

ON MEASURING THE VARIANCE-AGE PROFILE OF LIFETIME EARNINGS

By

Benjamin Eden  
University of California, Los Angeles  
and The Hebrew University

and

Ariel Pakes  
The Hebrew University  
and The Falk Institute

Discussion Paper Number 196  
Forthcoming in  
The Review of Economic Studies

#### ABSTRACT

This paper introduces an operationally useful measure of the uncertainty in different earnings paths. The measure, which we call the variance-age profile of lifetime earnings, is operational both because it can be estimated with relative ease and because, under the assumptions of the life-cycle permanent-income theory of individual decision making, it can be used to define a preference ordering over the riskiness of alternative earnings paths. The paper concludes with an illustration which compares the uncertainty associated with the earnings paths of different schooling groups in the Israeli economy and discusses data requirements in greater depth.

+ +

ON MEASURING THE VARIANCE-AGE PROFILE OF LIFETIME EARNINGS

1. *INTRODUCTION*

The purpose of this paper is to introduce a measure of the uncertainty in different earnings paths. The measure we propose has two desirable properties. First, given appropriate data, it is relatively easy to estimate. Second, under the assumptions of the life-cycle permanent-income theory of individual decision making, it can be used to provide a partial preference ordering over alternative earnings paths. As a result, the measure presented here ought to prove helpful in analysing an assortment of decisions, be they individual decisions or policy alternatives, which have long-term effects on earnings. Examples include decisions which affect schooling, on-the-job training, unionization, migration, and sector of employment.

Our measure of the uncertainty in earnings paths is derived from the effect of this uncertainty on the distribution of lifetime income. Lifetime income is defined as the discounted sum of future earnings plus the value of the individual's assets. Since future earnings are not known with certainty lifetime income is a random variable. To demonstrate, we let  $T$  be the length of the planning horizon,  $W = [w_1, w_2, \dots, w_T]$  designates

the individual's earnings path,  $r$  is the *safe* interest rate, and  $A_t$  the known value of the individual's assets at  $t$ . Then the distribution of  $Y(t) = \sum_{j=t}^T w_j (1+r)^{j-t} + A_t$ , conditional on all information available at  $t$ , is the distribution of lifetime income at  $t$ ; and the variance in  $Y(t)$  is the variance in the present value of future earnings. It is this variance, and its resolution over time, that we measure. Our interest in the variance of lifetime earnings stems, in part, from the life-cycle permanent-income theory of individual decision making. In that theory, individual satisfaction is determined by the maximization of an intertemporal choice function subject to the random lifetime budget constraint that the discounted value of consumption expenditures cannot exceed  $Y(t)$ . Thus the uncertainty in earnings will affect the individual's welfare through its effects on  $Y(t)$ .

Although the concept of uncertainty in earnings paths has been discussed extensively in the literature, very little empirical work has been done on it. We do have some evidence on the dispersion of earnings among members of different population groups (see for example Becker, 1975), and there are a few panel-data studies of the stochastic process generating earnings (see Lillard and Weiss, 1979, and the literature cited therein). The problem with using the large unexplained portion of the cross-sectional dispersion of earnings to construct measures of the uncertainty in different earnings profiles is that individuals possess information on their probable future positions in the earnings distribution, that we, in our role as researchers, do not. Thus Becker concludes his discussion of the variance in the returns to college education with the question: "How much of this large variation in the gain from a college education can be anticipated due to known differences in ability, environment, etc., and, therefore,

should not be considered part of the *ex ante* risk?" (1975, p. 189). By allowing for unobservable individual-specific components of variation in earnings the panel-data studies may be more helpful in this context. The problem with using them to construct a measure of, say, the variance in  $Y(t)$  is that to do so one must make some very stringent assumptions on both the stochastic process generating earnings (a topic we know little about) and on the amount of information about that process available to the individual at different dates.

In this paper we suggest a different approach to measuring the uncertainty in different earnings paths. Rather than making assumptions on the process that generates earnings we make the behavioral assumption that the life-cycle theory of consumption determines consumption expenditures. We then show how the implications of the life-cycle theory for the movements in consumption expenditures can be used to obtain a meaningful measure of the uncertainty in different earnings paths without making any but the mildest of assumptions regarding the earnings process. This is the major advantage of our approach. On the other hand, we do assume that the life-cycle theory is an adequate description of the evolution of consumption expenditures once one allows for a disturbance process.

