INCOME v. LEISURE

TAXES

by

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Consider an economy where, à la Mirrless [1971], consumers have identical concave utility functions $U(C,-Q)$ over consumption C and labor time Q. Consumers differ, however, in maximal skill levels. A consumer who works Q hours with skill θ produces output (income) θQ. A consumer with maximal skill level θ may work at any skill level $\hat{\theta} \leq \theta$. Suppose that θ is distributed on the interval $[0,1]$ according to the cumulative distribution function $F(\theta)$.

A tax authority is interested in raising a prescribed level of revenue in such a way as to minimize the resulting loss in social welfare. Equivalently, it seeks to maximize revenue subject to the constraint that social welfare is held at a prescribed level. The authority is assumed not to observe maximal skill level but to monitor income and/or hours worked (or, equivalently, leisure). Intuitively, the authority can do better (or at least as well) by observing and, therefore, taxing both income and leisure than by taxing only one of the two. But whether it is preferable to tax leisure or income, given that only one can be taxed perhaps because of the prohibitive cost of monitoring both, may not be so clear. In this paper we address that question.

We began the analysis by considering a government that can tax both income and leisure. We then take up leisure and income taxes individually. In each case we start by establishing first and second order conditions for a tax optimum and then consider in more detail the special case where $U(C,-Q) = C - h(Q)$ and there are no income effects on the choice of how many hours to work. This case is of interest not only because of the unambiguous comparison it provides between income and leisure taxes but because of its analytic simplicity. Both the first and second order conditions for a tax optimum are particularly straightforward in this case. More importantly, it becomes an elementary exercise in control theory to solve for the optimum tax schedule explicitly. One conclusion we draw from solving for these schedules is that, depending on shape of the distribution F, it may be desirable to treat a class of different consumers (those with different θ's) identically.[1] In contrast with this result, we find, when introducing risk aversion (that is, taking $U(C,-Q)=V(C-h(Q))$ where V is strictly concave), that not even horizontal equity is desirable; that is, through the introduction of random taxes, it may pay to treat identical consumers differently. This result differs from the usual propositions about random taxation (see Weiss [1976], for example) in that it does not depend on inducing consumers to work harder than they would under deterministic schedules.

Finally, in section IV we compare the two tax schemes. We show that as long as individuals' labor supply curves are not backward bending, income taxes are unambiguously better (or at least as good) as leisure taxes. This is true for a large class of social welfare functions, including the utilitarian and Rawlsian ones. (Actually, for all but the Rawlsian social welfare function, we require the additional hypothesis that leisure be a normal good.) Conversely, if an increase in the wage rate results in so great an increase in leisure that consumption declines, the ranking is reversed: leisure taxes are unambiguously superior.

I. INCOME AND LABOR HOURS BOTH OBSERVABLE

When income and labor hours are both monitorable and, hence, taxable the only potential barrier to obtaining a fully optimal tax equilibrium is the possibility that consumers will choose not to work at their maximal skill levels. This incentive is present unless more skilled individuals have an opportunity to achieve higher utility levels. Formally, tax equilibria are limited to consumption-labor schedules $(C(\theta),Q(\theta))$ having the property

$$(1) \quad \frac{d}{d\theta} U(C(\theta),-Q(\theta)) \geq 0^{2}$$

Tax revenue from an individual of skill level θ, $R(\theta)$, must satisfy the aggregate feasibility constraint

$$(2) \quad \int_0^1 R(\theta)dF(\theta) \leq \int_0^1 [\theta Q(\theta) - C(\theta)]dF(\theta)$$

As Mirrlees [1980], Dasgupta and Hammond [1978] and Maskin [1979] have noted, (1) will be satisfied when tax revenue is maximized subject to a <u>maxi-min</u> welfare constraint (in fact $\frac{d}{d\theta} U(C(\theta),-Q(\theta)) = 0$), that is, when one solves the program

$$(3) \quad \begin{cases} \max \int_0^1 R(\theta)dF(\theta) \\ \\ \text{subject to } \min U(C(\theta),-Q(\theta)) \geq \bar{U} \text{ and condition (2).} \end{cases}$$

For example, if $U(C,-Q) = C - h(Q)$, implying that there are no income effects on labor supply, program (3) is solved by taking $\tilde{Q}(\theta)$ so that $\theta = h'(\tilde{Q}(\theta))$ for each θ and $C(\theta)$ so that $U(\tilde{C}(\theta),-Q(\theta)) = \bar{U}$. Then $\tilde{R}(\theta) = \theta\tilde{Q}(\theta) - \tilde{C}(\theta)$. Tax revenue, $\tilde{R}$, can therefore be expressed as the following function of Q and Y where Y is income:

$$R = \hat{R}(Q,Y) = \begin{cases} \tilde{R}(\theta), \text{ if } (Q,Y) = (\tilde{Q}(\theta), \theta\tilde{Q}(\theta)) \text{ for some} \\ \infty, \text{ otherwise} \end{cases}$$

As Mirlees also points out, however, utility will be a declining function of θ when leisure is a normal good, if social welfare is utilitarian, that is, when one solves the program,

$$(4) \quad \begin{cases} \max \int_0^1 R(\theta)dF(\theta) \\ \text{subject to } \int_0^1 U(C(\theta),-Q(\theta))dF(\theta) \geq \bar{U} \\ \text{and } \int_0^1 C(\theta)dF(\theta) = \int_0^1 [\theta Q(\theta)-R(\theta)]dF(\theta). \end{cases}$$

In such cases, therefore, only a second best tax scheme is possible. Indeed, as Maskin [1979] shows, the <u>only</u> social welfare functions for which a full optimum can be achieved regardless of preferences are of the maxi-min variety (L-shaped indifference curves). Nonetheless in the special "no income effects" case a redistribution of wealth has no effect on the utilitarian sum of utilities. Thus the maxi-min full optimum is also the utilitarian optimum.

II. LEISURE TAXES

If the authority can observe hours worked only, it must set a tax schedule as a function of Q, $R = R_L(Q)$. Given schedule $R = R_L(Q)$, let $\langle Q(\theta), R(\theta)\rangle = \langle Q(\theta), R_L(Q(\theta))\rangle$ be the utility maximizing choice of a consumer with maximal skill level θ. With a leisure tax each consumer has an incentive to work at his maximal skill. Then a necessary condition for $\langle Q(\theta), R(\theta)\rangle$ to be optimal for skill level θ is that $\hat{\theta} = \theta$ maximizes $U(\theta Q(\hat{\theta})-R(\hat{\theta}),-Q(\hat{\theta}))$. Writing the marginal utilities of consumption and leisure as U_1 and U_2, the first order condition of the maximization is

$$(5)\quad \frac{d}{d\hat{\theta}} U(\theta Q(\hat{\theta})-R(\hat{\theta}),-Q(\hat{\theta}))$$
$$= U_1(\theta Q(\hat{\theta})-R(\hat{\theta}),-Q(\hat{\theta}))\,[\theta Q'(\hat{\theta})-R'(\hat{\theta})] - U_2(\theta Q(\hat{\theta})-R(\hat{\theta}),-Q(\hat{\theta}))\,[Q'(\hat{\theta})]$$
$$= 0 \quad \text{at } \hat{\theta} = \theta \text{ for all } \theta.$$

Solving for $R'(\theta)$ we obtain

$$(6)\quad R'(\theta) = [\theta - \frac{U_2(\theta Q-R,-Q)}{U_1(\theta Q-R,-Q)}]Q'(\theta)$$

Given (5), the rate at which utility varies with θ when leisure is taxed is

$$\frac{d}{d\theta} U(\theta Q(\theta)-R(\theta),-Q(\theta)) = Q(\theta)U_1 > 0$$

Since utility is constant in the full optimum the latter is never <u>attainable</u> when only leisure is taxable. More precisely, we have:

<u>Proposition 1:</u> If only a leisure tax is available, the authority cannot raise as high a level of revenue as if both hours and income are taxable.

