COMPETITIVE PRICE ADJUSTMENT AND LINKAGE TO THE MONEY SUPPLY

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I.

Recently I have applied an argument by Grossman and Stiglitz (1980) to an economy with no auctioneer and argued that in equilibrium, some sellers will advertise prices which are not based on updated information. To see this point, consider the problem of a seller who has to advertise a price for his product in the presence of costly information about demand. He can costlessly observe the prices advertised by other sellers in his neighborhood, and he can also spend some resources to find out the actual demand. If the price advertised by other sellers in the neighborhood (the market price) is always based on updated information, the seller will never invest in information gathering and will always follow the market. Therefore, to maintain incentive to collect information, there must be a positive probability that the price advertised in a given neighborhood will not be based on updated information. In an economy composed of many such neighborhoods, we will therefore have some neighborhoods which advertise a price that is based on updated information (informed neighborhoods) and some which advertise a price that is based on "old" information (uninformed neighborhoods). The implications of this general result are discussed in Eden (1981a) for the case in which real shocks cause the fluctuations in demand and in Eden (1981b) for the case of monetary disturbances.

In the model that deals with monetary disturbances, there is a one period lag in the transmission of advertised prices to the buyers, and sellers advertise dollar prices on the basis of their information with respect to the end of period demand. The only source of uncertainty is changes in the money supply, and sellers may get information, at some cost, about the end of period money supply, say, by "bribing" politicians who plan the change in the money supply or by investing resources in forecasting.

The analysis does not require the assumption that the cost of observing the current money supply (that is, the cost of observing the end of period money supply at the end of the period) is prohibitively expensive. 1/ And, indeed, for some definition of the money supply, such as the base, it is reasonable to assume that the current realization of the money supply can be observed, with very little effort, since it is published on a weekly basis.

But, if the current level of the base can be easily observed, and if there is some relationship between the end of period base and demand conditions, then we may expect that sellers will advertise a price which is contingent on the base. For example: "I will sell the good for $\$P_1$ if the base is M_1, for $\$P_2$ if the base is $M_2, \ldots,$ **and for** $\$P_n$ if the base is M_n." The question is why such contingent pricing is not observed?

One reason may be that advertising a vector of contingent price may be prohibitively expensive if the number of possible realizations of the money supply (base) is large. But, it will be shown that if everyone advertises a contingent price, then, in equilibrium, one will be able to express his contingent price by a scalar that I shall call a linked price, where a linked price of 1 means \$1.1 if the money supply went up by 10% during the period and \$1.2 if the money supply went up by 20% during the period.

Thus, at the end of the period, a unit of linked price is equal to M_t/M_{t-1} dollars, where M_{t-1} is the beginning of the period money supply and M_t is the end of period money supply. Note that a unit of linked price can also be expressed as $100/M_{t-1}$ percentage of M_t. It is convenient to choose the units of the money supply such that $M_{t-1} = 100$. In this case, a unit of a linked price is equal to 1% of M_t.

One can view the adoption of linked prices as a change in the unit of account. The new unit is a percentage of the current money supply (and prices are stated in terms of percentage of the current money supply per physical unit).

To examine linked prices from the practical point of view, it may be helpful to imagine a supermarket which advertises prices at the beginning of the year. Each day, the registers (mini computers) are fed with the figure of the current money supply. If the money supply is 10 percent above the beginning-of-the-year level, a price tag of $1.00 means $1.10; and if the money supply is 20 percent above the beginning-of-the-year level, a price tag of $1.00 means $1.20. Thus, the price tag, say, of tomatoes, will change in response to real shocks (changes in the real supply and demand for tomatoes), but not in response to monetary shocks. Buyers who wish to know the prices relative to other supermarkets (comparative shopping) can do better under a system of linked prices since, in general, inflation will not depreciate the value of their memory with respect to prices in other supermarkets. [2/] However, if a buyer wishes to control his nominal spending, he will have to carry a pocket calculator and to multiply each price tag by the appropriate factor. To make things easy for this kind of buyer, the supermarket should post a sign that tells buyers the multiplication factor.

Thus, it seems that linked prices can be implemented quite easily. This brings us back to the question of why linked prices are not used. The answer suggested here is that from the individual seller's point of view, it may be optimal to link his advertised price to the money supply only if other sellers follow the same practice. This point is obvious when considering the advantage of linked prices in comparative shopping. It is also obvious when considering the initial fixed cost of teaching the public the meaning of linked prices. Here I will show that even in the absence of transaction costs, and even in a world which is populated with people that have no trouble in understanding the concept of linked prices, it is not clear whether an individual seller can make money by the adoption of linked prices, when all other sellers use dollar prices.

II.

A model of competitive price adjustment was developed in Eden (1981a, 1981b). I shall use a condensed version of this model.

