ON THE VALUE OF COMMUNICATION IN NON-CONFLICT SITUATIONS

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August 1980
Working Paper No. 173
Department of Economics
University of California, Los Angeles

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In the absence of cooperation or strategic communication, non-conflict situations occur (1) when individuals share the same basic preference orderings over all social states, or (2) when everyone prefers certain noncooperative solution points to other noncooperative solution points. In either situation, a question arises as to whether or not a particular form of noncooperative interaction exists that permits sufficient communication that unambiguously inferior allocations are weeded out of the solution set.

We shall indeed find that a Stackelberg-von Neumann-Morgenstern "perfect information" solution possesses this weeding ability while a Cournot-Nash solution does not.

Regarding the first kind of non-conflict situation, the result helps explain why individuals facing a common, "team" payoff freely communicate without acting in a secretive manner. Also, applied to a single individual's intertemporal optimization plan, the result helps explain the infrequency of the self-disciplining of consumption available through devices such as Christmas clubs, such discipline being theoretically desirable only when the decisions concerning a future consumption level would be made with more information if it were made now than if it were left for some future date.

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*The author benefitted substantially from discussions with Jack Hirshleifer, Marius Schwartz, and Bob Williams.

Regarding the second type of non-conflict situation, the result shows that underinvestment traps and Pareto nonoptimal Pigouvian tax solution cannot occur under noncooperative interaction if each individual can observe the investments preceding his own. Nevertheless the second application is of little immediate empirical relevance because once communication sufficient to achieve an unambiguous perfect information solution is permitted, the communication of certain strategic commitments should also be permitted and, we shall see, once such partially-cooperative communication exist, underinvestment traps and Pareto nonoptimal Pigouvian tax solutions again emerge except in extreme cases. However, where we find institutions effectively eliminating these partially cooperative strategies, we do find an empirical application of our general optimality result for the second kind of non-conflict situation.

I

A convenient description of the first non-conflict situation has the utilities of each of the individuals in a group represented by monotone increasing functions of a common, continuous, real-valued function of individual actions, $f(x_1,\ldots,x_n)$, with the action, x_i, of the $i^{\underline{th}}$ individual, $i = 1,\ldots,n$, chosen from a compact set of feasible actions, X_i.

If the individuals in this situation <u>independently</u> choose their actions, each selecting an x_i that maximizes f for given $x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$, the resulting, Cournot-type, solution set may obviously contain many local maxima that are not global maxima. There would be nothing to guarantee the achievement of a globally maximal value of f. The source of the problem is that there is no communication between the decision makers and therefore no resulting "coordination" of their activities.

To represent perfect communication, or perfectly "coordinated" decision making, we assume "perfect information" in the von Stackelberg-von Neumann-Morgenstern sense, meaning that the individuals choose their actions in sequence, where individual 1 chooses first and communicates his action to the rest of the individuals before they move, individual 2 chooses next, similarly communicating his chosen actions to individuals 3, ..., n, before they move, etc. The nth individual will choose an action just as he did in the game without communication, choosing an x_n in X_n that maximizes $f(x_1,\ldots,x_{n-1},x_n)$ for the previously given, chosen, values of $x_1, \ldots, x_{n-1}$. We first show that an unambiguous solution to the above game always exists.

