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THE LIQUIDITY PREMIUM AND THE SOLIDITY PREMIUM

by

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THE LIQUIDITY PREMIUM AND THE SOLIDITY PREMIUM

Real interest yields on short-term bonds are generally observed to be lower than the real yields on longer-term bonds of equivalent default risk. That is, the term structure of real interest rates is normally ascending. Given an ascending term structure, the implicit forward short-term rate of interest (for any specified future date) must be higher than the current short-term interest rate. Nevertheless, there is no evidence of any upward trend in real interest rates over time. It follows that implicit forward short-term rates must also be higher than the actual short-term rates that will be realized in the future, on average. This difference, between the implicit forward short-term rate and the mathematical expectation of the future short-term rate, is called the liquidity premium.

The positive liquidity premium therefore follows if we accept two stylized facts: 1) an ascending term structure at any moment of time, and 2) historical stationarity of interest rates over time. Since historical stationarity might seem to need no special justification, explaining the liquidity premium reduces to explaining why the term structure is normally ascending. A number of analysts have indeed, explicitly or implicitly, taken this route [see Reuben A. Kessel (1965)]. It is true that the term structure is usually ascending, and that real interest rates seem to have fallen within a narrow range throughout human history. However, non-ascending term structures are not rare. As for the historical stationarity of interest rates, there seems to be no strong ground on which to postulate such a degree of uniformity. The empirical evidence is far from clear

on this. There have of course been periods when realized real interest rates have been noticeably higher or lower than longer-term historical averages. I therefore do not adopt the stylized facts as my point of departure. Instead, my intention is to identify the forces explaining the liquidity premium as such, independent of the shape of the term structure and the level of interest rates. Doing so will illuminate not only the determination of the liquidity premium but of the term structure as well.

Hicks [1946] and Keynes [1930] argued that it is risk-aversion which causes the forward rate to be greater than the expected future rate. Possible future variations in interest rates will affect the values of long-term bonds more than short-term bonds. Consequently, risk-averse investors have to be induced by higher yields to hold the "less liquid" longer-term bonds.

The Keynes-Hicks view has been criticized, and properly so, for its over-emphasis on capital-value risk as opposed to income risk. For example, someone concerned only to lock in a more-distant-future flow of income could simply make a long-term investment commitment, and then be entirely unconcerned about possible interest-rate variations and consequent fluctuations in capital values at intermediate dates. For such an individual, a premium might be required to induce him to hold a shorter-term instrument.

Such a consideration led Modigliani and Sutch [1966] to the notion of the "preferred habitat". They argued that because of personal variations in individual motives to save or to consume at different dates (due, for example, to life-cycle considerations), different investors would typically be concerned about consumption risk at different dates. Thus, nothing of a high order of generality could be said about the sign of the liquidity premium.

While the Modigliani-Sutch criticism of Keynes and Hicks is well-founded, their terminology may incorrectly suggest that each investor is typically concerned only with income risk at some single "habitat" date, or even that an individual's temporal consumption plans are exogenous data rather than endogenous variables. In the models to be presented here, individuals are concerned about income risks at every date (up to their planning horizons), and their temporal consumption plans are choice variables that depend upon relative prices and interest rates.

I will be employing a general-equilibrium model to analyze how individuals' time-and-state distributed endowments, preferences (for consumption over time and also for risk-bearing), beliefs (and the timing of information arrival affecting beliefs), and productive opportunities all contribute to the shaping of the term structure of real interest rates and the liquidity premium. I will first use a simplified world of pure exchange to highlight certain elements of the general theory of the liquidity premium. I will show, in particular, how the amount of and the timing of anticipated information arrival affect the liquidity premium. I will then introduce intertemporal productive transformations. Here the interaction of production with information about the realized magnitude of representative endowments (e.g., good crop or bad crop) affects beliefs about consumption in a systematic way, and plays a critical role in determining the liquidity premium.

However, in a world where risk and risk-aversion apply to incomes at all dates, the liquidity premium can be fully understood only when juxtaposed against the "solidity premium" (McCulloch [1973] and Bailey [1964]). The solidity premium is the difference between the forward discount (as this term is used, for example, in quotations of U.S. Treasury Bills) and the expected future discount. While it might be thought that a negative liquidity premium

($L < 0$) implies a positive solidity premium ($S > 0$), and vice versa, this is in general not the case.

