.96	1.96	1.96	1.96	1.96	1.96
AGGR LEVEL: 2	NOBS: 823	1.83	2.00	2.34	2.06	2.17	2.08
AGGR LEVEL: 5	NOBS: 329	2.05	2.35	1.95	2.35	1.77	2.09
AGGR LEVEL: 10	NOBS: 164	2.41	2.74	2.92	2.48	1.99	2.51
AGGR LEVEL: 20	NOBS: 82	2.62	2.16	2.32	2.16	1.94	2.24

F LEVEL: .97	RUN:	I	II	III	IV	V	AVG
AGGR LEVEL: 1	NOBS: 1646	2.02	2.02	2.02	2.02	2.02	2.02
AGGR LEVEL: 2	NOBS: 823	1.93	2.00	2.18	2.05	2.04	2.04
AGGR LEVEL: 5	NOBS: 329	2.05	2.16	1.97	2.22	1.86	2.05
AGGR LEVEL: 10	NOBS: 164	2.35	2.46	2.33	2.31	1.86	2.26
AGGR LEVEL: 20	NOBS: 82	2.25	2.29	2.04	2.15	2.12	2.17

F LEVEL: .99	RUN:	I	II	III	IV	V	AVG
AGGR LEVEL: 1	NOBS: 1646	2.00	2.00	2.00	2.00	2.00	2.00
AGGR LEVEL: 2	NOBS: 823	1.98	2.03	2.03	2.00	2.06	2.02
AGGR LEVEL: 5	NOBS: 329	2.05	2.06	2.05	2.16	1.85	2.04
AGGR LEVEL: 10	NOBS: 164	2.12	2.19	2.18	2.17	2.05	2.14
AGGR LEVEL: 20	NOBS: 82	2.27	2.04	2.21	2.22	1.93	2.14

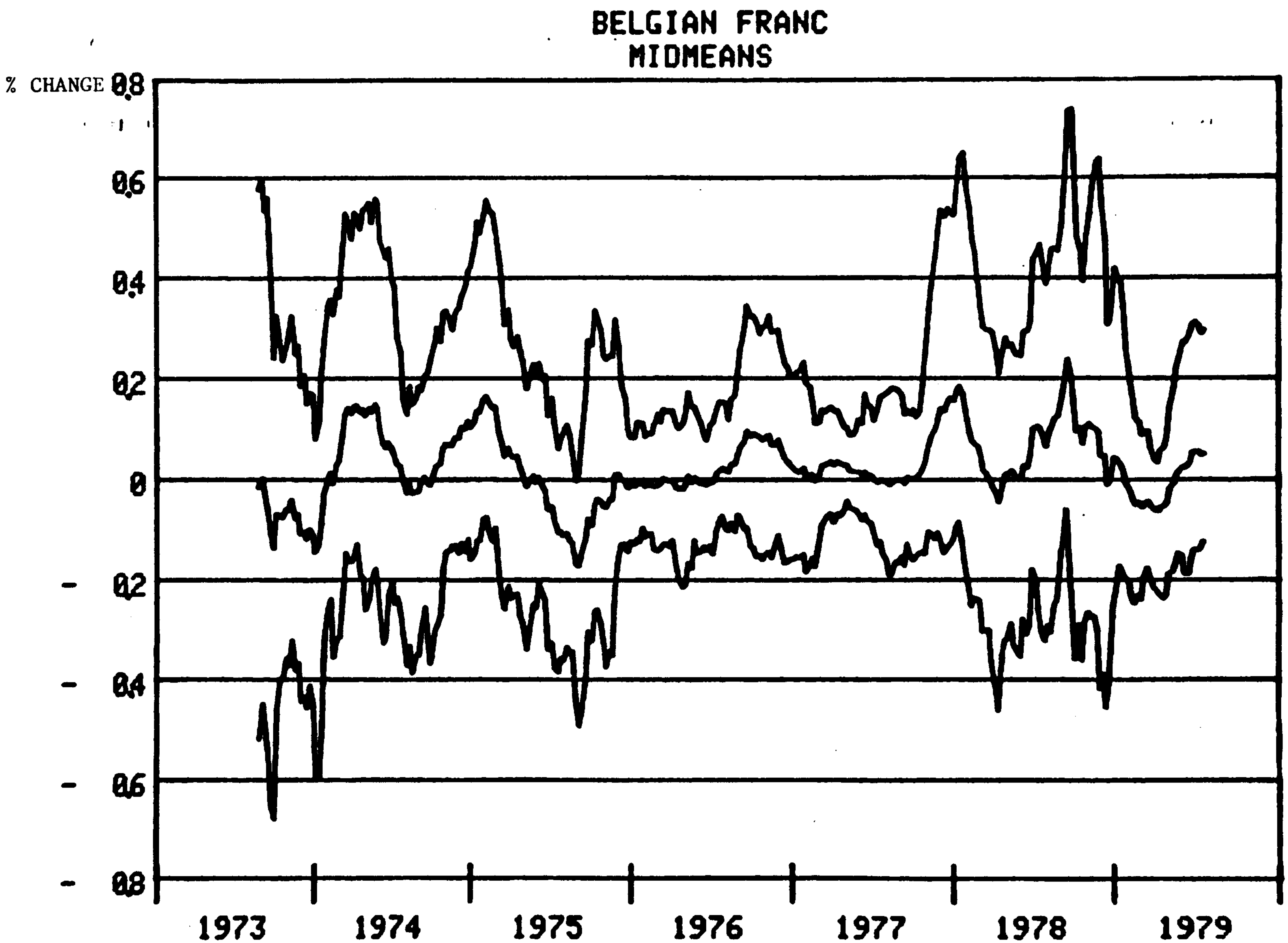
***** N2R *****

F LEVEL: .95	RUN:	I	II	III	IV	V	AVG
AGGR LEVEL: 1	NOBS: 1646	2.25	2.25	2.25	2.25	2.25	2.25
AGGR LEVEL: 2	NOBS: 823	2.10	2.07	1.99	1.80	2.10	2.01
AGGR LEVEL: 5	NOBS: 329	2.08	1.95	1.73	2.17	1.73	1.93
AGGR LEVEL: 10	NOBS: 164	1.59	1.94	1.80	2.40	1.76	1.90
AGGR LEVEL: 20	NOBS: 82	1.83	1.59	1.77	1.94	1.45	1.72

F LEVEL: .97	RUN:	I	II	III	IV	V	AVG
AGGR LEVEL: 1	NOBS: 1646	2.20	2.20	2.20	2.20	2.20	2.20
AGGR LEVEL: 2	NOBS: 823	2.16	2.05	2.03	1.84	2.14	2.04
AGGR LEVEL: 5	NOBS: 329	2.09	1.99	1.89	2.14	1.93	2.01
AGGR LEVEL: 10	NOBS: 164	1.68	2.16	1.79	2.30	1.82	1.95
AGGR LEVEL: 20	NOBS: 82	1.90	1.69	1.81	1.68	1.51	1.72

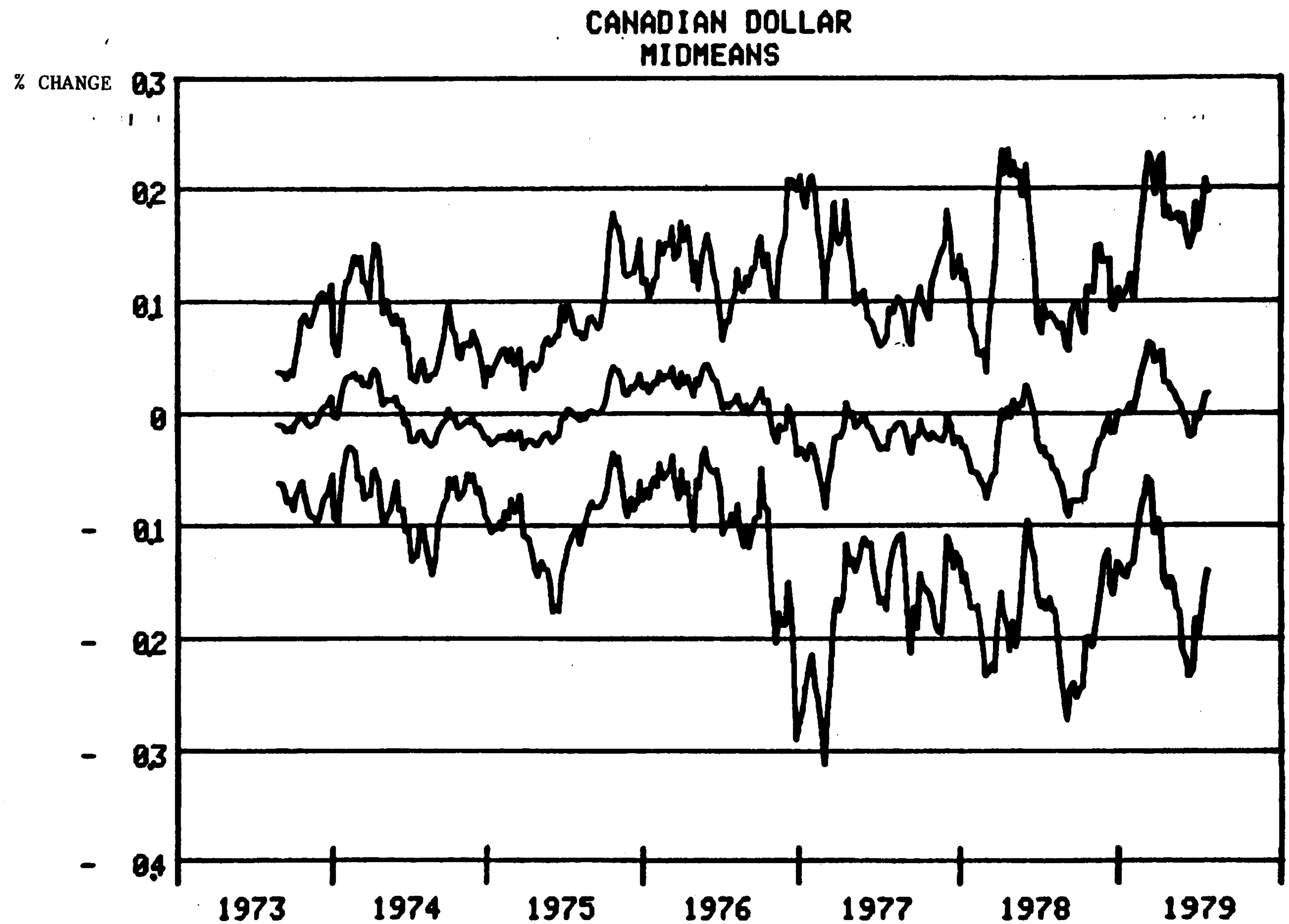
F LEVEL: .99	RUN:	I	II	III	IV	V	AVG
AGGR LEVEL: 1	NOBS: 1646	2.09	2.09	2.09	2.09	2.09	2.09
AGGR LEVEL: 2	NOBS: 823	2.04	2.07	2.04	1.96	2.05	2.03
AGGR LEVEL: 5	NOBS: 329	2.02	1.99	1.97	2.03	1.92	1.99
AGGR LEVEL: 10	NOBS: 164	1.85	2.08	1.89	2.08	1.93	1.97
AGGR LEVEL: 20	NOBS: 82	2.01	1.87	2.02	1.90	1.78	1.92

