

TOWARD A THEORY OF
COMPETITIVE PRICE ADJUSTMENT

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I. Introduction

In his 1959 essay "Toward a Theory of Price Adjustment" Arrow argued that "there exists a logical gap in the usual formulations of the theory of perfectly competitive economy, namely, that there is no place for a rational decision with respect to prices as there is with respect to quantities." (See also Koopmans [1957].) This gap is especially felt in macroeconomics where the basic framework is that of perfect competition and many results depend on the existence of some lag in the adjustment of prices. Here I propose a first step toward a theory of price adjustment in a competitive environment.^{1/} I shall consider here a simple exchange economy with no money. The application to the question of the real effects of changes in the money supply is discussed in a separate paper (Eden [1980]).

The main cost of adjustment considered here is the cost of information about changes in aggregate excess demand. The question is who will gather and pay for the information. This public good problem is analyzed in a model in which the artificial entity of the auctioneer is replaced by firms (sellers) who seek information about excess demand and announce the prices of their products. The public good aspect of the problem emerges when there is a unique optimal price which would be announced on the basis of "old" information shared by all participants. Then if a single firm buys "new" information and announces a different price all firms which are aware of this revised price will rightly think that it is based on updated information and will therefore follow the lead. In this case the announced price transmits all the relevant new information to all firms and the firm that updates the information therefore provides that

^{1/} For other attempts see the recent symposium on perfect competition in the Journal of Economic Theory, April 1980. For some literature on monopolistic price adjustment, see Gordon and Haynes (1970), Barro (1972) and Sheshinski and Weiss (1977, 1979).

public good which in the Walrasian model is provided by the auctioneer.^{2/}

The question is which firm will undertake the function of the auctioneer? The solution suggested here is that all firms will adopt a mixed strategy in which they undertake this function with a certain probability. It is shown however that there is always a positive probability that the prices advertised in a particular market will not be based on updated information, since otherwise there would be no incentive to buy information.

In the second section I describe the process of advertising prices for the case in which the aggregate demand that sellers in a particular market face is exogenously given and infinitely elastic. I then discuss the type of framework required to justify such demand curves and show that, in general, the Walrasian price is not the average of advertised prices.

II. The Sellers' Problem

I start with a simple environment in which there are n identical firms (sellers) each with a given supply of one unit of a certain good. The good is valueless to sellers and the cost of selling a unit is zero. The aggregate demand for this good is infinitely elastic at the random price θ , where θ may take two values: θ_1 or θ_2 with equal probability of occurrence, as in Figure 1. (We may think of the international demand from the point of view of a small country.) It is assumed that the percentage difference in prices is not large in the sense that $\theta_1 < \theta_2 < 2\theta_1$. (We may think of a short period during which drastic changes in prices are not expected.) It is assumed that all firms maximize expected profits.

^{2/} A similar external effect is analyzed in Grossman and Stiglitz (1976, 1980). They show that if prices reveal all the information there cannot exist a Walrasian equilibrium.

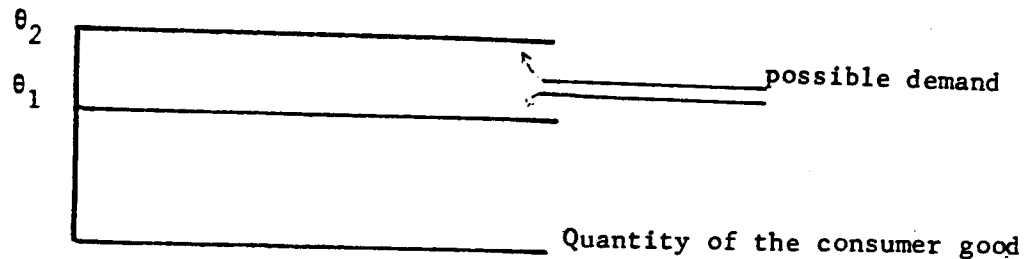


Figure 1

Before the opening of the market each seller can buy information, at the cost of x dollars, about the actual realization of θ . Then sellers place advertisements which state the price of their merchandise in the local newspaper. The newspaper is then published and circulated. Sellers may costlessly observe the price advertised by other sellers and based on this observation they may wish to revise their price. In this case they will place new advertisements and a new issue of the newspaper will appear. The process comes to an end when no one wishes to make a further revision of his price. Only then actual trading take place at the prices that were advertised in the last issue of the newspaper. These prices will be called final prices.^{3/} (The assumption that actual transactions take place at the final or equilibrium prices is also used by Walras. Here, however, sellers replace the auctioneer in determining the final prices.) It is impossible to change prices after the beginning of actual trading.^{4/} It is impossible to sell the good without advertising in

^{3/} Formally, let P^i be the final price advertised by seller i . Given the information that seller i may have about the realization of θ and given $(p^1, \dots, p^{i-1}, p^{i+1}, \dots, p^n)$, the price P^i maximize seller's i expected profit.

^{4/} Barro (1972) and Sheshinski and Weiss (1977, 1979) show that a monopoly will not choose to adjust its price continuously when there are some direct fixed costs for changing the price, such as the cost of transmitting the information about the change to consumers. These considerations may be applied here to determine the length of the period in which advertised prices are fixed.

the newspaper and it is impossible to buy information about the realization of θ after the opening of the market (i.e., after the first issue of the newspaper appears).

The payoff matrix for the individual seller is:

	$\theta = \theta_1$	$\theta = \theta_2$
$P = \theta_1$	θ_1	θ_1
$P = \theta_2$	0	θ_2

where P is his final price. Thus the assumption that $\theta_2 < 2\theta_1$ implies that on the basis of the initial prior distribution of θ , the price θ_1 promises the highest expected profit.

The seller's main problem is whether to buy the information about the realization of θ . As a first step for solving this problem I shall characterize a Nash strategy^{5/} with respect to the behavior of sellers after the opening of the market (i.e., after the decision whether to buy the information was already made).

Claim 1: The following is a Nash strategy: (a) if you have bought the information and observed $\theta = \theta_i$, advertise the price θ_i ($i=1,2$); (b) if you have not bought the information, advertise the price θ_1 unless you observe that the price θ_2 has been advertised. In this case advertise the price θ_2 .

^{5/} If all other firms follow a Nash strategy it is optimal for the individual firm to do the same.

To prove this claim, note that θ_2 will be advertised if and only if someone has actually observed $\theta = \theta_2$. It is therefore optimal for the uninformed seller to follow someone who advertise θ_2 . If the uninformed seller observes that no one has advertised θ_2 he may conclude that either no one has bought the information or someone has actually observed $\theta = \theta_1$. Given the observation that no one has advertised θ_2 the probability that $\theta = \theta_1$ is therefore greater than $1/2$.^{6/} Since we assume $2\theta_1 > \theta_2$ it is therefore optimal to advertise θ_1 in this case. Thus given that the informed sellers follow the strategy (a), it is optimal for the uninformed sellers to follow (b). Since the informed seller cannot do better than (a), this complete the proof.

Armed with Claim 1 we can compute the expected profit when buying and when not buying the information. If firm j buys the information it will advertise the observed realization θ_i . The expected profit in this case is

$$(1) \quad r_j = \theta_2/2 + \theta_1/2 - x = \bar{\theta} - x$$

where x is the cost of information and $\bar{\theta}$ is the expected value of θ . If firm j does not buy the information it may still reap the benefits if some other firm buys the information.