The term structure of interest rates depends upon, among other things, the riskiness of individuals' endowments and their aversion to risk, but reveals only the relative prices of certainty claims at different maturity dates. The pattern of L and S more specifically measures the relative riskiness of alternative maturity strategies. For example, it might be that the least risky way to assure a unit of income at a given future date would be simply to buy a bond maturing at that date; the term structure tells us the returns on such simple strategies. But, less obviously, the least risky way might instead be to buy a shorter-term instrument and plan on rolling it over, or to buy a longer-term instrument and plan on liquidating it before it matures. L and S provide measures of the riskiness of such complex strategies, as we shall see, one from the viewpoint of consumption income in the nearer future and the other for the farther future. As between any pair of future dates, therefore, the combination of the two is needed for a full description of the riskiness of alternative maturity strategies. (More generally, in a setting with many future consumption-production dates there will be L and S measures for each pair of dates, but the analysis here will not get into that level of complication.)

I. INTRODUCTION AND PREVIEW

We can cover all the essentials of the problem by dealing with a simple three-date model (dates 0, 1, and 2 years from the present). The current short-term real interest rate ${}_0r_1$ is defined in:

$$\frac{{}_0P_1}{{}_0P_0} \equiv \frac{1}{1+{}_0r_1} \quad (1)$$

Here ${}_0P_1$ is the price quoted today (date-0) of a unit claim to real income to be received next year (at date-1). ${}_0P_0$ is the price today of a unit of real income today; I will ordinarily take current income of any date as the numeraire commodity for prices quoted at that date, so that ${}_0P_0 \equiv 1$. The interest rate denoted ${}_0r_1$ is then the rate quoted at date-0, today, for discounting one-year future claims into their current (present-value) equivalent.

Analogously, the current long-term (2-year) interest rate ${}_0R_2$ is defined in:

$$\frac{{}_0P_2}{{}_0P_0} \equiv \frac{1}{(1+{}_0R_2)^2} \quad (2)$$

Here ${}_0P_2$ is the price quoted today of a claim to income to be received at date-2, and ${}_0R_2$ is the corresponding long-term interest rate for discounting such claims into their present-value equivalent. But the price ratio on the L.H.S. of equation (2) can also be written in another way that serves to define the forward short-term rate ${}_0r_2$:

$$\frac{{}_0P_2}{{}_0P_0} \equiv \frac{1}{(1+{}_0R_2)^2} \equiv \frac{1}{(1+{}_0r_1)(1+{}_0r_2)} \quad (3)$$

The notation ${}_0r_2$ indicates that this interest rate, though short-term in that it discounts income claims from date-2 back only to the previous year (date-1), is a rate quoted (or else implicit in other quoted prices and interest rates) at date-0.

As the economy moves historically through time, at date-1 an actual short-term rate ${}_1r_2$ will come into existence for discounting date-2 claims. This is the future short-term rate, defined in terms of the prices and interest rate quoted at date-1:

$$\frac{{}_1P_2}{{}_1P_1} \equiv \frac{1}{1+{}_1r_2} \quad (4)$$

(Here, since date-1 income becomes the numeraire for all claims quoted at date-1, it must be that ${}_1P_1 \equiv 1$.) Viewed from date-0, however, this actual future short-term rate ${}_1r_2$ will be uncertain. This brings us, finally, to the formal definition of the liquidity premium L:

$$L \equiv {}_0r_2 - E({}_1r_2) \quad (5)$$

The liquidity premium is the excess, on average, of the known forward short-term rate of interest over the unknown future short-term rate.

The simple algebra of the "discount" concept is completely analogous.

The current short-term real discount ${}_0d_1$ is defined in:

$$\frac{{}_0P_1}{{}_0P_0} \equiv 1+{}_0d_1 \quad (6)$$

Interest and discount are thus related as:

$$1+{}_0d_1 \equiv \frac{1}{1+{}_0r_1} \quad (7)$$

Evidently, positive interest will correspond to negative discount.