There are N identical buyers. It may be useful to think of the buyers as members of the old generation in an overlapping generations model. N is large and will be treated as a real number. At the beginning of period t, each buyer holds M_{t-1} dollars. At the end of period t (Friday), each buyer may get a transfer payment of $(\alpha-1)M_{t-1}$ dollars, $(\alpha>1)$. It is assumed that either all buyers get a transfer payment or no one gets it. The probability of a transfer payment is 1/2. Thus, the money supply per buyer, at the end of period t, M_t, is either $M_t = M_{t-1}$ or $M_t = \alpha M_{t-1}$. It is assumed that changes in the money supply are the only source of uncertainty and that at the beginning of period t, the probability distribution of M_t is known. Further, it is possible to buy information about the realization of M_t at the beginning of the period (Saturday). This may be done by "bribing" a politician to let you know whether a transfer payment is being planned, or by investing resources in forecasting. [3/]

At the end of the period, buyers exchange some or all of their nominal balances for a single consumption good. The demand of each buyer is given by

$$d(P_t, M_t) \tag{1}$$

where P_t is the dollar price of consumption and M_t is his holdings of nominal balances. It is assumed that (1) is homogeneous and we can therefore define

$$D(P_t/M_t) \equiv d(P_t/M_t,\ 1). \tag{2}$$

Thus, demand is a function of the linked price, P_t/M_t.

There are H identical producers-sellers. It may be useful to think of the sellers as members of the young generation in an overlapping generations model. H is large and will be treated as a real number. There is a one period lag in production, and production decisions are made at the beginning of the period. It is assumed that if a seller knows (with certainty) that he will sell at his advertised price P_t, and he knows (with certainty) that the realization of the money supply is NM_t, he will produce

$$s(P_t, M_t) \tag{3}$$

units of consumption. It is assumed that s( ) is homogeneous and we can therefore define

$$S(P_t/M_t) \equiv s(P_t/M_t, 1). \tag{4}$$

It is further assumed that under conditions of uncertainty the seller will use all the information which is available to him at the beginning of the period to compute some (weighted) average, $\overline{P_t/M_t}$, and supply according to

$$S(\overline{P_t/M_t}). \tag{5}$$

Thus, $\overline{P_t/M_t}$ is a certainty equivalent linked price. I shall use $P_t/\overline{M}$ to denote the average of P_t/M_t when the seller is certain that he will sell at his advertised price, but is uncertain with respect to the money supply. In this case, $\overline{M}$ is some average of M_{t-1} and αM_{t-1}.

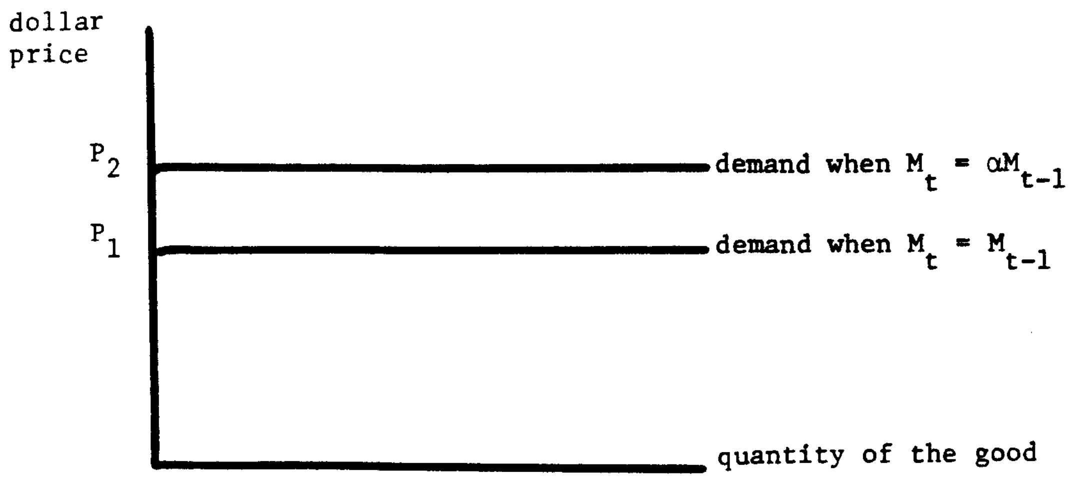
The specification in (5) assumes that some average of P_t/M_t is a sufficient statistic for the purchasing power of P_t dollars in the future. This is true, for example, in the overlapping generations model which is used in Eden (1980). In this model, individuals live for two periods, work in the first period, and consume only in the second period. Members of the old generation get the transfer payment in proportion to their holdings of nominal balances. Since in equilibrium the old generation spends all its money, one who holds 1% of the current money supply will be able to buy, on average, 1% of future output. It is also assumed that the money supply is serially independent and, therefore, in equilibrium, output is serially independent. Thus, evaluating $1 as a percentage of the current money supply is a sufficient statistic for its purchasing power in the future.

The sellers are distributed over many neighborhoods, n sellers per neighborhood. The number of sellers in each neighborhood, n, is large. Each seller can observe the prices which are advertised in his neighborhood at the beginning of the period (Sunday), before he commits himself to a final price. There is a one period lag in transmitting information about prices to buyers and to sellers in other neighborhoods. (Thus, each seller must advertise a final price at the beginning of the period.) Sellers make production decisions after observing the final price in their neighborhood (Monday) and take the final price into account when computing the certainty equivalent linked price. At the end of the period, information about prices (all over the economy) is costless. Transactions are costless. Buyers are served on a first-come, first-served basis, and there is no rationing for those who actually get served. Thus, whenever there is more than one price, all buyers will try to buy at the cheapest price, but typically not all of them will succeed.