Assume that firm j believes that other firms buy information independently of each other and that each of the competing firms buys the information with probability q . The subjective probability that no other firm will buy the information is thus

^{6/} Let B denote the event that no one has advertised the price θ_2 and let $\text{prob}(B)$ denote the probability of this event. Let $\text{prob}(\theta = \theta_1 | B)$ denote the probability of $\theta = \theta_1$ and B . Let μ denote the probability that at least someone has bought the information. Since under the above strategy $(\theta = \theta_1 | B)$ will occur if either no one has bought the information and $\theta = \theta_1$ or if someone has actually observed $\theta = \theta_1$, it follows that $\text{prob}(\theta = \theta_1 | B) = (1 - \mu)/2 + \mu/2 = 1/2$. Using Bayes theorem, the probability that $\theta = \theta_1$ given B is: $\text{prob}(\theta = \theta_1 | B) = \text{prob}(\theta = \theta_1 \cap B) / \text{prob}(B) = (1/2) / \text{prob}(B) \geq 1/2$.

$$(2) \quad (1-q)^{n-1}$$

Using Claim 1, firm j 's subjective probability that the final price will be θ_2 is equal to the probability that at least one firm will buy the information times the probability that $\theta = \theta_2$, which is: $[1-(1-q)^{n-1}]/2$. The probability that θ_1 will be advertised is equal to the probability that no one will buy the information, plus the probability that at least someone will buy the information and observe $\theta = \theta_1$, which is: $(1-q)^{n-1} + [1-(1-q)^{n-1}]/2$. The expected profits of firm j when it does not buy the information is thus

$$(3) \quad R_j = [1-(1-q)^{n-1}]\theta_2/2 \\ + [1-(1-q)^{n-1}]\theta_1/2 + \theta_1(1-q)^{n-1} \\ = [1-(1-q)^{n-1}]\bar{\theta} + \theta_1(1-q)^{n-1} .$$

An alternative way of computing (3) is by observing that since prices transmit all relevant information, the expected revenue of an uninformed firm must be equal to the expected revenue of an informed firm. The uninformed firm will therefore enjoy an expected revenue of $\bar{\theta}$ if at least one firm buys the information and a revenue of θ_1 otherwise.

Finally, if firm j decides to buy the information with probability q_j its expected profit will be

$$(4) \quad \pi_j(q_j, q) = q_j r_j + (1-q_j)R_j .$$

It can be shown that

$$(5) \quad \frac{\partial \pi_j}{\partial q_j} = (\bar{\theta} - \theta_1)(1-q)^{n-1} - x .$$

$(\bar{\theta} - \theta_1)$ is the increase in expected revenue that each firm will experience if at least one firm buys the information. I shall therefore refer to this term as the social value of information per firm (SVI). When this term is multiplied by the probability that no other firm will buy the information, we get the increase in expected revenue that a particular firm will experience if it buys the information or the private value of information (PVI). (Note that $PVI < SVI$.) The derivative (5) tells us that when the private value of observing θ is greater than the cost of doing so, $(\frac{\partial \pi_i}{\partial q_j} > 0)$, the firm will choose $q_j = 1$. When $PVI < x$ the firm will choose $q_j = 0$ and when $PVI = x$ the firm will be indifferent with respect to the choice of q_j .

To characterize an equilibrium in this environment I shall define the probability q^* to be a Nash solution if given $q_i = q^*$ for all $i \neq j$, $q_j = q^*$ is optimal for firm j , where $j = 1, \dots, n$. The possible Nash solutions are:

$$(6) \quad \text{if } SVI < x \text{ then } q^* = 0$$

$$(7) \quad \text{if } SVI \geq x \text{ then } q^* = 1 - \left(\frac{x}{SVI}\right)^{1/n-1}$$

where $SVI = \bar{\theta} - \theta_1$. Substituting (6) into (5) leads to $\frac{\partial \pi_i}{\partial q_j} < 0$ for all j , and substituting (7) into (5) leads to $\frac{\partial \pi_i}{\partial q_j} = 0$ for all j , thus q^* is optimal from the point of view of all firms.

Under (6) the probability that no one will buy the information is unity while under (7) this probability is larger than the limit

$$(8) \quad \lim_{n \rightarrow \infty} (1 - q^*)^n = \lim_{n \rightarrow \infty} \left(\frac{x}{SVI}\right)^{n/n-1} = \frac{x}{SVI}$$

In any case the probability that the "market" will not observe θ is greater than zero. This result is rather intuitive, since when the "market" is always informed there is no incentive to buy information. (See Grossman and Stiglitz [1976, 1980].) The discussion up to this point can be conveniently summarized by the following claim:

Claim 2: (a) If all firms follow the strategy described in Claim 1 and independently buy the information with probability $q = q^* = \max[1 - (\frac{x}{SVI})^{1/n-1}, 0]$, then it is optimal for the individual firm to do the same. (b) The probability that no firm will buy the information is greater than $\min(1, \frac{x}{SVI})$ and approaches this limit when the number of firms goes to infinity.

The Nash strategy in Claim 2 is not unique. Another solution is when a certain firm buys the information with probability one and other firms never buy the information. In this case if the information - buying firm makes normal profits, others must make above normal profits. In a more general model, this will induce entry of new firms and drive the information-buying firm out of business. The same objection holds for other asymmetric solutions. Another source of non-uniqueness may arise from an alternative to Claim 1. However this alternative yields less expected profits to all firms.^{7/}

^{7/} If $2\theta_1 - \epsilon < \theta_2 < 2\theta_1$, where ϵ is a small positive number and if the information cost is not prohibitive then the following can be shown to be a Nash strategy: (a) if you have bought the information and observed $\theta = \theta_i$, advertise the price θ_i ($i=1,2$); (b) if you have not bought the information, advertise the price θ_2 unless you observe that the price θ_1 has been advertised. The proof relies on the observation that under this strategy, given that no one has advertised θ_1 the probability that $\theta = \theta_2$ is greater than $1/2$. See footnote 6 for a similar argument.

Claim 2 is extended in Appendix 1 to the more general case in which θ may have many realizations and in Appendix 2 to a case in which the aggregate demand is downward sloping. In what follows I shall argue that in a truly competitive environment the assumption of an infinitely elastic demand curve is appropriate.

III. A competitive general equilibrium

The above model allows for a single seller to have a noticeable influence on the probability distribution of the final price. For example, the distribution of the final price will be different if he buys the information with certainty, since in this case the final price which is advertised by all sellers will always be based on updated information. In this sense the individual seller is not small relative to the economy. Here, I use the previous analysis to construct a model in which the individual seller cannot influence the probability distribution of the final prices which are advertised in the economy. In this sense, this section describes a truly competitive economy.

The power of a single seller to influence the final price is due to the assumption that his advertised prices are observed by all other sellers before they commit themselves to a final price.^{8/} Here I assume that before the commitment to a final price only a small fraction of all sellers can observe the prices which are advertised by an individual seller. Thus I consider an economy in which sellers are located in many markets, where a

^{8/} This power does not disappear when the number of sellers goes to infinity, since the probability that no one will buy the information approaches strictly positive limit.

market is defined as a group of sellers who, in effect, share information. In this set-up the individual seller may influence only the distribution of the final price in his market. Since each market is small relative to the economy, the individual seller's influence on the distribution of the final prices in the economy, is negligible.

This framework will be used to compare the Walrasian price and the average of all advertised final prices. This comparison is of special interest in view of the practice of assuming that firms advertise their best guess with respect to the Walrasian price. (See Brunner, Cukierman and Meltzer [1980] and Green and Laffont [1980] for models which employ this assumption). It is also important to examine the generally accepted hypothesis that, on average, prices "behave" as if there were an auctioneer.