The forward discount is defined in:

$$\frac{{}_0P_2}{{}_0P_1} \equiv 1 + {}_0d_2 \equiv \frac{1}{1 + {}_0r_2} \quad (8)$$

And the actual future discount is defined in:

$$\frac{{}_1P_2}{{}_1P_1} \equiv 1 + {}_1d_2 \equiv \frac{1}{1 + {}_1r_2} \quad (9)$$

This brings us to the formal definition of the solidity premium, S.

$$S \equiv {}_0d_2 - E({}_1d_2) \quad (10)$$

Of course, S can also be expressed in terms of interest rates, or in terms of price ratios:

$$S \equiv \frac{1}{1 + {}_0r_2} - E\left(\frac{1}{1 + {}_1r_2}\right) \equiv \frac{{}_0P_2}{{}_0P_1} - E\left(\frac{{}_1P_2}{{}_1P_1}\right) \quad (11)$$

A similar easy development from (5) leads to an expression for L in terms of price ratios:

$$L \equiv \frac{{}_0P_1}{{}_0P_2} - E\left(\frac{{}_1P_1}{{}_1P_2}\right) \quad (12)$$

Although there is a kind of inverse relationship, it will be evident that S is neither the reciprocal nor the negative of L.

The intuitive interpretation that makes these formalisms useful can be developed as follows. Think of the liquidity premium L as measuring the difference, as of the farther-future date (date-2), between the sure return (principal plus interest) on a long-term bond and the mean return on a short-term bond rolled-over at the uncertain future short-term rate. For, it follows

by elementary manipulations that:

$$(1+r_1)^L \equiv (1+R_2)^2 - (1+r_1)E(1+r_2) \quad (13)$$

As for S, think of it as measuring the difference, as of the nearer-future date (date-1), between the sure return on a short-term bond and the mean return on a long-term bond liquidated at the uncertain future discount. For:

$$S(1+R_2)^2 \equiv \frac{(1+R_2)^2}{1+r_2} - E\left(\frac{(1+R_2)^2}{1+r_2}\right) \equiv (1+r_1) - (1+R_2)^2 E(1+d_2) \quad (14)$$

Thus, $L > 0$ implies an excess yield, on average, of long-term over short-term instruments compared at date-2. $S > 0$, on the other hand, indicates an excess yield, on average, of short-term over long-term instruments compared at date-1. Since the dates at which the comparisons are made differ, L and S need not have opposite signs.

II. LIQUIDITY PREMIUM AND SOLIDITY PREMIUM IN PURE EXCHANGE

This section analyzes the forces underlying the liquidity premium and solidity premium, using an explicit contingent-claim model of income uncertainty at near-future (date-1) and far-future (date-2) dates. The present date is assumed free of uncertainty; at date-0 each individual is supposed to have a specific known endowment \bar{c}_0 of the real income commodity ("corn"). But at date-1 his endowment will be the probability distribution $(\bar{c}_{11}, \dots, \bar{c}_{1E}; \pi_{11}, \dots, \pi_{1E})$, where $e = 1, \dots, E$ indicates the state of the world at the earlier date and π_{1e} represents the associated probability (assessed at date-0). Similarly, at date-2 the endowment will be $(\bar{c}_{21}, \dots, \bar{c}_{2S}; \pi_{21}, \dots, \pi_{2S})$, where $s = 1, \dots, S$ is the index for states of the world at the later date and π_{2s} is the associated probability in terms of beliefs at date-0. In this section pure exchange is assumed: there are no productive opportunities (e.g., storage) for physically transforming income of one date into income of any other date.

Suppose at date-0 there are complete markets for claims to consumption contingent upon states of the world, at any date.¹ Then the current price of a claim to income at date-1 contingent upon state- e can be denoted ${}_0P_{1e}$, and similarly ${}_0P_{2s}$ is the current price of a claim to income at date-2 contingent upon state- s . Certainty claims to income as of any given date can be purchased by buying a full complement of the corresponding contingent claims. Thus:

$$\begin{aligned} {}_0P_1 &\equiv \sum_e {}_0P_{1e} \\ {}_0P_2 &\equiv \sum_s {}_0P_{2s} \end{aligned} \tag{15}$$