FIGURE C7



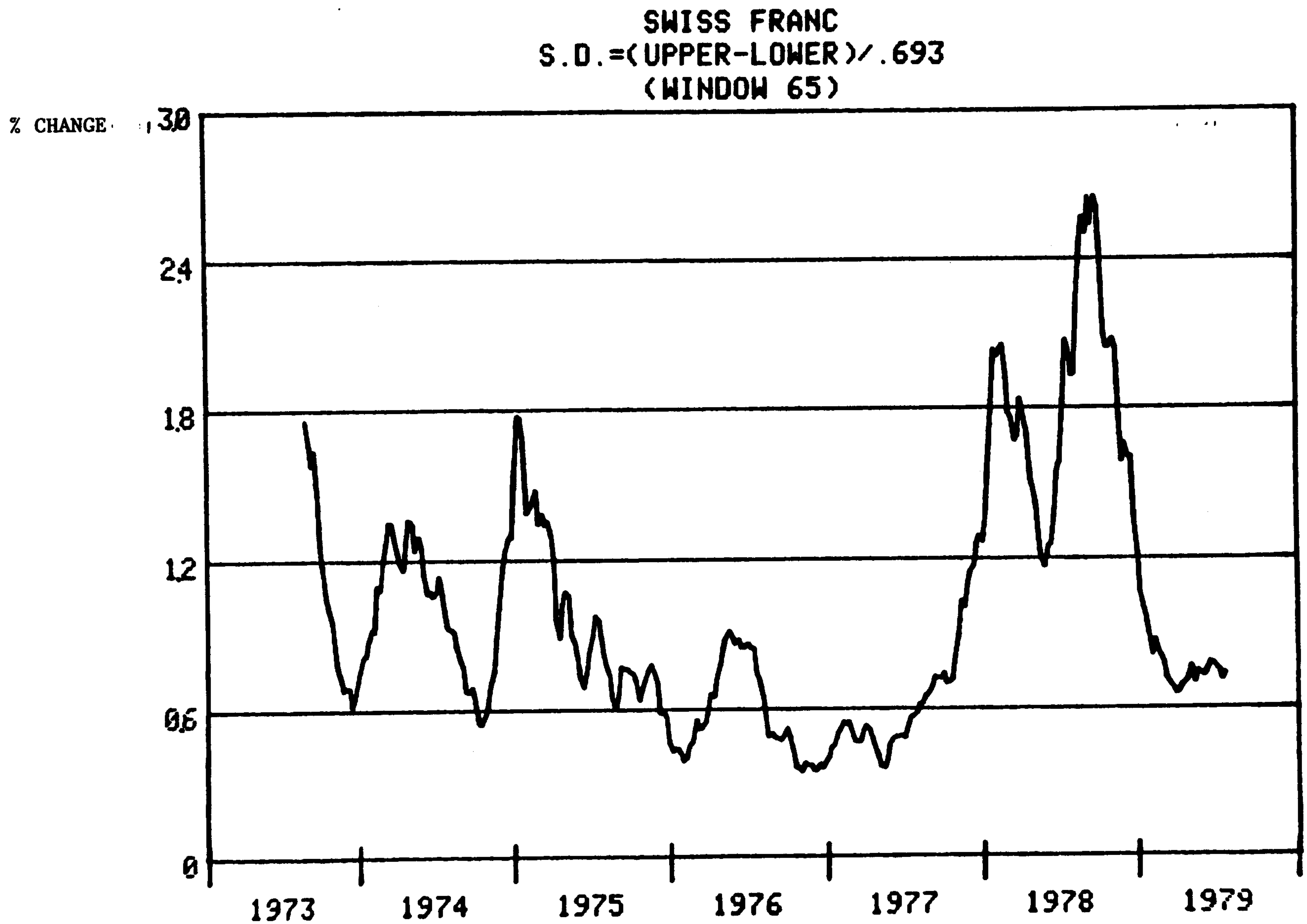
65-day window

FIGURE C9



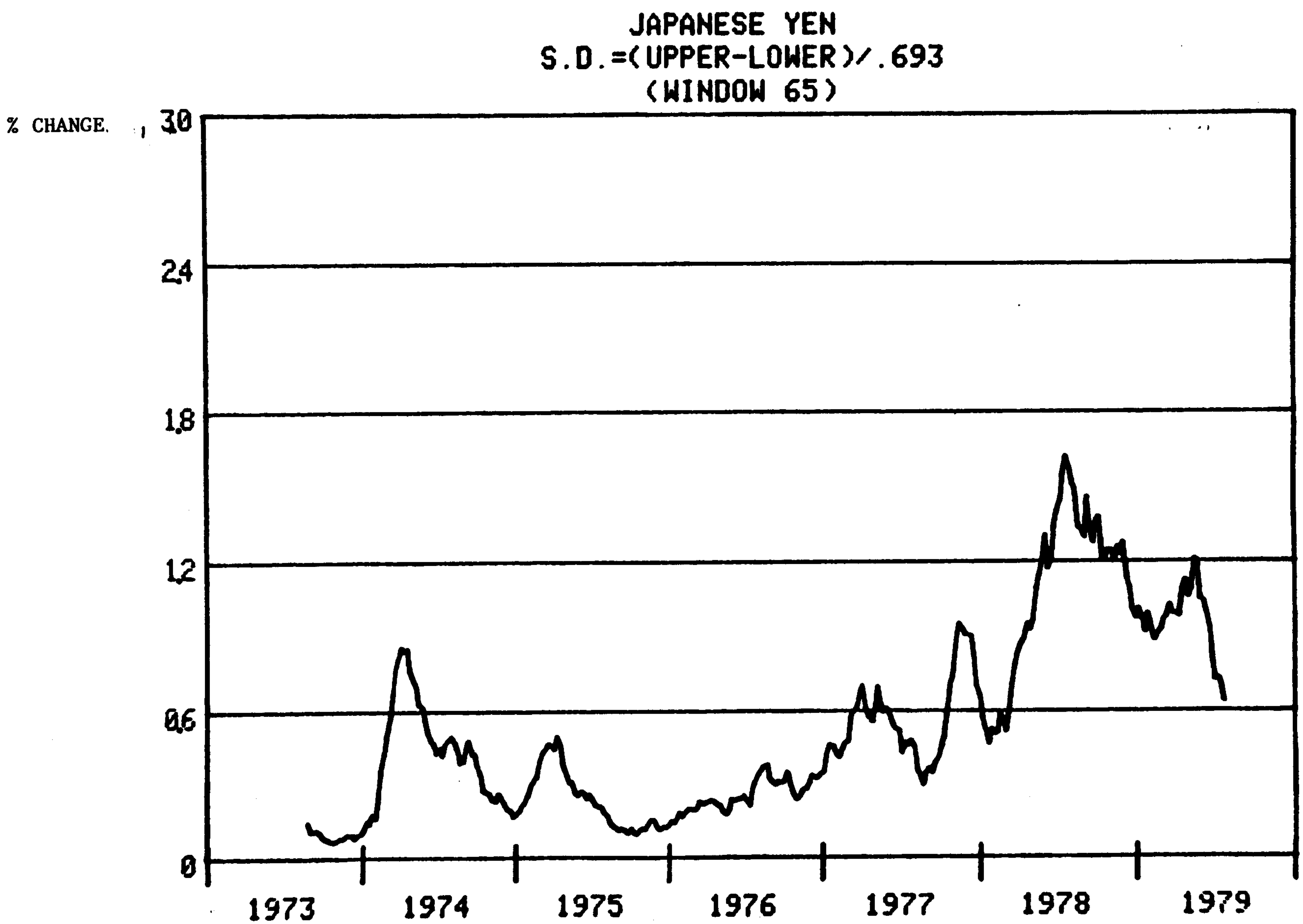
65-day window

FIGURE D2



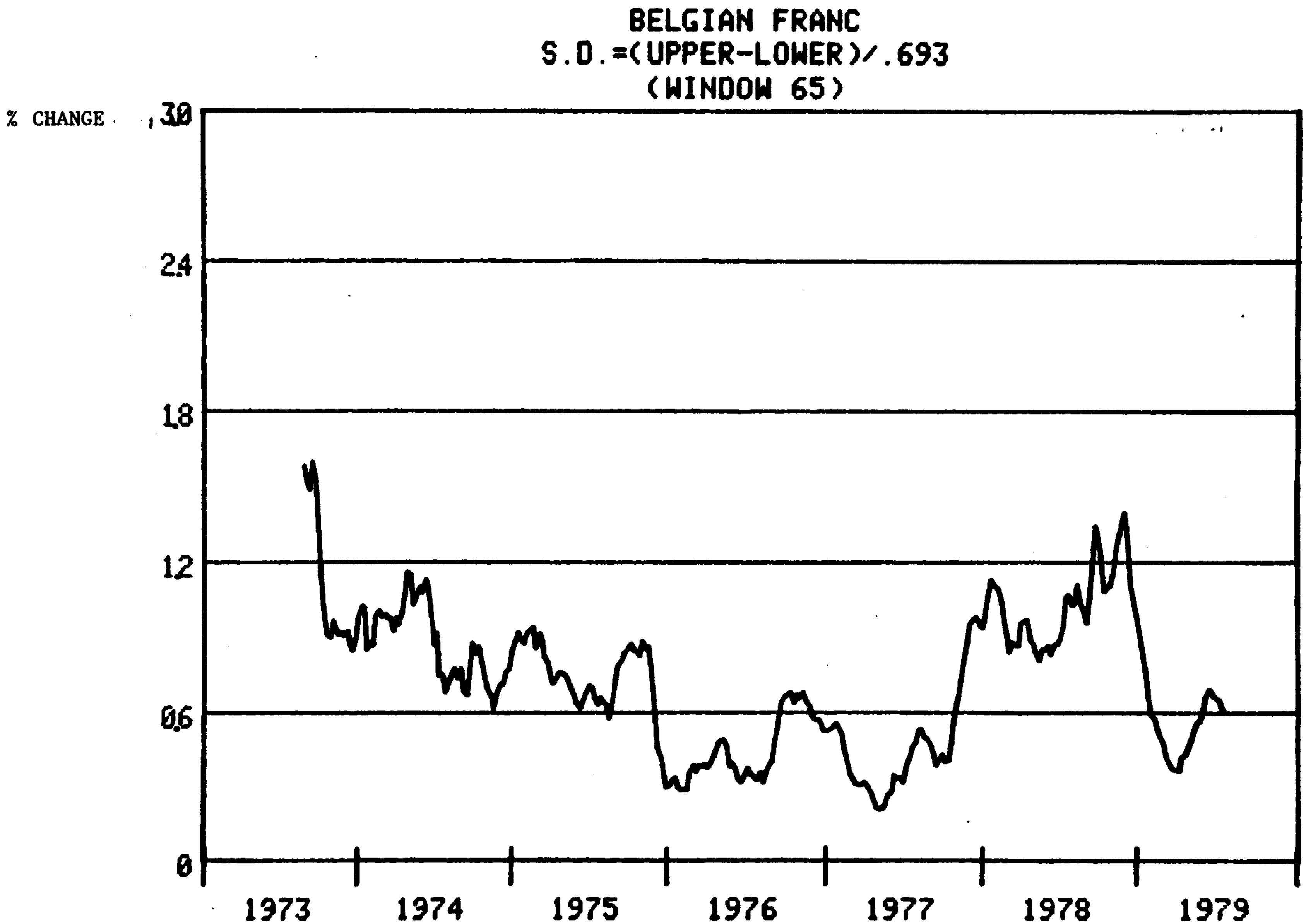
65-day window