I consider a simple, single period, two goods exchange economy in which there are many sellers and buyers. Each seller is endowed with a unit of good, Y, and each buyer is endowed with a unit of good, Z. It is assumed that buyers and sellers have the same Von Neumann-Morgenstern utility function defined on quantities of the two goods according to:

$$(9) \quad u(y,z)$$

where $u(\)$ is quasi concave and differentiable. The number of sellers, H, is known but the number of buyers, N, is a random variable and may be either N_1 or N_2 with equal probability. Both N and H are large numbers and will be treated as real numbers. It is assumed that $(N_2 - N_1) > 0$ and is not large in a sense that will be discussed later.

It is possible to buy the information about the realization of N for x units of Y. It is possible to buy the information only before the process of advertising prices begins. The process of advertising the price of Y in terms of Z (P) is carried out by many newspapers. (Strictly speaking

I need a continuum of newspapers.) For simplicity it is assumed that each newspaper serves n sellers. The process of advertising prices by a single newspaper is similar to the one described in the previous section. During this process each seller can costlessly find out the prices which are being advertised by the $n-1$ other sellers which use his newspaper. He cannot observe the process of advertising prices by other newspapers. When all n sellers are satisfied with their advertised prices, the final edition of the newspaper is ready. The final editions of all the newspapers appear simultaneously on Sunday morning. From this point in time the information about the prices in all the newspapers is costlessly available to all agents. Trading starts on Monday morning at these final prices. Buyers make orders by phone and sellers satisfy the orders on a first come first served basis. It is assumed that the phone calls and the delivery are costless.^{9/}

Each seller decides on the amount of Y that he will sell as a function of the final price, P , by solving

$$(10) \quad \max_y u(y,z) \quad \text{s.t.} \quad Py + z = PE$$

where E is the amount of Y that he has before trading occurs (that is, unity if the seller has not bought the information and $1 - x$ if he has).

When n is large Claim 2 implies that the number of sellers that will actually buy the information is small. I shall therefore ignore the expenditure on information in calculating aggregate supply and assume for this purpose that $E = 1$ for all sellers. Let $S(P)$ be the solution to (10), when

^{9/} In general, cost of delivery leads to a difference between the price received by sellers and the price paid by buyers. Such costs can be accommodated without changing the basic nature of the analysis.

$E = 1$. The aggregate supply of Y is then given by $HS(P)$, where H is the total number of sellers. The demand of a buyer when facing a single price, P , is the solution, $D(P)$, to

$$(11) \quad \max_y u(y, z) \quad \text{S.T. } PY + z = 1.$$

The aggregate demand may be either $N_1 D(P)$ or $N_2 D(P)$. The corresponding Walrasian prices will be denoted by (\bar{P}_1, \bar{P}_2) as in Figure 2.^{10/}

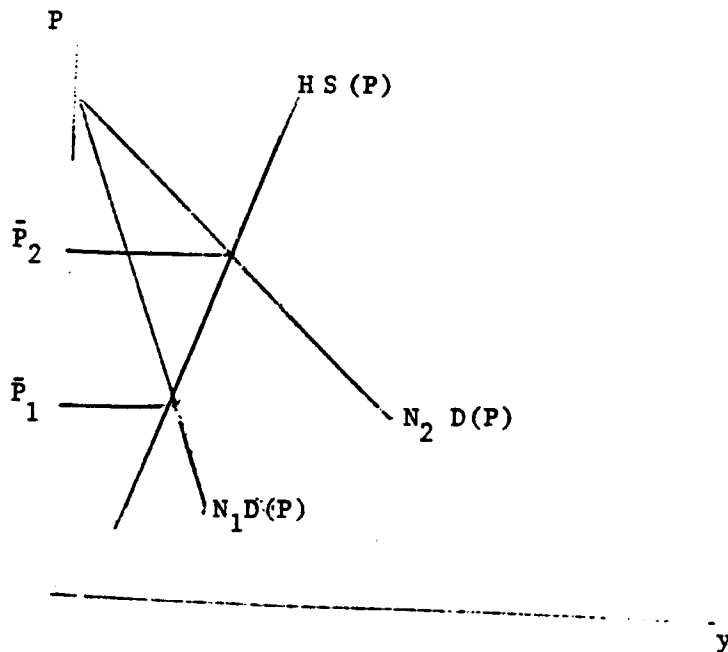


Figure 2

^{10/} It is assumed that the supply is upward sloping (i.e., that the substitution effect dominates the income effect) and that the demand curve is downward sloping.

I shall proceed by describing the behavior of the system under certain expectations. I shall then argue that in equilibrium the postulated expectations are roughly correct. It is assumed that each seller views the aggregate demand that his market (i.e., the group of the n sellers who use his newspaper to advertise prices) faces as contingent on the realization of N and infinitely elastic at the prices:

$$(12) \quad \begin{array}{ll} \bar{P}_1 & \text{if } N = N_1 \\ P_2^* & \text{if } N = N_2 \end{array} ,$$

where \bar{P}_1 is the Walrasian price for the case $N=N_1$ but P_2^* may be different than the Walrasian price for the case $N=N_2$; \bar{P}_1 plays the role of θ_1 and P_2^* plays the role of θ_2 in the previous section. Similar to the assumption made there it is assumed that $\bar{P}_1 \leq P_2^* \leq 2\bar{P}_1$. Thus the change in the number of buyers is not too large in the sense that it does not lead to an increase of more than 100% in the price.

Given the expectations (12), the discussion in the first section leads to Claim 3.

Claim 3: The following is a Nash strategy: (a) buy the information about the realization of N with probability q^* - to be calculated shortly; (b) if you have bought the information, advertise the prices (12); (c) if you have not bought the information, advertise the price \bar{P}_1 unless you observe that the price P_2^* has been advertised. In this case advertise the price P_2^* .

The probability q^* is calculated in appendix 3, where it is shown that when x is not too large there exists an internal solution, $0 < q^* < 1$. Here I shall consider this case of internal solution.

I shall define a newspaper as uninformed if none of the n sellers who use the newspaper has bought the information about the realization of N . The probability that a given newspaper will be uninformed is

$$(13) \quad (1 - \mu) = (1 - q^*)^n .$$

Since the number of newspapers is large I shall use the law of large numbers and assume that a fraction μ , ($0 < \mu < 1$) of all newspapers is informed, where μ is calculated from (13) and is non-random.

When $N = N_1$, Claim 3 implies that all newspapers will advertise $P = \bar{P}_1$ where

$$(14) \quad HS(\bar{P}_1) = N_1 D(\bar{P}_1) .$$

The assumption that there are many newspapers (i.e., that n is small relative to H) and the assumption that delivery and information about final prices are costless insure that in the case $N = N_1$, no single group of n sellers will be able to sell at $P > \bar{P}_1$, but all will be able to sell their entire supply at this price. Thus the sellers' expectations regarding the demand in the case $N = N_1$ are roughly correct.^{10/} When $N = N_2$ the sellers' expectations can be rationalized (in the sense of Muth [1961]) by the same considerations, provided that P_2^* clears the residual market.

Specifically, Claim 3 implies that when $N = N_2$, $(1 - \mu)H$ (uninformed) sellers will advertise \bar{P}_1 and μH (informed) sellers will advertise P_2^* . From (14) it follows that the uninformed sellers will satisfy the demand of $(1 - \mu)N_1$ buyers [since $(1 - \mu)N_1 D(\bar{P}_1) = (1 - \mu)HS(\bar{P}_1)$]. The informed sector will thus face the demand of the remaining $N_2 - (1 - \mu)N_1$ buyers. Clearing the residual market therefore requires

^{10/} Arrow (1959) points out that market clearing is essential for the "price taking" assumption.

$$(15) \quad [N_2 - (1 - \mu)N_1] D(P_2^*) = \mu HS(P_2^*) .$$

Note that when P_2^* satisfies (15), a single group of n sellers will not be able to sell at $P > P_2^*$ and will be able to sell all its supply at $P = P_2^*$. Thus under (15) the sellers' expectations regarding the demand when $N = N_2$ are roughly correct.