I shall start by allowing sellers to advertise dollar prices only. (The case in which the advertised price may be contingent on the money supply will be discussed later.) Under this restriction, it will be shown that in equilibrium, each seller views the aggregate demand that his neighborhood faces as infinitely elastic at a dollar price, P_t, which is contingent on the money supply, and is given by

(6) $$P_t = P_1, \text{ if } M_t = M_{t-1}; \text{ and}$$
$$P_t = P_2, \text{ if } M_t = \alpha M_{t-1}.$$

These expectations are illustrated by Figure 1.



The equilibrium demand for the output produced in a given neighborhood when advertising is restricted to dollar prices

Figure 1.

The expectations (6) imply that a seller who advertises P_2 will not be able to sell when $M_t = M_{t-1}$. Advertising P_1, on the other hand, ensures selling. The linked price that the seller will get as a function of his advertised price and the realization of M_t is given in Table 1,

	$M_t = M_{t-1}$	$M_t = \alpha M_{t-1}$	expected linked price
advertised dollar price = P_1	P_1/M_{t-1}	$P_1/\alpha M_{t-1}$	$P_1/2M_{t-1} + P_1/2\alpha M_{t-1}$
advertised dollar price = P_2	0	$P_2/\alpha M_{t-1}$	$P_2/2\alpha M_{t-1}$

Table 1.

where the expected linked price is calculated under the prior distribution of M_t. Table 1, together with the objective function of the sellers, can be used to determine the optimal price under the prior distribution of M_t. It can be shown that if α is not too large, the difference between $P_2/\alpha M_{t-1}$ and P_1/M_{t-1} is not too large and it is optimal to advertise the lower price, P_1. Here I shall assume that this is, in fact, the case. (The case in which α is large, and on the basis of the prior distribution of M_t it is optimal to advertise the higher price, is discussed in Eden [1981b].)

At the beginning of the period, each seller faces a choice between buying information about the realization of M_t (and advertising the "correct" price according to [6]) or following the price advertised by other sellers in his neighborhood. Specifically, it is assumed that on Saturday morning, each seller can buy the information. Then, sellers place advertisements which state the price of their merchandise in the local newspaper. The newspaper is published and sellers may costlessly observe the prices advertised by other sellers in their neighborhood. Based on this observation, they may wish to revise their prices. In this case, they will place new advertisements and a new issue of the newspaper will appear. The process comes to an end when no one wishes to make a further revision of his price. The final issue of the newspaper is circulated all over the economy. It is assumed that the above process does not take real time, and the final issue appears at the beginning of the period (Sunday). It takes one period to transmit information about final prices to buyers all over the economy. At the end of the period (Friday), actual trading takes place at the final prices.

The good must be sold at the end of the period. Otherwise, it will evaporate. This constraint does not allow enough time for buyers to engage in an extensive search and, therefore, it is impossible to sell the good without advertising a price in the newspaper. It is also assumed that it is impossible to buy information about the realization of M_t after the first issue of the local newspaper appears. The assumption of a lag in transmitting information about prices to buyers implies that during the period sellers cannot change prices.

Under the assumption that (α is not too large and, therefore) on the basis of the prior distribution of M_t, it is optimal to advertise the lower price, P_1, the sellers' behavior with respect to buying information and advertising prices can be described by

Claim 1: The following is a Nash strategy [4/]: (a) buy the information about the realization of M_t with probability q^* (to be discussed shortly); (b) if you have bought the information, advertise the price P_1 if $M_t = M_{t-1}$ and advertise the price P_2 if $M_t = \alpha M_{t-1}$; (c) if you have not bought the information, advertise the price P_1 unless you observe that the price P_2 has been advertised. In this case, advertise the price P_2.

To show this claim, note that P_2 will be advertised if and only if someone has actually observed $M_t = \alpha M_{t-1}$. If no one has observed $M_t = \alpha M_{t-1}$, then by assumption it is optimal to advertise P_1. The calculation of q^*, for a simple case, is carried out in the Appendix. The probability that no one in a given neighborhood will buy information is

$$(7) \qquad 1-\mu = (1-q^*)^n.$$

Since Claim 1 implies that the final price is the same for all sellers in a given neighborhood, profits will be roughly the same for all the sellers in the neighborhood. Therefore, if an individual seller is certain that at least someone will get the information, he will not have much incentive to spend resources on information gathering. Thus, it is reasonable to assume that the probability (7) is bounded away from zero. [5] This probability can be unity if the cost of information is prohibitively expensive. Here I shall assume that the cost is not prohibitive and that μ is also bounded away from unity. (Thus, $0 \ll \mu \ll 1$.) This implies that $0 < q^* < 1$. Thus, sellers use a mixed strategy to determine whether to buy information. They therefore must be indifferent between being uninformed and becoming informed. Note also that when n goes to infinity, the probability q^* must go to zero since otherwise (7) will go to zero.

