

COMPETITIVE PRICE ADJUSTMENT  
TO CHANGES IN THE MONEY SUPPLY

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## ABSTRACT

This model applies the analysis of competitive price adjustment in Eden [forthcoming], to the case in which changes in the money supply are the only source of uncertainty. It is shown that, if the possible changes in the money supply are not large the economy will exhibit a positive relationship between money and real output but no involuntary unemployment. If the possible changes in the money supply are large the economy may still exhibit a positive relationship between money and real output while allowing for involuntary unemployment.

## I. Introduction

In a related paper [Eden, forthcoming]. I discuss the question of who will gather and pay for information which is necessary for announcing equilibrium prices. This public good problem is analyzed in a competitive environment in which the artificial entity of the auctioneer is replaced by firms (sellers). The public good aspect of the problem emerges when there is a unique optimal price which would be announced on the basis of "old" information shared by all participants. If a single firm buys "new" information and updates its price, all firms which are aware of this revised price will rightly think that it is based on "new" information and will therefore follow the lead. The firm that updates the information therefore provides that public good which in the Walrasian model is provided by the auctioneer.

The question is which firm will undertake the function of the auctioneer? The solution suggested in Eden [forthcoming] is that all firms will adopt a mixed strategy in which they undertake this function and buy information with a certain probability. It is argued that to maintain an incentive to buy information there must be a strictly positive probability that no one will buy information and therefore the price advertised in a particular market will not always reflect updated information. (See Grossman and Stiglitz [1976, 1980] for a similar argument in a Walrasian setting.)

Here I propose to apply the above analysis of competitive price adjustment to the case in which the disturbance is a change in the money supply.

To make the paper self-contained I summarize the part of Eden [forthcoming] which describes the process of advertising prices under the assumption that expectations are exogenously given. I then consider an overlapping generations model in which (rational) expectations are determined in equilibrium, and analyse the real effects of changes in the money supply.

It is shown that, depending on the choice of a parameter that governs the evolution of the money supply, the model can produce two regimes: If the possible changes in the money supply are not large the economy will exhibit a positive relationship between money and real output but no involuntary unemployment. If the possible changes in the money supply are large the economy may still exhibit a positive relationship between money and real output while allowing for involuntary unemployment.

The results under the first regime are similar to the results in Lucas [1972] and the literature on the rational expectations equilibrium approach to the business cycles which followed Lucas' article. (See Barro [1979a] for a summary.) The results under the second regime have a Keynesian flavour. However, there are some differences between each regime and its counterpart in the literature. These differences will be discussed in the concluding section.

## II. The Sellers' Problem

I start with a simple environment in which there are  $n$  identical sellers each with a given supply of one unit of a certain good. The good is valueless to sellers and the cost of selling a unit is zero. The aggregate demand for this good is infinitely elastic at the random price  $\theta$ , where  $\theta$  may take two values:  $\theta_1$  or  $\theta_2$  ( $\theta_2 > \theta_1$ ) with equal probability of occurrence, as in Figure 1. (We may think of the international demand from the point of view of a small country.) It is assumed that all sellers maximize expected profits.

Before the opening of the market each seller can buy information, at the cost of  $x$  dollars, about the actual realization of  $\theta$ . At the opening of the market all sellers gather and each must announce a price (first round price). Sellers can then costlessly observe the prices announced by others and revise their announced prices. The second round prices are determined when no one wishes to announce a further revision. These final prices are advertised in a newspaper. Buyers get the information about prices from the newspaper and place orders by phone. It is impossible to change prices after they are advertised. It is impossible to advertise prices and to sell the good without attending the gathering, and it is impossible to buy information about the realization of  $\theta$  after the opening of the market. Thus as in the Walrasian mechanism it is assumed that transactions take place at the final prices. Here, however, sellers replace the auctioneer in announcing prices. The payoff matrix for the individual seller is:

	$\theta = \theta_1$	$\theta = \theta_2$
$p = \theta_1$	$\theta_1$	$\theta_1$
$p = \theta_2$	0	$\theta_2$

Table 1

where  $p$  is the seller's advertised price.

I start by analyzing the case in which  $\theta_2 < 2\theta_1$ . Thus, on the basis of the initial prior distribution of  $\theta$ , it is optimal to announce the lower price  $\theta_1$ . The seller's main problem is whether to buy the information about the realization of  $\theta$ . As a preliminary step for analyzing this problem it is helpful to characterize a Nash strategy<sup>1/</sup> with respect to announcing prices at the sellers' gathering.

**Claim 1:** The following is a Nash strategy: (a) if you have bought the information and observed  $\theta = \theta_i$ , announce in the first round the price  $\theta_i$  ( $i=1,2$ ); (b) if you have not bought the information, announce in the first round the price  $\theta_1$ ; (c) in the second round announce the highest of the prices announced in the first round.