To establish conditions under which the first order condition is also sufficient we use (6) to rewrite the derivative of $U(\theta Q(\hat{\theta})-R(\hat{\theta}),-Q(\hat{\theta}))$ as

$$(7)\quad \frac{d}{d\theta} U(\theta Q(\hat{\theta})-R(\hat{\theta}),-Q(\hat{\theta})) =$$

$$U_1(\theta Q-R,-Q)[(\theta-\hat{\theta}) + (\frac{U_2(\hat{\theta}Q-R,-Q)}{U_1(\hat{\theta}Q-R,-Q)} - \frac{U_2(\theta Q-R,-Q)}{U_1(\theta Q-R,-Q)})]Q'$$

Differentiating (7) with respect to θ and evaluating at $\hat{\theta} = \theta$, we obtain the second order condition

$$(8)\quad \frac{d^2U}{d\theta^2} = Q'[-(U_1)^2+U_1U_{12}Q-U_2U_{11}Q]/U_1 \leq 0$$

Inequality (8) is a condition necessary for $\hat{\theta} = \theta$ to solve the consumer's maximization problem. This inequality is also sufficient if $U(\theta Q(\hat{\theta})-R(\hat{\theta}),-Q(\hat{\theta}))$ is non-decreasing for $\hat{\theta} < \theta$ and non-increasing for $\hat{\theta} > \theta$.

Such is the case when Q' does not vanish and $U \in \mathcal{U}^*$, where $\mathcal{U}^*$ is the class of all concave twice differentiable, monotonically increasing utility functions for which the ordinary labor supply curve is upward sloping (or, more precisely, not backward bending). To see this consider any $(C^*,Q^*) > (0,0)$ and choose a "wage" w and lump sum tax T such that (C^*,Q^*) is optimal for a consumer facing the budget constraint $C = wQ - T$. That is, (C^*,Q^*) satisfies the first order condition

$$(9)\quad wU_1(wQ^*-T,-Q^*) - U_2(wQ^*-T,-Q^*) = 0$$

Differentiating (9) with respect to w we obtain

$$U_1 + wU_{11}(Q^*+w\frac{dQ^*}{dw}) - 2wU_{12}\frac{dQ^*}{dw} - U_{12}Q^* + U_{22}\frac{dQ^*}{dw} = 0$$

Solving for $\frac{dQ^*}{dw}$ and substituting for w from (9) yields

$$(10) \quad \frac{dQ^*}{dw} = U_1[-(U_1)^2 + U_1U_{12}Q^* - U_1U_{12}Q^*]/[U_2^2U_{11} - 2U_1U_2U_{12} + (U_1)^2U_{22}]$$

The denominator on the right hand side of (10) is negative since U is concave. Then if $U \in \mathcal{U}^*$, so that $dQ^*/dw \geq 0$, we must have

$$-(U_1)^2 + U_1U_{12}Q - U_2U_{11}Q \leq 0 \text{ everywhere.}$$

It follows immediately that for any $\hat{\theta}$, $\bar{Q}$, $\bar{R}$

$$\frac{\partial}{\partial\hat{\theta}}\left[\theta - \hat{\theta} + \frac{U_2(\hat{\theta}\bar{Q}-\bar{R},-\bar{Q})}{U_1(\hat{\theta}\bar{Q}-\bar{R},-\bar{Q})} + \frac{U_2(\hat{\theta}\bar{Q}-\bar{R},-\bar{Q})}{U_1(\hat{\theta}\bar{Q}-\bar{R},-\bar{Q})}\right]$$

is less than or equal to zero. Then, from (7)

$$(\theta - \hat{\theta})\frac{dU}{d\hat{\theta}}(\hat{\theta}Q(\hat{\theta})-R(\hat{\theta}),-Q(\hat{\theta})) \geq 0 \ .$$

We therefore have

<u>Proposition 2</u>: If $U \in \mathcal{U}^*$ (upward sloping labor supply) then $Q'(\hat{\theta}) \geq 0$ is a necessary and sufficient condition for the existence of a function $R(\theta)$ such that $\hat{\theta} = \theta$ maximizes $U(\theta Q(\hat{\theta})-R(\hat{\theta}),-Q(\hat{\theta}))$.

Of course, one subclass of $\mathcal{U}^*$ of particular interest consists of the no income effect utility functions $U(C,-Q) = C - h(Q)$. In this case, equation (6), defining $R'(\theta)$ becomes

$$(11) \quad R'(\theta) = [\theta-h'(Q(\hat{\theta}))]Q'(\hat{\theta})$$

Inequality (8) becomes

(12) $Q'(\ ) \geq 0$

Integrating (11) we find

(13) $R(\theta) = \int_0^\theta [t - h'(Q(t)]Q'(t)\, dt + R_0$

It is now a straightforward matter to solve for the optimal tax schedule for a particular social welfare function. We will go through this exercise for the maxi-min and utilitarian functions. When social welfare is maxi-min, one has the program

(14)
$$\begin{cases} \max\limits_{(Q(\cdot) \geq 0} \int_0^1 \{\int_0^\theta [tQ'(t)-h'(Q(t))Q'(t)]dt + R_0\}dF(\theta) \\ \text{subject to } Q'(\theta) \geq 0 \\ \text{and } U(-R_0,0) = -R_0 - h(0) \geq \bar{U} \\ \text{(Utility of a consumer with zero skill is at least } \bar{U}). \end{cases}$$

Scale the utility function so that $h(0) = 0$. Then, integrating the maximand in (14) by parts, we obtain

(15)
$$\begin{cases} \max\limits_{Q(\cdot) \geq 0} \int_0^1 [J(\theta)Q(\theta) - h(Q(\theta))]dF(\theta) + R_0 \\ \text{subject to } R_0 = -\bar{U} \text{ and } Q'(\theta) \geq 0 \\ \text{where } J(\theta) = \dfrac{F(\theta) + F'(\theta) - 1}{F'(\theta)}\ . \end{cases}$$

Suppose that $h'(0) = 0$ and for some Q_0 $\lim\limits_{Q \to Q_0} h'(Q) = \infty$. Then if one solves program (15), ignoring the "incentive compatibility" constraint $Q' \geq Q$, one obtains the solution

$$(16) \quad \begin{cases} J(\theta) = h'(Q(\theta)), & \text{for all } \theta \text{ such that } J(\theta) \geq 0 \\ Q(\theta) = 0, & \text{for all } \theta \text{ such that } J(\theta) < 0. \end{cases}$$

Differentiating $J(\theta) = h'(Q(\theta))$ with respect to θ, we have $J'(\theta) = h''(Q(\theta))Q'(\theta)$. By assumption $U(C,-Q)$ is concave so that $h''(Q)$ is positive. Therefore, if $J(\theta)$ is increasing in θ, (16) automatically satisfies the condition $Q'(\theta) \geq 0$ and hence solves the leisure tax problem. More precisely, if $J(\theta)$ is monotonically increasing in θ, then there exists a unique θ_0 such that $J(\theta_0) = 0$. Then the revenue maximizing labor supply schedule, $Q^*(\theta)$, is given by

$$(17) \quad \begin{cases} Q^*(\theta) = 0, & \theta \geq \theta_0 \\ J(\theta) = h'(Q^*(\theta)), & \theta > \theta_0. \end{cases}$$

<u>Example 1</u>: Suppose that $h(Q) = Q^2$ and that θ is distributed uniformly on $[0,1]$.

Then, $J(\theta) = 2\theta - 1$, and so, from (17), the optimal Q^* satisfies

$$Q^*(\theta) = \begin{cases} 0 & , \ \theta \leq \frac{1}{2} \\ \theta - \frac{1}{2} & , \ \theta \geq \frac{1}{2} \end{cases}$$

From (13), $R^*(\theta) = \frac{1}{8} - \frac{1}{2}(1-\theta)^2 + R_0$. Combining the expressions for Q^* and R^*, we obtain the optimal leisure tax,

$$R_L^*(Q) = R_0 + \frac{1}{2}(Q-Q^2).$$

Notice that the marginal tax rate on hours worked declines for all Q. Moreover since $Q^*(1) = \frac{1}{2}$ the most skilled have a zero marginal tax rate. It is of some interest to compare the revenue generated from this tax with that generated from an optimum tax on leisure and income both. In the former case, we have

$$\int_0^1 R^*(\theta)dF = \int_{\frac{1}{2}}^1 [\frac{1}{8} - \frac{1}{2}(1-\theta)^2]d\theta + R_0 = \frac{1}{24} + R_0$$

In the latter case the tax authority can extract all the income produced by the workers without sacrificing efficiency. Then $Q(\theta) = \frac{1}{2}\theta$, $R(\theta) = \frac{1}{4}\theta^2 + R_0$ and total revenue is

$$\int_0^1 \tilde{R}(\theta)dF = \int_0^1 \frac{1}{4}\theta^2 d\theta + R_0 = \frac{1}{6} + R_0$$

Thus, in the case where $R_0 = 0$ (no lump sum subsidy or tax) revenue is quadrupled by introducing a tax on income as well as leisure.