The existence of an optimal x_n for the last mover is assured by the compactness of X_n and the continuity of f (Weierstrass theorem). There may be several such maximizing values of x_n. We shall let $x_n^*(x_1,\ldots,x_{n-1})$ represent n's solution correspondence. Since x_n^* is going to be so picked, individual n-1 will attempt to pick an x_{n-1} that maximizes, for given $x_1, \ldots, x_{n-2}$, the function $f(x_1, \ldots, x_{n-2}, x_{n-1}, x_n^*(x_1, \ldots, x_{n-2}, x_{n-1}))$. Since the value of f for a given x_{n-1} is the same regardless of the value of x_n subsequently chosen from the non-empty image set of $x_n^*(x_1, \ldots, x_{n-1})$, the actual choice by n from this set is a matter of indifference to n-1 as well as to n and therefore does not affect the choice by n-1. Momentarily assuming the existence of a maximizing solution for individual n-1, an assumption validated in the next paragraph, the maximization yields another non-empty correspondence, $x_{n-1}^*(x_1, \ldots, x_{n-2})$. Similarly, individual n-2 attempts to pick, prior to the choices of n-1 and n-2, an x_{n-2} that maximizes, for given $x_1, \ldots, x_{n-3}$, $f(x_1,\ldots, x_{n-3}, x_{n-2}, x_{n-1}^*(x_1,\ldots, x_{n-3}, x_{n-2}), x_n^*(x_1, \ldots, x_{n-3}, x_{n-2}, x_{n-1}^*(x_1, \ldots, x_{n-3}, x_{n-2})))$, etc. The solution set to this sequence of maximizations (x^*), may, of course contain several elements.

To prove that the set is non-empty, it is sufficient to prove that the response correspondences $x_{n-1}^*(\quad), \ldots, x_1^*(\quad)$ are non-empty. Again using the Weierstrass theorem, $x_{n-1}^*(\quad)$ is non-empty if the domain of the objective function variables controlled by n-1 (i.e., $(x_{n-1}, x_n^*(x_{n-1}))$) is compact. Since the domain of x_{n-1}, X_{n-1}, is compact by assumption we need only show that the range of $x_n^*(x_{n-1})$, or $\bigcup_{x_{n-1}\varepsilon X_{n-1}} x_n^*(x_{n-1})$, is compact. This is done in the following three steps. First, because $(x_{n-1},x_n^*(x_{n-1}))$ maximizes a continuous, real-valued objective function in (x_{n-1},x_n) for a given x_{n-1}, we know that $x_n^*(x_{n-1})$ is upper-semicontinuous (Berge). Second, $x_n^*(x_{n-1})$ is closed for any given value of x_{n-1}. For suppose otherwise; then the set $x_n^*(x_{n-1})$ would not contain all of its limit points.

Call one of these excluded limit points z. Since X_n is closed, $z \in X_n$. And since z is not in $x_n^*(x_{n-1})$, $f(x_1,\ldots,x_{n-1},z) < f(x_1,\ldots,x_{n-1}, x_n^*(x_{n-1}))$. From these facts, it would follow that $\lim_{x_n^v \to z} f(x_1,\ldots,x_{n-1}, x_n^v(x_{n-1})) >$ $f(x_1^*,\ldots,x_{n-1},z)$, which contradicts the continuity of f. So $x_n(x_{n-1})$ is an upper-semicontinuous function with a closed image for any given x_{n-1}. We can now complete the proof by applying the result of Nikaido (Lemma 4.5) stating that such a function defined over a compact set produces a total image set, our $\bigcup_{x_{n-1} \in X_n} x_n(x_{n-1})$, which is compact. So $x_{n-1}^*(x_1,\ldots,x_{n-2})$ is non-empty. The same procedure can be repeated to show that $x_{n-2}^*(x_1,\ldots,x_{n-2})$ is non-empty, etc.

This completes our existence proof. The theorem should not be very surprising. A similar existence theorem, using a different argument and a slightly more restrictive technology, has been produced by Goldman. In any case, we are now prepared to discuss optimality.

In general, that is, when conflict may be present, perfect information solutions are not generally jointly efficient. Standard prisoner's dilemma games illustrate this simple fact. But we are dealing here with a non-conflict situation, where the possible payoffs do not permit the redistributional opportunities presented in a standard prisoner's dilemma game.[1]

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[1] An additional, well-known difficulty with perfect information solutions is that when a later mover is indifferent between several possible actions, prior movers -- not knowing which among the later mover's indifferent actions will actually be selected -- do not really know what to do. This difficulty also disappears in non-conflict situations because, as we have already indicated, when prior movers always share the indifference of later ones, the particular actions of later movers within their solution correspondences have no effect on the utilities or decisions of prior movers.

We now prove that a perfect information solution will always achieve a joint optimum in the above, non-conflict situation.