A neighborhood is defined as uninformed if no one of its n sellers has bought the information. Since we have a large number of neighborhoods, I shall use the law of large numbers and assume that a fraction, μ, of all neighborhoods is informed, where μ is calculated from (7) and is non-random.

Claim 1 implies that P_1 will be advertised if either no one in the neighborhood has bought the information or someone has bought it and observed $M_t = M_{t-1}$. Therefore, when $M_t = M_{t-1}$, all neighborhoods (informed and uninformed) will advertise the price P_1, but only the sellers who have bought the information will know the realization of M_t. Since when n is large, q* is small, the fraction of sellers who have actually bought the information is negligible. We can therefore assume that almost all sellers will use some average, $\overline{M}$, of M_{t-1} and αM_{t-1} as a deflator and will supply a total of $HS(P_1/\overline{M})$. The supply of the informed sector is thus $\mu HS(P_1/\overline{M})$ and the supply of the uninformed sector is $(1-\mu)HS(P_1/\overline{M})$.

It is assumed that P_1 clears the market for the case $M_t = M_{t-1}$. Thus, $HS(P_1/\overline{M}) = ND(P_1/M_{t-1})$; the informed sector satisfies the demand of μN buyers and the uninformed sector satisfies the demand of $(1-\mu)N$ buyers. This is illustrated by the solid lines in Figure 2, where the linked price, P/M, is plotted on the vertical axis, Q_o^u is the quantity supplied by the uninformed sector, and Q_o^I is the quantity supplied by the informed sector.

Note that market clearing rationalizes the sellers' expectations with respect to the demand that each neighborhood will face when $M_t = M_{t-1}$. In particular, since there are many neighborhoods, the sellers in each neighborhood will not be able to sell at $P > P_1$, and will be able to sell all their supply at the price P_1.

When $M_t = \alpha M_{t-1}$, informed neighborhoods will advertise P_2 while uninformed neighborhoods will advertise P_1. The uninformed sector will supply the same quantity as before, but since their linked price is now lower $(P_1/\alpha M_{t-1} < P_1/M_{t-1})$, each buyer will demand a larger quantity. The number of buyers that the uninformed sector will satisfy, N*, is therefore smaller than before (i.e., $N^* < (1-\mu)N$). The remaining N-N* buyers will have to go to the informed sector and buy at the higher price.