Thus I suggest an equilibrium solution in which sellers buy the information with probability q^* (calculated in appendix 3), informed newspapers advertise the price \bar{P}_1 if $N = N_1$ and advertise the price P_2^* if $N = N_2$, uninformed newspapers advertise the price \bar{P}_1 , and (\bar{P}_1, P_2^*) satisfies the market clearing conditions (14) and (15). This is an equilibrium in the sense that expectations are roughly correct and no one has an incentive to change his strategy with respect to buying information and with respect to advertising the price. The following defines the concept of equilibrium.

Definition: (q, P_1, P_2, P_3) is a competitive equilibrium vector if

(a) each seller views the aggregate demand that his market (i.e., the group of n sellers who use his newspaper) faces as contingent on the realization of N and infinitely elastic at the prices:

$$(*) \quad \begin{array}{ll} P_1 & \text{if } N = N_1 \\ P_2 & \text{if } N = N_2 \end{array} ;$$

(b) there exists a Nash strategy that dictates to sellers to buy the information about the realization of N with probability $q > 0$ and leads to the advertised prices (*) by informed newspapers and to the advertised price P_3 by uninformed newspapers;

(c) the price advertised by informed sellers clears the residual market, i.e., the market after uninformed sellers have completed all possible

trading at their advertised price.^{11/}

Note that (c) ensures that expectations are roughly correct and (b) ensures that in equilibrium there are no unexploited opportunities to increase expected utility. In particular, uninformed sellers cannot increase expected utility by becoming informed.

It is shown in appendix 3 that if the cost of information, x , is not too large and the demand and supply schedules are not too inelastic, then there exists an equilibrium vector in which $P_1 = P_3 = \bar{P}_1$ (where \bar{P}_1 is defined by [14]) and $P_2 = P_2^*$ (where P_2^* is defined by [15]).^{12/}

I shall now turn to examine the relationship between the average of the final prices which are advertised in the economy and the Walrasian price. When $N = N_1$, all advertise the Walrasian price. When $N = N_2$, the uninformed sector advertises the price \bar{P}_1 . Since (14) implies that $(1 - \mu)N_1 D(\bar{P}_1) = (1 - \mu)HS(\bar{P}_1)$, it will satisfy the demand of $(1 - \mu)N_1$ buyers. This is less than the number of buyers it will satisfy if it advertises the higher Walrasian price, \bar{P}_2 , $[(1 - \mu)N_1 < (1 - \mu)N_2]$. Therefore the residual number of buyers is larger relative to the case in which the uninformed sector advertises \bar{P}_2 , $[N_2 - (1 - \mu)N_1 > \mu N_2]$ and the price that

^{11/} The assumption that μ is non-random may be relaxed and the market clearing conditions may be substituted by the requirement that the demand will converge in probability to the supply.

^{12/} The cost of information is required to be not too large in order to ensure that some neighborhoods will be informed. When this cost is prohibitive sellers will choose a longer period in which advertised prices are fixed (see note 4).

The demand and the supply are required to be elastic in order to ensure that the informed sector will be able to clear the residual demand at a price $P_2^* < 2\bar{P}_1$. When this condition is violated each neighborhood may face a downward sloping demand curve.

clears the residual market is therefore higher than \bar{P}_2 , as in Figure 3.^{13/} Thus, $P_2^* > \bar{P}_2$ and therefore the Walrasian price is, in this case, a weighted average of the advertised prices: \bar{P}_1, P_2^* . However, in general, it will not be an arithmetic average and the weights will depend on the elasticities of supply and demand.

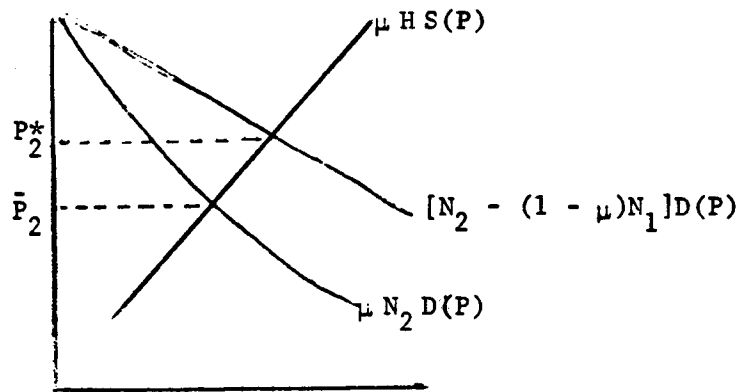


Figure 3

^{13/} Assuming that in the next period the market settles at the Walrasian price, this implies that an increase in demand leads to an overshooting in the price advertised by informed sellers. This may be related to the "profit taking" or "technical correction" phenomena in the stock exchange.

A different result is obtained for a case in which $(N_2 - N_1)$ is large. This case is examined in Appendix 3, where it is shown that a large uncertainty about the demand leads to a large difference between the two possible prices, which in this case are: the Walrasian price \bar{P}_2 if $N = N_2$ and the price P_1^* if $N = N_1$. The difference in the prices is such, that to the uninformed seller the high price promises a higher expected profit than the low price. The uninformed sellers will therefore follow a strategy of advertising the high price, \bar{P}_2 , unless they observe that the lower price, P_1^* , has been advertised. The informed sellers will advertise the price P_1^* if they observe $N = N_1$ and the Walrasian price \bar{P}_2 if they observe $N = N_2$. (In terms of the definition of competitive equilibrium: $P_2 = P_3 = \bar{P}_2$ and $P_1 = P_1^*$). It follows that when $N = N_1$ the uninformed sellers will not be able to sell at the high price, \bar{P}_2 , and the informed sellers will therefore satisfy the entire demand. This leads to $P_1^* > \bar{P}_1$ (= the Walrasian price for the case $N = N_1$).^{14/} Thus the Walrasian price must be lower than any average of the advertised prices: \bar{P}_2 and P_1^* . This result is not consistent with the hypothesis that firms advertise their best guess with respect to the Walrasian price, or with the hypothesis that on average prices "behave" as if there were an auctioneer.

The above discussion and the discussion in Appendix 1 suggest a general conclusion with respect to the "behavior" of final prices. On the basis of the initial prior, sellers advertise a Walrasian price which corresponds to a particular realization of the demand. (They choose the Walrasian price which maximizes expected profits on the basis of the initial prior.) If informed sellers observe a demand which is different than the one which was implicitly

^{14/} Thus in this case of large uncertainty prices will be "sticky" in the downward direction.

assumed by the uninformed sellers, they will be able to sell at a price which is higher than the Walrasian price for the observed demand. This is because at the "wrong" price the uninformed sellers will always satisfy a smaller number of buyers than at the market clearing price. (If the price is too low each buyer will buy a relatively large quantity and therefore a given supply will satisfy a smaller number of buyers. If the price is too high the uninformed will not sell at all.)

IV. Conclusions and Summary

An attempt was made to model the process of price adjustment in a competitive environment in which the cost of information about changes in demand is the major adjustment cost. When an uninformed seller can observe the prices which are advertised by other sellers, he will try to identify and to follow those sellers who have bought the information about the actual realization of the demand. It is assumed that the objective functions and the prior beliefs of all sellers are known and therefore all sellers can calculate the optimal price that will be advertised on the basis of the initial prior. It is therefore possible to conclude that sellers who advertise a price which is different than the optimum under the initial prior, have bought information about actual demand. This provides an identification of the informed seller. The implication is that the informed seller will end up sharing the benefits of the information with other sellers.