To prove this claim, note that  $\theta_2$  will be announced if and only if someone has actually observed  $\theta = \theta_2$ . If no one has observed  $\theta = \theta_2$ , then the assumption that  $\theta_2 < 2\theta_1$  implies that it is optimal to announce  $\theta_1$ . Note that the final announced price is the same for all sellers.

Armed with Claim 1 we can compute the expected profit when buying and when not buying the information. If seller  $j$  buys the information he will announce the observed realization  $\theta_j$ . The expected profit in this case is

$$(1) \quad r_j = \theta_2/2 + \theta_1/2 - x = \bar{\theta} - x,$$

where  $x$  is the price of information and  $\bar{\theta}$  is the expected value of  $\theta$ . If seller  $j$  does not buy the information he may still reap the benefits if some other seller buys the information.

Assume that seller  $j$  believes that other sellers buy information independently of each other, each with probability  $q$ . The subjective probability that no other seller will buy the information is thus

$$(2) \quad (1-q)^{n-1}.$$

Using Claim 1, seller  $j$ 's subjective probability that the final price announced in the market will be  $\theta_2$  is equal to the probability that at least one seller will buy the information times the probability that  $\theta = \theta_2$ , which is:  $[1-(1-q)^{n-1}]/2$ . The probability that  $\theta_1$  will be announced is equal to the probability that no one will buy the information, plus the probability that at least someone will buy the information and observe  $\theta = \theta_1$ , which is:  $(1-q)^{n-1} + [1-(1-q)^{n-1}]/2$ . The expected profits of seller  $j$  when he does not buy the information is thus

$$(3) \quad \begin{aligned} R_j &= [1-(1-q)^{n-1}]\theta_2/2 \\ &\quad + [1-(1-q)^{n-1}]\theta_1/2 + \theta_1(1-q)^{n-1} \\ &= [1-(1-q)^{n-1}]\bar{\theta} + \theta_1(1-q)^{n-1}. \end{aligned}$$

An alternative way of computing (3) is by observing that since prices transmit all relevant information, the expected revenue of an uninformed seller

must be equal to the expected revenue of an informed seller. The uninformed seller will therefore enjoy an expected revenue of  $\bar{\theta}$  if at least one seller buys the information and a revenue of  $\theta_1$  otherwise.

Finally, if seller  $j$  decides to buy the information with probability  $q_j$  his expected profit will be

$$(4) \quad \pi_j(q_j, q) = q_j r_j + (1 - q_j) R_j.$$

Taking a partial derivative of (4) leads to:

$$(5) \quad \frac{\partial \pi_j}{\partial q_j} = (\bar{\theta} - \theta_1)(1 - q)^{n-1} - x.$$

$(\bar{\theta} - \theta_1)$  is the increase in expected revenue that each seller will experience if at least one seller buys the information. I shall therefore refer to this term as the social value of information per seller (SVI). When this term is multiplied by the probability that no other seller will buy the information, we get the increase in expected revenue that a particular seller will experience if it buys the information or the private value of information (PVI). (Note that  $PVI < SVI$ .) The derivative (5) tells us that when the private value of observing  $\theta$  is greater than the cost of doing so, ( $\partial \pi_j / \partial q_j > 0$ ), the seller will choose  $q_j = 1$ . When  $PVI < x$  the seller will choose  $q_j = 0$  and when  $PVI = x$  the seller will be indifferent with respect to the choice of  $q_j$ .

To characterize an equilibrium in this environment I shall define the probability  $q^*$  to be a Nash solution if given  $q_i = q^*$  for all  $i \neq j$ ,  $q_j = q^*$  is optimal for seller  $j$ , where  $j = 1, \dots, n$ . The possible Nash



solutions are:

$$(6) \quad \text{if } SVI < x, \text{ then } q^* = 0$$

$$(7) \quad \text{if } SVI \geq x, \text{ then } q^* = 1 - \left(\frac{x}{SVI}\right)^{1/n-1}$$

where  $SVI = \bar{\theta} - \theta_1$ . Substituting (6) into (5) leads to  $\partial \pi_j / \partial q_j < 0$  for all  $j$ , and substituting (7) into (5) leads to  $\partial \pi_j / \partial q_j = 0$  for all  $j$ , thus  $q^*$  is optimal from the point of view of all sellers. Note that under (7),  $q^*$  goes to zero when  $n$  goes to infinity. Thus, the typical seller is uninformed.<sup>2/</sup>

The probability that no one will buy the information is given by

$$(8) \quad (1-q^*)^n = \min\left[\left(\frac{x}{SVI}\right)^{n/n-1}, 1\right].$$

This probability is strictly positive.<sup>3/</sup> The intuition is that a strictly positive probability is required to maintain an incentive to buy information.