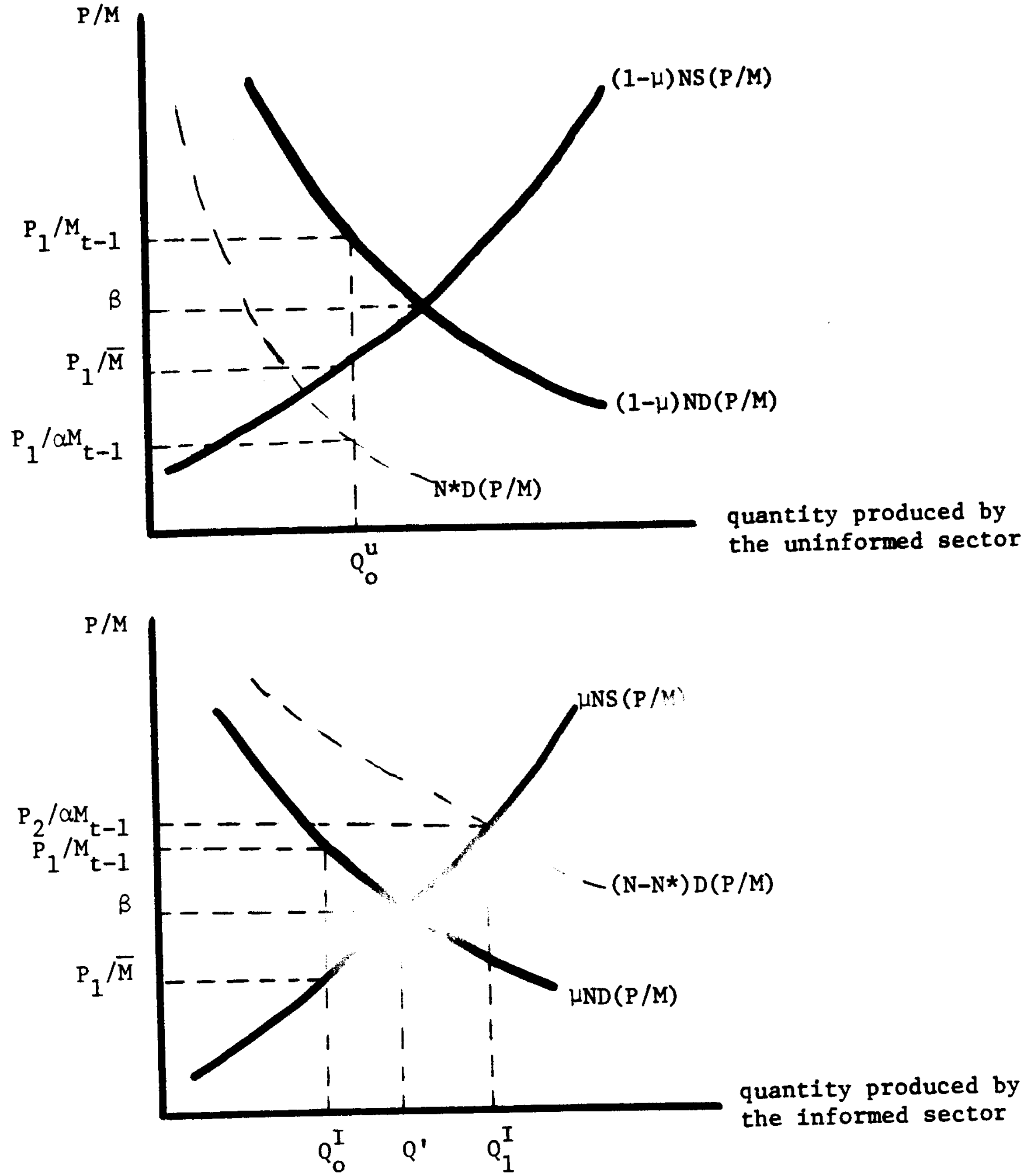

P/M
(1-μ)NS(P/M)
P_1/M_{t-1}
β
$P_1/\bar{M}$
(1-μ)ND(P/M)
$P_1/\alpha M_{t-1}$
N*D(P/M)
quantity produced by the uninformed sector
Q_o^u
P/M
μNS(P/M)
$P_2/\alpha M_{t-1}$
P_1/M_{t-1}
(N-N*)D(P/M)
β
$P_1/\bar{M}$
μND(P/M)
quantity produced by the informed sector
Q_o^I
Q'
Q_1^I

Figure 2

At the beginning of the period, sellers in the informed sector know that $M_t = \alpha M_{t-1}$, since P_2 is advertised only if someone has observed this realization. The informed sector will therefore supply $\mu HS(P_2/\alpha M_{t-1})$ units. It is assumed that P_2 clears the residual market; that is, $(N-N^*)D(P_2/\alpha M_{t-1}) = \mu HS(P_2/\alpha M_{t-1})$. Since there are many informed neighborhoods, each individual neighborhood will not be able to sell at $P > P_2$, but will be able to sell its entire supply at P_2. This rationalizes the sellers' expectations with respect to the demand in the case $M_t = \alpha M_{t-1}$.

Thus, the above is a description of equilibrium in the sense that expectations are correct and agents do not have an incentive to deviate from the equilibrium strategy. In particular, uninformed sellers do not have an incentive to become informed.

The equilibrium solution for the case $M_t = \alpha M_{t-1}$ is illustrated by the intersection of the dashed demand curves with the supply curves in Figure 2. It is clear that the output produced in the informed sector is larger when $M_t = \alpha M_{t-1}$. The intuitive explanation is that when $M_t = M_{t-1}$, sellers are not sure about the realization of the money supply and they defalte by $\overline{M} > M_{t-1}$. Therefore, they see a lower linked price than the buyers. This discrepancy is eliminated when $M_t = \alpha M_{t-1}$ and the suppliers in the informed sector are able to infer the realization of the money supply. Thus, even if the informed sector faced the same real demand, the equilibrium quantity would increase to Q'. The increase from Q' to Q_1^I is due to the spillover from the uninformed sector, which leads to an increase in the real demand.

To sum up, the production of the uninformed sector does not depend on the realization of the money supply, while the production of the informed sector is higher when $M_t = \alpha M_{t-1}$. This implies a positive relationship between money and total production.

III.

Another possible equilibrium emerges when sellers are allowed to advertise a price which is contingent on the money supply. The most general form is when each seller advertises a vector which states a different dollar price for each different realization of M_t. When there are many possible realizations of M_t, advertising such a vector is likely to be prohibitively expensive. In this case, however, sellers do not need to advertise a vector. It is enough if they advertise a linked price rather than a nominal price. Specifically, if at the end of the period the realization of M_t can be costlessly observed and all sellers advertise that they will charge a percentage β of the actual money supply (i.e., the Walrasian price - see Figure 2), each individual seller will face a demand curve which is infinitely elastic at the linked price, β, and will not be able to do any better than advertising this linked price. Output in this case will be independent of the money supply.