Due to the public good nature of the information content in announced prices, sellers adopt a mixed strategy in which they buy the information about changes in aggregate demand with a certain probability. As the number of sellers in a given market increases, the probability that a single seller will

buy the information approaches zero. (Thus sellers always watch the prices advertised by their close competitors but almost never watch aggregate magnitudes.) To maintain an incentive to buy information, the probability that the price which is finally advertised in a particular market is not based on updated information must be strictly positive. This result requires only a strictly positive cost of information and does not require any assumption about its magnitude.

An implication of the result that the final price advertised in a given market cannot always be based on updated information is that a single seller can have a noticeable effect on the probability distribution of the final price in that market. (If, for example, an individual seller decides to buy the information with probability one, the final price in the market will always be based on updated information.) The ability of the individual seller to affect the final price will disappear in a competitive economy which is composed of many markets (where a market is defined as a group of sellers each of whom can observe the price which is advertised by the others before he commits himself to a final price). Since in this case a change in the final price in a given market will have only a negligible effect on the distribution of final prices in the economy.

Since, in a given market, the probability that no one will buy the information is strictly positive, not all markets will advertise a price which is based on updated information. It is shown that in general, the Walrasian price is not an average of advertised prices. Thus our model yields results which are different from the implication of the hypothesis that firms advertise their best guess with

respect to the Walrasian price. Moreover, it does not support the hypothesis that on average prices "behave" as if there were an auctioneer.

The model suggests a generalization with respect to the "behavior" of final prices. On the basis of the initial prior, uninformed sellers advertise a Walrasian price which corresponds to a particular realization of the demand. If informed sellers observe a demand which is different than the one which was implicitly assumed by uninformed sellers, they will be able to sell at a price which is higher than the Walrasian price for the observed demand.

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Appendix 1An infinitely elastic demand curve with many possible realizations: the single market case

I consider here the case in which a single market faces an infinitely elastic demand curve at the price θ , where θ may have z possible realizations: $(\theta_1, \dots, \theta_z)$. The realization θ_i occurs with a given probability, $\text{prob}(\theta = \theta_i)$. (I shall use the notation: prob [a given event] to denote the probability that a given event will occur). Without limiting generality I shall assume that $\theta_1 < \theta_2 < \dots < \theta_z$.

It is assumed that for some $1 \leq k \leq z$

$$(A1.1) \quad \theta_k \text{ prob}(\theta \geq \theta_k) > \theta_i \text{ prob}(\theta \geq \theta_i) \quad \text{for all } i \neq k,$$

where $\text{prob}(\theta \geq \theta_i) = \sum_{j=i}^z \text{prob}(\theta = \theta_j)$. Thus before the opening of the market, the price θ_k promise the highest expected profit from the point of view of an uninformed seller. In the special case in which θ may take two possible realizations with equal probability of occurrence, the assumption $\theta_2 > 2\theta_1$ is equivalent to the assumption: $\theta_1 \text{ prob}(\theta \geq \theta_1) > \theta_2 \text{ prob}(\theta \geq \theta_2)$. Thus θ_k plays the same role as the one played by θ_1 in the text.

To accommodate the general case Claim 1 should be modified by

Claim 4: The following is a Nash strategy:

(a) if you have bought the information and observed $\theta = \theta_i$, advertise the price θ_i ($i=1, \dots, z$); (b) if you have not bought the information, advertise the price θ_k unless you observe that another price, θ_i ($i \neq k, 1 \leq i \leq z$), has been advertised. In this case advertise the price θ_i . (If no other price has been advertised, or many other prices have been advertised or the other price that has been advertised is not equal to any possible realization of θ , stick to the price θ_k .)

To show this claim let us first consider the uninformed seller's point of view. Given that all sellers follow the above strategy,

$\theta_i, (i = k)$ will be advertised only when someone has actually observed $\theta = \theta_i$. In this case it is optimal for the uninformed seller to advertise θ_i . If no one has advertised a price other than θ_k then the uninformed seller may conclude that either someone has bought the information and observed $\theta = \theta_k$, or that no one has bought the information.

To find the price that maximizes expected profit given the observation that no one has advertised a price which is different than θ_k , I introduce some additional notation:

A_i = the event that $\theta \geq \theta_i$;

B = the event that no one has advertised a price which is different than θ_k ;

μ = the probability that at least someone has bought the information;

$\text{prob}(A_i \cap B)$ = the probability that both A_i and B will occur;

$\text{prob}(A_i | B)$ = the probability of A_i given B .

Since all sellers advertise θ_k either if no one has bought the information or if someone has observed $\theta = \theta_k$, it follows that for $i \leq k$

$$(A1.2) \text{ prob}(A_i \cap B) = (1-\mu)\text{prob}(A_i) + \mu \text{prob}(\theta = \theta_k).$$

And for $i > k$

$$(A1.3) \text{ prob}(A_i \cap B) = (1-\mu)\text{prob}(A_i)$$

Using (A1.1) together with (A1.2) and (A1.3) yield

$$(A1.4) \theta_k \text{ prob}(A_k \cap B) / \text{prob}(B) > \theta_i \text{ prob}(A_i \cap B) / \text{prob}(B)$$

Using Bayes theorem (A1.4) implies that

$$(A1.5) \theta_k \text{ prob}(A_k | B) > \theta_i \text{ prob}(A_i | B).$$

Thus given the observation that no one has advertised a price which is different than θ_k , the price θ_k maximizes expected profits and it is therefore optimal for the uninformed seller to advertise θ_k .

Thus we have shown that given the strategy of the informed seller the strategy of the uninformed is optimal. It can be easily seen that the informed seller cannot do any better than advertising what he has directly observed. This completes the proof.

The calculation of the Nash probability with which the individual seller will buy the information, q^* , is similar to the calculation in the text, where here

$$SVI = \sum_{i=1}^z \theta_i \text{prob}(\theta = \theta_i) - \theta_k \text{prob}(\theta \geq \theta_k) \quad .$$

Appendix 2

The single market analysis for a case of an elastic downward sloping demand curve

I consider a simple case of a market which is similar to the one described in Section II, but the aggregate demand which it faces is downward sloping rather than infinitely elastic. For the sake of concreteness we may assume that, as in Section III, each buyer has the same demand schedule, $D(P)$, and there is uncertainty about the number of buyers, N . It is assumed that N may take two possible values: N_1 or N_2 with equal probability of occurrence. The aggregate demand is thus given by $N_1D(P)$ or $N_2D(P)$ as in Figure 4. $\theta_i, (i=1,2)$ are the Walrasian prices. It is assumed that $D(P_{max}) = 0$ and $P_{max} < 2\theta_1$. See Figure 4. This assumption is necessary to insure that the final price advertised in the market will be the same for all sellers. (The case in which two prices are advertised is more complex and is currently under investigation by Dan Peled and myself).