In the case where  $\theta_2 \geq 2\theta_1$ , it is optimal under the initial prior to advertise  $\theta_2$  rather than  $\theta_1$ . Therefore,

Claim 2: When  $\theta_2 \geq 2\theta_1$ , then the following is a Nash strategy: (a) if you have bought the information and observed  $\theta = \theta_1$ , announce in the first round the price  $\theta_1$  ( $i = 1, 2$ ); (b) if you have not bought the information, announce in the first round the price  $\theta_2$ ; (c) in the second round announce the lowest of the prices announced in the first round.

To prove this claim, note that  $\theta_1$  will be announced if and only if someone has actually observed  $\theta = \theta_1$ . If no one has observed  $\theta = \theta_1$ , then the assumption that  $\theta_2 \geq 2\theta_1$  implies that it is optimal to announce  $\theta_2$ .

The probability  $q^*$  can be calculated in a way which is similar to the calculations for the case  $\theta_2 < 2\theta_1$ . In general, I shall refer to the case in which on the basis of the initial prior it is optimal to advertise the lower price ( $\theta_2 < 2\theta_1$ , in the above example) as regime A and to the case in which on the basis of the initial prior it is optimal to advertise the higher price as regime B.

### III. Adjustment to Changes in the Money Supply

The analysis in the previous section will be applied to a well-known model of money. The purpose is to show that depending on the choice of a parameter that governs the evolution of the money supply, the model can produce two regimes. Under regime A the model can generate a positive relationship between money and real output but no involuntary unemployment. Under regime B it can generate a positive relationship between money and real output while allowing for involuntary unemployment.

I use a simple version of an overlapping generations model [Samuelson, 1958] in which each generation lives for two periods, produces only in the first period a single non-storable good and consumes only in the second period. All individuals (in all generations) have the same, Von Neumann-Morgenstern, utility function

$$(9) \quad V = u(\ell) + c,$$

where  $\ell$  is first period leisure ( $0 \leq \ell \leq 1$ ) and  $c$  is second period consumption. Thus, the utility function exhibits risk neutrality with respect to second period consumption. The function  $u(\ )$  is assumed to be monotone concave and differentiable. Each individual can produce according to the linear production function:

$$(10) \quad c = L,$$

where  $L = 1 - \ell$ , is the amount of labor input. There is a one period lag in production. Thus, labor input is invested during the period, while consumption occurs at the end of the period.

Money is the only asset and it is typically used by the old generation (the buyers) to buy the consumption good from the young generation (the sellers or the producers). Changes in the money supply are the only source of uncertainty. It is assumed that the additional money is distributed in proportion to the holdings of money and individuals are aware of this fact. (Thus if you hold 1% of the money supply you get a transfer payment of 1% of the newly printed money.)

It is assumed that the money supply at time  $t$ ,  $M_t$ , may be either  $M_{t-1}$  or  $\alpha M_{t-1}$  with equal probability of occurrence. The parameter  $\alpha$  is assumed to be greater than unity. Economic agents who are born at time  $t$  know the probability distribution of  $M_t$  and may buy information (from a member of the old generation) about the actual realization of  $M_t$ .

It is assumed that the sellers (members of the young generation) are distributed over a large number of neighborhoods,  $n$  sellers per neighborhood. It is assumed that the number of sellers in each neighborhood,  $n$ , is also large. Each seller can observe the prices which are advertised in his neighborhood at the beginning of the period, before he commits himself to a final price. He can observe the prices which are advertised in other neighborhoods only at the end of the period after he is committed to a price. (Thus, there is a one period lag in transmitting information about prices to sellers in other neighborhoods.) The process of advertising prices in each neighborhood is similar to the one described in the previous section. At the beginning of the period each seller can buy the information about the actual realization of  $M_t$  for a percentage  $x$  of the actual money supply.<sup>4/</sup> Then all the neighborhood's sellers gather

and each announces a (first round) price. After all the sellers have observed the prices announced in the first round, they announce second round prices which are then advertised in a newspaper. (It is impossible to change prices during the period. It is impossible to advertise prices and to sell the good without attending the neighborhood's gathering and it is impossible to buy information after the gathering has already started.)

Since there is a one period lag in production and in the transmission of information about prices which are advertised in other neighborhoods, each producer chooses his labor input before he learns about the prices which are advertised in other neighborhoods. At the end of the period, after the production process has been completed, sellers start to receive orders, by phone, from buyers all over the economy. These orders are satisfied on a first come first served basis. It is assumed that delivery is costless and that by this time (the end of the period) buyers are perfectly informed about all advertised prices.