FIGURE D4



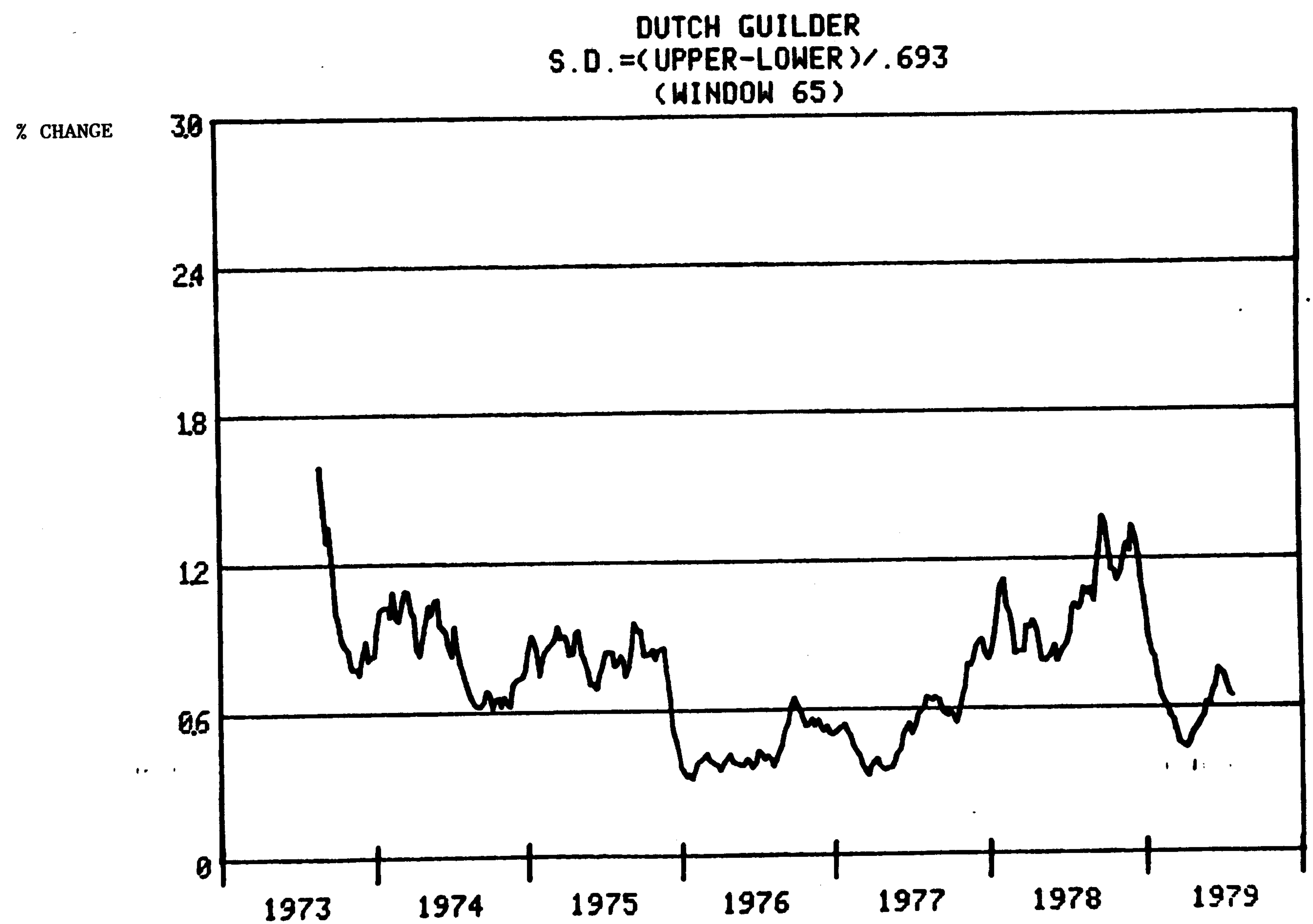
65-day window

FIGURE D6



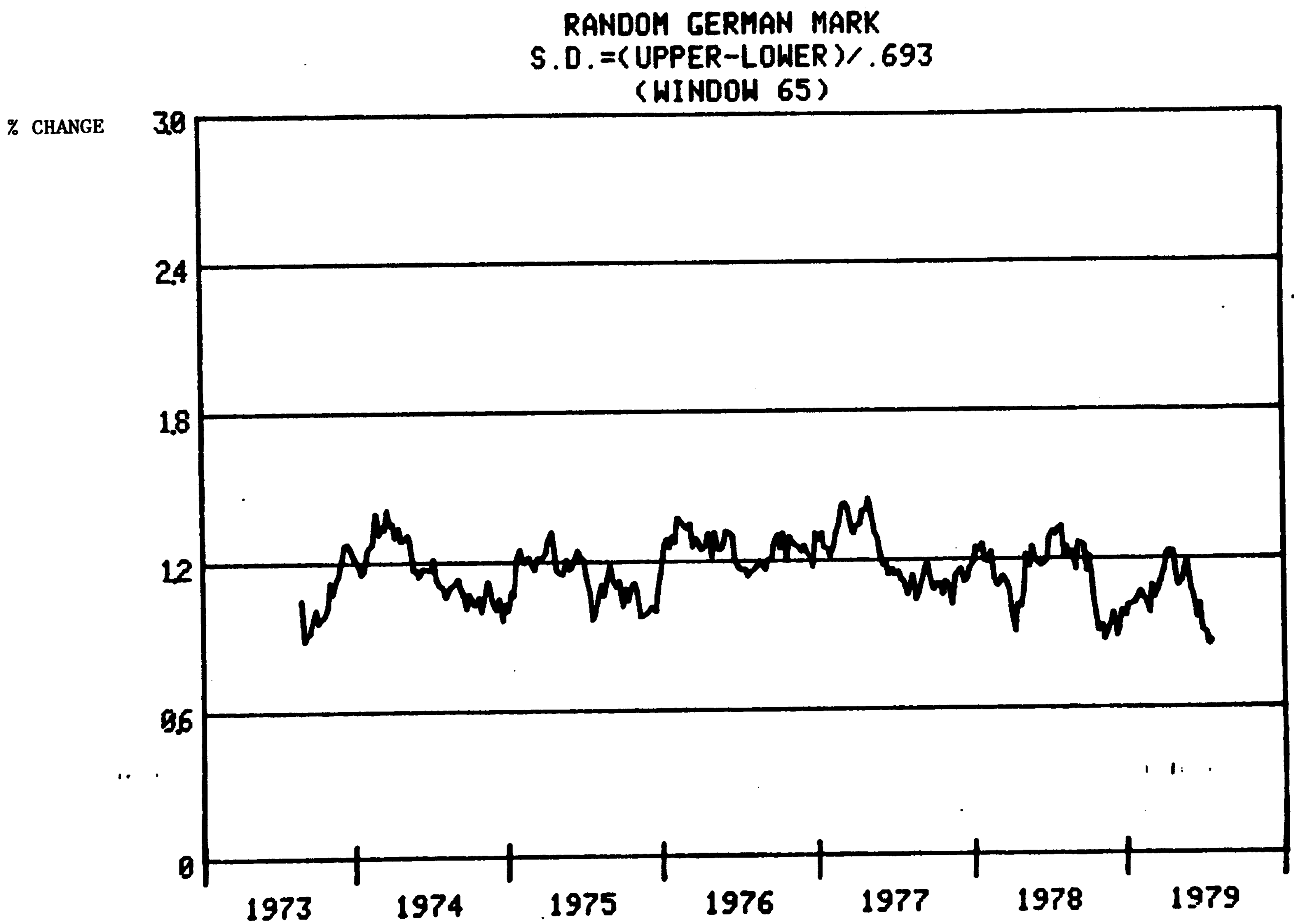
65-day window

FIGURE D8



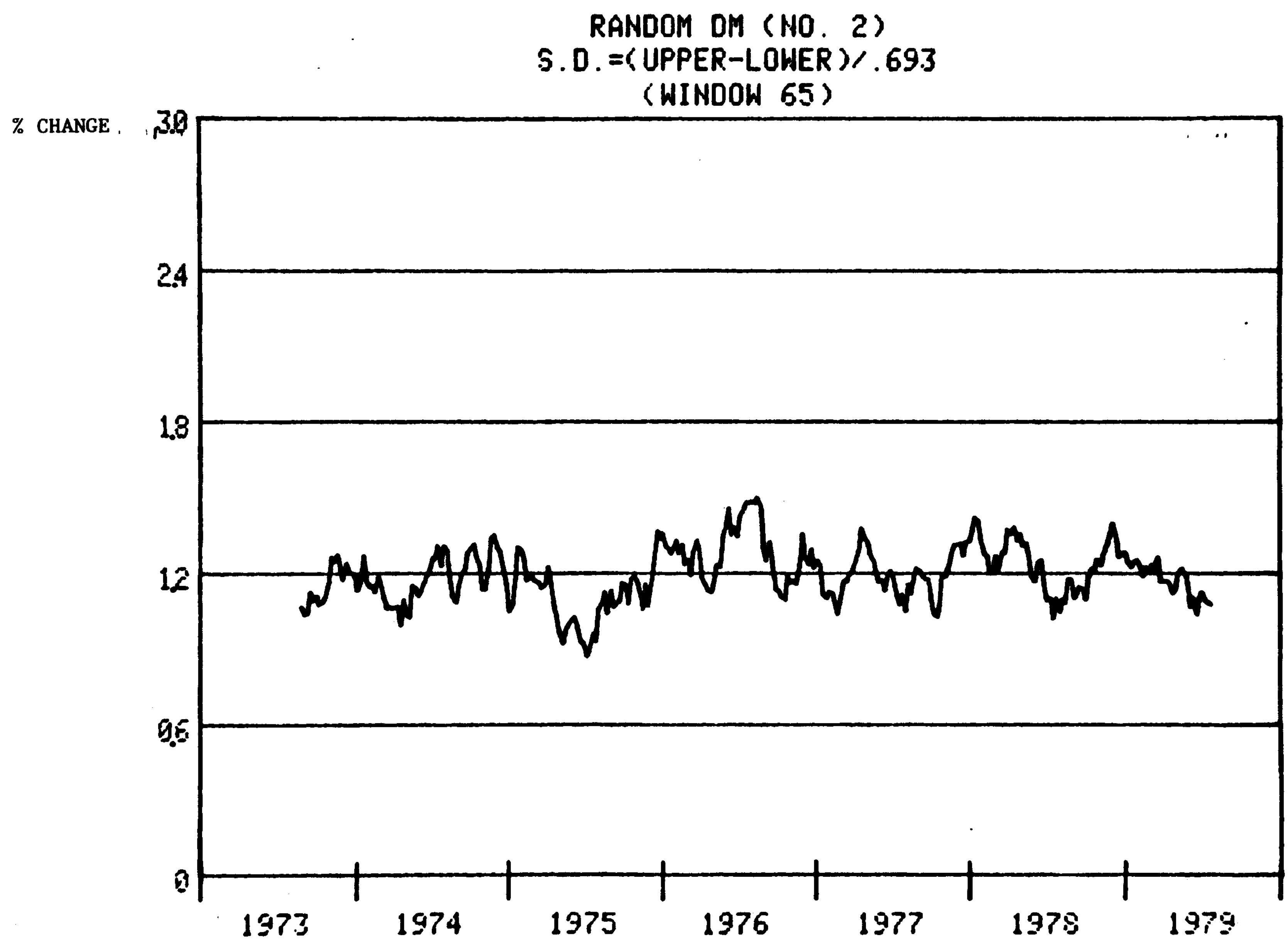