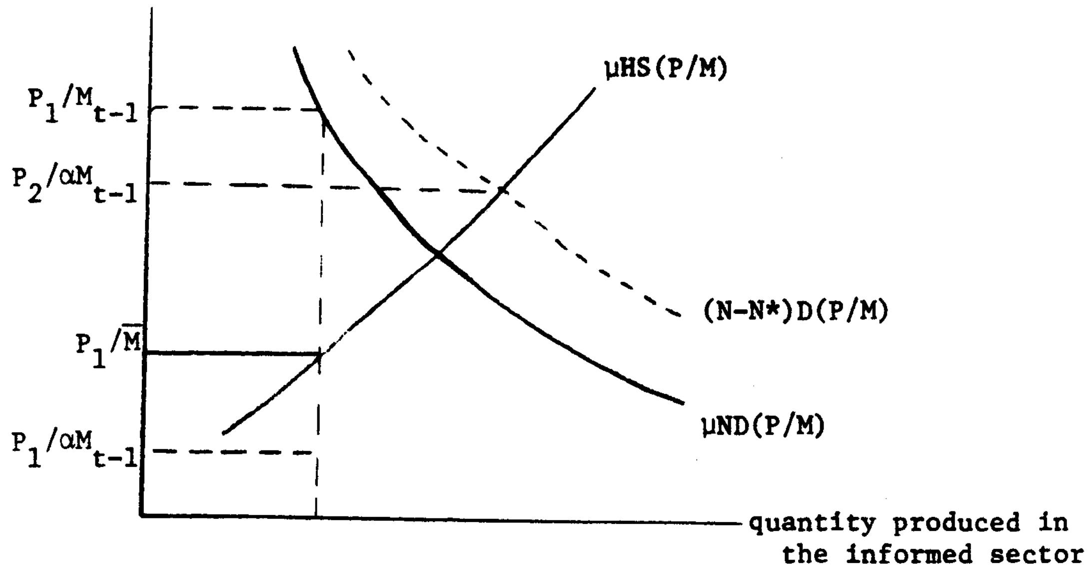
Why don't we observe the use of linked prices? One explanation may be that it is costly to observe the current money supply. This does not seem plausible in view of the good statistics on high-powered money which are published on a weekly basis in various newspapers. Another explanation may view the adoption of a new unit of account as changing a language. Esperanto may be a more efficient way of communicating than English; but if all other people speak English, it does not pay the individual to learn Esperanto.

Indeed, in our model it is not clear whether an individual seller can do better by advertising a linked price if all other sellers advertise a dollar price. [6/]

The incentive to advertise a linked price may emerge when all the sellers in the neighborhood advertise the dollar price P_1 and the individual seller does not know the realization of the money supply. In this case, if $P_2/\alpha M_{t-1} > P_1/M_{t-1}$ and $P_2 \geq \alpha P_1$, then advertising the linked price, P_1/M_{t-1}, is better than advertising the dollar price P_1, since it implies selling at $\$P_1$ if $M_t = M_{t-1}$ and at $\$\alpha P_1$ if $M_t = \alpha M_{t-1}$. This case corresponds to the one described in Figure 2. However, $P_2/\alpha M_{t-1} < P_1/M_{t-1}$ is possible. (See Figure 3.) In this case, when all advertise dollar prices, an individual who considers advertising a linked price will face a demand which is infinitely elastic at

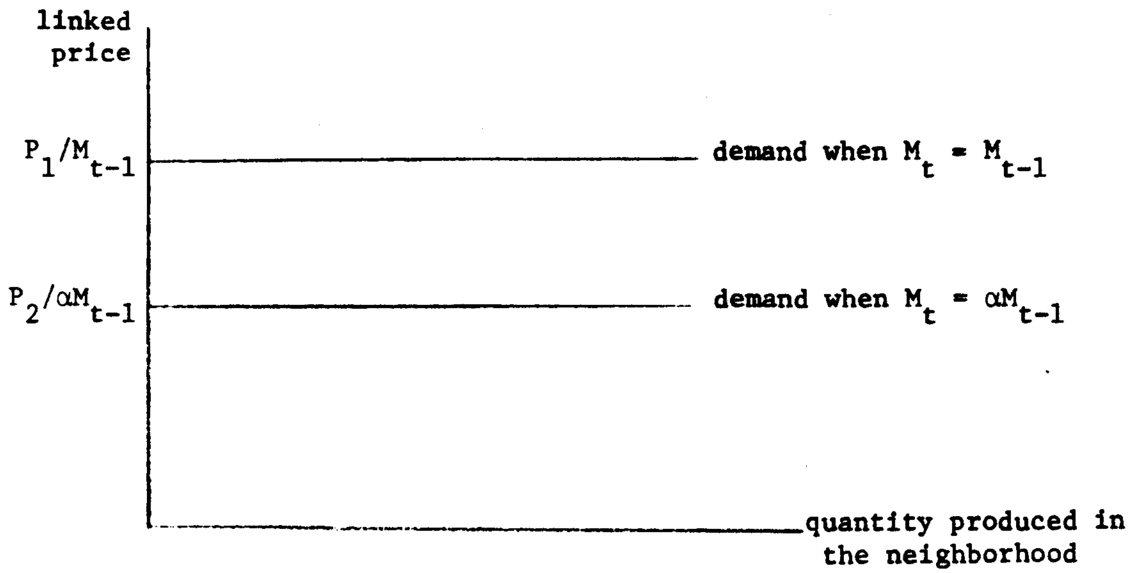
$$(8) \qquad \begin{aligned} &P_1/M_{t-1} \quad \text{if} \quad M_t = M_{t-1} \\ &P_2/\alpha M_{t-1} \quad \text{if} \quad M_t = \alpha M_{t-1}. \end{aligned}$$

This demand is illustrated by Figure 4 for the case $P_2/\alpha M_{t-1} < P_1/M_{t-1}$.