Returning to the solution of the general program, (14), we note that $J(\theta)$ will be monotonically increasing if the density $F'(\theta)$ does not drop off too rapidly ($J''(\theta)$ has the same sign as $(F')^2 + F''(1-F)$). If $J(\theta)$ is not monotonically increasing, then the constraint $Q' \geq 0$ is sometimes binding. The optimal labor schedule can be conveniently found as an application of the maximum principle. From (15) we have the Hamiltonian

$$(18) \quad H(\theta,Q,\lambda,\nu,R_0) = J(\theta)Q(\theta)-h(Q(\theta)) + R_0 - \nu R_0 - \lambda Q'(\theta)$$

The complementary slackness conditions yield

$$\lambda(\theta) \geq 0 \text{ and } \lambda(\theta) > 0 \rightarrow Q'(\theta) = 0.$$

From the first order conditions

$$\lambda'(\theta) = -\frac{\partial H}{\partial Q} = -[J(\theta) - h'(Q(\theta))].$$

Suppose, for simplicity, that $J(\theta)$ has only one non-monotonic interval as in Figure 1. In that case $\lambda' = 0$ cannot hold everywhere because it is incompatible with $Q'(\theta) \geq 0$. Thus, there exists an interval $[\theta_1, \theta_2]$ where $\lambda > 0$, and hence $Q'(\theta) = 0$. Since $\lambda(\theta) = 0$ for $\theta < \theta_1$, and $\theta > \theta_2$ we must have $\int_{\theta_1}^{\theta_2} \lambda'(\theta)d\theta = 0$. Thus the interval $[\theta_1, \theta_2]$ is determined by the condition

$$(19) \qquad \int_{\theta_1}^{\theta_2} [J(\theta)-h'(Q(\theta))]dF = 0$$

Because Q is constant in $[\theta_1, \theta_2]$, (19) can be rearranged to obtain

$$J(\theta_1) = \frac{\int_{\theta_1}^{\theta_2} J(\theta)dF}{F(\theta_2) - F(\theta)_1} = J(\theta_2).$$

More generally, for any $J(\theta) = \theta - (1-F(\theta))/F'(\theta)$ there is a unique partition of the unit interval $[0,\theta_0]$, ... $[\theta_i, \theta_{i+1}]$, ... $[\theta_{2n}, 1]$, satisfying

(A) $J(\theta_0) = 0$ and for all θ, $\theta < \theta_0 \rightarrow \int_{\theta}^{\theta_0} J(x)dF(x) \leq 0$

(B) $J(\theta)$ is increasing on $\bigcup_{i=0}^{n} [\theta_{2i}, \theta_{2in}]$ and $J(\theta_{2i-1}) = J(\theta_{2i})$ $i = 1, \ldots, m$

(C) For all $\theta \in [\theta_{2i-1}, \theta_{2i}]$ $i = 1, \ldots, m$

$$\int_{\theta_{2i-1}}^{\theta} [J(x) - J(\theta_{2i-1})]dF(x) \geq 0$$

and the constraint is binding at $\theta = \theta_{2i}$.

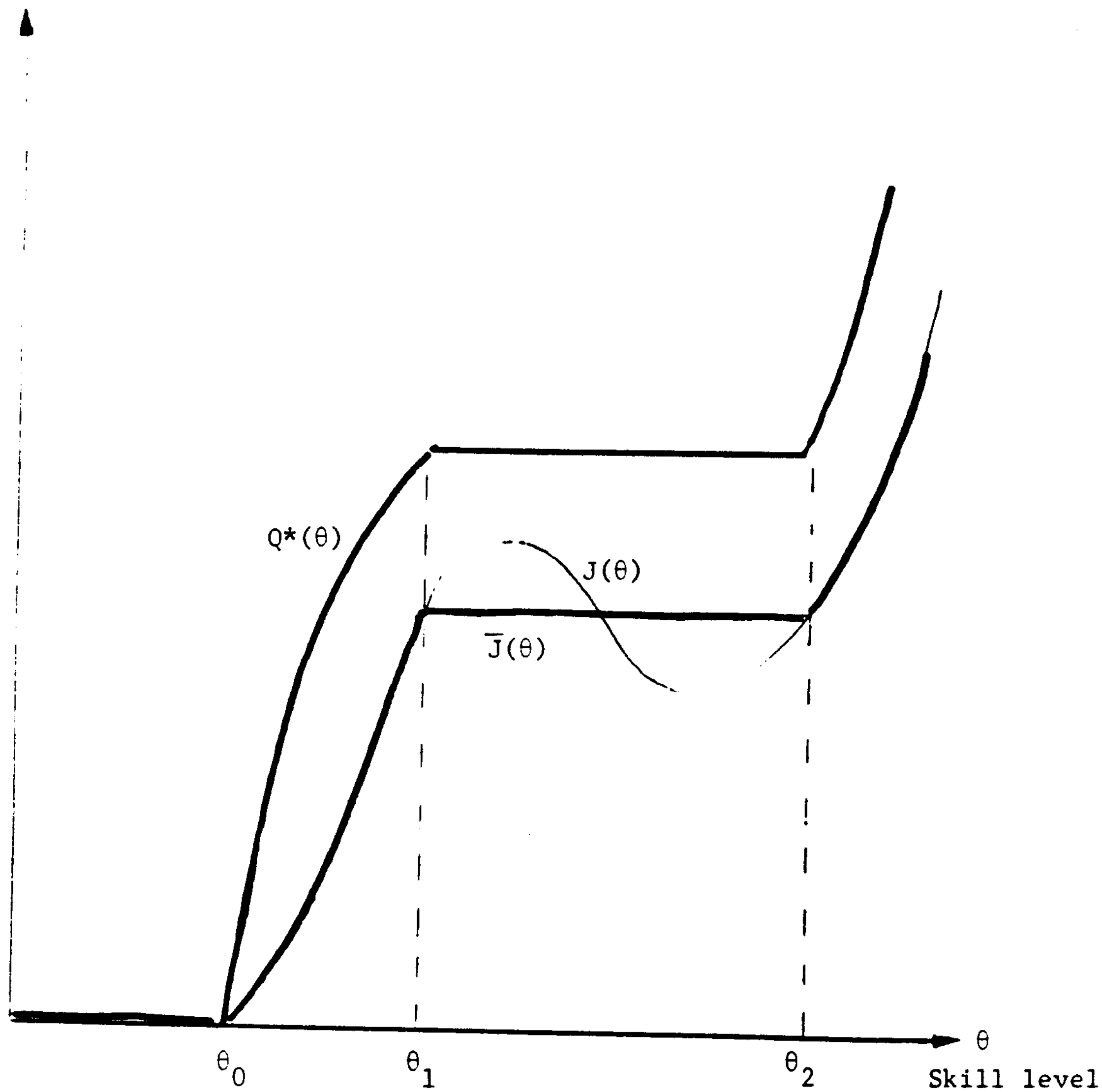

Fig. 1: Optimal Labor Supply

Then we may define $J^*(\theta)$, a monotonically increasing continuous function, as follows:

$$J^*(\theta) = \begin{cases} 0, & \theta \leq \theta_0 \\ J(\theta), & \theta \in \bigcup_{i=0}^{m} [\theta_{2i}, \theta_{2in}] \\ J(\theta_{2i-1}), & \theta \in \bigcup_{i=1}^{m} [\theta_{2i-1}, \theta_{2i}] \end{cases}$$

Extending the arguments above we have

Proposition 3: When the government taxes leisure to maximize revenue subject to the maxi-min constraint

$$U(-R_0, 0) - \bar{U} \geq 0$$

and $U(C,-Q) = C-h(Q)$, the solution $\langle Q^*(\theta), R^*(\theta)\rangle$ is given by

(a) $h'(Q^*(\theta)) = J^*(\theta)$

(b) $R^*(\theta) = \int_0^{\theta} \frac{dQ^*(x)}{dx}[x-J(x)]dx - \bar{U}$

Together (a) and (b) implicitly define $R_L^*(Q)$, the optimal leisure tax.

The proof is a straightforward verification that the above conditions satisfy the first order conditions for a maximum.[3] From the convexity of the problem the first order conditions suffice for a maximum.