To introduce our measure we must first describe how the variance in  $Y(t)$  evolves over the life cycle. It is clear that as time passes information accumulates and the uncertainty associated with any given earnings path is likely to decrease. One can provide a more concrete description of the time resolution of the uncertainty in lifetime earnings by defining  $v_{t+1}^2 = \text{var}[Y(t)](1+r)^2 - \text{var}[Y(t+1)]$  for  $t = 0; 1, \dots, T-1$ . The  $v_{t+1}^2$  is the decrease in the variance of the present value of future

earnings that occurs as a result of information that accumulates during period  $t$ ; or, equivalently, it is the variance in the innovation of lifetime income that is realized during period  $t$ . Since the earnings path,  $W$ , is known with certainty at the end of the planning horizon,  $\text{var}[Y(T)] = 0$  and the initial or total variance in lifetime income is just the discounted sum of the variances realized over the life-cycle. That is, it follows by recursion that  $\text{var}[Y(0)] = \sum_{j=1}^T (1+r)^{-2j} v_j^2$ . The vector  $V^2 = [v_1^2, \dots, v_T^2]$  provides a description of both the initial uncertainty associated with any earnings path and of how that uncertainty is realized over time. We shall call  $V^2$  the variance-age profile of lifetime earnings. If its elements are large there is a high degree of uncertainty in the associated earnings stream while if  $V^2 = [v_1^2, 0, 0, \dots, 0]$  all of that uncertainty is resolved by the end of the first period.

This paper provides a method of estimating the  $V^2$  vectors associated with different earnings paths. Eden (1980) has shown how these estimates can be used to compare riskiness, and therefore provide a partial preference ordering over alternative earnings paths. As one might expect, earnings paths with lower total variance, and more of that variance realized in earlier periods, will be preferred over alternatives.<sup>1</sup>

The next section shows how the logical implications of the life-cycle consumption model can be used to estimate  $V^2$  in a world of homogeneous preferences and perfectly measured variables. This part of the paper is closely related to recent work on predicting aggregate consumption by Hall (1978) and Hayashi (1980). Section 3 adds the appropriate disturbances and describes the identification scheme. In Section 4 we present some illustrative calculations of the variance-age profiles for different schooling groups in the Israel economy and discuss data

requirements in greater depth. A brief summary closes the paper.

2. USING THE LIFE-CYCLE MODEL TO MEASURE  $V^2$

In the life-cycle consumption model each individual at every  $t$  plans a random consumption vector for the remainder of the planning horizon by maximizing an intertemporal objective function subject to the random lifetime budget constraint  $Y(t) = \sum_{j=t}^T C_j / (1+r)^{j-t}$ , where  $C_j$  is consumption in period  $j$ . This section will show how the difference between the consumption planned for period  $t+1$  in period  $t$ , and actual consumption in period  $t+1$ , can be used as an indicator of  $V_{t+1}^2$ .

We begin by associating with every random earnings path a new vector  $E = [\xi_0, \xi_1, \dots, \xi_T]$ .  $\xi_0$  is defined to be the individual's expectation on the value of his lifetime income at the beginning of the planning horizon, while  $\xi_{t+1}$  is the change in the expected value of lifetime income that occurs because of information which is available in period  $t+1$  but not in period  $t$ . That is, letting  $E_t$  denote the mathematical expectations operator conditional on all information available at  $t$ , and noting that  $A_{t+1} = A_t(1+r) + w_t - C_t$ , where  $w_j, C_j$  for all  $j \leq t$  are known with certainty in period  $t$  and thereafter, we have

$$\begin{aligned} \xi_0 &= E_0 Y(0) \\ \xi_{t+1} &= E_{t+1} Y(t+1) - E_t Y(t+1) = (E_{t+1} - E_t) \sum_{j=1}^T w_j (1+r)^{t+1-j} \end{aligned} \quad (1)$$

for  $t = 0, 1, \dots, T-1$ .

Since the  $\xi_{t+1}$  are the revisions in the individual's expectations on future earnings, they ought not to be correlated with any information

available to the individual at time  $t$ . Thus provided that the information set in period  $t$  is contained in the information set in period  $t + \tau$ , for all  $t \geq 0$ ,  $\tau > 0$ , one can prove (see appendix) that the  $\xi_t$  associated with any earnings path are mutually uncorrelated and that the variance of  $\xi_t$  equals  $v_t^2$  as defined in the introduction. (The assumption here is that individuals do not forget information relevant to the prediction of future earnings. It implies that  $E_j(E_{t+1} - E_t)w_q = 0$ , for all  $j \leq t$  and for all  $q$ .)