Once the passage of time reveals the state of the world at date-1, individuals will in general revise their beliefs about the probabilities of the date-2 states. (That is, the advent of state- e is not only an income-event but also generally

an information-event.) The revised probabilities can be denoted $\pi_{s,e}$. Then the price of a certainty claim to date-2 consumption, quoted at date-1 after state-e has obtained, can be expressed as:

$${}_{1e}P_2 \equiv \sum_s {}_{1e}P_{2s,e} \quad (16)$$

We can define the future short-term interest rate ${}_{1e}r_2$, contingent upon state-e having been realized at date-1, in:

$$\frac{{}_{1e}P_2}{{}_{1e}P_1} \equiv \frac{1}{1+{}_{1e}r_2} \quad (17)$$

As usual, the denominator on the L.H.S. would be unity, since it is the price of the numeraire commodity current at date-1 after state-e obtains.

The expected future rate of interest, in terms of probability beliefs at date-0, can then be written:

$$E(1+{}_1r_2) \equiv E\left(\frac{{}_1P_1}{{}_1P_2}\right) \equiv \sum_e \pi_e \frac{{}_{1e}P_1}{\sum_s {}_{1e}P_{2s,e}} \quad (18)$$

Recalling equation (12), the liquidity premium L can be expressed in terms of the underlying bundles of contingent claims as:

$$L \equiv \frac{{}_0P_1}{{}_0P_2} - E\left(\frac{{}_1P_1}{{}_1P_2}\right) \equiv \frac{\sum_e {}_0P_{1e}}{\sum_s {}_0P_{2s}} - \sum_e \pi_e \frac{{}_{1e}P_1}{\sum_s {}_{1e}P_{2s,e}} \quad (19)$$

To press further, I shall have to say something more about the forces underlying the determination of prices. First of all, I will assume away any differences of beliefs in the economy: everyone agrees as to the probabilities π_e , π_s , and $\pi_{s,e}$. Let every individual make choices under uncertainty so as to maximize expected utility $U = E(V)$, where $V(c_0, c_1, c_2)$ is his "cardinal", risk-averse, preference-scaling function for dated consumption income vectors.² In addition to the standard

postulate of state-independence of the utility function, I will also be making the simplifying assumption of time-independence: that the V function is separable in the variables c_0, c_1 , and c_2 . While this limits the generality of our results, nonseparable tastes (i.e., allowing for possible intertemporal preference complementarities) would be a second-order effect that can only be accommodated by rather burdensome notation.³

If the social endowments of income are positive in every state, and if everyone assigns infinite negative utility to zero consumption at any date, an interior solution is guaranteed in which each individual holds positive amounts of every contingent claim c_{1e} and c_{2s} . Then in equilibrium at date-0 each individual's expected marginal utilities will be proportional to the prices:

$$\frac{v_0}{0^P 0} = \frac{\pi_e v_{1e}}{0^P 1e} = \frac{\pi_s v_{2s}}{0^P 2s} \quad (20)$$

where $v_0 \equiv \partial U / \partial c_0$, $\pi_{1e} v_{1e} \equiv \partial U / \partial c_{1e}$, and $\pi_{2s} v_{2s} \equiv \partial U / \partial c_{2s}$.⁴ This equation reveals, therefore, the relation of prices to preferences and endowments (which together determine the marginal utilities) and to probability beliefs.

Substituting from (20) into (19) leads to:

$$L = \frac{\sum_e \pi_e v_{1e}}{\sum_s \pi_s v_{2s}} - \sum_e \pi_e \frac{v_{1e}}{\sum_{s,e} \pi_{s,e} v_{2s}} \quad (21)$$

Or, in a more condensed notation:

$$L = \frac{E(v_1)}{E(v_2)} - E \frac{v_1}{E_e(v_2)} \quad (22)$$

Here E_e indicates the expectation (of date-2 marginal utility) conditional upon state-e at date-1. (That is, calculated in terms of the $\pi_{s,e}$ probability beliefs.)

Expectations symbolized simply as E are taken with respect to beliefs at date-0.