Sellers serve customers on a first come first served basis. Buyers are perfectly informed about advertised prices and, since there are no transaction costs, they will always buy at the lowest price unless the lowest price sellers are stocked out. It is also assumed that the number of sellers, n , is large. These assumptions imply that (a) if $N=N_i$ and all sellers advertise the market clearing price θ_i , then each individual seller faces an infinitely elastic demand curve at the price θ_i ; (b) if $N=N_i$ and all other sellers advertise $P > \theta_i$ then the individual seller faces an infinitely elastic demand curve at the price $P - \epsilon$, where ϵ is infinitesimal; (c) if $N=N_i$ and all other sellers advertise $P < \theta_i$, then the individual seller can sell all his output at a price which is close to P_{max} . (Since at $P < \theta_i$ there will be some unsatisfied buyers who will pay the

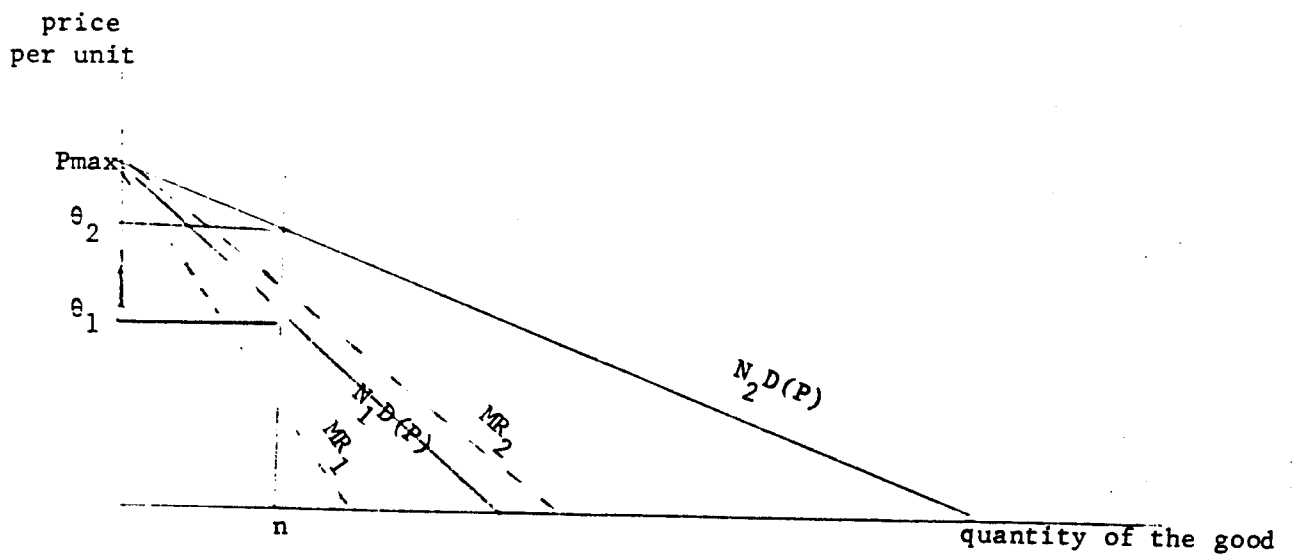


Figure 4

maximum price for the good.) I shall also assume, for simplicity, that if $N=N_i$ and all sellers advertise $P > \theta_i$, the aggregate demand is distributed equally among sellers. Finally, I shall assume that the demand is sufficiently elastic so that the marginal revenue is zero at a quantity which is larger than the supply, n . (See the MR_i schedules in Figure 4.)

Under these assumptions it will be shown that there exists a Nash strategy in which the informed sellers advertise the Walrasian price.

Claim 5: The following is a Nash strategy: (a) buy the information with probability q - to be calculated shortly; (b) if you have bought the information and observed $N=N_i$, advertise the Walrasian price, θ_i ($i = 1, 2$); (c) if you have not bought the information, advertise in the first issue of the newspaper the price θ_1 ; (d) if you have not bought the information and observe that all other sellers advertise a single price, P , advertise the price:

$P - \epsilon$ if $P > \theta_1$ and $P \neq \theta_2$, where ϵ is infinitesimal;

P if $P = \theta_1$, or $P = \theta_2$;

$P_{\max} - \epsilon$ if $P < \theta_1$, where ϵ is small.

(e) if you have not bought the information and you observe that other sellers advertise more than one price, advertise the price which is advertised by the smallest number of sellers (i.e., follow the minority).

The above strategy describes the reaction of the individual seller to the advertisements of other sellers. It instructs informed sellers to advertise the Walrasian price. It instructs uninformed sellers to follow the

minority when others advertise more than a single price; to cut the price when others advertise a single non-Walrasian price which is higher than θ_1 ; to advertise a price which is close to P_{\max} when others advertise a single price which is lower than θ_1 and to follow the others if they all advertise the same Walrasian price.

Note that if all sellers follow this strategy, the final price θ_1 will be advertised if no one has bought the information and the Walrasian price will be advertised if at least someone has bought the information. The probability q can therefore be calculated by (6) and (7) in the text. Here I shall assume that the cost of information is not prohibitive and therefore (7) is the appropriate solution. It implies that q^* goes to zero as the number of sellers goes to infinity. The informed sellers are therefore, almost surely, a minority in the class of all sellers.

To show the claim, I shall start from (d). If all other sellers advertise $P > \theta_2$ then there must be an excess supply. In this case the individual seller will not sell at all if he advertises a price which is higher than P , he will sell $1/n$ of the demand if he advertises P and he will sell his entire supply if he advertises $P - \epsilon$. The last alternative is clearly the best.

If all others advertise the price, P such that $\theta_1 < P < \theta_2$, then the uninformed may assume that the probability of excess demand is equal to the probability of excess supply (i.e., he cannot infer whether $N=N_1$ or $N=N_2$). If there is an excess demand he may sell his supply for a price which is close to P_{\max} . His expected profits in this case will be somewhat less than $P_{\max}/2$ (the probability of having an excess demand times the price). The alternative is to sell his entire supply with certainty at the price of $P - \epsilon$. This alternative is better since we assume $P_{\max}/2 < \theta_1$.

If all other sellers advertise $P < \theta_1$ the uninformed seller may conclude that there must be an excess demand and therefore he will advertise a price which is close to P_{max} .

If all other sellers advertise the Walrasian price, θ_2 , the uninformed seller will use (c) to conclude that someone must actually have observed $N=N_2$. The demand that he faces is therefore infinitely elastic at the price θ_2 and the optimal reaction is to advertise θ_2 .

If all other sellers advertise θ_1 , the uninformed seller cannot know whether it is because someone has actually observed $N=N_1$ or because no one has bought the information. If the uninformed seller advertises θ_1 he will sell his entire supply with certainty. If he advertises a price which is higher than θ_1 he will sell only if $N=N_2$. The alternative is therefore to advertise a price which is close to P_{max} (say, $P_{max} - \epsilon$). The probability that $N=N_2$ given the observation that all other sellers advertised θ_1 is less than $1/2$ (see footnote 6). The expected profit of advertising $P_{max} - \epsilon$ is therefore less than $P_{max}/2$ which by assumption is less than θ_1 . The uninformed seller's best choice is thus to advertise θ_1 in this case. Thus we have shown (d).

To show (e) note that (7) implies that the informed sellers are almost surely a minority in the class of all sellers. If the uninformed seller observes that in the first issue of the newspaper most of the sellers advertised θ_1 and a minority advertised θ_2 he will use (b) to conclude that the minority has actually bought the information and observed $N=N_2$. The best reaction is therefore to advertise θ_2 . If he observes that in some issue of the newspaper the minority advertise a price which

is different than θ_2 he may assume that he is being manipulated by the informed sellers. For example, an informed seller who observed $N=N_1$ may advertise θ_2 and after everyone has followed him, cut the price by a small amount. By following the minority he will prevent such manipulations.