65-day window

FIGURE D10



65-day window

FIGURE D12



65-day window

FIGURE E2

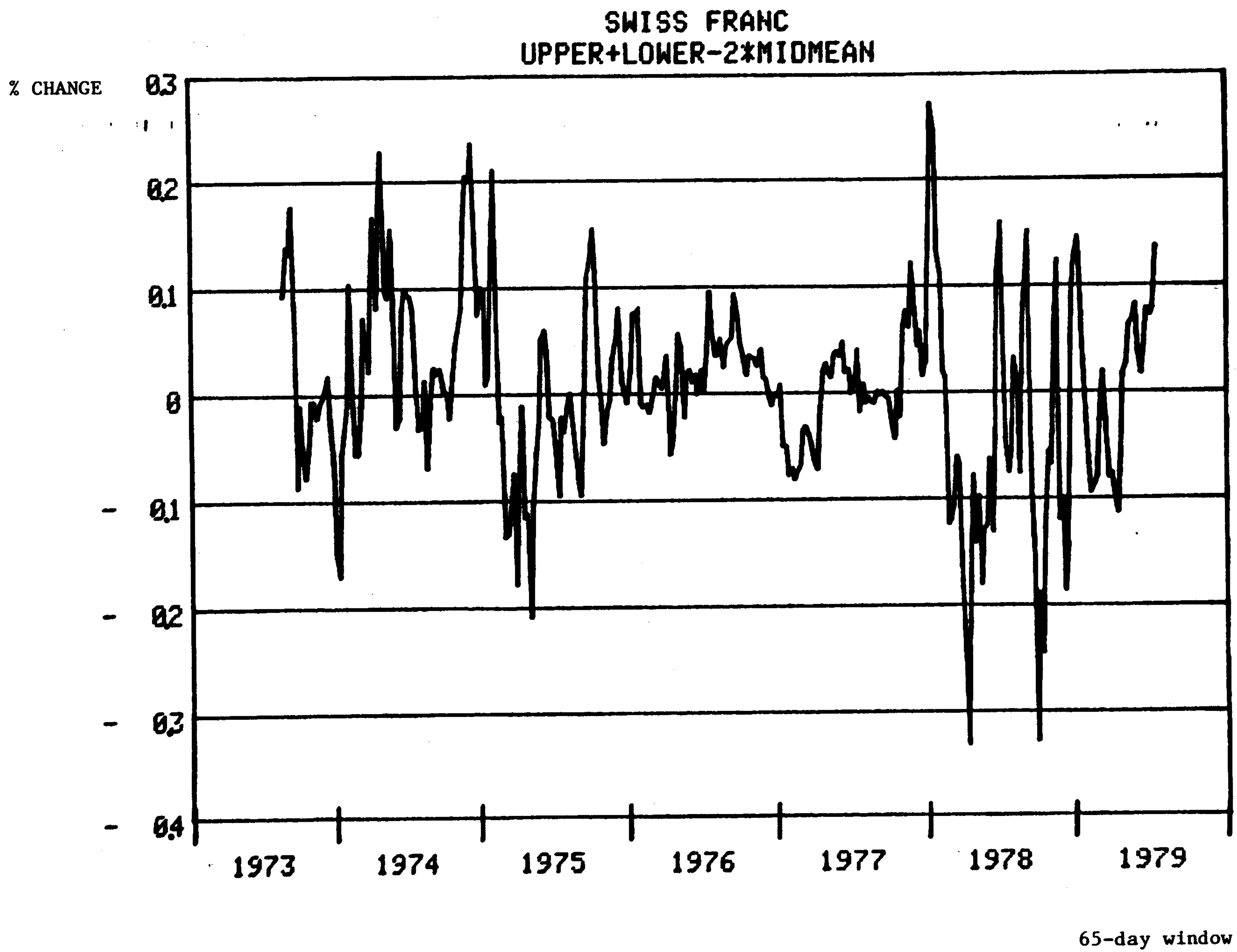
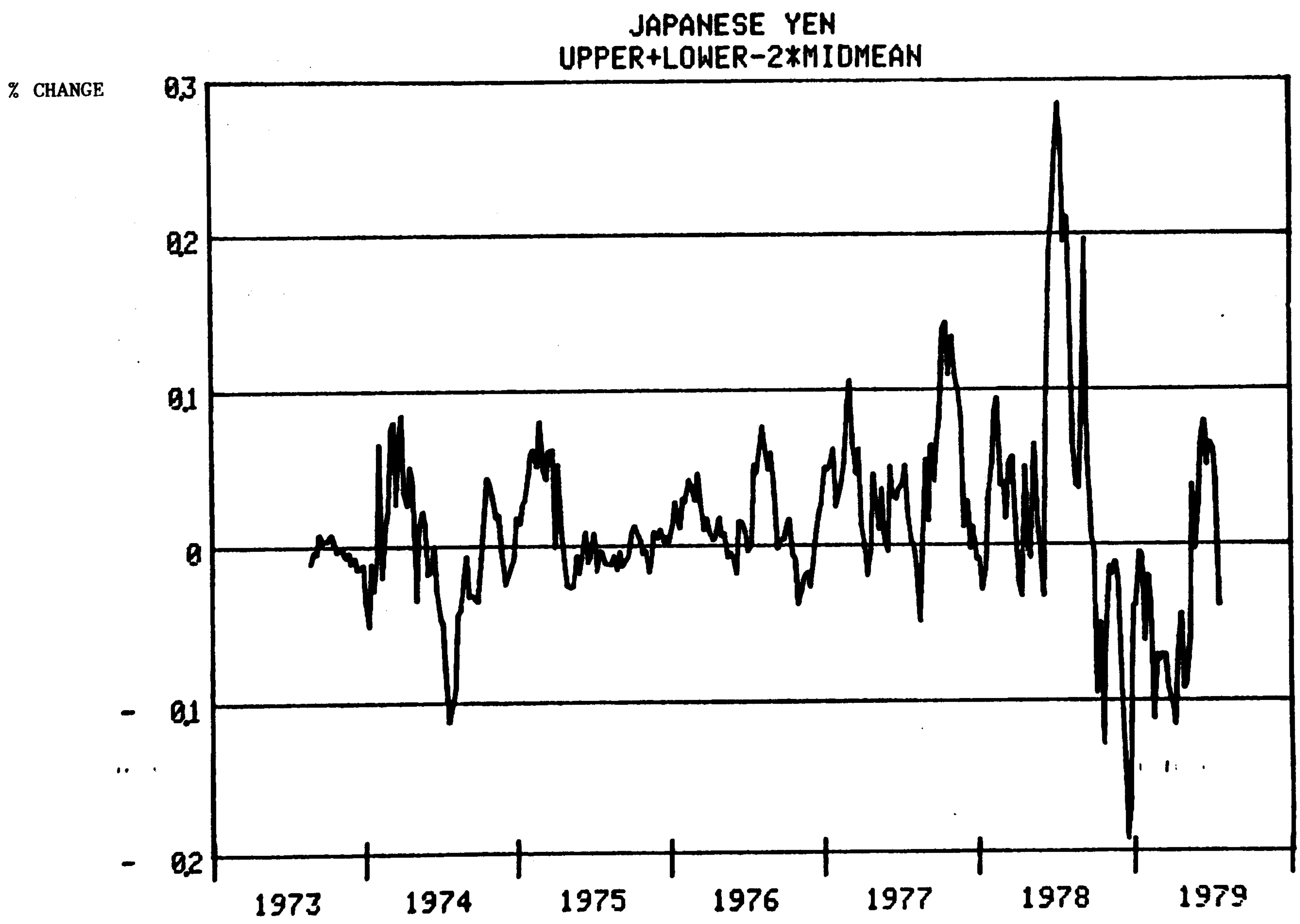
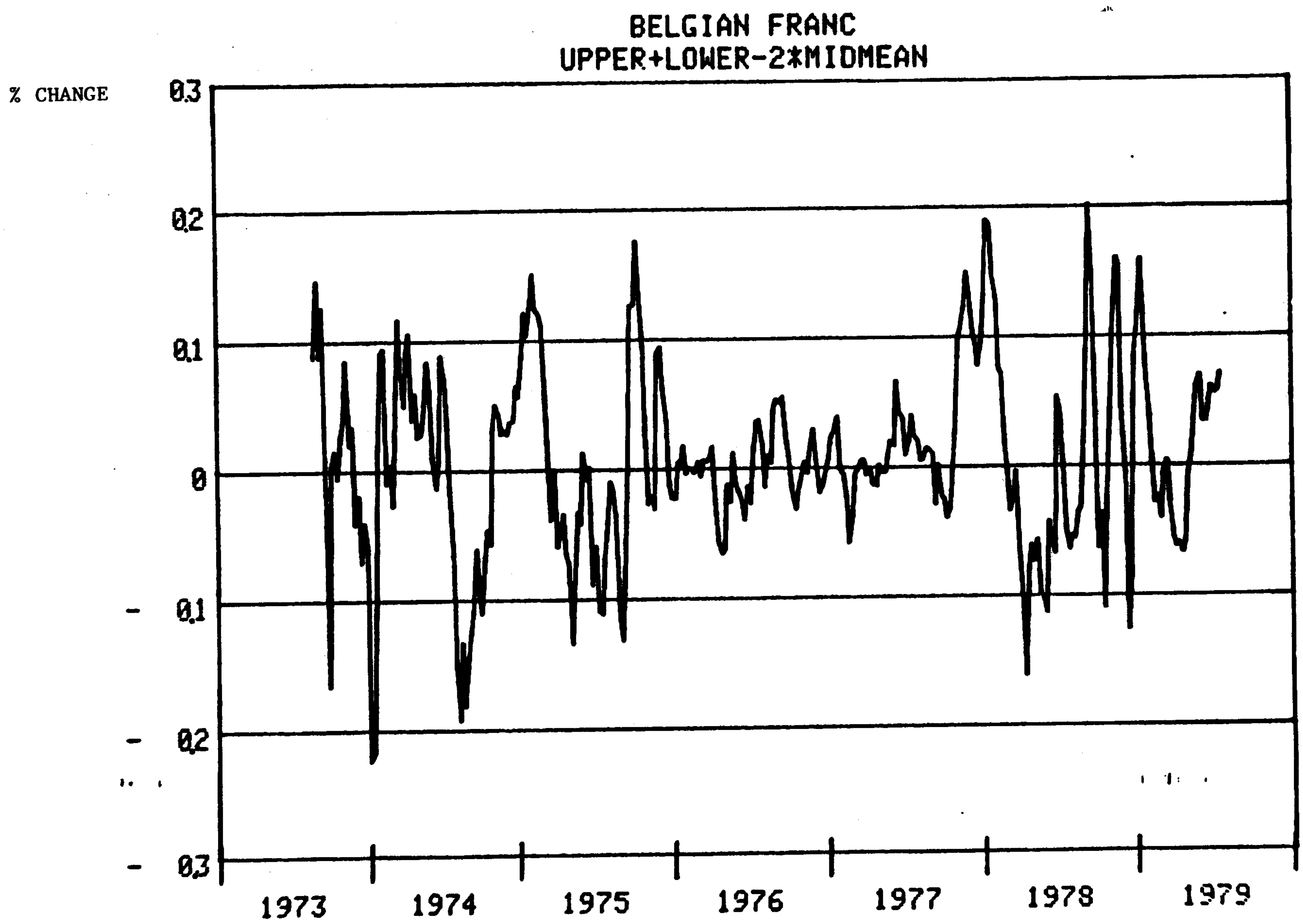