Our problem, therefore, becomes one of estimating the variances of the  $\xi_t$ . To see how the life-cycle permanent-income theory of consumption allows one to do this, assume that the distribution of consumption paths chosen under  $\bar{E}$  will be the same as those chosen under the random earnings path which defines  $\bar{E}$ . Since the first two moments of the lifetime budget constraint are the same under both vectors for all  $t$  by construction, a sufficient condition for this assumption is that the distribution of lifetime income be determined by these two moments.<sup>2</sup> Next define  $C_{t+1}^*$  to be the consumption planned in period  $t$  for period  $t+1$  if the realization of  $\xi_{t+1}$  is 0, and  $C_{t+1}$  to be actual consumption. Since  $\xi_{t+1}$  is the addition to expected lifetime income over the preceding period it is reasonable to assume that<sup>3</sup>

$$C_{t+1} - C_{t+1}^* = \beta_{t+1} \xi_{t+1}, \quad (2)$$

where  $0 < \beta_{t+1} < 1$ .

Suppose that we had measures of actual and planned consumption for individual  $i$ , and assume for the moment that neither the model nor the measures contain any errors. Equation (2) implies that in this case we



can derive an unbiased estimator of  $\beta_{t+1}^2 v_{t+1,i}^2$  by squaring  $C_{t+1,i} - C_{t+1,i}^*$ . There will, in general, be a distribution of  $v_{t+1,i}^2$  among members of the population and we shall be interested in estimating (and comparing) the average values of this variance, say  $\bar{v}_{t+1}^2$ , in different population groups. To do so we require a sample of  $N$  individuals of the same age who are randomly drawn from the group of interest. Given such a sample, and some mild regularity conditions on the distributions of the  $\xi_{t+1,i}$ , equation (2) ensures that

$$\text{plim} N^{-1} \sum_{i=1}^N (C_{t+1,i} - C_{t+1,i}^*)^2 = \beta_{t+1}^2 \bar{v}_{t+1}^2 . \quad (3)$$

A sufficient condition for (3) is that the variance of  $v_{t+1,i}^2$  be bounded from above. It can be used to identify  $\bar{v}_{t+1}^2$  provided that  $C_{t+1}^*$  and  $\beta_{t+1}$  can be identified. (For notational simplicity we shall henceforth use  $\bar{v}_{t+1}^2$  to refer to the average value of  $v_{t+1,i}^2$  among a given group in the population.)

To identify  $C_{t+1}^*$  we assume that the consumer's maximization problem at every  $t$  is

$$\max_{C_t, \dots, C_T} E_t \sum_{j=0}^{T-t} (1 + \delta)^{-j} U(C_{t+j}) , \quad (4)$$

subject to

$$Y(t) = \sum_{j=0}^{T-t} (1 + r)^{-j} C_{t+j} , \quad (5)$$

where  $U(\cdot)$  is a strictly concave one-period utility function, and  $\delta$  is the rate of subjective time preference. It can be shown (see Hall, 1978)

that the optimum consumption programme in this case will satisfy the first-order condition

$$E_t U'(C_{t+1}) = \frac{1 + \delta}{1 + r} U'(C_t) . \quad (6)$$

As noted by Hall (1978), equation (5) implies that consumption in period  $t+1$  should be predicted by  $C_t$  alone. We approximate (5) by

$$C_{t+1}^* = E_t C_{t+1} = \alpha + \lambda C_t . \quad (7)$$

This approximation is good if  $U'$  is close to linear. In this case  $\lambda = (1 + \delta)/(1 + r)$  and  $\alpha = \alpha_0(\lambda - 1)$ . A reasonable prior seems to be  $\alpha = 0$  and  $\lambda = 1$ . We shall use this prior later.<sup>4</sup>

### 3. DISTURBANCE TERMS AND THE IDENTIFICATION SCHEME

This section will provide an identification scheme when we allow for differences between the latent values of the variables that appear in the model and their observed values. Econometrically the resulting model belongs to a class of latent-variable models discussed in detail by Jöreskog (1973). Our presentation of the maximum-likelihood estimates for the model is similar to that of Chamberlain (1977).