I shall proceed by describing the behavior of the system under certain expectations. I shall then argue that in equilibrium the postulated expectations are roughly correct. It is assumed that at time  $t$  each seller views the aggregate demand that his neighborhood faces as contingent on the money supply and infinitely elastic at  $P$ , where

$$(11) \quad \begin{aligned} P &= P_1, \text{ if } M_t = M_{t-1}; \text{ and} \\ P &= P_2, \text{ if } M_t = \alpha M_{t-1}. \end{aligned}$$

Note that if the old generation always spends all its money then one who at time  $t$  holds \$1 will be able to buy, on average,  $(1/M_t)y_{t+1}$  units

of consumption at time  $t + 1$ , where  $y_{t+1}$  is the aggregate real output at time  $t + 1$ .<sup>5/</sup> Since the utility function is linear in second period consumption, the seller can be viewed as if he tries to maximize the expected share of the current money supply (= the expected share of future output). It is therefore convenient to use percentage of the current money supply as the basic unit of account.

Given the expectation (11) the payoff per unit of labor, in terms of percentage of the current money supply, is

	$M_t = M_{t-1}$	$M_t = M_{t-1}$
$P = P_1$	$P_1/M_{t-1}$	$P_1/M_{t-1}$
$P = P_2$	0	$P_2/M_{t-1}$

Table 2

This payoff matrix is used in the appendix to calculate the Nash probability,  $q^*$ , with which the individual seller will buy information about the money supply. The calculation of  $q^*$  is similar to the calculations of this variable in section II. A minor difference is that here the information about the money supply is useful for choosing the labor supply as well as for advertising the price. I shall consider here the case of internal solution,  $0 < q^* < 1$ .

I shall define a neighborhood as uninformed if none of the  $n$  sellers in the neighborhood has bought the information about the realization of the money supply. The probability that no one in a given neighborhood will buy the information is

$$(12) \quad 1 - \mu = (1 - q^*)^n.$$

Since we have a large number of neighborhoods I shall use the law of large numbers and assume that a fraction  $\mu$  of all neighborhoods are informed, where  $\mu$  is calculated from (12) and is non-random.

Table 2 can be used to show that under the initial prior it is optimal to announce the lower price  $P_1$  if

$$(13) \quad (P_2 - P_1)/\alpha M_{t-1} < P_1/M_{t-1}.$$

(Otherwise it is optimal to announce the higher price,  $P_2$ . The inequality (13) plays the same role as the inequality  $\theta_2 < 2\theta_1$ , in Section II.) The inequality (13) will hold, in equilibrium, for magnitudes of  $\alpha$  which are not too large (and the reverse will be true for large  $\alpha$ 's). I shall start by assuming (13). This is the case of regime A.

Under this regime a version of Claim 1 is applicable, namely:

Claim 3: The following is a Nash strategy: (a) buy the information about the realization of the money supply with probability  $q^*$  (see the appendix for the calculation of  $q^*$ ); (b) if you have bought the information announce (11) in the first round (that is,  $P = P_1$  if  $M_t = M_{t-1}$  and  $P = P_2$  if  $M_t = \alpha M_{t-1}$ ); (c) if you have not bought the information announce  $P_1$  in the first round; (d) in the second round announce the highest out of all prices announced in the first round.

Thus in neighborhoods in which at least one seller has bought the information, (11) will be advertised. In neighborhoods in which no one has bought the information,  $P = P_1$  will be advertised. Therefore, when

$M_t = M_{t-1}$  the price  $P_1$  will be advertised in all neighborhoods; when  $M_t = \alpha M_{t-1}$ ,  $P_2$  will be advertised in informed neighborhoods and  $P_1$  in uninformed neighborhoods.

Since the probability that an individual seller will buy the information goes to zero when  $n$  goes to infinity (see [7]), the fraction of sellers, in any given neighborhood, who actually bought the information is negligible. A typical seller will therefore calculate the expected payoff only on the basis of the price which was advertised in his neighborhood. Since Claim 3 implies that the higher price will be advertised only if someone has actually observed  $M_t = \alpha M_{t-1}$  the payoff per unit, in terms of percentage of the money supply, given that  $P_2$  was advertised (given  $P_2$ ) is:

$$(14) \quad w_2 = P_2 / \alpha M_{t-1}$$

The price  $P_1$  may be advertised either if no one has bought the information or if someone has bought the information and observed  $M_t = M_{t-1}$ . Therefore a seller who observes that the price  $P_1$  was advertised will not be able to figure out the realization of the money supply with certainty. Let  $\phi$  be the probability that  $M_t = M_{t-1}$  given  $P_1$ . The expected payoff given  $P_1$  is:

$$(15) \quad w_1 = \phi P_1 / M_{t-1} + (1-\phi) P_1 / \alpha M_{t-1}$$

Let  $L(w_i)$  be the labor supply of a producer who expects a payoff of  $w_i$  ( $i = 1, 2$ ). That is,  $0 \leq L(w_i) \leq 1$ , maximizes  $u(1-L) + w_i L E y_{t+1}$ , where  $E y_{t+1}$  denotes the expected value of aggregate real income at time  $t+1$ , given information which is available at time  $t$ . The concavity of  $u(\cdot)$  can be used to show that  $L'(\cdot) \geq 0$  (i.e., that the labor supply curve is upward sloping.)