SWISS FRANC
UPPER+LOWER-2*MIDMEAN
% CHANGE
0.3
0.2
0.1
0
- 0.1
- 0.2
- 0.3
- 0.4
1973
1974
1975
1976
1977
1978
1979
65-day window

FIGURE E4



65-day window

FIGURE E6



65-day window

FIGURE E9

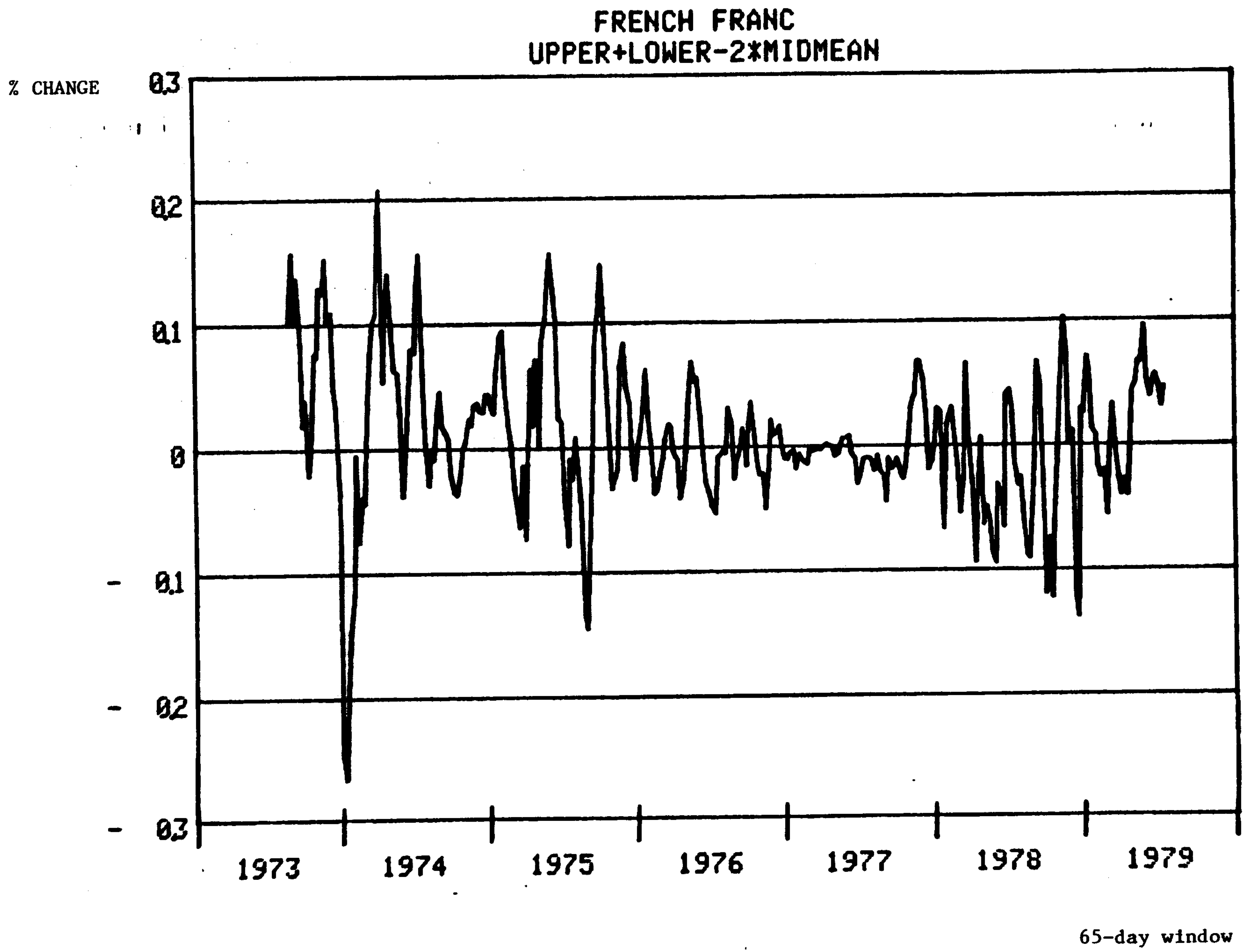

FRENCH FRANC
UPPER+LOWER-2*MIDMEAN
% CHANGE
0.3
0.2
0.1
0
- 0.1
- 0.2
- 0.3
1973
1974
1975
1976
1977
1978
1979
65-day window

TABLE 7

ESTIMATES OF THE EXPONENT α: DIRECT AGGREGATION

***** WGR *****

F LEVEL: 0.95

AGGR LEVEL	NOBS	ZSTAT	ALPHA
1	1640	3.494	1.372
2	920	3.405	1.393
5	329	3.661	1.329
10	164	3.445	1.392
20	92	2.429	1.974

F LEVEL: 0.97

AGGR LEVEL	NOBS	ZSTAT	ALPHA
1	1640	4.328	1.452
2	920	4.243	1.466
5	329	4.047	1.499
10	164	4.292	1.459
20	92	3.454	1.632

F LEVEL: 0.99

AGGR LEVEL	NOBS	ZSTAT	ALPHA
1	1640	6.636	1.575
2	920	5.938	1.629
5	329	7.193	1.535
10	164	5.933	1.629
20	92	3.906	1.960

***** SHR *****

F LEVEL: 0.95

AGGR LEVEL	NOBS	ZSTAT	ALPHA
1	1640	3.673	1.326
2	920	3.365	1.399
5	329	4.046	1.254
10	164	3.250	1.434
20	92	2.873	1.575

F LEVEL: 0.97

AGGR LEVEL	NOBS	ZSTAT	ALPHA
1	1640	4.757	1.399
2	920	4.280	1.460
5	329	4.948	1.362
10	164	3.595	1.599
20	92	3.470	1.628

F LEVEL: 0.99

AGGR LEVEL	NOBS	ZSTAT	ALPHA
1	1640	7.569	1.509
2	920	7.361	1.523
5	329	7.141	1.539
10	164	5.295	1.699
20	92	4.009	1.843

***** DUR *****

F LEVEL: 0.95

AGGR LEVEL:	1	NOBS:	1640	ZSTAT:	3.276	ALPHA:	1.426
AGGR LEVEL:	2	NOBS:	820	ZSTAT:	3.249	ALPHA:	1.435
AGGR LEVEL:	5	NOBS:	329	ZSTAT:	3.779	ALPHA:	1.302
AGGR LEVEL:	10	NOBS:	164	ZSTAT:	3.092	ALPHA:	1.492
AGGR LEVEL:	20	NOBS:	92	ZSTAT:	2.565	ALPHA:	1.752