Following Friedman (1957) and others, it is assumed that *measured* consumption,  $c_t$ , is related to the latent consumption variable which appears in the model,  $C_t$ , by

$$c_t = C_t + \varepsilon_t , \quad (8)$$

where  $\epsilon_t$  is a zero mean disturbance term which is assumed to be uncorrelated with  $C_t$  and to have a constant variance over two-year intervals. It arises as a result of mismeasurement and differences in preferences. Substituting (7) and (2) into (8) we obtain,

$$c_{t+1} = \alpha + \lambda c_t + \beta_{t+1} \xi_{t+1} + \epsilon_{t+1} - \lambda \epsilon_t, \quad (9)$$

Comparing (9) with (2) one finds that allowing for disturbance terms has the implication that the difference between observed consumption in period  $t+1$  and the consumption expected in period  $t+1$  given the information in period  $t$ ,  $\alpha + \lambda c_t$ , is not a perfect indicator of  $\beta_{t+1} \xi_{t+1}$ , since it may also be a result of a nonzero realization of  $\epsilon_{t+1} - \lambda \epsilon_t$ . In addition, the disturbance term is, by construction, correlated with  $c_t$ . Therefore, an ordinary least-squares regression on (9) will provide inconsistent estimates of  $\alpha$  and  $\lambda$ .

One way of solving this problem is to assume that

$$\text{cov}(\epsilon_t, w_t) = \text{cov}(\epsilon_{t+1}, w_t) = 0. \quad (10)$$

Although (10) has been used extensively by other researchers, its reasonableness depends on the way the data have been collected.<sup>5</sup> Since the latter is an empirical issue we discuss it, and alternative methods of identifying  $\alpha + \lambda c_t$ , in the next section. Given (10), however, we note that, since  $\xi_{t+1}$  is uncorrelated with any information available at  $t$  (including  $w_t$ ) and  $c_t$  is a function of  $w_t$ ,  $\alpha$ , and  $\lambda$  can be identified by using  $w_t$  as an instrument on  $c_t$  in (9).

To separate the two sources of the difference between  $c_{t+1}$  and

$\alpha + \lambda c_t$ , namely  $(\varepsilon_{t+1} - \lambda \varepsilon_t)$  and  $\beta_{t+1} \xi_{t+1}$ , we use the property that  $c_t$  is correlated with  $\varepsilon_{t+1} - \lambda \varepsilon_t$ , but not with  $\xi_{t+1}$ . Letting a circumflex over a parameter indicate its estimated value and defining  $e_{t+1} = c_{t+1} - \hat{\alpha} - \hat{\lambda} c_t$  (that is,  $e_{t+1}$  is the difference between observed consumption and the estimate of planned consumption) we have

$$e_{t+1} = \beta_{t+1} \xi_{t+1} - \lambda \varepsilon_t + \varepsilon_{t+1} + O(N^{-1}), \quad (11)$$

where  $O(N^{-1}) = [(\alpha - \hat{\alpha}) + (\lambda - \hat{\lambda})c_t]$ . Since  $O(N^{-1})$  converges to zero with sample size,  $N$ , it does not affect the maximum-likelihood estimates, and can be ignored if  $N$  is large.

If it is assumed that  $\lambda = 1$  (this assumption is discussed and tested in the next section), it follows that

$$\text{var}(c_{t+1}) = \beta_{t+1}^2 v_{t+1}^2 + 2\sigma_{\varepsilon}^2(1 - \rho)$$

and (12)

$$\text{cov}(e_{t+1}, c_t) = -\sigma_{\varepsilon}^2(1 - \rho),$$

where  $\text{var}(\varepsilon) = \sigma_{\varepsilon}^2$  and  $\text{cov}(\varepsilon_t, \varepsilon_{t+1}) = \rho\sigma_{\varepsilon}^2$ . Let  $S(x, y)$  denote the sample covariance of  $x$  and  $y$ ; then (12) implies that

$$\text{plim}[S(e_{t+1}, e_{t+1}) + 2S(c_t, e_{t+1})] = \beta_{t+1}^2 v_{t+1}^2. \quad (13)$$

Finally, to identify  $v_{t+1}^2$ , we need to estimate  $\beta_{t+1}$ . This can be done by using the relationship between  $\beta_{t+1}$  and  $\lambda$ . This relationship

is derived by Hall (1978), who shows that

$$\beta_{t+1} = [1 + \frac{\lambda}{1+r} + \dots + (\frac{\lambda}{1+r})^{T-t-1}]^{-1} . \quad (14)$$

Thus, a complete identification of the model requires information about the planning horizon,  $T$ , and the interest rate,  $r$ .