The expectations that  $P = P_1$  if  $M_t = M_{t-1}$ , can be rationalized if  $P_1$  satisfies the market clearing condition:<sup>6/</sup>

$$(16) \quad P_1 NL(w_1) = M_{t-1},$$

where  $N$  is the number of people born at time  $t$ . (16) rationalizes the seller's expectations for the case  $M_t = M_{t-1}$ , since it insures that the group of sellers in each neighborhood will be able to sell all its supply at  $P_1$  but (if each neighborhood is small relative to the economy) will not be able to sell at  $P > P_1$ .

Expectations regarding the case  $M_t = \alpha M_{t-1}$  can be rationalized by the same considerations, provided that  $P_2$  clears the residual market. Specifically, when  $M_t = \alpha M_{t-1}$ ,  $(1-\mu)N$  sellers will advertise  $P = P_1$  and will sell  $(1-\mu)NL(w_1)$  units for the total amount of  $(1-\mu)M_{t-1}$  dollars (see [16]). The nominal purchasing power that is left in the hands of the old generation after the trade with uninformed neighborhoods is completed is thus,  $\alpha M_{t-1} - (1-\mu)M_{t-1}$ . Therefore if  $P_2$  satisfies

$$(17) \quad P_2 \mu NL(w_2) = \alpha M_{t-1} - (1-\mu)M_{t-1},$$

the group of sellers in each informed neighborhood will be able to sell all its supply at  $P_2$  but will not be able to sell at  $P > P_2$ . This rationalizes the expectations for the case  $M_t = \alpha M_{t-1}$ .

Thus expectations are rationalized by market clearing conditions, where the informed sector plays the central role of clearing the residual market. In general,

Definition  $(q, P_1, P_2, P_3)$  is an equilibrium vector for time  $t$  if

(a) each seller views the aggregate demand that his neighborhood faces as contingent on the money supply and infinitely elastic at

$$(*) \quad \begin{aligned} P &= P_1, \text{ if } M_t = M_{t-1}; \text{ and} \\ P &= P_2, \text{ if } M_t = \alpha M_{t-1}; \end{aligned}$$

(b) sellers form rational expectations about the purchasing power of money in the next period;

(c) there exists a Nash strategy that dictates (to sellers) to buy the information about the realization of  $M_t$  with probability  $q > 0$  and leads to the advertised prices  $(*)$  in informed neighborhoods and to the advertised price  $P_3$  in uninformed neighborhoods;

(d) the price advertised in informed neighborhoods clears the residual market. (Where the residual market is what is left after uninformed sellers have completed all the possible trading at their advertised price.)

Note that (d) ensures that expectations are roughly correct and (c) ensures that in equilibrium there are no unexploited opportunities to increase expected utility. In particular, uninformed sellers cannot increase expected utility by becoming informed.

In terms of this general definition, the equilibrium solution which was suggested for regime A, satisfies  $P_3 = P_1$ . Thus, uninformed neighborhoods advertise the price which is expected for the lower realization of the money supply. (The equilibrium solution which will be suggested for regime B, satisfies  $P_3 = P_2$ .)

The existence of this type of equilibrium, for the two possible regimes, is discussed in the appendices to Eden [forthcoming, 1980]. I shall now argue that when  $M_t = \alpha M_{t-1}$ , producers in informed neighborhoods will

increase their supply while producers in uninformed neighborhoods will produce the same amount, all relative to the case  $M_t = M_{t-1}$ .

I shall start by showing that if  $L(w_1) = L(w_2)$  then the payoff per unit of labor must be larger when  $M_t = \alpha M_{t-1}$  (i.e.  $w_2 > w_1$ ).

Since when  $P_1$  is advertised an uninformed seller cannot infer (with certainty) the realization of the money supply,  $P_2 \geq \alpha P_1$  is sufficient to ensure that  $w_2 > w_1$  (see [14] and [15]). To show that  $P_2 \geq \alpha P_1$ , note that  $\alpha > 1$  and  $\mu < 1$  can be used to show that

$$(18) \quad \alpha M_{t-1} - (1-\mu)M_{t-1} > \alpha \mu M_{t-1} .$$

Substituting (18) in (17) leads to

$$(19) \quad P_2 \mu N L(w_2) > \alpha \mu M_{t-1}$$

and if  $L(w_1) = L(w_2)$  then we can substitute (16) to get  $P_2 > \alpha P_1$ .