F LEVEL: 0.97

AGGR LEVEL:	1	NOBS:	1640	ZSTAT:	4.147	ALPHA:	1.484
AGGR LEVEL:	2	NOBS:	820	ZSTAT:	4.222	ALPHA:	1.470
AGGR LEVEL:	5	NOBS:	329	ZSTAT:	4.205	ALPHA:	1.473
AGGR LEVEL:	10	NOBS:	164	ZSTAT:	3.730	ALPHA:	1.566
AGGR LEVEL:	20	NOBS:	92	ZSTAT:	4.113	ALPHA:	1.490

F LEVEL: 0.99

AGGR LEVEL:	1	NOBS:	1640	ZSTAT:	6.442	ALPHA:	1.590
AGGR LEVEL:	2	NOBS:	820	ZSTAT:	5.909	ALPHA:	1.631
AGGR LEVEL:	5	NOBS:	329	ZSTAT:	6.385	ALPHA:	1.594
AGGR LEVEL:	10	NOBS:	164	ZSTAT:	5.306	ALPHA:	1.698
AGGR LEVEL:	20	NOBS:	92	ZSTAT:	4.612	ALPHA:	1.762

***** FRA *****

F LEVEL: 0.95

AGGR LEVEL:	1	NOBS:	1640	ZSTAT:	3.993	ALPHA:	1.264
AGGR LEVEL:	2	NOBS:	820	ZSTAT:	3.793	ALPHA:	1.299
AGGR LEVEL:	5	NOBS:	329	ZSTAT:	4.044	ALPHA:	1.254
AGGR LEVEL:	10	NOBS:	164	ZSTAT:	3.327	ALPHA:	1.410
AGGR LEVEL:	20	NOBS:	92	ZSTAT:	2.742	ALPHA:	1.639

F LEVEL: 0.97

AGGR LEVEL:	1	NOBS:	1640	ZSTAT:	4.959	ALPHA:	1.361
AGGR LEVEL:	2	NOBS:	820	ZSTAT:	4.919	ALPHA:	1.390
AGGR LEVEL:	5	NOBS:	329	ZSTAT:	4.866	ALPHA:	1.373
AGGR LEVEL:	10	NOBS:	164	ZSTAT:	3.829	ALPHA:	1.543
AGGR LEVEL:	20	NOBS:	92	ZSTAT:	3.121	ALPHA:	1.741

F LEVEL: 0.99

AGGR LEVEL:	1	NOBS:	1640	ZSTAT:	9.298	ALPHA:	1.469
AGGR LEVEL:	2	NOBS:	820	ZSTAT:	7.178	ALPHA:	1.535
AGGR LEVEL:	5	NOBS:	329	ZSTAT:	9.199	ALPHA:	1.475
AGGR LEVEL:	10	NOBS:	164	ZSTAT:	5.704	ALPHA:	1.649
AGGR LEVEL:	20	NOBS:	92	ZSTAT:	3.296	ALPHA:	2.001

***** ITR *****

F LEVEL: 0.95

AGGR LEVEL:	1	NOBS:	1640	ZSTAT:	5.595	ALPHA:	1.054
AGGR LEVEL:	2	NOBS:	820	ZSTAT:	4.626	ALPHA:	1.166
AGGR LEVEL:	5	NOBS:	328	ZSTAT:	5.493	ALPHA:	1.064
AGGR LEVEL:	10	NOBS:	164	ZSTAT:	4.535	ALPHA:	1.179
AGGR LEVEL:	20	NOBS:	82	ZSTAT:	4.930	ALPHA:	1.126

F LEVEL: 0.97

AGGR LEVEL:	1	NOBS:	1640	ZSTAT:	7.800	ALPHA:	1.120
AGGR LEVEL:	2	NOBS:	820	ZSTAT:	6.236	ALPHA:	1.228
AGGR LEVEL:	5	NOBS:	328	ZSTAT:	7.533	ALPHA:	1.137
AGGR LEVEL:	10	NOBS:	164	ZSTAT:	6.938	ALPHA:	1.178
AGGR LEVEL:	20	NOBS:	82	ZSTAT:	6.075	ALPHA:	1.243

F LEVEL: 0.99

AGGR LEVEL:	1	NOBS:	1640	ZSTAT:	14.298	ALPHA:	1.245
AGGR LEVEL:	2	NOBS:	820	ZSTAT:	11.276	ALPHA:	1.335
AGGR LEVEL:	5	NOBS:	328	ZSTAT:	13.565	ALPHA:	1.266
AGGR LEVEL:	10	NOBS:	164	ZSTAT:	9.075	ALPHA:	1.427
AGGR LEVEL:	20	NOBS:	82	ZSTAT:	8.049	ALPHA:	1.484

***** NOR *****

F LEVEL: 0.95

AGGR LEVEL:	1	NOBS:	1640	ZSTAT:	2.215	ALPHA:	2.182
AGGR LEVEL:	2	NOBS:	820	ZSTAT:	2.381	ALPHA:	1.928
AGGR LEVEL:	5	NOBS:	328	ZSTAT:	2.179	ALPHA:	2.253
AGGR LEVEL:	10	NOBS:	164	ZSTAT:	2.171	ALPHA:	2.266
AGGR LEVEL:	20	NOBS:	82	ZSTAT:	2.064	ALPHA:	2.499

F LEVEL: 0.97

AGGR LEVEL:	1	NOBS:	1640	ZSTAT:	2.528	ALPHA:	2.123
AGGR LEVEL:	2	NOBS:	820	ZSTAT:	2.712	ALPHA:	1.959
AGGR LEVEL:	5	NOBS:	328	ZSTAT:	2.529	ALPHA:	2.122
AGGR LEVEL:	10	NOBS:	164	ZSTAT:	2.260	ALPHA:	2.441
AGGR LEVEL:	20	NOBS:	82	ZSTAT:	2.660	ALPHA:	2.000

F LEVEL: 0.99

AGGR LEVEL:	1	NOBS:	1640	ZSTAT:	3.214	ALPHA:	2.023
AGGR LEVEL:	2	NOBS:	820	ZSTAT:	3.213	ALPHA:	2.023
AGGR LEVEL:	5	NOBS:	328	ZSTAT:	3.505	ALPHA:	1.939
AGGR LEVEL:	10	NOBS:	164	ZSTAT:	3.202	ALPHA:	2.027
AGGR LEVEL:	20	NOBS:	82	ZSTAT:	2.829	ALPHA:	2.161

***** JAR *****

F LEVEL: .95

AGGR LEVEL	NOIS	RUN: I	II	III	IV	V	AVG
1	1646	1.02	1.02	1.02	1.02	1.02	1.02
2	823	1.21	1.30	1.33	1.31	1.22	1.27
5	329	1.42	1.98	1.55	1.50	1.59	1.61
10	164	1.53	1.80	1.49	1.42	1.62	1.57
20	82	1.71	1.93	2.44	1.45	1.58	1.82

F LEVEL: .97

AGGR LEVEL	NOIS	RUN: I	II	III	IV	V	AVG
1	1646	1.11	1.11	1.11	1.11	1.11	1.11
2	823	1.33	1.42	1.43	1.42	1.34	1.39
5	329	1.49	2.01	1.51	1.56	1.67	1.65
10	164	1.71	1.87	1.62	1.56	1.70	1.69
20	82	1.72	1.57	2.12	1.63	1.69	1.75

F LEVEL: .99

AGGR LEVEL	NOIS	RUN: I	II	III	IV	V	AVG
1	1646	1.31	1.31	1.31	1.31	1.31	1.31
2	823	1.52	1.58	1.59	1.59	1.52	1.56
5	329	1.66	1.81	1.68	1.66	1.74	1.71
10	164	1.70	1.70	1.82	1.72	1.83	1.76
20	82	1.80	1.71	1.80	1.74	1.80	1.77

***** DUR *****

F LEVEL: .95

AGGR LEVEL	NOIS	RUN: I	II	III	IV	V	AVG
1	1646	1.43	1.43	1.43	1.43	1.43	1.43
2	823	1.48	1.52	1.58	1.47	1.59	1.53
5	329	2.08	1.62	1.64	1.66	1.74	1.75
10	164	1.59	1.67	1.71	1.54	2.04	1.71
20	82	1.54	1.87	1.97	1.36	2.16	1.78

F LEVEL: .97

AGGR LEVEL	NOIS	RUN: I	II	III	IV	V	AVG
1	1646	1.48	1.48	1.48	1.48	1.48	1.48
2	823	1.56	1.57	1.63	1.48	1.65	1.58
5	329	1.87	1.70	1.82	1.71	1.87	1.79
10	164	1.70	1.70	1.68	1.63	2.09	1.76
20	82	1.64	1.85	1.94	1.43	2.09	1.79

F LEVEL: .99

AGGR LEVEL	NOIS	RUN: I	II	III	IV	V	AVG
1	1646	1.59	1.59	1.59	1.59	1.59	1.59
2	823	1.65	1.67	1.67	1.56	1.69	1.65
5	329	1.84	1.74	1.77	1.77	1.86	1.80
10	164	1.74	1.85	1.74	1.80	1.97	1.82
20	82	1.78	1.93	1.90	1.60	2.17	1.88

***** FRR *****

F LEVEL: .95

AGGR LEVEL	NOIS	RUN: I	II	III	IV	V	AVG
1	1646	1.26	1.26	1.26	1.26	1.26	1.26
2	823	1.37	1.52	1.40	1.44	1.49	1.45
5	329	1.54	1.64	1.87	1.75	1.65	1.69
10	164	1.66	1.56	2.15	2.35	1.51	1.85
20	82	1.70	1.27	2.84	1.95	1.65	1.88

F LEVEL: .97

AGGR LEVEL	NOIS	RUN: I	II	III	IV	V	AVG
1	1646	1.36	1.36	1.36	1.36	1.36	1.36
2	823	1.44	1.50	1.46	1.44	1.53	1.48
5	329	1.69	1.60	1.84	1.72	1.69	1.71
10	164	1.70	1.63	2.31	2.04	1.60	1.86
20	82	1.59	1.38	1.90	1.91	1.81	1.72

F LEVEL: .99

AGGR LEVEL	NOIS	RUN: I	II	III	IV	V	AVG
1	1646	1.47	1.47	1.47	1.47	1.47	1.47
2	823	1.61	1.66	1.59	1.62	1.64	1.62
5	329	1.79	1.76	1.83	1.81	1.75	1.79
10	164	1.77	1.78	1.81	2.02	1.81	1.84
20	82	1.83	1.69	2.04	1.90	1.96	1.88

currencies yield slope coefficients very close to (k-1)/k, intercepts near 0, and R^2 near $(k-1)^2/k^2$.) Therefore, $\sigma_k^2 = R^2\sigma_k^2 + \sigma_e^2$, so $\sigma_k^2 = \sigma_e^2/(1-R^2)$, suggesting the estimate $\hat{\sigma}_k^2 = \hat{\sigma}_e^2/(1-(\frac{k-1}{k})^2)$, where $\hat{\sigma}_e^2$ is the usual variance estimator of the N-k observations $(r_{kt+1} - \frac{k-1}{k} r_{kt})$. The values reported in Table 2 were obtained in this manner.