Thus it was shown that if all others follow (b) - (c) it is optimal for the uninformed seller to do the same. I now turn to show that it will also be optimal from the informed seller's point of view. The informed seller knows that if he advertises a non-Walrasian price which is greater than θ_1 the uninformed sellers will first follow him (this is implied by [e]) and then cut his price (implied by [d]). His viable alternatives are therefore limited to the Walrasian prices θ_1 and θ_2 .

The informed seller also knows that he will get $1/n$ of the aggregate profit (since by assumption the demand is divided equally among all sellers). His problem is thus to choose a price out of the pair (θ_1, θ_2) which maximizes aggregate profits. Under the assumptions about the elasticity of demand advertising θ_1 if $N=N_1$ maximizes aggregate profits. Thus (b) is optimal given (c) - (e). This completes the proof of Claim 5.

Appendix 3

An Addendum to Section III

This Appendix supports the discussion on competitive general equilibrium, in the third section of the paper. I shall first calculate the probability with which each seller buys the information. I shall then use these calculations to examine the conditions under which a competitive equilibrium exists for the case in which the difference in demand is not large. Finally, I shall examine the case in which the difference in demand is large.

The calculation of q^* for Claim 3

The only difference between the following calculations and the calculations in section II is that here I shall not use the assumption of risk neutrality.

Given the expectations (12), the expected utility of sellers when buying the information is (compare with [1]):

$$(A3.1) \quad r_j(P_2) = G(P_2, 1-x)/2 + G(\bar{P}_1, 1-x)/2,$$

$$\text{where } G(P,E) = \max_y u(y, PE - Py).$$

Assuming that other sellers buy the information with probability q , the expected utility of seller j when not buying the information is (compare with [3]):

$$(A3.2) \quad R_j(P_2, q) = [1 - (1 - q)^{n-1}][G(P_2, 1) + G(\bar{P}_1, 1)]/2 \\ + (1 - q)^{n-1} G(\bar{P}_1, 1)/2 .$$

The expected utility of seller j when buying the information with probability q_j is thus

$$(A3.3) \quad V(q_j, q) = q_j r_j(P_2^*) + (1-q_j)R_j(P_2^*, q)$$

and ^{15/}

$$(A3.4) \quad \frac{\partial V}{\partial q_j} = r_j(P_2^*) - R_j(P_2^*, q) \quad .$$

Nash equilibrium requires that q^* will satisfy

$$(A3.5) \quad \frac{\partial V(q^*, q^*)}{\partial q_j} \leq 0, \text{ with equality when } q^* > 0.$$

Combining (A3.4) and (A3.5) leads to the condition for internal solution:

$$(A3.6) \quad r_j(P_2^*) - R_j(P_2^*, q^*) = 0 \quad .$$

To show that if the cost of information is not too large, then there exists an internal solution, note that when other sellers buy the information with certainty (i.e., $q = 1$) the expected utility when not buying the information, $R_j(P_2^*, 1)$, is higher than the expected utility when buying the information, $r_j(P_2^*)$. The reason is that in this case the information can be obtained, for free, by observing the prices which are advertised by others. See Figure 5, where the solid line represents the level of $r_j(P_2^*)$ and the broken line represents R_j as a function of q .

^{15/} When x is small we can use a linear approximation to (A3.4)

$$\frac{\partial V}{\partial q_j} = (1-q)^{n-1} \text{SVI} - x$$

where here $\text{SVI} = G(P_2^*, 1)/2 + G(\bar{P}_1, 1)/2 - G(\bar{P}_1, 1) \quad .$

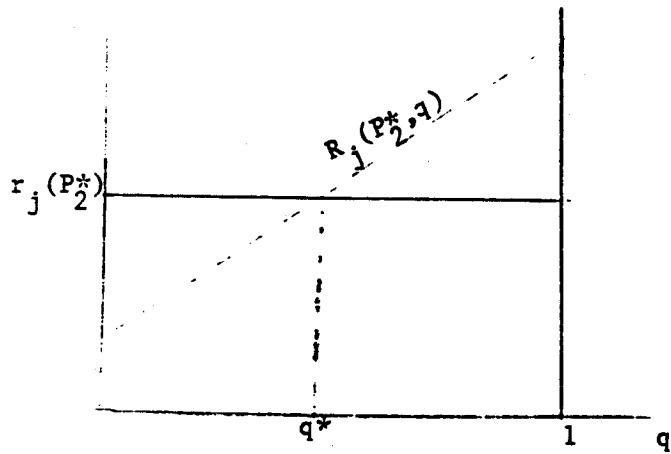


Figure 5

Furthermore, since the benefits of the information are shared by all the participants in the market, the expected utility when not buying the information is an increasing function of the probability that "others" buy the information. Thus $\frac{\partial K_j}{\partial q} > 0$ and the broken line in Figure 5 is upward sloping. Finally when $q = 0$ and x is not too large the expected utility when not buying the information, $R_j(P_2^*, 0)$, is smaller than when buying the information, $r_j(P_2^*)$, and we will have an internal solution, $0 < q^* < 1$. When x is large ("prohibitive") we will have a corner solution, $q^* = 0$.

The existence of equilibrium when $N_2 - N_1$ is not large

I will show that if x is not too large and the demand and supply schedules are not too inelastic, then there exists an equilibrium in which $P_1 = P_3 = \bar{P}_1$. (Thus the uninformed advertise the Walrasian price which corresponds to $N = N_1$). Note that Claim 1 implies (h) in the definition of competitive equilibrium. That is given the expectation (12), there exists a Nash strategy that leads to the advertised prices: (12) in informed newspapers and \bar{P}_1 in uninformed newspapers. The probability q^* and the residual market clearing price, P_2^* , are determined simultaneously. It therefore remains to be shown that there exists a solution (P_2^*, q^*) to the following two equations:

$$(A3.7) \quad r_j(P_2) - R_j(P_2, q) = 0$$

$$(A3.8) \quad (\mu N_1 + \eta) D(P_2) = \mu HS(P_2)$$

where $\eta = N_2 - N_1$ and $1 - \mu = (1 - q)^n$.

To construct the locus of points which solve (A3.7) note that when $q = 0$, (A3.7) becomes

$$(A3.9) \quad G(P_2, 1-x)/2 + G(\bar{P}_1, 1-x)/2 = G(\bar{P}_1, 1)$$

It can be shown that the solution to (A3.9) is $P_2 = \bar{P}_1 + \epsilon(x)$ where ϵ is a decreasing function of x and $\lim_{x \rightarrow 0} \epsilon(x) = 0$. It will be assumed that x is not too large in the sense that $\bar{P}_1 + \epsilon(x) < \bar{P}_2$. When P_2 goes to infinity the probability q that solves (A3.7) goes to unity. The locus of points which solves (A3.7) is thus given by the solid line, AA in Figure 6.