Example 2:

To illustrate the possibility that individuals with different skill levels may be treated alike under optimal leisure taxation we seek some distribution function $F(\theta)$ for which the density drops rapidly over some subinterval. This effect is captured most easily if $F(\theta)$ has a downward kink. Suppose, for example, that

$$F(\theta) = \begin{cases} \frac{3}{2}\theta, & 0 \leq \theta \leq \frac{1}{2} \\ \frac{1}{2}\theta + \frac{1}{2}, & \frac{1}{2} \leq \theta \leq 1 \end{cases}$$

Then

$$J(\theta) = \begin{cases} 2\theta - \frac{2}{3}, & 0 \leq \theta < \frac{1}{2} \\ 2\theta - 1, & \frac{1}{2} < \theta < 1 \end{cases}$$

If in addition $h(Q) = Q^2$ then from Proposition 3:

$$Q^*(\theta) = \begin{cases} 0, & \theta \leq \frac{1}{3} \\ \theta - \frac{1}{3}, & \frac{1}{3} \leq \theta \leq \frac{7-\sqrt{3}}{12} \\ \frac{1}{4} - \frac{\sqrt{3}}{12}, & \frac{7-\sqrt{3}}{12} \leq \theta \leq \frac{3}{4} - \frac{\sqrt{3}}{12} \\ \theta - \frac{1}{2}, & \frac{3}{4} - \frac{\sqrt{3}}{12} \leq \theta \leq 1 \end{cases}$$

Example 3:

An even simpler example is a useful aid in developing some intuition for the results summarized in Theorem 3. Suppose that there are only three skill levels $\theta \in \{\alpha, \beta, \gamma\}$. Each consumer has an after tax utility of

$$U_\theta(Q,R) = \theta Q - h(Q) - R$$

Suppose (Q_α, R_α) depicted in Figure 2 is optimal for $\theta = \alpha$. In order to maximize the tax take from those with skill level γ the authority announces

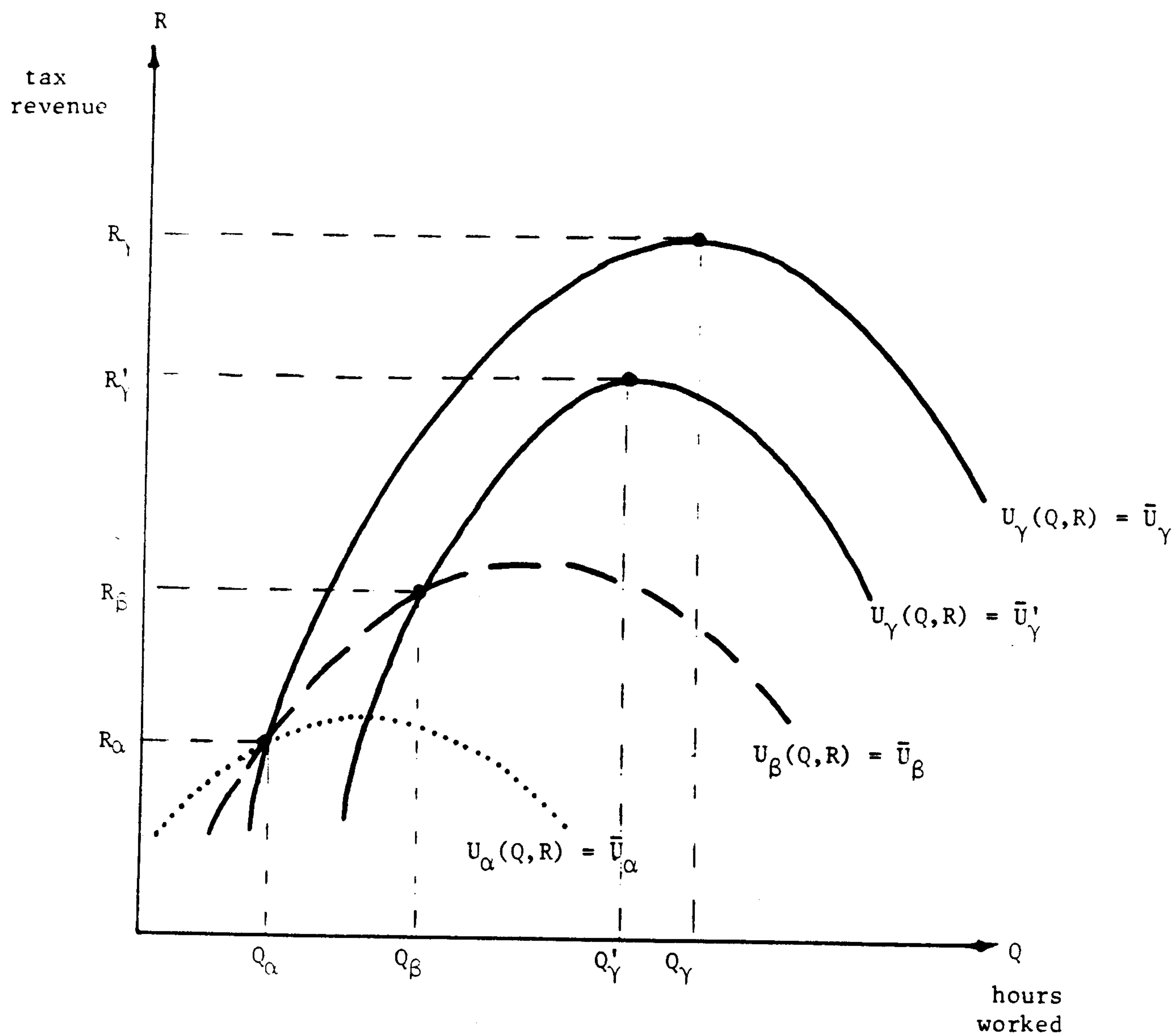

Fig. 2: Optimal leisure taxes with three skill classes

the alternative (Q_α,R_α). (Any higher value of R_γ than that depicted would lead the most skilled to work Q_α hours). Since those with skill level θ_β have indifference curves which are of intermediate steepness, they will choose the alternative (Q_α, R_α).

The authority now considers extracting more from type β. The best it can do is choose some alternative (Q_α,R_α) on the indifference curve $U_\beta(Q,R) = U_\beta(Q_\alpha,R_\alpha)$. The gain is the extra revenue $R_\beta - R_\alpha$ received from each individual of type β. However as the figure illustrates this gain comes at the cost of lowering the tax paid by type γ from R_γ to R'_γ. Clearly the relative frequency of the two types in the population is a critical factor in determining the desirability of separating out all three skill levels. If there are sufficiently few of type β separation of types β and γ will not be optimal. Moreover, as long as there are sufficiently many of type α it will not be optimal to tax only type γ.

We turn now to optimal tax schedules for the utalitarian social welfare function. From (3) the tax authority seeks to maximize revenue

$\int_0^1 R(\theta)dF(\theta)$ subject to the constraints,

$$\int_0^1 U(C(\theta),-Q(\theta)dF(\theta) = \int_0^1 C(\theta)df(\theta) - \int_0^1 h(Q(\theta))dF(\theta) \geq \bar{W}$$

and $$\int_0^1 C(\theta)dF(\theta) = \int_0^\infty [\theta Q(\theta)-R(\theta)]dF(\theta)$$

Combining these two constraints, it is required that

$$\int_0^1 [\theta Q(\theta)-R(\theta)-h(Q(\theta))]dF(\theta) \geq \bar{W}$$

From Theorem 2, the authority must also satisfy the constraint

$$Q'(\theta) \geq 0$$

From (15) $\int_0^1 R(\theta)dF(\theta) = \int_0^1 [J(\theta)Q(\theta)-h(\theta))]dF(\theta) + R_0$

Then assuming that a consumer of zero skill level cannot be taxed we have the additional constraint, $R_0 \leq 0$. Combining all the above we must solve the following program.

$$(20)\quad \left\{ \begin{array}{l} \max\limits_{Q(\theta) \geq 0} \int_0^1 [J(\theta)Q(\theta)-h(Q(\theta))+R_0]dF(\theta) \\ \text{subject to } Q' \geq 0, \int_0^1 [\theta Q(\theta)-J(\theta)Q(\theta)-R_0]dF(\theta) = \bar{W} \\ \text{and } R_0 \leq 0, \\ \text{where, again, } J(\theta) = (F(\theta)+\theta F'(\theta)-1)/F'(\theta) \end{array} \right.$$

The Hamiltonian for this problem is

$$\begin{aligned} H(\theta,Q(\theta), \lambda(\theta), \mu) &= J(\theta)Q(\theta)-h(Q\ (\theta))+R_0 - \lambda(\theta)Q'(\theta) + \mu(\theta Q-J(\theta)Q(\theta)-R_0) \\ &= (J(1-\mu) +\mu\theta)Q - h(Q) + (1-\mu)R_0 - \lambda Q'. \end{aligned}$$

From the first order conditions, one may easily verify the following result.