Of course:

$$E(E(v_2)) = \sum_e \pi_e E_e(v_2) = E(v_2) \quad (23)$$

In parallel, we develop a similar expression for S. Starting from equation (11), the price-ratio version of S, we obtain in a similar fashion the analog of equation (22):

$$S = \frac{E(v_2)}{E(v_1)} - E \frac{E(v_2)}{v_1} \quad (24)$$

Certain simple relations must hold between the signs of L and S, following Jensen's Inequality, which may be written

$$E(1/x) \geq 1/E(x)$$

(where equality holds only for non-random x). In particular, taking x as the random variable $\frac{1^P_1}{1^P_2}$, it can be shown that L and S can never both be positive.⁵

Furthermore, only if $\frac{1^P_1}{1^P_2}$ is not random can L and S both be zero.⁶

Thus, the possible cases of interest are:

- 1) L < 0, S < 0
- 2) L > 0, S < 0
- 3) L < 0, S > 0

(Note: In each of these cases, either of the inequalities -- but not both -- can be more generally, weak.)

For the economic interpretation of these various cases, it will be illuminating to see how L and S depend upon certain covariances between dated marginal utilities

and contingent exchange ratios between near-future and far-future claims. Rearranging the expression for L in equation (22):⁷

$$L = \frac{1}{E(E(v_2))} [E(v_1) - E\left(\frac{v_1}{E(v_2)}\right) E(E(v_2))] \\ = \frac{1}{E(v_2)} \left\{ E\left(\frac{v_1}{E(v_2)} E(v_2)\right) - E\left(\frac{v_1}{E(v_2)}\right) E(E(v_2)) \right\}$$

which is just:

$$L = \frac{1}{E(v_2)} \text{Cov}\left(\frac{v_1}{E(v_2)}, E(v_2)\right) \quad (25)$$

And similarly:

$$S = \frac{1}{E(v_1)} \text{Cov}\left(\frac{E(v_2)}{v_1}, v_1\right) \quad (26)$$

The first covariable in (25), $\frac{v_1}{E(v_2)}$, can be seen from equations (17), (18), and (20) to equal $1 + {}_{1e}r_2$, one plus the interest rate contingent on the occurrence of state e at date-1. Similarly, its reciprocal, $\frac{E(v_2)}{v_1}$, the first covariable in equation (26), is one plus the contingent discount.

From equation (25) we can see that L is positive when the covariance between contingent future interest rates and date-2 marginal utility is positive, or, roughly speaking, when the covariance between interest rates and date-2 consumption is negative. This is the same thing as saying that $L > 0$ when the value of a rolled-over short-term bond covaries against consumption at date-2, which makes the strategy of holding short-term bonds a form of insurance with respect to date-2.

Similarly, from equation (26) we infer that $S > 0$ when the covariance between the future discount and marginal utility at date-1 is positive, or essentially when the value of a liquidated long-term bond covaries against date-1 consumption.

Recall that $L > 0$ implies $S \leq 0$, and $S > 0$ implies $L \leq 0$. The interpretation is that if the short-term bond's rolled-over value covaries against date-2 consumption, the long-term bond's liquidated value must covary with date-1 consumption. That is, if the short-maturity strategy provides insurance from the viewpoint of date-2, the long-maturity strategy must enlarge risk at date-1. However, if either L or S is negative, the other need not be positive. Which is to say that if short-term bonds rolled-over are risk-enlarging at date-2, this does not imply that liquidated long-term bonds provide insurance at date-1. Both maturity strategies may in fact be risk-enlarging at their non-maturity dates.

The possible sign combinations of L and S correspond to the following economic scenarios. For simplicity, I will describe the strong-inequality cases only. (Equality cases arise only when either we learn nothing about one of the two dates, or when the interest rate is not random.)

1) $L > 0$, $S < 0$: Here the short-term bond rolled over provides insurance against consumption risk as of date-2. Such a short-maturity strategy is actually risk-reducing as of date-2 (because of the negative covariance between the contingent future short-term rate and the date-2 endowment), and is thus even less risky than holding a riskless long-term bond maturing at date-2. The long-term bond is, however, risk-enlarging if liquidated at date-1, where the short-term bond is riskless. Thus, the short-term strategy has superior risk characteristics, viewed from either date. Hence, if the long-term bond is to be held it must, on

average, pay off more when it matures at date-2 ($L > 0$) or if liquidated at date-1 ($S < 0$).