Suppose x^* were not a global maximum point. Then there would be an x^o, say a global maximum point, such that $f(x^o) > f(x^*)$. Had individual n been presented with $x_1^o, \ldots, x_{n-1}^o$, he would have picked x_n^o (i.e., $x_n^*(x_1^o, \ldots, x_{n-1}^o) = x_n^o$); and x^o would have resulted instead of x^*. So n was not presented with $(x_1^o, \ldots, x_{n-1}^o)$. It also follows that if individual n-1 had been presented with $x_1^o, \ldots x_{n-2}^o$, he would have picked x_{n-1}^o; for $x_n^*(x_1^o, \ldots, x_{n-1}^o) = x_n^o$ and $f(x^o) > f(x^*)$. So n-1 was not presented with $x_1^o, \ldots, x_{n-2}^o$. Similarly, n-2 was not presented with $x_1^o, \ldots, x_{n-3}^o$, etc. up to individual 1. But individual 1 has no excuse. He must have not maximized his utility. For, according, to the above sequence, wherein $x^* \neq x^o$ implies $x_1^* \neq x_1^o$, if he had picked $x_1^* = x_1^o$, then x^* would have equalled x^o and his utility would have been higher. So the supposition that $x_1^* \neq x^o$ contradicts the assumption of individually rational choice. The solution must be a global maximum.

What this shows is that communication can only help individuals in a non-conflict situation. When there is only a common goal, there is no incentive to withhold information. "Coordination," meaning the sequential communication of one's own rational choice, is sufficient to obtain a global optimum in this case. The result explains why people in non-conflict, "team" situations openly communicate. At the same time, it provides a test for whether a given situation is, in fact, a team situation: If individuals are observed to act in a secretive way, withholding all of their information, then they are in a conflict situation.

Since our entire set of "individuals" can be viewed as a single "individual" acting at different moments in time, the common f being the single individual's intertemporal utility function, our result says that an individual who always knows his current and future opportunities and preferences will always choose his globally optimal plan by simple optimizing at every point in time given his past decisions. While this is inconsistent with the observation that people sometimes act so as to impose discipline on their future consumption choices, say by joining a Christmas club or by hiring personal financial managers that impose a series of consumption budget constraints over time, a little introspection indicates that individuals exhibiting such behavior also see themselves as "getting carried away" at certain points in the future, at which times they would myopically ignore certain, then-future, consequences of their then-current actions. Our theorem therefore serves to explain why we observe individuals investing to self-discipline their future consumption choices only when they would otherwise suffer from future myopia. We are intentionally avoiding here the Strotz-Pollak representation of the problem as one in which the individual has different overall objective functions, or f's, at different points in time. It is, we believe, nonsensical to assume this to hold for a single, rational individual with consistently perfect foresight. When foresight becomes blurred at times, as in the Christmas club example, the true objective function should not be altered; its just that decisions are different under different information structures. A practical advantage of our formulation is that it leads to a less ambiguous choice of time to make a decision with respect to a particular action (viz., the time of the action unless superior information exists at an earlier time).