#### 4. AN EXAMPLE: ESTIMATES OF $V^2$ FOR DIFFERENT SCHOOLING GROUPS

In order to illustrate the use of our technique and to consider relevant estimation problems, we estimated the  $V^2$  vectors associated with the earnings paths of high school and university graduates in Israel. The data were gathered by the Israeli Central Bureau of Statistics and cover families (both spouses present) who were interviewed about their consumption decisions in 1963/64 and again in 1964/65. In each case the husband was between the ages of 21 and 65 and had at least five years of schooling.<sup>6</sup> The variables  $w_t$  and  $c_t$  were defined as the wage income of the male head of household from his primary labour activity and the household's consumption expenditures, respectively.

There are two reasons to doubt the instrumental variable estimates of equation (9) from our data set. First, in any particular year the *ex-ante* expectations of the distribution of  $\xi_{t+1}$  may not equal the *ex-post* distribution of its realizations, since as a result of a macro (or a year) effect we might sample only a portion of the distribution of  $\xi_{t+1}$ . When the GNP is below its expected average, consumers will tend to experience negative realizations of  $\xi_{t+1}$  and will adjust by choosing lower (than average) levels of  $C_{t+1}$ . In this case, if we limit the sample to one period,

we will obtain a downward bias in the estimate of both  $C_{t+1}^*$  and  $v_{t+1}^2$ . Similarly, when the labour market for the group we are interested in is unexpectedly buoyant, we will overestimate  $C_{t+1}^*$  and again underestimate  $v_{t+1}^2$ . An econometric solution to this problem requires data over many years. Our data, however, consists of observations over only two years. The second reason for doubting the instrumental variable estimates is that the measure of consumption in our data does not adequately account for the consumption services rendered by durables, and this may cause some correlation between  $\epsilon_t$  and earnings. More carefully gathered data, or separate information on components of consumption expenditures, both issues we come back to shortly, could help resolve this problem. An alternative to using the information in the data to estimate  $\alpha$  and  $\lambda$  is to impose the prior that  $r \approx \delta$ , so that  $\alpha = 0$  and  $\lambda = 1$ . We first estimated (9) by instrumental variables for each of the groups to be discussed below. The joint hypothesis that  $(\alpha, \lambda) = (0, 1)$  was always accepted for each group and for the sample as a whole. For the most aggregated grouping of our data, the estimates of  $\lambda$  were 1.05 and 1.06 with standard errors of 0.09 and 0.14, respectively, while there was slightly greater variation for the more disaggregated groupings. Given these results and our reservations about the precise limits of the instrumental variable estimators from this data set, we proceed by presenting the variance components estimates based on the prior that  $(\alpha, \lambda) = (0, 1)$ . As one might expect, the variance component estimates for the case of free parameters were similar.

Since the variance-age profile of lifetime earnings is a new concept we start by presenting measures of it for an average member of the sample. Table I presents the estimated variance components and some relevant moments.

Note that the variance in wages,  $S_w^2 = \frac{1}{2}\text{var}(w_t) + \frac{1}{2}\text{var}(w_{t+1})$ , is always greater than the variance in consumption,  $S_c^2 = \frac{1}{2}\text{var}(c_t) + \frac{1}{2}\text{var}(c_{t+1})$ .

This is consistent with life-cycle consumption behaviour. Further, note that  $S_w^2$  is about 50 percent larger in the older age group. This does not imply, however, that more uncertainty is resolved at older ages, since, over time, individuals accumulate information with respect to their positions in the cross-sectional distribution of earnings. The variance of  $c_{t+1} - c_t$  also increases with age but this increase, as one would expect, is much less than the increase in  $S_w^2$ . To obtain the variance of

$\beta_{t+1} \xi_{t+1}$ ,  $\text{var}(c_{t+1} - c_t)$  must be purged of observed consumption changes caused by mismeasurement. Line 4 in Table I presents the estimates of  $\sigma_\epsilon^2(1 - \rho)$ , while line 5 uses these estimates to calculate  $\beta^2\sigma_\xi^2$ . Since  $\sigma_\epsilon^2(1 - \rho)$  is much larger for the older age group,  $\beta^2\sigma_\xi^2$  is larger for the younger group. Table II uses (13) and (14) to calculate  $\beta_t$  and  $\sigma_\xi^2$  assuming  $T = 70$ , under alternative assumptions on  $r$ .