P_1/M_{t-1}
$P_2/\alpha M_{t-1}$
$P_1/\overline{M}$
$P_1/\alpha M_{t-1}$
$\mu HS(P/M)$
$(N-N^*)D(P/M)$
$\mu ND(P/M)$
quantity produced in the informed sector

Figure 3



The demand in terms of linked prices as perceived by the individual seller when all other sellers in his neighborhood advertise the dollar price P_1.

Figure 4.

The linked price that the seller will get as a function of his action and the realization of the money supply is described in Table 2,

	$M_t = M_{t-1}$	$M_t = \alpha M_{t-1}$	expected linked price
advertised linked price = P_1/M_{t-1}	P_1/M_{t-1}	0	$\emptyset P_1/M_{t-1}$
advertised linked price = $P_2/\alpha M_{t-1}$	$P_2/\alpha M_{t-1}$	$P_2/\alpha M_{t-1}$	$P_2/\alpha M_{t-1}$

Table 2

where $\emptyset$ is the probability that $M_t = M_{t-1}$ given that no one in the neighborhood has advertised the higher dollar price, P_2. Note that Claim 1 implies $\emptyset > 1/2$.

The probability $\emptyset$ and the seller's objective function will determine whether it is better to advertise the higher linked price P_1/M_{t-1} and risk the possibility of not selling or the lower linked price which ensures selling. If the higher linked price turns out to be the choice, then the dollar price P_1 is even better since it promises an expected linked price of $\emptyset P_1/M_{t-1} + (1-\emptyset)P_1 \alpha M_{t-1}$. (See Table 1 for the case $\emptyset = 1/2$).

Thus, it is possible that the individual seller will not benefit from advertising a linked price if all other sellers are advertising a dollar price. Adopting linked prices may therefore require cooperation among sellers. [7/]

FOOTNOTES

1/ This assumption is used by Lucas (1972) and the literature on the equilibrium approach to the business cycle which followed this article.

2/ This advantage of linked prices seems quite important. For example, in Israel, which is subject these days to triple-digit inflation, one often hears the complaint that it is difficult to know whether a given price is relatively expensive.

3/ It is more realistic to assume that by investing resources you can get a signal rather than the actual realization. I do not expect that the basic result will be sensitive to this modification.

4/ If all other sellers follow a Nash strategy, it is optimal for the individual seller to do the same.

5/ As we will see, the informed seller will do better in planning production than the uninformed seller in the case in which the price does not reveal the realization of the money supply. The probability (7) may be zero only if the value of information about the money supply for the specific purpose of planning production is greater than the cost of information. See the discussion in the Appendix for the requirement $R_2 > r$.

6/ He can certainly do better if he advertises a fully contingent price, like "I will charge $\$P_1$ if $M_t = M_{t-1}$ and $\$P_2$ if $M_t = \alpha M_{t-1}$." But such contingent prices may be prohibitively expensive.

7/ For a discussion of the policy issues, see Eden (1979).

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Appendix to Section II.

To calculate q* we need to specify the objective function of the sellers. To consider a simple example, I assume that the sellers live for two periods, produce only in the first period, and consume only in the second period. All sellers have the same Von Neumann Morgenstern utility function

$$\text{(A1)} \quad U(\ell, c)$$

where ℓ is first period leisure ($0 \leq \ell \leq 1$) and c is second period consumption. The function U( ) is assumed to be strictly monotone, strictly quasi-concave and differentiable.

In the first period, each seller has an excess to a linear production function

$$\text{(A2)} \quad c = L \equiv 1-\ell$$

where L is the supply of labor. It is assumed that one who holds 1% of the money supply at time t will be able to buy 0.01 $\tilde{y}_{t+1}$ units of consumption at time t+1, where $\tilde{y}_{t+1}$ is a random variable. It costs X units of labor to buy the information about the realization of M_t. (i.e., X is the time it takes to find out the planned transfer payment.)

To calculate the expected utility of a seller who buys the information, let $\bar{L}_1$ solve

$$\text{(A3)} \quad \max_{0 \leq L \leq 1} EU[1-L,\ P_1(L-X)\ \tilde{y}_{t+1}/M_{t-1}]$$

and let $\bar{L}_2$ solve

$$\text{(A4)} \quad \max_{0 \leq L \leq 1} EU[1-L,\ P_2(L-X)\ \tilde{y}_{t+1}/\alpha M_{t-1}],$$

where expectations in (A3) and (A4) are taken over the probability distribution of $\tilde{y}_{t+1}$.