The intuition for this result can be improved by considering the hypothetical case in which sellers in uninformed neighborhoods advertise the price  $\alpha P_1$ . In this case if  $M_t = \alpha M_{t-1}$ , they will sell their supply for the total amount of  $(1-\mu)\alpha M_{t-1}$  dollars; sellers in informed neighborhoods will face a residual purchasing power of  $\mu \alpha M_{t-1}$  dollars and, if labor supply is the same,  $\alpha P_1$  will clear the residual market. Since sellers in uninformed neighborhoods advertise a price which is lower than  $\alpha P_1$  the residual purchasing power is larger and the residual market clearing price is higher.

Thus we have shown that under the assumption of inelastic labor supply, the expected payoff in informed neighborhoods is higher when  $M_t = \alpha M_{t-1}$ . If the labor supply curve is upward sloping, we will get an equilibrium solution in which both the labor supply and the expected payoff are higher. The labor supply and the expected payoff in uninformed neighborhoods do not depend on the actual realization of the money supply. (Since the choice of labor supply is done before the producers observe the prices in other neighborhoods.) Thus the net effect of money on output is positive.

I shall now analyze regime B, under which  $\alpha$  is large and the reverse of (13) holds. In this case on the basis of the initial prior (11) it is optimal for the uninformed seller to announce the higher price,  $P_2$ , rather than the lower price,  $P_1$ . We should therefore get an equilibrium solution in which sellers in uninformed neighborhoods advertise the price which corresponds to their expectation for the case  $M_t = \alpha M_{t-1}$ . I.e.,  $P_3 = P_2$ . If  $M_t = \alpha M_{t-1}$  all neighborhoods will advertise  $P_2$  and the market clearing condition is therefore

$$(20) \quad P_2 \text{NL}(w_2) = \alpha M_{t-1} ,$$

where here  $w_2 = \phi P_2 / \alpha M_{t-1}$  (see Table 2) and  $\phi$  is the probability that  $M_t = \alpha M_{t-1}$  given that  $P_2$  was advertised. If the money supply fails to increase sellers in uninformed neighborhoods will not be able to sell at all, and the residual market clearing condition is therefore

$$(21) \quad P_1 \mu \text{NL}(w_1) = M_{t-1} ,$$

where here  $w_1 = P_1 / M_{t-1}$ .

To examine the effect of money on traded output note that if labor supply is inelastic (i.e.,  $L' = 0$ ), then  $\mu < 1$  implies that the traded output will be larger when  $M_t = \alpha M_{t-1}$ . But under the same assumption (20) and (21) can be used to show that the expected payoff to producers in informed neighborhoods is larger when  $M_t = M_{t-1}$  (i.e.,  $w_1 > w_2$ ).<sup>7/</sup> We shall therefore get an equilibrium solution in which both the labor supply and the expected payoff in informed neighborhoods are larger. Thus, when  $M_t = M_{t-1}$  not all neighborhoods will sell but those who sell will sell more. The net effect of money on output, under regime B, is therefore ambiguous.

In a more complete model the inability of sellers in uninformed neighborhoods to sell at their advertised price, will eliminate their demand for inputs, once they realize that they have made a mistake. This will lead to involuntary unemployment. To elaborate on this point I shall introduce the distinction between variable and fixed factors of production.

#### IV. Fixed and Variable Factors of Production

It seems that the easiest way to allow for involuntary unemployment is by changing the assumption about the information lag. Instead of assuming that sellers learn about prices in other neighborhoods with one period lag, I shall assume here that this lag is less than a period but more than half of a period, say three quarters of a period. In this case, producers can choose the supply of labor in the last quarter on the basis of information about the prices which were advertised in all the neighborhoods, but at the last quarter they do not have enough time to transmit a new price.

I shall refer to the labor supply during the first three quarters as the fixed factor of production and to the labor supply during the last quarter as the variable factor of production. Thus, the fixed factor is chosen on the basis of the price which was advertised in a single neighborhood while the variable factor is chosen on the basis of prices which were advertised in all the neighborhoods.

Under regime B, the higher price,  $P_2$ , is advertised in uninformed neighborhoods. If at the beginning of the fourth quarter, sellers in uninformed neighborhoods observe that in some neighborhoods the advertised price is  $P_1$ , they will infer that  $M_t = M_{t-1}$  and that it is impossible to sell at  $P_2$ . They will therefore choose not to work at all during the fourth quarter. This unemployment is involuntary in the sense that if they could have transmitted a lower price they would have chosen to work.

To gain some insight to the nature of the involuntary unemployment it may be useful to think of the producer in the first three quarters and the producer in the last quarter as separate entities: a firm and a worker.

The source of the problem is in firms that advertise a price which is too high rather than in workers that ask for a wage rate which is too high, since the firms will not pay any positive wage rate after they have realized that they have made a mistake. (For a more complete model that allows for the distinction between firms and workers and for a fuller analysis of the effect of money on the supply of the variable factor, see an earlier version of this paper, Eden [1980].)