7. Moving Statistics

The precise definition of the k-period moving midmean MMM_t^k from Cleveland and Kleiner (1975) is as follows. For $t \geq k$, consider the set $R_t^k = \{r_t, r_{t-1}, \ldots, r_{t-k+1}\}$, and let $X_t^k = (X_1, \ldots, X_k)$ be the ordered set with the same elements as R_t^k, arranged in increasing order. Define E_t^k, the sample inverse distribution function, to be x_i at $i -.5/k$, $i = 1, \ldots, k$; to be x_1 at 0 and x_k at 1; use linear interpolation to define E_t^k elsewhere. For $0 \leq a < b \leq 1$, define $I_t^k(a,b) = \frac{1}{b-a} \int_a^b E_t^k(s)ds$. Then $MMM_t^k = I_t^k(.25,.75)$. For large k, this definition of MMM_t^k is virtually indistinguishable from the 25%-trimmed mean, but for smaller k it is preferable since it smoothly interpolates between borderline observations (i.e., observations nearest the 25th and 75th percentiles).

It would seem logical to define UMM_t^k and LMM_t^k to be I(.625,.875) and I(.125,.475) respectively. However, such a definition would make $\hat{\mu}_t(k) = MMM_t^k$ and $\hat{\sigma}_t(k) = c\ (UMM_t^k - LMM_t^k)$ rather strongly mutually dependent. For instance, if μ_t shifts, $\hat{\sigma}_t$ will be biased upwards for $\sim k/2$ subsequent periods. To avoid this, one first centers the data $(r_t \rightarrow r_t' = r_t - MMM_t^k)$ and then defines $R_t^{\prime k}$, $X_t^{\prime k}$, $E_t^{\prime k}$, and $I_t^{\prime k}$ (a,b) as before using the centered data. Then $UMM_t^k = I_t^{\prime k}(.625,.875) + MMM_t^k$ and $LMM_t^k = I_t^{\prime k}(.125,.475) + MMM_t^k$. It turns out that UMM_t^k (LMM_t^k) is slightly biased as an estimator of the 75th (25th) percentile. In independent samples from a standard normal population, the asymptotic expectation of UMM_t^k (LMM_t^k) is .693 (-.693), the 75.6 (24.4)

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FOOTNOTES

[1]Not to be confused with leptospirosis, a disease of humans and domesticated animals caused by spirochetes of the genus leptospira, or leptonecrosis, decay of the phloem tissues in plants.

[2]If $\alpha = 2$, the distribution is normal with mean d and standard deviation $c/\sqrt{2}$. If $\alpha = 1$, the distribution is Cauchy centered at d with semi-interquartile range c. If $1 < \alpha < 2$, the stable distribution is leptokurtotic and has no known elementary expression for its density or cumulative distribution function.

[3]Table 2 uses the k-period standard deviation s_k to estimate scale. Although the scale estimator $\hat{c}_k$ (described in the following paragraph of the text) might be superior for present purposes, the s_k estimates are more germane for issues discussed in Section VI.

[4]Because of the stability property. Actually, as Fama and Roll (1971) point out, a slight bias in the tables (due to rounding off) would cause the $\hat{\alpha}$ to drift downward slightly as k increases.

[5]Actually, five different random permutations were used and the results averaged in order to reduce errors. The informal arguments of the text are not affected, but if one wished to perform quantitative tests on the table entries, this averaging should be taken into account. The separate results for each random permutation (as well as results for replacing 97 by 95 and 99 in the α-estimator) are included in the Appendix; all tell basically the same story.

FIGURE A1

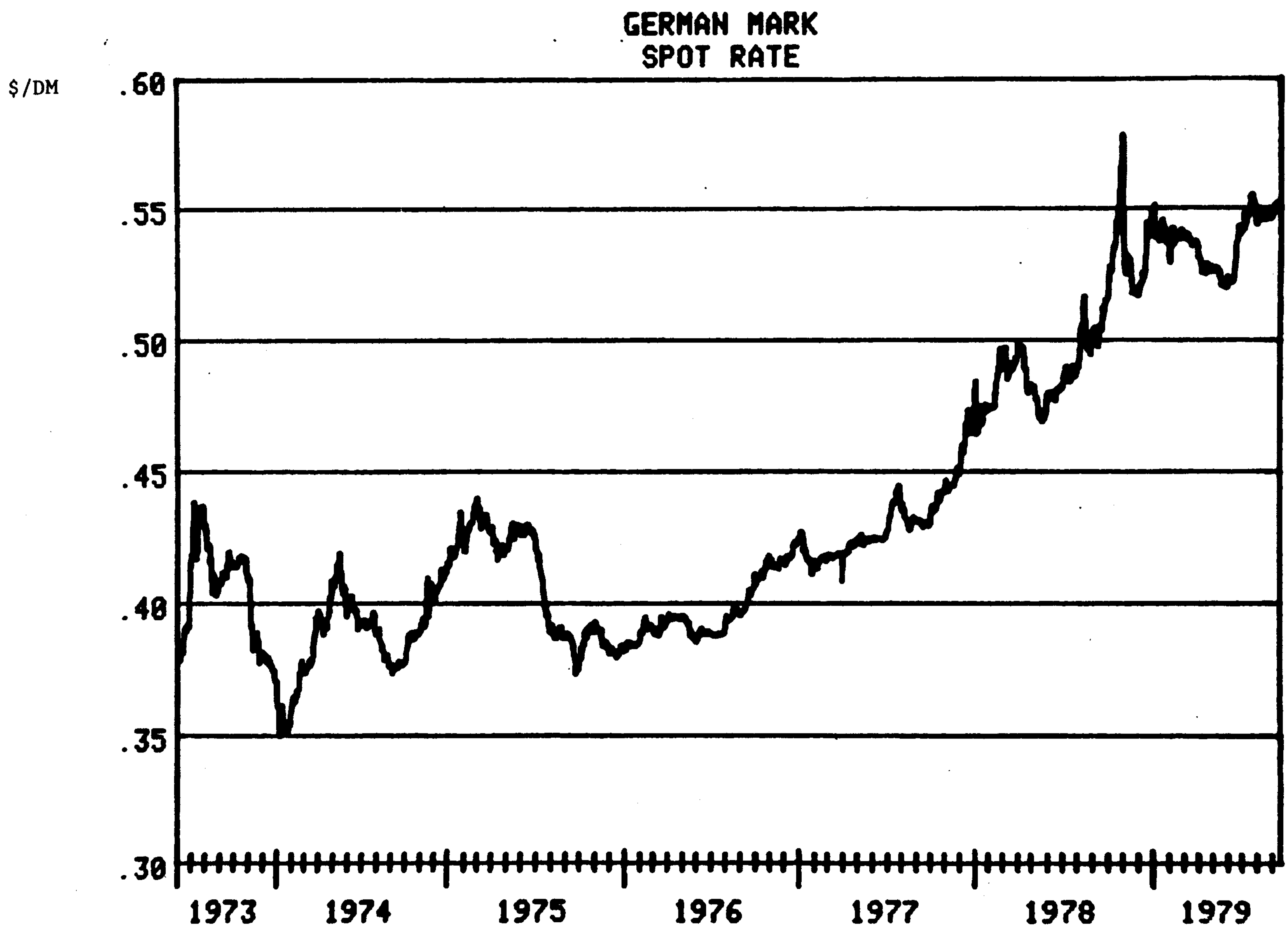
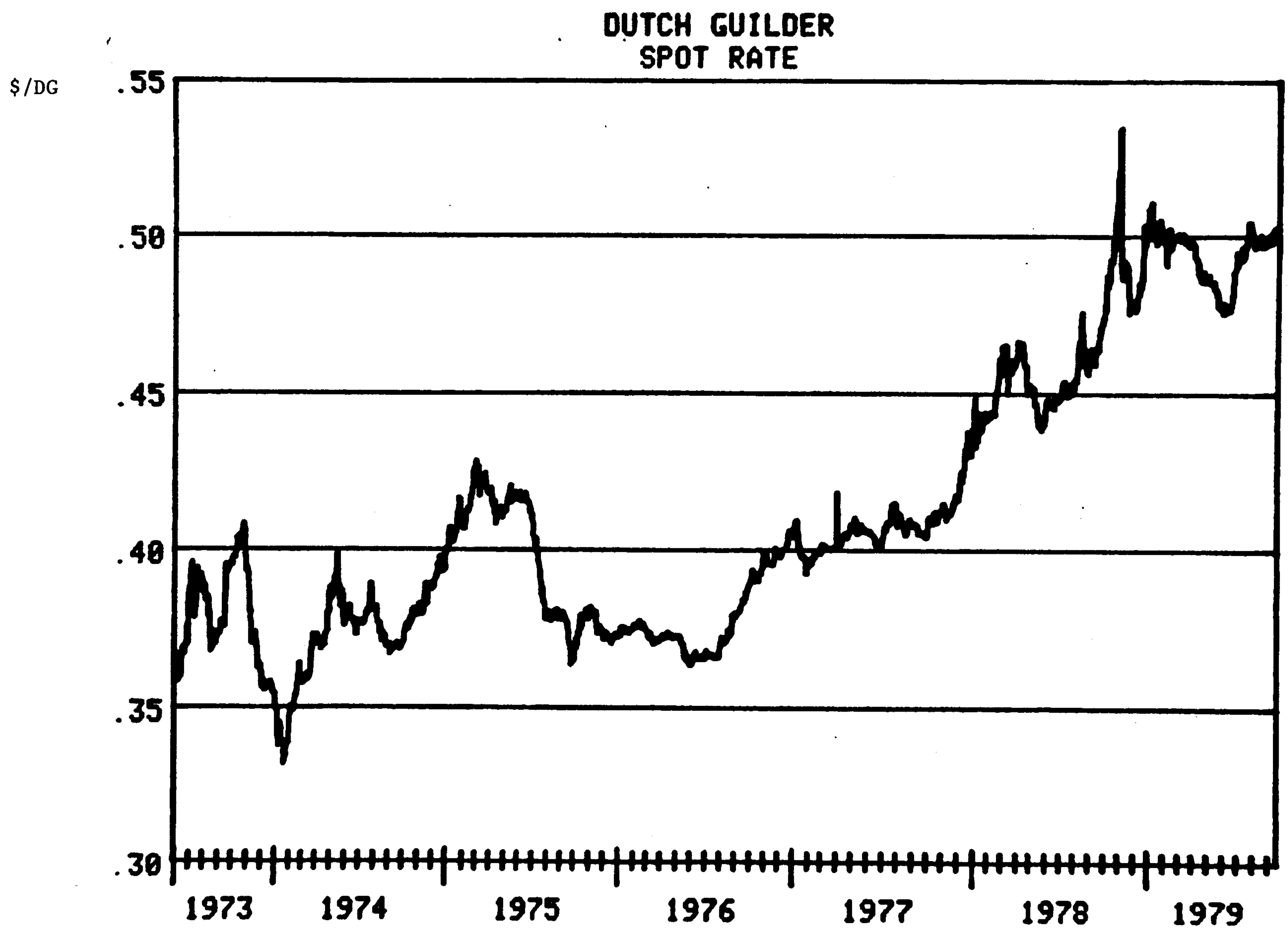