The main conclusion from these tables is that more information on lifetime earnings is accumulated per unit of time early on in the life cycle. The standard errors of the estimated variance components indicate, however, that this conclusion cannot be held too firmly. The major cause of the lack of precision in the estimates of  $\beta^2\sigma_\xi^2$  is the magnitude of  $\sigma_\epsilon^2(1 - \rho)$  (the variance of  $\beta^2\sigma_\xi^2$  is an increasing, convex function of this term). Data bases with measures of components of consumption expenditure (such as expenditures on durables, nondurables, and one-time health expenditures) and information on family background characteristics (such as changes in household composition) would do better on this count. Such information could be used to construct measures of consumption which would have a lower variance in the measurement error component of  $\epsilon_t$  and

would allow one to introduce an explicit model for transitory changes in consumption into (13), thereby causing a further decrease in  $\sigma_{\epsilon}^2(1 - \rho)$ .<sup>7</sup> For example, had we been able to decrease the variance of the disturbance in equation (9) to equal that of  $\beta\xi$ , and if all other parameters remained unchanged, the standard error of  $\beta^2\sigma_{\xi}^2$  would have fallen from 12,124 to 2,374 in the older age group, and from 6,832 to 2,431 in the younger (Table I). Note also that the variance of  $\beta^2\sigma_{\xi}^2$  is of the order  $1/N$  so that if, under these same assumptions, the size of the sample were increased to 5000, the standard errors would decrease further to 716 and 613, respectively.

Next we compare the variance-age profile of lifetime income experienced by those who went to college with that for individuals who only attended high school. Each educational group was split into an older and a younger age group. Table III presents the relevant moments together with sample size for each group. All of the general comments made above apply to these figures as well; in particular,  $S_c^2$  is always less than  $S_w^2$  and  $\sigma_{\epsilon}^2(1 - \rho)$  increases with age in both educational groups and is always large, causing imprecise estimates of  $\beta^2\sigma_{\xi}^2$ . Comparing the point estimates of  $\beta^2\sigma_{\xi}^2$  across groups, we find that among those who did not go to college more of the variance of lifetime earnings is realized per unit of time in the earlier part of the life cycle, while among those who did go to college the opposite is true. That is, the college-educated have to wait relatively longer to acquire information on their lifetime earnings. Therefore, even if the total variance in lifetime earnings were the same for both groups, the variance-age profile of those who did not attend college would be preferred to the profile for those who did (see Eden, 1980). In fact, however, the total variance in lifetime earnings is also higher for the college-educated.



Again, the standard errors of our estimates are large and as a result any conclusions from them should be considered as preliminary.<sup>8</sup>

##### 5. SUMMARY

This paper has introduced an operational procedure for measuring the uncertainty in alternative earnings paths. The measure we propose is obtained by partitioning the randomness in the initial discounted value of lifetime earnings into  $T$  mutually uncorrelated, sequentially realized random variables and using the implications of the life-cycle permanent-income theory of consumer behaviour to estimate the variance in each of these deviates. Use of consumption data and the life-cycle theory allows us to measure the variance in lifetime earnings, and the time-path of the resolution of this variance, without making any but the mildest of assumptions on how earnings are generated. This is the major virtue of our approach. On the other hand, our procedure does rely on the assumptions underlying the life-cycle theory of consumption. Incorporating appropriate disturbance terms can account for deviations from strict life-cycle behaviour but if these deviations are large and if the data does not allow one to build an explicit model of how they are generated, then the measures we propose are likely to be imprecise.

Two avenues of research are likely to result in improvements in our measure of the variance-age profile of lifetime earnings. Additional information on the reasons for deviations from pure life-cycle consumption behaviour ought to enable one to provide more reliable estimates of  $V^2$ . Also, a more restrictive and, one hopes, testable assumption about the earnings process would allow one to use the information in both observed

consumption and the earnings profiles to estimate  $V^2$ .

The advantages of using consumption and earnings data simultaneously extend beyond the measurement of  $V^2$ . The implication of the life-cycle theory that we have used in this paper is that differences in the distribution of consumption expenditures (either over time for a given individual or in the cross section) ought to reflect differences in the distribution of lifetime income. That is a useful implication for any study which requires a measure of lifetime income. The alternative method of providing such a measure is to make assumptions on the earnings process which enable one to construct it from earnings data, or, if possible, from a combination of earnings and assets data. Once one has both consumption and earnings data one should be able to test the hypotheses underlying both procedures.