Under regime A, the lower price,  $P_1$ , is advertised in uninformed neighborhoods. Sellers in these neighborhoods will therefore always sell at their advertised price. The discussion in Section III suggests that the net effect of money on the supply of the fixed factor (i.e., labor supply during the first three quarters) is positive.

To examine the effect of money on the variable factor, note that if after observing all prices, sellers in uninformed neighborhoods infer that  $M_t = \alpha M_{t-1}$  they will revise their expected payoff downward from (15) to  $P_1/\alpha M_{t-1}$ . If they infer that  $M_t = M_{t-1}$  they will revise their expected payoff upward from (15) to  $P_1/M_{t-1}$ . Thus the relationship between the money supply and the expected payoff in uninformed neighborhoods is negative and therefore the relationship between the money supply and the supply of the variable factor in uninformed neighborhoods is negative. The relationship between the money supply and the supply of the variable factor in informed neighborhoods is positive, since in these neighborhoods no new information is revealed by observing the prices in other neighborhoods and therefore the variable factor will behave in the same way as the fixed factor (see Section III). The net effect of money on the supply of the variable factor is therefore ambiguous.

### V. A Generalization and Conclusions

It may be worthwhile to outline the case in which the money supply has many possible realizations. Let  $M_t$  take the possible realization  $M_{ti}$  with probability  $\lambda_i$ , ( $i = 1, \dots, z$ ). -- In equilibrium, each seller views the aggregate demand that his neighborhood faces as contingent on the money supply and infinitely elastic at  $P$ , where

$$(22) \quad P = P_i, \text{ if } M_t = M_{ti}$$

(and  $P_i$  clears the residual market when  $M_t = M_{ti}$ ).

Without limiting generality assume that on the basis of the initial prior distribution it is optimal to announce  $P_K$ .<sup>8/</sup> Then, it can be shown (see Appendix 1 in Eden [forthcoming] that

Claim 4: The following is a Nash strategy:

- (a) if you have bought the information and observed  $M_t = M_{ti}$ , advertise the price  $P_i$  ( $i = 1, \dots, z$ );
- (b) if you have not bought the information advertise the price  $P_K$  unless you observe that another price,  $P_i$  ( $i \neq K$ ), has been announced. In this case advertise the price  $P_i$ .

Thus, the price which corresponds to the sellers' expectations for the case  $M_t = M_{tK}$  is advertised in uninformed neighborhoods. I shall refer to  $M_{tK}$  as the anticipated money supply. If the money supply is above its anticipated level, sellers in informed neighborhoods will experience a higher expected payoff and will increase the supply both of the fixed and of the variable factors while sellers in uninformed neighborhoods will not change their supply of the fixed factor and will reduce their supply of the variable factor.



This case corresponds to regime A. If the money supply is below its anticipated level, sellers in uninformed neighborhoods will not be able to sell at the price  $P_K$ . The variable factor in these neighborhoods will experience involuntary unemployment while the output produced by the fixed factor will not sell (in a more complete model it may be labeled involuntary inventories). This will be at least partially offset by the increase in output in informed neighborhoods. This is regime B.

It seems that the above analysis is some mixture of Lucas [1972] with the Keynesian model. As in Lucas' model we can have a positive effect of money on real output under rational expectations. Here however this effect does not require real shocks and it does not require the assumption that information about changes in the money supply is prohibitively expensive. As in the Keynesian model we can have something like a "downward stickiness" of prices and we can have involuntary unemployment. Here, however, the prices are endogenous and the level from which prices start to be "downward sticky" is determined in equilibrium rather than historically given.

Appendix

The Calculation of  $q^*$  for Regime A in Section III

I will show that if the cost of information is not prohibitive, then there exists an internal solution to the probability with which the individual seller buys the information.

Table 2 can be used to show that if seller  $j$  buys the information, his expected utility is:

$$(A1) \quad r_j = \left[ \max_{0 < L < 1} u(1-L) + L E_t y_{t+1} (P_2 / \alpha M_{t-1} - x) \right] / 2 \\ + \left[ \max_{0 < L < 1} u(1=L) + L E_t y_{t+1} (P_1 / M_{t-1} - x) \right] / 2$$

where  $E_t y_{t+1}$  is the expected aggregate real output in time  $t+1$ . Since  $M_t$  are serially independent it can be shown that  $y_{t+1}$  are serially independent and therefore  $E_t y_{t+1}$  is the same for both realizations of the money supply.

If seller  $j$  does not buy the information and the price  $P_1$  is advertised, he will solve:

$$(A2) \quad \max_{0 < L < 1} u(1-L) + L w_1 E_t y_{t+1}$$

where  $w_1$  is the expected payoff in terms of percentage of the money supply given that  $P_1$  is advertised in the neighborhood (given  $P_1$ ). The definition of  $w_1$  in (15) can be made more precise by expressing the probability  $\phi$  in terms of  $q$  (the probability with which a typical seller buys the information).