Since Claim 1 implies that a seller who buys the information will advertise P_1 if he observes $M_t = M_{t-1}$ and P_2 if he observes $M_t = \alpha M_{t-1}$, his expected utility will be

(A5) $r = (A3)/2 + (A4)/2 = EU[1-\bar{L}_1, P_1(\bar{L}_1-X)\ \tilde{y}_{t+1}/M_{t-1}]/2 + EU[1-\bar{L}_2, P_2(\bar{L}_2-X)\tilde{y}_{t+1}/\alpha M_{t-1}]/2$

To calculate the expected utility of a seller who does not buy the information, consider the case in which the seller observes that P_1 has been advertised. In this case he solves

(A6) $\max\limits_{0 \le L \le 1} EU[1-L, P_1(L-X)\ \tilde{y}_{t+1}/\tilde{M}_t]$

where here expectations are taken over the conditional joint probability distribution of $\tilde{y}_{t+1}$, and the end of period money supply, $\tilde{M}_t$. The condition is the fact that the price P_1 has been advertised. I will denote the solution to (A6) by L_1^*.

Claim 1 implies that when no one buys the information, the price P_1 will be advertised. The expected utility of an uninformed seller, given that no one else is informed, is therefore

(A7) $R_1 = EU[1-L_1^*, P_1L_1^*\ \tilde{y}_{t+1}/M_{t-1}]/2 + EU[1-L_1^*, P_1L_1^*\ \tilde{y}_{t+1}/\alpha M_{t-1}]/2$

When someone else buys the information, the "correct" price will be advertised. The expected utility of the uninformed seller in this case is thus

(A8) $R_2 = EU[1-L_1^*, P_1L_1^*\ \tilde{y}_{t+1}/M_{t-1}]/2 + EU[1-\bar{L}_2, P_2\bar{L}_2\ \tilde{y}_{t+1}/\alpha M_{t-1}]/2$

Note that when P_2 is advertised, the uninformed seller can infer the realization of the money supply and therefore solve the problem (A4).

If other sellers buy information independently, each with probability q, then the probability that no one of the other n-1 sellers will buy the information is $(1-q)^{n-1}$. The probability that at least someone out of the other n-1 sellers will buy it is $[1-(1-q)^{n-1}]$. The expected utility of a seller who does not buy the information is therefore

(A9) $R(q) = (1-q)^{n-1}R_1 + [1-(1-q)^{n-1}]R_2$.

To justify a mixed strategy, the sellers must be indifferent between being uninformed and becoming informed. Thus, we should look for q such that

(A10) $R(q) = r$.

Note that $R_1 < R_2$ and R is strictly increasing in q. Furthermore, $R = R_2$ when $q = 1$ and $R = R_1$ when $q = 0$. Therefore, if $R_2 > r$ and $R_1 < r$, there will be a unique, interior solution to (A10) as in Figure 5.

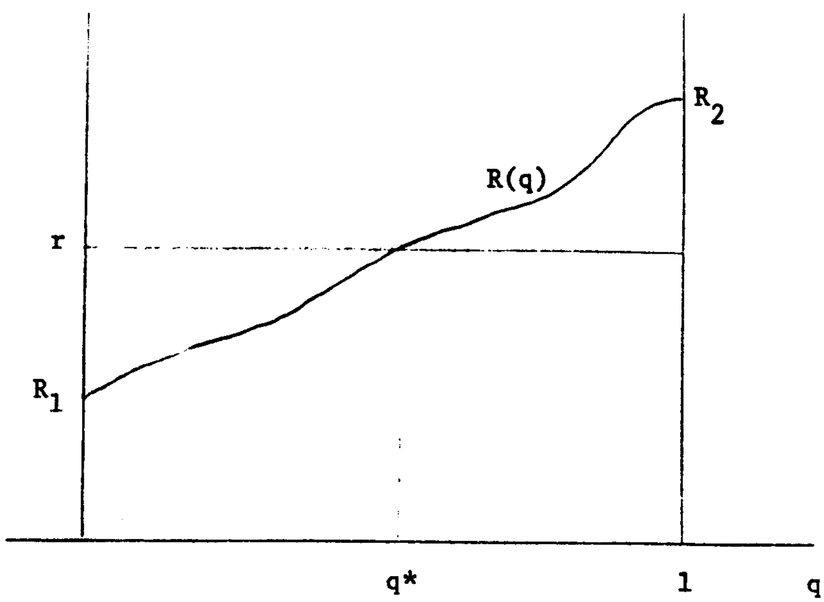

Figure 5

To ensure that $R_1 < r$, we need to assume that X is not too large (prohibitive). The requirement $R_2 - r > 0$ can be written as

$$\text{(A11)}\quad EU[1-L_1^*,\ P_1L_1^*\ \tilde{y}_{t+1}/M_{t-1}]$$
$$-EU[1-\bar{L}_1,\ P_1(\bar{L}_1-X)\ \tilde{y}_{t+1}/M_{t-1}]$$
$$>EU[1-\bar{L}_2,\ P_2(\bar{L}_2-X)\ \tilde{y}_{t+1}/\alpha M_{t-1}]$$
$$-EU[1-\bar{L}_2,\ P_2\bar{L}_2\ \tilde{y}_{t+1}/\alpha M_{t-1}]$$

The right hand side of (A11) must be negative. The sign of the left hand side is ambiguous since L_1^* is different than $\bar{L}_1$. Roughly speaking, it will be positive if the expected utility is not very sensitive to the choice of L or if X is not too small.