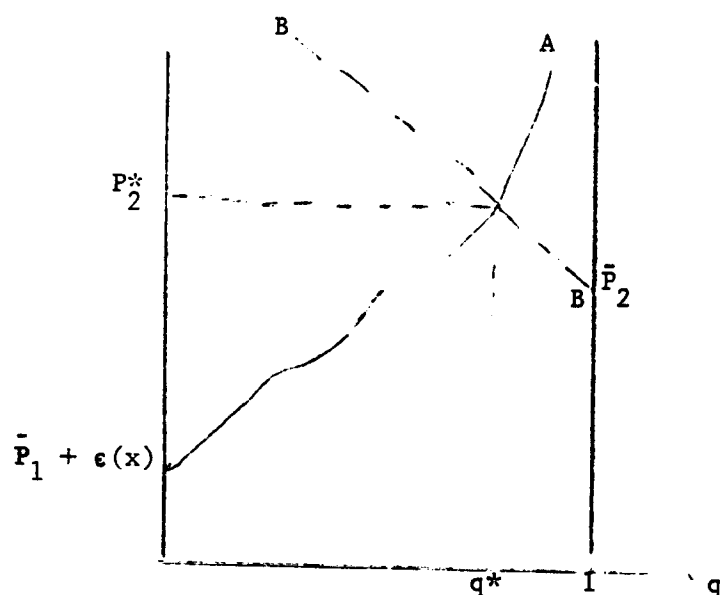


Figure 6

To construct the locus of points which solve (A3.8), I substitute $1 - \mu$ into (A3.8) to get

$$(A3.10) \quad \{[1 - (1-q)^n] N_1 + \eta\} D(P_2) = [1 - (1-q)^n] HS(P_2)$$

When $P_2 = \bar{P}_2$ the probability q that solves (A3.10) is unity (from the definition of \bar{P}_2). The discussion in the text (see Figure 3) implies that when the probability q is less than unity, P_2 which solves (A3.10) is

greater than \bar{P}_2 . Thus the locus (P_2, q) which solves (A3.10) must lie above \bar{P}_2 as illustrated by the broken line, BB, in Figure 6. Finally to ensure that $P_2^* < 2\bar{P}_1$, the BB curve should not be too inelastic. This can be ensured by assuming that the demand, $D(P)$, and the supply, $S(P)$, are not too inelastic.^{16/}

The case in which $N_2 - N_1$ is large

As a preliminary step it may be helpful to modify the strategy in Claim 1 to the case in which the difference between the two possible prices is large, i.e., $\theta_2 > 2\theta_1$. In this case on the basis of the initial prior, the price θ_2 promises higher expected profits and therefore the uninformed will advertise this price rather than θ_1 . Formally,

Claim 6: The following is a Nash strategy: (a) if you have bought the information and observed $\theta = \theta_1$, advertise the price θ_i ($i = 1, 2$); (b) if you have not bought the information, advertise the price θ_2 unless you observe that the price θ_1 has been advertised. In this case advertise the price θ_1 .

To prove this claim, note that θ_1 will be advertised if and only if someone has actually observed $\theta = \theta_1$. If no one has observed $\theta = \theta_1$ then the assumption that $\theta_2 > 2\theta_1$ implies that it is optimal to advertise θ_2 .

The suggested equilibrium solution for the case in which the difference between the two possible realizations of the demand is large is therefore $P_2 = P_3 = \bar{P}_2$ (the Walrasian price, \bar{P}_2 , is advertised by the uninformed sellers) and $P_1 = P_1^*$,

^{16/} The slope of the BB curve is given by

$$\frac{dq}{dP} = \frac{\mu HS'(P) - (\mu N_1 + \eta)D'(P)}{n(1-q)^{n-1} [N_1 D(P) - HS(P)]} < 0 .$$

where $\bar{P}_1 < P_1^* < \bar{P}_2/2$. (The price which is advertised by the informed sellers when they observe $N = N_1$ is greater than the Walrasian price for $N=N_1$ and less than half of the Walrasian price for $N=N_2$).

Claim 6 can be applied to show the existence of a Nash strategy that leads to the advertised prices:

$$P_1^* \text{ if } N = N_1 \text{ and } \bar{P}_2 \text{ if } N = N_2$$

by informed newspapers and to the advertised price \bar{P}_2 by uninformed newspapers. Thus we have shown (b) in the definition of competitive equilibrium. To show (c), I shall start by calculating the probability with which each seller buys the information.

The expected utility of seller j when he buys the information is

$$(A3.11) \quad r_j(P_1) = G(\bar{P}_2, 1 - x)/2 + G(P_1, 1 - x)/2 .$$

When seller j does not buy the information but someone else in his market does, seller j will advertise (and sell) at either \bar{P}_2 or P_1^* depending on the realization of N . When no one in his market buys the information he advertises \bar{P}_2 and may either sell at this price (if $N = N_2$) or not sell at all. Assuming that other sellers in his market buy the information with probability q , the expected utility of seller j when not buying the information is therefore

$$(A3.12) \quad R_j(P_1, q) = [1 - (1-q)^{n-1}][G(\bar{P}_2, 1) + G(P_1, 1)]/2 \\ + (1-q)^{n-1}[u(1, 0) + G(\bar{P}_2, 1)]/2 .$$

The condition for an internal solution, $0 < q < 1$, is (compare with [A3.6])

$$(A3.13) \quad r_j(P_1) = R_j(P_1, q)$$

Since when $N=N_1$, the uninformed sellers do not sell, the residual market clearing condition in this case is given by

$$(A3.14) \quad N_1 D(P_1) = \mu HS(P_1)$$

where $\mu = 1 - (1 - q)^n$. It remains to be shown that there exists a solution (q^*, P_1^*) to (A3.13) and (A3.14).

When $q = 0$, (A3.13) becomes

$$(A3.15) \quad G(\bar{P}_2, 1 - x)/2 + G(P_1, 1 - x)/2 = u(1, 0)/2 + G(\bar{P}_2, 1)/2 .$$

The price P_1 that solves (A3.15) can be made close to zero by assuming that x is sufficiently small. Further when P_1 goes to infinity q goes to unity. Thus the locus of points that solves (A3.13) can be described by the solid line AA in Figure 7.

It can be shown that the locus of points that solves (A3.14) is downward sloping and at $q = 1$, $P_1 = \bar{P}_1$. This is illustrated by the broken line BB in Figure 7. Further, the more elastic the demand $D(P)$ and the supply $S(P)$ are, the more elastic the locus BB will be. Therefore to ensure a solution $P_1^* < \bar{P}_2/2$, I assume that the demand and the supply are not too inelastic.

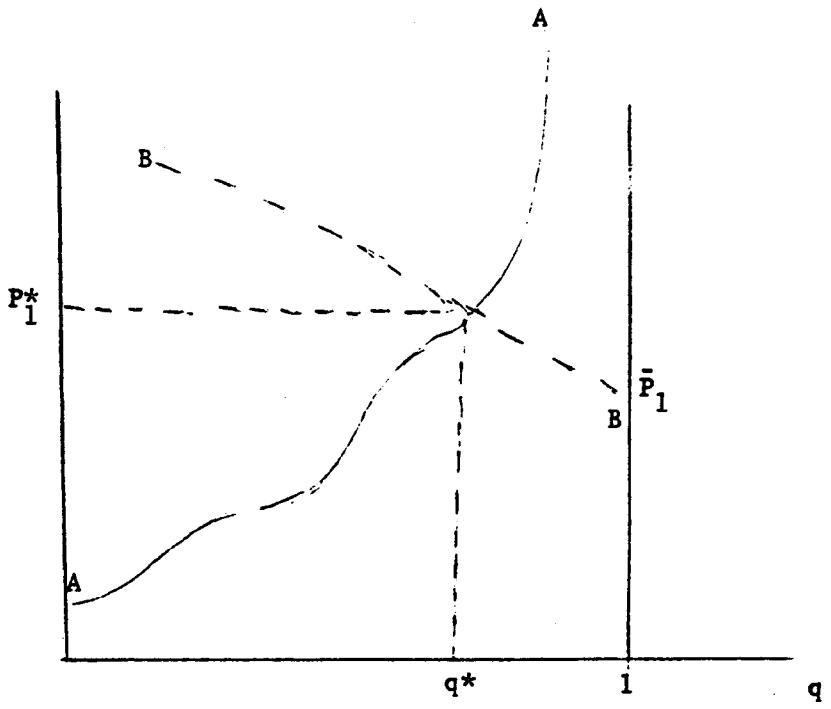


Figure 7