APPENDIX: THE DERIVATION OF  $\Xi$

For simplicity we assume that the real interest rate is zero. From (1) in the text we have

$$\xi_{t+1} = E_{t+1} \sum_{j=1}^T w_j - E_t \sum_{j=1}^T w_j = E(L|I_{t+1}) - E(L|I_t), \quad (A.1)$$

where  $L = \sum_{j=1}^T w_j$ , that is  $L$  is lifetime earnings, and  $I_{t+1}$  is the information set in period  $t+1$  for  $t = 0, 1, \dots, T-1$ .

It is assumed that  $I_t$  is contained in  $I_{t+\tau}$  for all  $t \geq 0$  and  $\tau \geq 0$ . Then, after conditioning both sides of (A.1) on  $I_{t-j}$ , for  $0 < j < t+1$ , and passing through another expectations operator, one obtains

$E(\xi_{t+1}|I_{t-j}) = 0$ . Similarly, further use of the double expectations operator proves that  $\text{cov}(\xi_i, \xi_q|I_{t-j}) = 0$  for all  $i, j, q = 1, \dots, T$ , and  $i \neq q$ .

(A.1) may be solved recursively for  $E(L|I_T) = L$ , that is,

$$L = \xi_T + \xi_{T-1} + \dots + \xi_1 + \xi_0^* \quad (A.2)$$

where  $\xi_0^* = E(L|I_0) = \xi_0 - A_0$ .

From (A.2) it follows that  $L$  can be partitioned into the sum of  $T$  mutually uncorrelated, sequentially realized random variables and a constant term. The definitions of  $V^2$  and  $\text{var}[Y(0)]$  given in the text follow immediately.

We wish to acknowledge with thanks the helpful comments of Gary Chamberlain, Zvi Griliches, Reuben Gronau, an anonymous referee, and the editor of this journal, and the financial assistance of the Falk Institute in Jerusalem and the National Science Foundation (Grant No. 73-05374). All errors are ours.

*NOTES*

<sup>1</sup> This result does not depend on any cardinal properties of the intertemporal choice function, such as its concavity, but rather just on the simple point that information is useful in the sense that it allows one to plan more accurately. Strictly speaking, our use of Eden's (1980) criterion requires either that lifetime income be distributed normally, or that the intertemporal objective function be additive in a quadratic instantaneous utility function.

<sup>2</sup> This is not a necessary condition since the intertemporal objective function may be such that all consumption programmes are determined by the first two moments of  $Y(t)$  regardless of the distribution of  $Y(t)$ . See Levhari and Srinivasan (1969) for a discussion of this point.

<sup>3</sup> Equation (2) is exact for Friedman's (1957) permanent-income theory of the consumption function (see Hayashi, 1980), and will always be a good approximation to the life-cycle theory of consumption provided that the difference between actual and expected consumption is small relative to expected consumption (see Hall, 1978).

<sup>4</sup> The precise formula for  $C_{t+1}^*$  will depend on the form of the utility function (see Hall, 1978). For our data, equation (7) with  $\lambda = 1$  and  $\alpha = 0$  seemed adequate (see the discussion below). It is interesting to

note that recent work by Hall and Mishkin (1980) on a larger sample of consumers comes to the same conclusion regarding the appropriate formula for  $C_{t+1}^*$ . Although Hall and Mishkin are not concerned with the same set of issues as we are and make quite different assumptions from us, their paper is the only other research we are aware of that uses micro panel data on movements in consumption expenditures as an indicator of innovations in lifetime earnings.

<sup>5</sup> See, for example, Liviatan (1961, 1963). Liviatan's (1961) analysis of the demand for individual products uses  $c_{t+1}$  as an indicator of permanent income in period  $t+1$ , an idea similar to the one used here.

<sup>6</sup> The data are described more fully in Israel (1967). We are grateful to Reuben Gronau for allowing us to use his key for this data set.

<sup>7</sup> The addition of variables to equation (9) would necessarily decrease the variance of the estimate of  $\beta^2 \sigma_{\xi}^2$  but would have a cost in terms of an increase in the variance of  $\lambda$ . Since, in our case, the regression coefficients are not of primary importance, the trade-off seems worth while.

<sup>8</sup> Two caveats should be entered here. First, in this early version of our analysis, we have ignored problems induced by self-selection. That is, an individual who goes (does not go) to college may expect to experience a more desirable variance-age profile as a result of going (not going) to college than a random member of the population would. In addition, there is the question of the stability of these profiles. To make inferences about individual decision-making from the information contained in a single cross section one must assume that an individual who enters college at the age of twenty expects to experience at the age of thirty the same variance which is experienced now by a person who matriculated ten years ago. However, neither of these problems is new

and they also appear in the more traditional estimates of the first-order moments of labour income streams.