GERMAN MARK
SPOT RATE
$/DM
.60
.55
.50
.45
.40
.35
.30
1973
1974
1975
1976
1977
1978
1979

FIGURE A3



DUTCH GUILDER
SPOT RATE
$/DG
.55
.50
.45
.40
.35
.30
1973
1974
1975
1976
1977
1978
1979

FIGURE A5

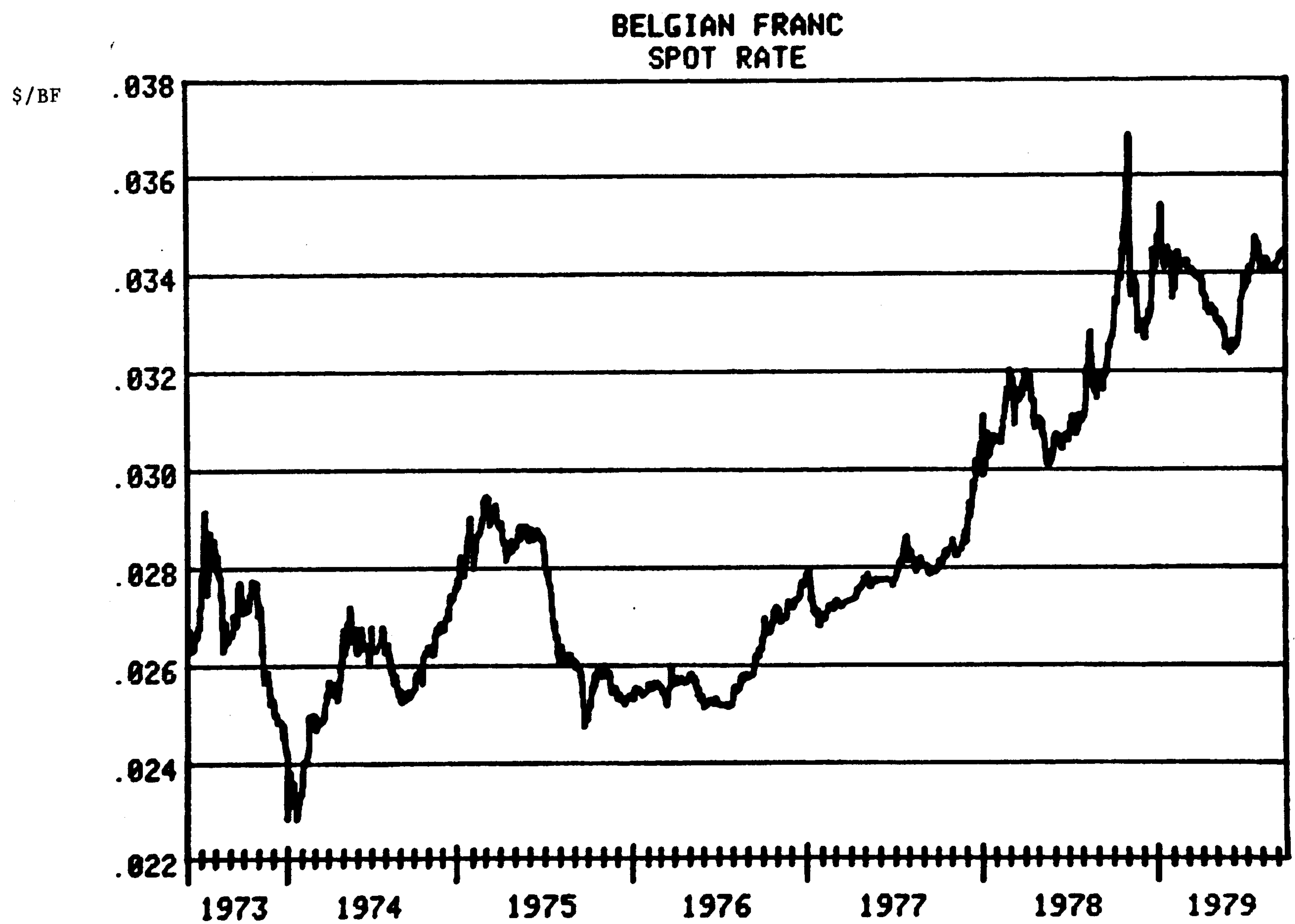
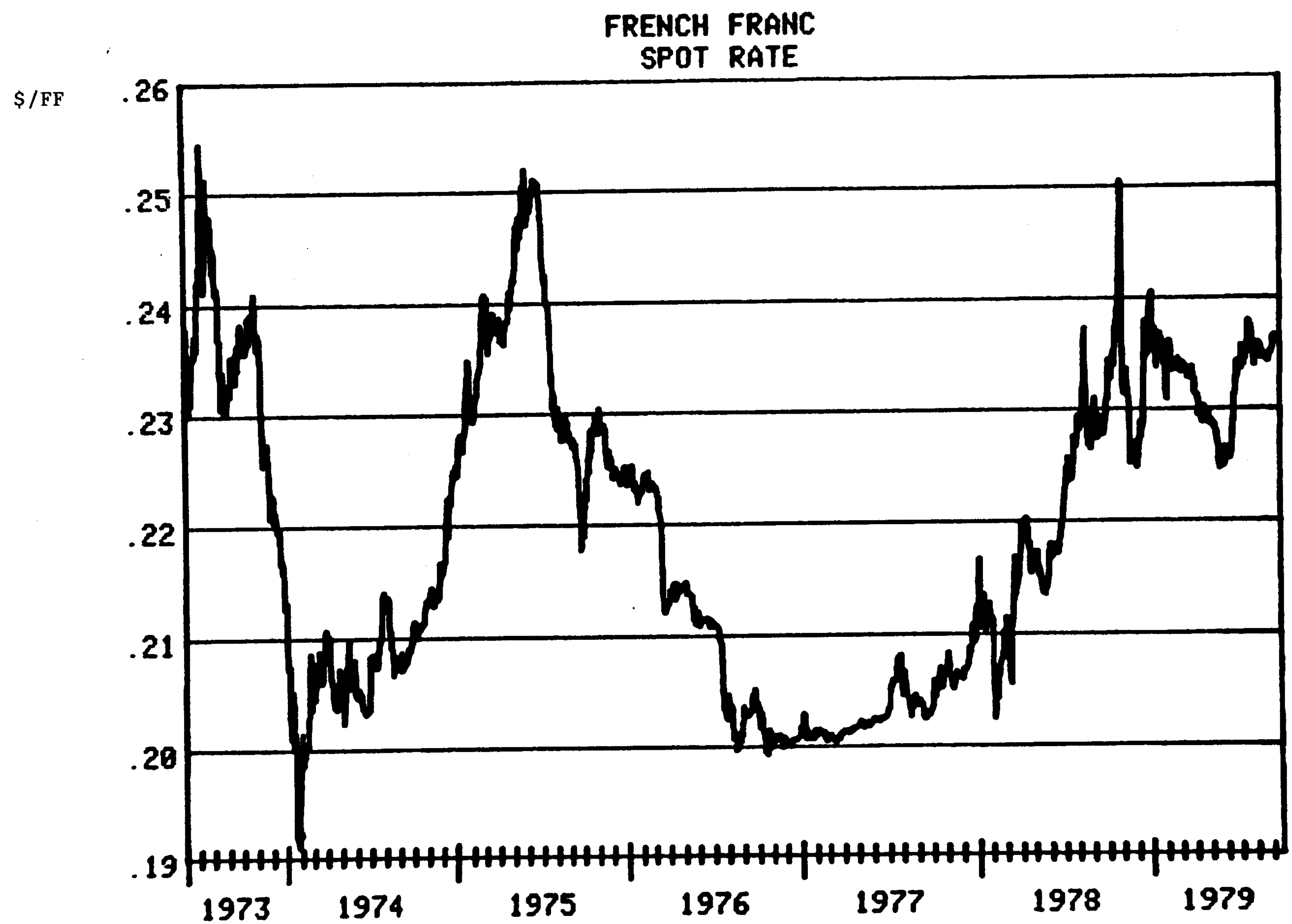

BELGIAN FRANC
SPOT RATE
$/BF
.038
.036
.034
.032
.030
.028
.026
.024
.022
1973
1974
1975
1976
1977
1978
1979

FIGURE A7



FRENCH FRANC
SPOT RATE
$/FF
.26
.25
.24
.23
.22
.21
.20
.19
1973
1974
1975
1976
1977
1978
1979