<u>Proposition 4</u>: When the tax authority taxes leisure to maximize revenue subject to a utilitarian social welfare constraint (that is, when the authority solves program (27)), the solution $\langle Q^*(\theta), R^*(\theta)\rangle$ is as in Proposition 3 except that $J(\theta)$ is replaced by

$$\bar{J}(\theta) = \mu\theta + (1-\mu)J(\theta) \quad , \quad 0 \leq \mu \leq 1 \ .$$

To understand the role played by the shadow value μ consider the special case in which $J(\theta)$ is everywhere increasing hence $\bar{J}(\theta) = J(\theta)$, $\theta \geq \theta_0$.

Note that when $\mu = 1$ the modified condition (a) of Proposition 3, $\bar{J}(\theta) = h'(Q^*(\theta))$, becomes

$$(21) \quad \theta = h'(Q^*(\theta)), \qquad \theta \geq \theta_0$$

Noting once again that after tax utility $U = Q - R_L(Q) - h(Q)$, it follows immediately that (21) is satisfied for all Q if and only if $dR_L(Q)/dQ = 0$. Thus $\mu = 1$ corresponds to the case in which the constraint $R_0 \leq 0$ is not binding and so no revenue is raised.

As the constraint on the sum of utilities is weakened, μ declines and hence $\bar{J}(\theta) = J(\theta) + \mu(\theta - J(\theta))$ also declines for all θ. Given the concavity of h(Q) it follows from condition (a) of Proposition 3 that labor supply is lower for all θ. Moreover, there is some $\theta_1 > 0$ such that $Q^*(\theta) = 0$ for all $\theta \leq \theta_1$. At the other limit, we have $\mu = 0$ and $J^*(\theta) = J(\theta)$. Of course the solution is then exactly as in Proposition 3 for the special case in which the minimum utility, $\bar{U}$, is zero.

III. INCOME TAXES

If the authority can observe only income, Y it must determine a tax schedule, $R_I(Y)$, where $Y = \theta Q$. Given such a schedule let $\langle Y(\theta), R(\theta)\rangle = \langle Y(\theta), R_I(Y(\theta))\rangle$ be the optimal choice of a consumer with maximal skill θ. As with a leisure tax each consumer has an incentive to work at his maximal skill. Then a necessary condition for $\langle Y(\theta), R(\theta)\rangle$ to be optimal for skill level θ is that $\hat{\theta} = \theta$ maximize $U(Y(\hat{\theta}) - R(\hat{\theta}), - Y(\hat{\theta})/\theta)$. The first order condition of the maximization is

$$(22)\quad \frac{dU}{d\theta} = U_1(Y(\hat{\theta})-R(\hat{\theta}),-Y(\hat{\theta})/\theta)[Y'(\hat{\theta})-R'(\hat{\theta})] - U_2(Y(\hat{\theta})-R(\hat{\theta}),-Y(\hat{\theta})/\theta)[Y'(\hat{\theta})/\theta]$$

$$= 0 \quad \text{at} \quad \hat{\theta} = \theta \text{ , for all } \theta.$$

Solving for $R'(\theta)$ we obtain

$$(23)\quad R'(\theta) = [\theta - \frac{U_2(Y-R,-Y/\theta)}{U_1(Y-R,-Y/\theta)}]\frac{Y'(\theta)}{\theta}$$

From (23) it is easy to establish the counterpart of Proposition 1.

<u>Proposition 5</u>: If only an income tax is available, the authority cannot raise as high a level of revenue as when both income and leisure taxes are feasible.

Furthermore, to establish sufficient conditions we substitute for $R'(\hat{\theta})$ in (23) obtaining

$$(24)\quad \frac{dU}{d\theta} = U_1(Y-R,Y/\hat{\theta})\,[\frac{U_2(Y-R,-Y/\hat{\theta})}{U_1(Y-R,-Y/\hat{\theta})}\frac{Y'(\hat{\theta})}{\hat{\theta}} - \frac{U_2(Y-R,-Y/\theta)}{U_1(Y-R,-Y/\theta)}\frac{Y'(\hat{\theta})}{\theta}]$$

Differentiating (24) with respect to $\hat{\theta}$ and evaluating at $\hat{\theta} = \theta$ we obtain the second order condition

(25) $Y'(\theta)[-U_2U_1\theta + U_1U_{22}Y - U_2U_{12}Y] \leq 0$

Inequality (25) is a condition necessary for $\hat{\theta} = \theta$ to solve the consumer's maximization problem it is also sufficient if $\hat{\theta} = \theta$ solves (22) uniquely. Such will be the case if $Y'(\theta)$ does not vanish and $U \varepsilon \mathcal{U}^{**}$, where $\mathcal{U}^{**}$ consists of all concave, monotonically increasing, twice differentiable utility functions for which an increase in the wage rate raises consumption. The argument for why this is so parallels the argument for the class $\mathcal{U}^*$ with leisure taxes. It turns on showing that $U \varepsilon \mathcal{U}^{**}$ implies

(26) $-U_2U_1\theta + U_1U_{22}Y - U_2U_{12}Y \leq 0.$

We have

Proposition 6: If $U \varepsilon \mathcal{U}^{**}$ (an increase in the wage rate raises consumption) then $Y'(\theta) \geq 0$ is a necessary and sufficient condition for the existence of a function $R(\theta)$ such that $\hat{\theta} = \theta$ maximizes $U(Y(\hat{\theta}) - R(\hat{\theta}), -Y(\hat{\theta})/\theta)$.

If $U \not\varepsilon \mathcal{U}^{**}$ an increase in the wage rate lower consumption. Since utility cannot fall with a rise in the wage rate it follows that hours worked, Q, must also fall. Then $U \varepsilon \mathcal{U}^*$ and we have therefore established that

(27) $\mathcal{U}^* \subset \mathcal{U}^{**}.$

The labor supply curve certainly cannot be backward-bending in a neighborhood of a zero wage rate. To deal with backward-bending supply curves, therefore, we shall suppose that the minimum skill level, $\underline{\theta}$, is positive. By reversing the inequalities in (25) and (26) one obtains virtually the converse of Proposition 6.

Proposition 7: If U is a utility function such that for $\theta \geq \underline{\theta}$ an increase in the wage rate lowers consumption, then $Y'(\theta) \leq 0$ is a necessary and sufficient condition for the existence of a function $R(\theta)$ such that $\hat{\theta} = \theta$ maximizes $U(Y(\hat{\theta})-R(\hat{\theta}),-Y(\hat{\theta})/\theta)$.

As we did for leisure taxes, we next examine the solution to the optimal income tax problem when preferences exhibit no income effects. In this case, equation (23) becomes

$$(28) \quad R'(\theta) = Y'(\theta) - h'(Y/\theta)Y'(\theta)/\theta$$

Reintegrating (28) we obtain

$$(29) \quad R(\theta) = Y(\theta) - h(Y/\theta) - \int_0^\theta h'\left(\frac{Y(\theta)}{t}\right) \frac{Y(\theta)}{t^2}\, dt + R_0$$

when social welfare is maxi-min, one solves the program

$$(30) \quad \max_{Y(\theta)} \int_0^1 [Y(\theta)-h(Y/\theta) - \int_0^\theta h'\left(\frac{Y(t)}{t}\right) \frac{Y(t)}{t^2}\, dt + R_0]dF$$

subject to $-R_0 \geq \bar{U}$ and $Y'(\theta) \geq 0$.