II

We now generalize the environment by allowing basic preferences that permit general conflict. That is, we now describe preferences by allowing each individual to possess a unique, continuous, utility function, $U_i(x_1, \ldots, x_n)$, $i = 1, \ldots, n$. It is well-known that Cournot-Nash solution sets may contain points that are worse for everyone than other solution points. Indeed, there are important economic examples of such multiple noncooperative equilibria. Economic underdevelopment, for example, can be understood as a situation in which different potential producers of complementary collective-good inputs ("social overhead factors"), although able to internalize all of the incremental benefits flowing from their separate inputs, each find it unprofitable to produce their particular input only because none of the other potential suppliers have produced theirs; each of the producers would provide his input if only the others did the same (Thompson). For example, suppose a particular region requires a port in a particular location, a mine in another location and a road connecting the port with the mine before any one of these three inputs generates positive net social benefits; once any pair of these inputs are present, supplying the third is highly profitable, both privately and socially. One possible Cournot solution to the corresponding producer interaction problem has a zero supply of each of these inputs: Each potential input supplier provides none of this input when the others provide none of theirs. Of course, another possible Cournot solution point has each potential supplier providing his input and earning a positive profit. A similar example occurs when Pigouvian taxes are applied in attempting to correct external diseconomies. Since any of several possible allocations satisfying all of the marginal conditions for a local Pareto optimum qualifies as an equilibrium under a Cournot interaction with Pigouvian taxes while one of these allocations may be globally Pareto superior to others, Pigouvian taxes are generally insufficient to achieve globally Pareto optimal allocations (Thompson-Batchelder). A simple empirical example is provided by a pollution externality (occurring in Slippery Rock Creek, Pa.) in which both acid-creating coal mines and alkaline-creating limestone miners both dump their waste into a stream and thereby neutralize each other's effluent, creating no significant externality when operating together even though any one mine, appearing alone, pollutes the water and kills millions of fish (See Harder). One Cournot equilibrium has no mining on the stream and understandably heavy Pigouvian taxes. But another, Pareto superior, Cournot equilibrium has both types of mines present and no Pigouvian taxes.

The possibility of multiple equilibria of this sort is removed from these economies examples once the Cournot assumption is replaced with a von-Stackelberg-von Neuman-Morgenstern assumption of "perfect information" with respect to the investment behavior of others. In the above empirical example, if the mining investment-entry decisions were sequential, either one of our miners would be willing to enter first, as he knows that his investment will inspire the other to enter, the latter's entry being induced by the absence of positive Pigouvian taxes once the former has entered. In the slightly more complex, and much more important, underdevelopment example, any one of the input suppliers, say the port producer, will enter first because he knows that a second input supplier, say the road producer, will then enter because the latter knows that once a port and road are supplied, the miner will enter.

All this suggests that "coordination," or perfect information interaction, serves to rule out the non-conflict situation in which some non-cooperative solution outcomes are uniformly inferior to other solution outcomes.

To obtain a perfect information solution, we first have individual n maximize $\underset{x_n}{U_n}(x_1, \ldots, x_{n-1}, x_n)$, setting up a dependence of x_n^* on $x_1, \ldots, x_{n-1}$, expressed in the correspondence, $x_n^*(x_1, \ldots, x_{n-1})$; individual n-1 then, in attempting to choose an x_{n-1} that maximizes $U_{n-1}(x_1, \ldots, x_{n-1}, x_n^*(x_1, \ldots, x_{n-1}))$, may find that his maximizing solution is ambiguous in that it depends on the particular value of x_n^* chosen from the $x_n^*(x_1, \ldots, x_{n-1})$ correspondence for any given x_{n-1}. To remove this "assignment," or "selection," ambiguity, we now let n-1 decide among these x_n values. (Our motivation underlying this procedure will be discussed in Section III below.) So n-1 may choose not only among the x_{n-1} in X_{n-1}, but also among several possible values of x_n satisfying n's response correspondence, $x_n^*(x_1, \ldots, x_{n-1})$. Individual n-1's choice solution is therefore described as the set, (x_{n-1}^*, x_n^{**}), that maximizes, w-r-t x_{n-1} and x_n^*, $U_{n-1}(x_1,\ldots,x_{n-2}, x_{n-1}, x_n^*(x_1,\ldots,x_{n-1}))$, where x_n^{**} is a subset of $x_n^*(x_1,\ldots,x_{n-1}^*)$. Note that n-1's solution set, besides presenting some possible ambiguities in his own actions for earlier movers, may fail to resolve ambiguities in n's moves in that n-1 may also be indifferent between several x_n^* belonging to $x_n^*(x_1,\ldots,x_{n-1}^*)$. So n-2's solution is the set $(x_{n-2}^*, x_{n-1}^{**}, x_n^{***})$ that maximizes, w-r-t x_{n-2}, x_{n-1}^*, and x_n^{**},

$U_{n-2}(x_1,\ldots,x_{n-3},x_{n-2},x^*_{n-1}(x_1,\ldots,x_{n-2}),\ x^{**}_n(x_1,\ldots,x_{n-2},x^*_{n-1}(x_1,\ldots,x_{n-2})))$, where x^*_{n-1} is a particular value of $x^*_{n-1}(x_1,\ldots,x_{n-2})$ and x^{**}_n is a particular value of $x^{**}_n(x_1,\ldots,x^*_{n-1})$.