Thus,

$$(A3) \quad w_1 = [1 - (1-q)^{n-1} / 2] P_1 / M_{t-1} + [(1-q)^{n-1} / 2] P_1 / \alpha M_{t-1}$$

If seller  $j$  does not buy the information and the price  $P_2$  is advertised, he will solve:

$$(A4) \max_L u(1-L) + L E y_{t+1} P_2 / \alpha M_{t-1}.$$

The expected utility when not buying the information is a function of the probability  $q$  and will be denoted by  $R_j(q)$ . It can be computed by multiplying (A2) by the probability that  $P_1$  will be advertised and (A4) by the probability that  $P_2$  will be advertised. Thus,

$$(A5) R_j(q) = (A2) \{ [1 - (1-q)^{n-1}] / 2 + (1-q)^{n-1} \} + (A4) [1 - (1-q)^n] / 2$$

The expected utility of seller  $j$  when buying the information with probability  $q_j$  is therefore

$$(A6) \Pi(q_j, q) = q_j r_j + (1-q_j) R_j(q)$$

and the seller will maximize (A6) with respect to  $q_j$ . An internal solution requires:

$$(A7) R_j(q) = r_j.$$

To examine the condition for internal solution note that when someone else buys the information with certainty, you can get all the information by observing the price which is advertised in the neighborhood without spending  $x$ . Thus  $R_j(1) > r_j$ . When  $q = 0$ , the price  $P_1$  will always be advertised. Therefore  $R_j(0) = \max_L u(1-L) + L w_1 E y_{t+1}$  and if  $x$  is not large (prohibitive) then  $R_j(0) < r_j$ . Since  $R_j(q)$  is continuous it follows that if  $x$  is not prohibitive there exists an internal solution,  $q^*$ , to (A7). This solution is also a Nash solution since under (A7),  $q^*$  is optimal for all sellers.

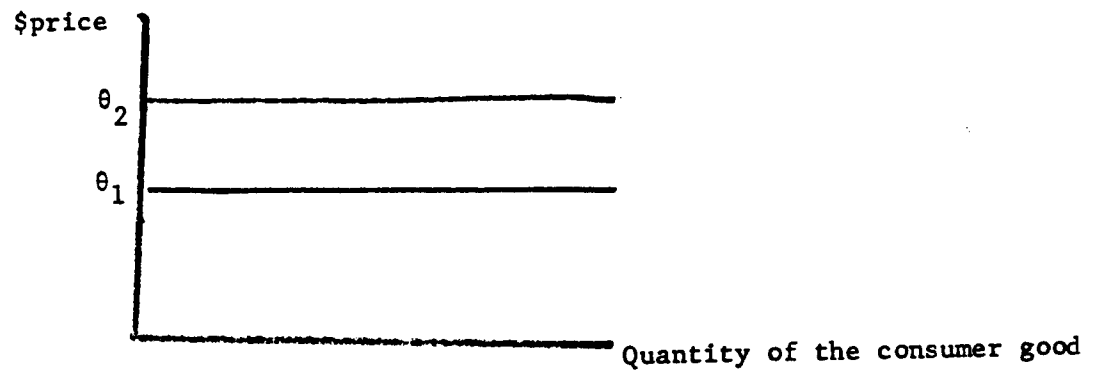


Figure 1

### Footnotes

<sup>1</sup>If all other sellers follow a Nash strategy it is optimal for the individual seller to do the same.

<sup>2</sup>For a similar result in a different model, see Barro [1979b].

<sup>3</sup>Under (6) the probability in (8) is unity. Under (7) this probability is strictly positive and when  $n$  goes to infinity it approaches the limit  $x/SVI$ .

<sup>4</sup>Thus the payment for the information is linked to the money supply. For example, it may be \$1 if  $M_t = M_{t-1}$  and  $\$a$  if  $M_t = \alpha M_{t-1}$ .

<sup>5</sup>This is due to the fact that newly printed money is distributed in proportion to the holding of "old" money. The words "on average" refer to the case when there is more than one price. In this case it is assumed that the probability of buying at the lower price is the same for all consumers.

<sup>6</sup>Since  $q^*$  is infinitesimal, the aggregate expenditure on information gathering is small relative to aggregate output. This expenditure is therefore ignored in the market clearing equations.

<sup>7</sup>(20) implies  $P_2/\alpha M_{t-1} = 1/NL$  and (21) implies  $P_1/M_{t-1} = 1/\mu NL$ .

<sup>8</sup>Thus,

$$P_K \sum_{i=K}^z \lambda_i > P_j \sum_{i=j}^z \lambda_i, \text{ for all } j \neq K.$$

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