Rewriting the maximand of (30) and forming the Hamiltonian, we obtain

$$H(\theta,Y,\psi) = [Y-h(\tfrac{Y}{\theta})]F'(\theta) - (1-F(\theta)h'(\tfrac{Y}{\theta}) \frac{Y}{\theta^2} + \psi Y'$$

If the first order conditions are to be sufficient for a maximum H must be concave in Y. Therefore suppose that $h'''(\cdot) \geq 0$. We have

Proposition 8: If the authority maximizes tax revenue subject to a maxi-min social welfare constraint (that is, solves (30)) the solution is given by $\{\langle Y(\theta),R(\theta)\rangle, \theta_1, \theta_2, \ldots, \theta_{2m+1}\}$ with $\theta_1 < \theta_2 < \ldots \theta_{2m+1}$, where

(a) $K(0,\theta) < 0, \quad \theta \leq \theta_1$

(b) $Y(\theta) = 0, \quad \theta \leq \theta_1$

(c) $\int_{0_{2i}}^{\theta_{2i+1}} K(Y(\theta),\theta)\,dF = 0$

(d) $Y(\theta) = Y(\theta_{2i}) \quad \theta_{2i} \leq \theta \leq \theta_{2i+1}$

(e) $K(Y/\theta),\theta) = 0$ otherwise

and $K(Y,\theta) = 1 - h'(\frac{Y}{\theta})\frac{1}{\theta} - \frac{1-F}{F'}\ \frac{1}{\theta^2}h'(\frac{Y}{\theta}) - \frac{1-F}{F'}\frac{Y^2}{\theta^3}\ h''(\frac{Y}{\theta})$

As in the previous section the solution when social welfare is utilitarian is formally very similar. We have

<u>Proposition 9:</u> If the authority maximizes tax revenue subject to a utilitarian social welfare constraint the solution is given by Proposition 8 except that $K(Y,\theta)$ is replaced by

$$L(Y,\theta) = K(Y,\theta) + \phi(\theta h'(\frac{Y}{\theta}) + h''(\frac{Y}{\theta}))\frac{1-F}{\theta^3 F'}), \qquad 0 \leq \phi \leq 1.$$

<u>Example 4</u>:

Suppose, as in Example 1, that $h(Y/\theta) = (Y/\theta)^2$ and that θ is distributed uniformly on [0,1]. Then

$$K(Y,\theta) = 1 - 2Y/\theta^3$$

Since K is increasing in θ there is complete separation of skill classes for all $\theta > \theta_1$. Moreover, since lump sum transfers have no effects on labor supply $\theta_1 = 0$. Then for all θ we have,

$$Y = \frac{1}{2}\,\theta^3$$

From (28)

$$R'(\theta) = Y'(\theta)\ (1-2Y/\theta^2)$$

Combining the two expressions and integrating we therefore have

$$R_I(Y) = Y - \frac{3}{8}(2Y)^{4/3} + R_0$$

Note that the marginal tax rate declines for all Y. Moreover, since $Y(1) = 1/2$, the most skilled have zero marginal tax rate. Finally, since θ is distributed uniformly,

$$\Pr\{\tilde{Y} < Y\} = \Pr\{\frac{1}{2}\theta^2 < Y\} = \Pr\{\theta < (2Y)^{1/3}\} = (2Y)^{1/3}$$

Then the total revenue raised by the tax authority is

$$\int_0^1 R_I(Y)d(2Y)^{-2/3} = 1/20 + R_0$$

Since we have already shown that, under the assumptions of Examples 1 and 4, the optimal leisure tax yields tax revenue equal to $1/24 + R_0$, it follows that the income tax is a more effective revenue raising mechanism. We shall generalize this observation in Proposition 12 below.

For this example the optimal tax schedule involves full separation of the different skill classes into different tax brackets. However, Proposition 8 and 9 make it clear that, as with leisure taxes, it may be desirable to treat a class of different consumers identically.

A contrasting conclusion can be drawn if one examines the possible advantages of random taxation when consumers are risk averse. Suppose that consumers' risk preferences can be captured by the von Neumann-Morgenstern utility function $U(C,-Q) = V(C-h(Q))$, where V is concave. Consider an optimal income tax, as in Proposition 9, of the form $\langle Y(\theta), R(\theta)\rangle$. We shall demonstrate that, provided consumers are sufficiently risk averse, the government can increase its tax revenue, subject to the maxi-min constraint by employing a random tax.

Proposition 10: Suppose $\langle Y(\theta), R(\theta)\rangle$ is an optimal maxi-min tax schedule for the utility function $U(C,-Q) = V(C-h(Q))$, where V is concave. Then if $-V''/V'$ is sufficiently large, there exists a random tax schedule that increases tax revenue of the form

$$\langle Y(\theta) + \varepsilon\theta^2, R(\theta) - T_1(\theta)\rangle \text{ with probability } \frac{1}{2}$$

$$\langle Y(\theta) - \varepsilon\theta^2, R(\theta) - T_2(\theta)\rangle \text{ with probability } \frac{1}{2}$$

where $T_1(\theta)$ and $T_2(\theta) \geq 0$.

Remark: Weiss [1976] has also studied the desirability of random taxation.[4] He requires a condition on preferences that ensures, in effect, that as taxes become more random, consumers choose to work harder. In our framework, such a condition is unnecessary because, rather than expressing the taxes as a function of Y, we express both taxes and income as random functions of θ. That is, professing one's skill level, rather than choosing one's income, becomes the consumer's choice variable. It is, in fact, simple to verify that, given our specification of the consumer's utility function, he will not work harder if taxes alone are random so that randomizing income is essential. However, note that in the random income schedule proposed in Proposition 10, the consumer does not, on average, work harder than under the deterministic schedule.

Proof: See appendix.

Since average labor supply is the same under the stochastic and deterministic schemes, the fact that randomization pays may seem puzzling. What is happening is that by introducing uncertainty in the income schedule and keeping the tax schedule fixed, the government can induce a sufficiently risk averse consumer to profess a $\hat{\theta}$ in excess of his actual skill level θ. But then the government can once again induce the consumer to select $\hat{\theta} = \theta$ by increasing the taxes collected when income is high.

Proposition 10 actually establishes a bit more than it claims. Note that the proof does not make use of the hypothesis that the schedule $\langle Y(\theta), R(\theta) \rangle$ is optimal. Thus the same conclusion about increasing tax revenue through randomization can be inferred for any other deterministic schedule as well.

IV. INCOME VS. LEISURE TAXES

So far, we have studied income and leisure taxes separately. In this concluding section, we compare their effectiveness. We begin by showing that the labor supply function induced by any leisure tax can be duplicated by some income tax if preferences belong to the class $\mathcal{U}^*$.

Proposition 11: Suppose that $U \in \mathcal{U}^*$ and that the leisure tax schedule $R = R(Q)$ gives rise to the schedule $\langle Q(\theta), R(\theta) \rangle$. Then there exists an income tax schedule $\bar{R} = \bar{R}(Y)$ that gives rise to $\langle \bar{Q}(\theta), \bar{R}(\theta) \rangle$, where $\bar{Q}(\theta) = Q(\theta)$.

Proof:

Suppose that the hypotheses of the theorem hold. From Proposition 2, $Q'(\theta) \geq 0$. Take $Y(\theta) = \theta Q(\theta)$. Then $Y'(\theta) \geq 0$. From (27), $U \in \mathcal{U}^{**}$. Therefore from Proposition 6, there exists an income tax schedule $R = R(Y)$ that gives rise to income schedule $Y(\theta)$.

Q.E.D.

From Proposition 11, we can establish that if preferences are such that the labor supply curve is upward sloping, then the optimal income tax is superior to the optimal leisure tax.

Proposition 12: If $U \in \mathcal{U}^*$ then the optimal income tax generates more revenue than the optimal leisure tax when the social welfare function is maximin.

Proof:

Under the stated hypothesis, suppose that $R = R_L(Q)$ is the optimal leisure tax. Express this schedule as $\langle Q(\theta), R(\theta) \rangle$.

From Proposition 11 there exists an income tax schedule $\bar{R} = \bar{R}_I(Y)$ that generates the same labor supply schedule as does $R_L(Q)$. Express such an income tax schedule as $\langle \tilde{Y}(\theta), \tilde{R}(\theta) \rangle$ where $\tilde{Y}(\theta) = \theta Q(\theta)$.

Because, from equation (23), $\tilde{R}(\theta)$ is determined only up to a choice of scalar, we may take $\tilde{R}(0) = R(0)$.

From (6) and (23) we have

$$(31) \quad R'(\theta) = [\theta - \frac{U_2(\theta Q-R,-Q)}{U_1(\theta Q-R,-Q)}]Q'(\theta)$$

and

$$(32) \quad \tilde{R}'(\theta) = [\theta - \frac{U_2(\theta Q-R,-Q)}{U_1(\theta Q-R,-Q)}](Q'(\theta) + \frac{Q(\theta)}{\theta})$$

First we establish that the optimal leisure tax function $R_L(Q)$ is non-decreasing. If not then there is some interval $\Theta = (\theta_1, \theta_2)$ such that,

(i) $R_L(Q) < R_L(Q(\theta_1)) \qquad Q \in (Q(\theta_1), Q(\theta_2))$,

(ii) either $F(\theta_2) = 1$ or $R_L(Q(\theta_2)) = R_L(Q(\theta_1))$.