A proof of the non-emptiness of the solution set is a straightforward extension of the argument of Part I. x^*_n exists for the same reason. Because U_{n-1} is continuous in x_n as well as x_{n-1}, to show that n-1's solution set is non-empty, we need only show that the feasibility set $\{x_{n-1},x^*_n(x_1,\ldots,x_{n-1})\}$ for any given $x_1,\ldots,x_{n-2}$ is compact. This has already been shown in Part I, as the proof there did not depend on the assumption of identical preferences. And, as above, the argument proceeds to the first individual, who resolves any of his own indifferences with an arbitrary choice. Although there may be several elements in the solution set, the solution is unambiguous in that everyone knows the choices that will follow his own.

Our main interest is in Pareto optimality and in particular in the fact that the perfect information solution set (in contrast to a Cournot-Nash solution set) cannot contain elements that are strictly Pareto inferior to other allocations that are also in the solution set. That is, the final solution set, $x^* = (x^*_1,x^{**}_2,\ldots,x^{**\ldots*}_n)$, contains no points that are strictly Pareto inferior to other points in the solution set.

The proof follows almost immediately from the definition of a solution. Consider a point $x' \varepsilon x^*$ and another point x'' with the property that $U_i(x'') < U_i(x')$ for all i. Since individual 1's particular choice uniquely determines the particular solution, and $U_1(x') > U_1(x'')$, individual 1 would only choose x''_1 over x'_1 if $U_1(x''_1,x_2(x''_1),\ldots) \geq U_1(x')$, in which case a choice of $x_1 = x''_1$ implies a particular solution unequal to x''. And since a choice of $x_1 \neq x''_1$ also implies a particular solution unequal to x", any rational choice by 1 rules out a particular solution equal to x". So x" cannot be in the solution set.

III

While the results in Section I have some direct empirical relevance, the empirical relevance of the results of Section II is much less direct, a remark we now attempt to justify.

First, our method of removing the ambiguity for prior movers posed by the indifference of a subsequent mover is to have a prior (the one closest to the indifferent mover) "buy" -- with negligible resources -- the choice of subsequent indifferent movers when he is not similarly indifferent. Thus, if, in the underdevelopment example, the potential road producer were indifferent between building and not building, but everyone else would profit if he built the road, another producer would "pay" the indifferent potential road producer a penny to induce him to produce. In a sense, this procedure is illegitimate in that it introduces some cooperation, which was ruled out by assumption. Nevertheless, what is introduced is a naturally costless form of cooperation in that it requires no significant commitment. There is no incentive for the potential road producer, having received his penny, not to reciprocate by producing his road so no substantial commitment is required to induce him to deliver the agreed upon decision once he has received his penny. Since significant commitments are not required to enforce the agreements, the agreements can be thought of as essentially costless as long as the parties can communicate. Our solution is still -- in essence -- non-cooperative.

But once such communications are admitted, the above argument requires that substantial commitments are <u>in</u>admissible. For such commitments would generate different, somewhat cooperative, interactions (Thompson-Faith). However, we believe that it is empirically unreasonable to allow communication without also enabling the individual's certain, substantial, commitment abilities. If commitment abilities were such that the relevant parties chose -- prior to any of their actions -- their final reaction functions among all technically feasible functions, a general optimality result would emerge (Thompson-Faith). (In the underdevelopment example, the first committer, having perfect commitment ability, would say: "I will supply my input if the others do, and I will take all of the net profits except those just sufficient to induce each of the others to produce; otherwise I will not produce." The others will rationally produce their inputs, and, ruling out solution indifference by the first committer, the solution set of outputs would maximize the joint profits of the input suppliers.) While this may be an adequate abstraction for describing strategic communication within legislatures in developed nations (Thompson-Faith), a much more realistic condition for describing sequences of overhead investments in underdeveloped areas is that the relevant commitment abilities preclude such a prior selection and communication of reaction functions. The reason is simply that underdevelopment is heavily concentrated in essentially tribal areas of the world, where normal sequences of social overhead investments would require an action by each of several, relatively rarely communicating, tribes. Here, relatively little prior cooperation is present. Now suppose one tribe builds a port, and a second tribe a road. The third tribe, being the only potential mine supplier in the area, would then be willing to devote resources up to the sum of the <u>total</u> economic values of the other two inputs in order to make a commitment <u>not</u> to supply the mine unless it received this sum.