REFERENCES

- BECKER, G.S. (1975), *Human Capital*, 2nd Ed. (National Bureau of Economic Research: Human Behavior and Social Institutions No. 5.) New York and London: Columbia University Press.
- CHAMBERLAIN, G. (1977), "An Instrumental Variable Interpretation of Identification in Variance Components and MIMIC Models", in P. Taubman (ed.) *Kinometrics: The Determinants of Socio-Economic Success Within and Between Families*, Amsterdam: North-Holland.
- EDEN, B. (1980), "Stochastic Dominance in Human Capital", *Journal of Political Economy*, 88 (7), 135-45.
- FRIEDMAN, M. (1957), *A Theory of the Consumption Function* (National Bureau of Economic Research: General Series No. 63.) Princeton: Princeton University Press.
- HALL, R.E. (1978), "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence", *Journal of Political Economy*, 86 (6), 971-987.
- HALL, R.E. and MISHKIN, F.S. (1980), *The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households* (Working Paper No. 505.) Cambridge Mass.: National Bureau of Economic Research.
- HAYASHI, F. (1980), "Utility Maximization, Rational Expectations, and Aggregate Consumption", unpublished Ph.D. dissertation, Harvard University.
- ISRAEL. Central Bureau of Statistics (1967), *Saving Survey 1963/64*. (Special Series No. 217.) Jerusalem.

- JÖRESKOG, K.G. (1973), "A General Method for Estimating a Linear Structural Equation System", in A.S. Goldberger and O.D. Duncan (eds.) *Structural Equation Models in the Social Sciences*, New York and London: Seminar Press, pp. 85-112.
- LEVHARI, D. and SRINIVASAN, T.N. (1969), "Optimal Servings under Uncertainty", *Review of Economic Studies*, 36 (2), 153-163.
- LILLARD, L.A. and WEISS, Y. (1979), "Components of Variation in Panel Earnings Data: American Scientists 1960-70", *Econometrica*, 47 (2), 437-454.
- LIVIATAN, N. (1961), "Errors in Variables and Engel Curve Analysis", *Econometrica*, 29, 336-362.
- LIVIATAN, N. (1963), "Tests of the Permanent-Income Hypothesis Based on a Reinterview Survey", in C.F. Christ and others, *Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld*, Stanford, Calif.: Stanford University Press, pp. 29-59.



TABLE I

*Estimated and sample moments for the younger and older age groups<sup>a/</sup>*

	Younger 21-44	Older 45-64
1. $S_w^2$	197,787	297,476
2. $S_c^2$	86,704	123,538
3. $\text{var}(c_{t+1} - c_t)$	74,404	111,917
4. $\sigma_\epsilon^2(1 - \rho)$	34,930 4,158	54,283 6,855
5. $\beta^2\sigma_\xi^2$	6,017 6,832	3,554 12,124
6. N	434	353

<sup>a/</sup> Small numerals below the coefficients are standard errors. All moments presented in this and other tables are in hundreds of 1963 IL per year.

TABLE II<sup>a/</sup>

$\beta_t$  and  $\sigma_{\xi}^2$

	$\beta_t$		$\sigma_{\xi}^2$	
	Younger t = 33	Older t = 55	Younger t = 33	Older t = 55
r = 0.10	0.09	0.11	742,876	438,815
r = 0.20	0.17	0.17	208,211	122,990
r = 0.30	0.23	0.23	113,748	67,191

<sup>a/</sup> Calculated using equations (13)-(14), with T = 70.

TABLE III

*Estimated and sample moments for age-education groups<sup>a/</sup>*

	High school		College	
	Younger	Older	Younger	Older
	25-44	45-59	25-44	45-59
1. $S_w^2$	164,693	247,191	261,852	451,981
2. $S_c^2$	69,899	105,896	100,332	149,215
3. $\text{var}(c_{t+1} - c_t)$	59,098	98,992	85,317	159,153
4. $\sigma_\epsilon^2(1 - \rho)$	28,733 5,281	48,104 10,538	26,604 9,070	59,786 17,492
5. $\beta^2\sigma_\xi^2$	5,566 7,811	3,122 16,791	38,195 17,165	45,848 30,028
6. N	176	112	96	80

<sup>a/</sup> The age groups differ from those in Table I because there were only about 5 observations in the 21-25 and 60-65 age groups in each educational class and they contributed a great deal of the  $\sigma_\epsilon^2(1 - \rho)$ .