Define $\bar{R}(Q) = \begin{cases} R_L(Q(\theta_1)) \quad , \quad Q \in [Q(\theta_1), Q(\theta_2)] \\ R_L(Q), \text{ otherwise.} \end{cases}$

Since $\bar{R}(Q) \geq R_L(Q)$ it follows that for all $\theta \notin \Theta$ the choice of Q will remain unchanged, that is

$$\bar{Q}(\theta) = Q(\theta), \quad \theta \notin \Theta$$

In particular, $\bar{Q}(\theta_1) = Q(\theta_1)$ and $\bar{Q}(\theta_2) = Q(\theta_2)$.

Moreover, since $U \in \mathcal{U}^*$ the slope of the indifference curve of the indirect utility function

$$u(Q,R,\theta) = U(\theta Q-R,-Q)$$

through any point $<Q,R>$ is greater for higher values of θ. Then for any $\theta > \theta_1$ and any $Q < Q(\theta_1)$, $<Q(\theta_1), R_L(Q(\theta_1))>$ is preferred over $<Q,R_L(Q)>$.

Similarly, for any $\theta < \theta_2$ and any $Q > Q(\theta_2)$, $<Q(\theta_2), R_L(Q(\theta_2))>$ is preferred over $<Q, R_L(Q(\theta_2))>$. It follows immediately that

$$\bar{R}(Q(\theta)) = R_L(Q(\theta_1)) \text{ for all } \theta \in \Theta$$

But by construction $\bar{R}(Q(\theta)) > R_L(Q(\theta))$ for $\theta \in \Theta$. Thus total tax revenue is higher under the tax function $\bar{R}(Q)$ contradicting the hypothesis that $R_L(Q)$ is optimal.

Next define $\Theta_m = (\theta_{2m-2}, \theta_{2m-1})$ to be the mth open interval over which $Q'(\theta) = 0$. Since $Q'(\theta) \geq 0$ we have from (31)

$$\frac{dR_L}{dQ}(Q(\theta)) = \theta - \frac{U_2(\theta Q-R,-Q)}{U_1(\theta Q-R,-Q)}, \quad \theta \notin \bigcup_m \Theta_m$$

Having established that $R_L(Q)$ is non-decreasing, we know that

$$(33) \quad Q(\theta)[\theta - \frac{U_2(\theta Q-R,-Q)}{U_1(\theta Q-R,-Q)}] \geq 0 \quad \theta \notin \bigcup_m \Theta_m$$

Furthermore, from (31) we know that over the interval Θ_m

$$<Q(\theta), R(\theta)> = <Q(\theta_{2m-2}), R(Q(\theta_{2m-1}))>$$

Since $U \in \mathcal{U}^*$, for all $\theta \in \Theta_m$ we also have

$$\frac{\partial}{\partial \theta}[\theta - \frac{U_2(\theta Q-R,-Q)}{U_1(\theta Q-R,-Q)}] = 1 - \frac{U_1 U_{12} Q - U_2 U_{11} Q}{U_1^2} \geq 0$$

Since (33) is satisfied at $\theta = \theta_{2m-2}$ it follows that

$$(34) \quad Q(\theta)[\theta - \frac{U_2(\theta Q-R,-Q)}{U_1(\theta Q-R,-Q)}] \geq 0, \theta \in \Theta_m$$

Finally, consider (31) and (32) as a pair of differential equations for $R(\theta)$ and $\tilde{R}(\theta)$. From (33) and (34) it follows immediately that

$$(35)\quad \frac{d\tilde{R}}{d\theta}(\theta), \geq \frac{dR}{d\theta}(\theta), \text{ whenever } \tilde{R}(\theta) = R(\theta).$$

Since by choice of initial condition

$$(36)\quad R(0) = R(0),$$

we must therefore have

$$\tilde{R}(\theta) \geq R(\theta), \text{ for all } \theta,$$

Q.E.D.

Modifying the above proof only slightly we now show that an income tax is superior to a leisure tax for a broad class of individualistic social welfare functions, when preferences are restricted to $\mathcal{U}^*$ and leisure is not an inferior good.

Proposition 13: If $U \in \mathcal{U}^*$ and leisure is not an inferior good, then the optimal income tax generates more revenue than the optimal leisure tax for any social welfare function of the form

$$(*)\qquad W = \int_0^1 \omega(U(C(\theta)-Q(\theta)))dF(\theta)$$

where ω is an increasing concave function.

Proof:

Let $\langle Q(\theta), R(\theta)\rangle$ be the optimal leisure tax schedule for a social welfare function as hypothesized. From the proof of Proposition 12 there is a family of income tax schedules $\langle\tilde{Y}(\theta), \tilde{R}(\theta)\rangle$ with the property that hours worked by each skill level are exactly the same as under the optimal leisure tax:

(37) $\tilde{Q}(\theta) = \frac{\tilde{Y}(\theta)}{\theta} = Q(\theta)$

Moreover, from condition (35), we also have, for each such $<\tilde{Y}(\theta), \tilde{R}(\theta)>$,

(38) $\tilde{R}(\theta) = R(\theta)$ at $\theta = \bar{\theta} \rightarrow \frac{d\tilde{R}(\theta)}{d\theta} \geq \frac{dR(\bar{\theta})}{d\theta}$

It follows that if the income tax function intersects the leisure tax function it does so from below.

Now consider the two social welfare weighted utility profiles

$$V^*(\theta) = \omega(U(\theta Q(\theta)-R(\theta),-Q(\theta)))$$

and

$$\tilde{V}(\theta) = \omega(U(\theta Q(\theta) - \tilde{R}(\theta),-Q(\theta)))$$

Given (37) and (38) it follows that if the two schedules intersect at $\bar{\theta}$

(39) $V^*(\theta) - \tilde{V}(\theta) > 0 \leftrightarrow \theta > \bar{\theta}$

Define $\tilde{V}_1(\theta) = \omega'(U)U_1(\theta\tilde{Q}(\theta)-\tilde{R}(\theta),-\tilde{Q}(\theta))$. Then given the concavity of U and ω it follows from (37) that for all $\theta \neq \bar{\theta}$

(40) $V^*(\theta) - \tilde{V}(\theta) \leq \tilde{V}_1(\theta)(\tilde{R}(\theta) - R(\theta))$

The proof will be complete if we can establish that

(41) $\tilde{V}_1(\theta) < \tilde{V}_1(\bar{\theta}) \leftrightarrow \theta > \bar{\theta}$

For then, combining (39) - (41) we have

$$V^*(\theta) - \tilde{V}(\theta) < \tilde{V}_1(\bar{\theta})(\tilde{R}(\theta)-R(\theta))$$

Hence

$$W^* - \hat{W} = \int_0^1 (V^*(\theta) - \tilde{V}(\theta))dF(\theta) < \tilde{V}_1(\bar{\theta}) \int_0^1 (\tilde{R}(\theta)-R(\theta))dF(\theta)$$

Therefore if the concave tax schedule $\tilde{R}(\theta)$ is chosen to achieve the same level of social welfare as the leisure tax, it generates more tax revenue.

It remains to establish (41). For $\theta > \bar{\theta}$ $\tilde{Q}(\theta) > \tilde{Q}(\bar{\theta})$. Then, from Shephard's Lemma, unless leisure is an inferior good, the marginal utility of consumption cannot rise in a move around the indifference curve $U(C,-Q) = \tilde{U}(\bar{\theta})$ from $\tilde{Q}(\bar{\theta})$ to $\tilde{Q}(\theta)$. Also $\tilde{U}(\theta) > \tilde{U}(\bar{\theta})$, therefore since U is concave, the marginal utility of consumption must decline in moving to the consumption bundle chosen by skill level θ. Since $\tilde{V}(\theta) = \omega(\tilde{U}(\theta))$ and ω is concave, it follows that the welfare weighted marginal utility of consumption, $\tilde{V}_1(\theta) = \omega'(\tilde{U})U_1$ is also decreasing with θ.

Q.E.D.

<u>Corollary 1</u>: If U exhibits no income effects, then for the class of social welfare functions described in Proposition 13, the optimal income tax generates more revenue than the optimal leisure tax.