from the other suppliers.[2] Of course, if such a commitment were achieved, the first two suppliers would make net losses on their original investments and would therefore not want to provide their inputs. Even if they successfully defended themselves from this attempt, the potentially high costs of doing so could easily make their net profits negative. The problem is that while the net economic surplus from the entire operation does not justify the substantial resource cost of making and communicating a prior commitment yielding the committer this surplus, the total economic value generated by a series of prior investments does justify the resource cost of making and communicating a commitment. Any partial series of investments in complementary social overhead capital may therefore generate not a continuation to an optimum as in the simple, perfect information model we have described, but rather a sufficiently large return to making and communicating commitments in attempting to acquire the rents of prior investors that the early investment become necessarily uneconomic. The resulting lack of production of complementary forms of social overhead capital gives us a non-Cournovian, transaction-costs-type explanation of underdevelopment through the underproduction of complementary social overhead inputs. The fact that economic development has traditionally flowed to the regions

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[2]While this "hold up" problem has long been recognized by men of affairs, it has received little attention from economic theorists. The first theoretical discussions we are aware of appears in Rothenberg. Some recent applications appear in Goldberg and in Alchian, Klein, and Crawford. Our contributions are to establish the conditions under which the problem exists and apply it to produce underdevelopment traps and inefficient Pigouvian equilibria. For example, it is not clear from the previous literature that such transaction costs increase with the value of the transaction so as to preclude the transaction regardless of the value of the transaction as long as this value remains below the costs of initial strategic communication.

with the greatest cohesiveness, or communication-ability but not to non-cohesive regions (Eisenstadt) provides some empirical support for our alternative form of underdevelopment trap -- one resulting from <u>more</u> rather than less communication than is found in ordinary "perfect information" interactions.

Our theoretical solution to the problem of inefficient Pigouvian equilibria under a Cournot-Nash assumption -- i.e., having the actors behave under a perfect information interaction -- also loses its empirical relevance once we recognize that cooperative interaction, which is initially too costly, may easily become profitable after a certain number of individuals have made their investments. In our Slippery Rock mining example, one miner, having seen the other enter, may now profitably commit himself not to enter (even though it is profitable to do so) unless he is paid (for his "acid neutralizing services") the value of his entry to the other miner. The first miner, realizing that such a payment would make his profits negative, does not enter and the inefficient Pigouvian equilibrium is restored.

Let us now consider a more important, more familiar example of potentially inefficient Pigouvian equilibria. Suppose that all but one landowner in a given area build houses on their land. The owner of the remaining parcel of land in the area would then rationally commit himself to build an externality-creating factory unless he is paid not to do so <u>even though</u> the Pigouvian taxes on the factory make operating it there unprofitable. Seeing this, the other landowners would then not all have built houses there in the first place even though they should. The possibility of such underinvestment inefficiencies in the real world has, we believe, led local policy makers and large land developers all over the country to reject Pigouvian taxation in favor of quantity constraints (i.e., zoning restrictions or restrictive covenants) that effectively eliminate the possibility of our inefficient, partially cooperative solution. With such restrictions in force, and other forms of partial cooperation too costly, our second model becomes empirically relevant. For this application it tells us that -- given appropriate zoning laws or restricting covenances -- decentralized private building equilibria contain no underinvestment traps even though there is some degree of complementarity between the structures in a given area.

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