Finally, by reversing the inequalities and interchanging leisure and income taxes, we can obtain the counterparts of Propositions 12 and 13 for downward sloping labor supply curves:

<u>Proposition 14</u>: If U is such that consumption is a declining function of the wage rate for wages greater than $\underline{\theta} > 0$ (implying that the labor supply curve is downward sloping), then the optimal leisure tax generates more revenue than the optimal income tax for any social welfare function of the form (*) (See Proposition 13). Moreover, this remains true for the limiting case where social welfare is maximin.

Appendix

Proof of Proposition 10.

Suppose that for each θ, income is either $Y(\theta) + \varepsilon\theta^2$ or $Y(\theta) - \varepsilon\theta^2$ (where $\varepsilon > 0$), each with probability one half. Let $R_1(\theta,\varepsilon)$ be the tax in the former case, and $R_2(\theta,\varepsilon)$ the tax in the latter. A consumer with maximal skill level θ faces the maximization problem

$$\text{(A1)}\qquad \max_{\hat{\theta}} \{\tfrac{1}{2}V(Y(\hat{\theta}) + \varepsilon\hat{\theta}^2 - R_1(\hat{\theta},\varepsilon) - h((Y(\hat{\theta}) + \varepsilon\hat{\theta}^2)/\theta) + \tfrac{1}{2}V(Y(\hat{\theta}) - \varepsilon\hat{\theta}^2 - R_2(\hat{\theta},\varepsilon) - h((Y(\hat{\theta}) - \varepsilon\hat{\theta}^2)/\theta)\}$$

Differentiating with respect to $\hat{\theta}$ and setting $\hat{\theta} = \theta$ we have

$$\text{(A2)}\qquad \tfrac{1}{2}V'(A)[Y' + 2\varepsilon\theta - \frac{\partial R_1}{\partial\theta} - \frac{dh}{d\theta}(Y/\theta+\varepsilon\theta) - (Y/\theta^2 + \varepsilon)h'(Y/\theta+\varepsilon\theta)]$$
$$\tfrac{1}{2}V'(B)[Y' - 2\varepsilon\theta - \frac{\partial R_2}{\partial\theta} - \frac{dh}{d\theta}(Y/\theta-\varepsilon\theta) - (Y/\theta^2+\varepsilon)h'(Y/\theta-\varepsilon\theta)]$$
$$= 0 \text{ for all } \theta$$

where

$$\text{(A3)}\qquad \begin{aligned} A &= Y + \varepsilon\theta^2 - R_1(\theta,\varepsilon) - h(Y/\theta + \varepsilon\theta) \\ B &= Y - \varepsilon\theta^2 - R_2(\theta,\varepsilon) - h(Y/\theta - \varepsilon\theta) \end{aligned}$$

Let

$$\text{(A4)}\qquad \begin{aligned} \frac{\partial R_1}{\partial\theta} &= Y' + 2\varepsilon\theta - \frac{dh}{d\theta}(Y/\theta+\varepsilon\theta) - (Y/\theta^2+\varepsilon)h'(Y/\theta+\varepsilon\theta) + S_1(\theta,\varepsilon) \\ \frac{\partial R_2}{\partial\theta} &= Y' - 2\varepsilon\theta - \frac{dh}{d\theta}(Y/\theta-\varepsilon\theta) - (Y/\theta^2-\varepsilon)h'(Y/\theta-\varepsilon\theta) - S_2(\theta,\varepsilon) \end{aligned}$$

Then, from (A2)

$$\text{(A5)}\quad V'(A)S_1(\theta,\varepsilon) = V'(B)S_2(\theta,\varepsilon)$$

Integrating (A4) we obtain

$$\text{(A6)}\quad \begin{aligned} R_1(\theta,\varepsilon) &= Y + \varepsilon\theta^2 - h(Y/\theta+\varepsilon\theta) - \int_0^\theta (Y/x^2+\varepsilon)h'(Y/x+\varepsilon x)dx + \int_0^\theta S_1(x,\varepsilon)dx \\ R_2(\theta,\varepsilon) &= Y - \varepsilon\theta^2 - h(Y/\theta-\varepsilon\theta) - \int_0^\theta (Y/x^2-\varepsilon)h'(Y/x-\varepsilon x)dx - \int_0^\theta S_2(x,\varepsilon)dx \end{aligned}$$

Expected tax payments by a consumer with skill θ are, therefore,

$$\text{(A7)}\quad R(\theta,\varepsilon) = \tfrac{1}{2}R_1(\theta,\varepsilon) + \tfrac{1}{2}R_2(\theta,\varepsilon)$$

Take

$$\text{(A8)}\quad S_1(\theta,\varepsilon) = K\theta\varepsilon$$

From (A5) $S_1(\theta,\varepsilon) - S_2(\theta,\varepsilon) = \left(\frac{V'(B) - V'(A)}{V'(B)}\right)K\theta\varepsilon$

Differentiating twice with respect to ε and evaluating at $\varepsilon = 0$ we have

$$\text{(A9)}\quad \frac{\partial}{\partial\varepsilon}(S_1-S_2)\Big|_{\varepsilon=0} = 0$$

and

$$\text{(A10)}\quad \frac{\partial^2}{\partial\varepsilon^2}(S_1-S_2)\Big|_{\varepsilon=0} = \frac{K\theta V''}{V'}\left(\frac{\partial B}{\partial\varepsilon} - \frac{\partial A}{\partial\varepsilon}\right)\Big|_{\varepsilon=0}$$

From (A3)

$$\text{(A11)}\quad \left(\frac{\partial B}{\partial\varepsilon} - \frac{\partial A}{\partial\varepsilon}\right)\Big|_{\varepsilon=0} = -2\theta^2 + 2\theta h'(Y/\theta) - \left(\frac{\partial R_2}{\partial\varepsilon} - \frac{\partial R_1}{\partial\varepsilon}\right)\Big|_{\varepsilon=0}$$

Differentiating (A6) with respect to ε and substituting for S_1 and S_2 using (A8) and (A9) we have

$$\text{(A12)}\quad \left(\frac{\partial R_2}{\partial \varepsilon} - \frac{\partial R_1}{\partial \varepsilon}\right)\Bigg|_{\varepsilon=0} = -2\theta^2 + 2\theta h'(Y/\theta) - K\theta^2 + 2\int_0^\theta [h'(Y/x) + (Y/x)h''(Y/x)]dx$$

Combining (A10) - (A12) yields

$$\text{(A13)}\quad \frac{\partial^2}{\partial \varepsilon}(S_1 - S_2)\Bigg|_{\varepsilon=0} = K\,\frac{V''}{V'}(K\theta^2 - 2\int_0^\theta (h'(Y/x) + (Y/x)h''(Y/x))dx)$$

Finally, differentiating (A7) twice and evaluating at $\varepsilon = 0$ we have

$$\text{(A14)}\quad \frac{\partial R}{\partial \varepsilon}\Bigg|_{\varepsilon=0} = 0$$

and

$$\text{(A15)}\quad \frac{\partial^2 R}{\partial \varepsilon^2}\Bigg|_{\varepsilon=0} = -\theta^2 h''(Y/\theta) - \int_0^\theta Yh'''(Y/x)dx + K\theta\frac{V''}{V'}\{\tfrac{1}{2}K\theta^2 - \int_0^\theta [h'(Y/x) + Y/xh''(Y/x)]dx\}$$

Assuming $h''(\cdot)$ is positive and bounded away from zero, the expression inside the braces is negative for sufficiently small K. Then if $-V''/V'$ is sufficiently large the right-hand side of (A15) is positive for all positive θ. It follows immediately that, under these conditions

$$\frac{\partial^2}{\partial \varepsilon^2}\int_0^1 R(\theta,\varepsilon)dF(\theta)\Bigg|_{\varepsilon=0} > 0$$

Thus expected revenue is increased beyond that of $\langle Y(\theta), R(\theta)\rangle$ if ε is slightly greater than zero.

Q.E.D.

Footnotes

[1]Mirrlees [1980] also discusses this possibility. By introducing the simplifying assumption of no income effects we are able to isolate necessary and sufficient conditions for such an outcome.

[2]Here we assume that the consumption and labor schedules are (almost everywhere) differentiable functions of θ. We shall maintain this assumption throughout except where expressly stated.

[3]A formal proof for a closely related control problem can be found in Maskin and Riley [1980].

[4]Stiglitz [1980] has also discussed the possible gains to randomizing taxes.

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