2) $L < 0, S > 0$: Here the long-term bond provides date-1 insurance, since there is positive covariance between contingent interest rates and date-1 consumption, while the short-maturity strategy is risk-enlarging at date-2. Hence, the long-term maturity strategy has superior risk characteristics as of either date, so in equilibrium, the short-term maturity strategy must, on average, pay off more either at date-1 ($S > 0$) or at date-2 ($L < 0$).

3) $L < 0, S < 0$: This seemingly paradoxical case is easily interpreted. Here each bond remains, of course, riskless as of its own maturity date, but is risk-enlarging as of its non-maturity date. So the short-term bond rolled over must, on average, pay off more at date-2 ($L < 0$) while the long-term bond liquidated must, on average, pay off more at date-1 ($S < 0$).

We can also explain the economics underlying the impossibility of the $L > 0, S > 0$ combination. $L > 0$ implies that the short-term strategy provides insurance at date-2, which means that high contingent interest rates must be associated with small expected consumption endowments at date-2. $S > 0$ implies that long-term bonds provide insurance at date-1, which means that high interest (discount) rates must be associated with large consumption endowments at date-1. But, if the realized state at date-1 turns out to be rich, while date-2 is likely to be poor, the contingent interest rate would have to be low, not high.

I will now decompose the covariances of equation (25) and (26) so as to highlight two underlying elements determining the signs and magnitudes of L and S : (1) the pattern of serial correlation of dated marginal utilities, or -- speaking more loosely -- the serial correlation of consumption over time and (2) the relative magnitude of two coefficients of variation, the first of date-1

marginal utility and the second of date-2 conditional expected marginal utility. There are two main forces affecting serial correlation: intertemporal productive transformations (e.g., storage) and new information. We shall see in the next section how intertemporal productive transformations tend to induce positive serial correlation. In this section, we shall see how arrival of information about date-2, either due to the mere realization of some state at date-1, or due to the arrival of some side-message, may affect the serial correlation either way. As for the coefficients of variation of the dated marginal utilities, these are reflections of the power of the date-1 information as pertaining to date-1 and to date-2. Since our knowledge about the date-1 state will be complete at date-1, the extent of the information gained is essentially measured by our date-0 uncertainty about the date-1 state -- the coefficient of variation of v_1 . But we may also at date-1 receive information about date-2. The coefficient of variation of $E_e(v_2)$ measures how much we expect (as of date-0) to revise (as of date-1) our initial beliefs about the date-2 state. Thus, both the relative size of the revisions (indicated by the coefficients of variation) and also the direction of dependence of the revisions (indicated by the serial correlation) affect L and S.

Rearranging (25) and (26), and simplifying notation by letting $x \equiv v_1$ and $y \equiv E_e(v_2)$:

$$\begin{aligned} L &= \frac{1}{E(y)} \text{Cov} \left(y, \frac{x}{y} \right) \\ S &= \frac{1}{E(x)} \text{Cov} \left(x, \frac{y}{x} \right) \end{aligned} \tag{32}$$

These expressions can be rearranged to obtain:⁸

$$L = E(x) \sigma_{1/y} \left(\left[\frac{\sigma_y}{E(y)} \rho_{y,1/y} - \frac{\sigma_x}{E(x)} \rho_{x,1/y} \right] \right) \tag{33}$$

$$S = E(y) \left(\sigma_{1/x} \left[\frac{\sigma_x}{E(x)} \rho_{x,1/x} - \frac{\sigma_y}{E(y)} \rho_{y,1/x} \right] \right) \quad (34)$$

Here σ_x and σ_y are respectively the standard deviation of date-1 marginal utility and the standard deviation of conditional expected date-2 marginal utility;

$\sigma_{1/x}$ and $\sigma_{1/y}$ are the standard deviations of their reciprocals; and ρ is the coefficient of correlation.

The sign of $\rho_{x,y}$ does not conclusively determine the sign of $\rho_{x,1/y}$, but the two will take on opposite signs, except in rather bizarre cases where one variable is highly skewed. For purposes of exposition, therefore, I shall carry on as though the signs of $\rho_{x,1/y}$ and $\rho_{y,1/x}$ are always the opposite of $\rho_{x,y}$. (As for $\rho_{y,1/y}$, the correlation of any variable with its own reciprocal is always negative.)

Subject to the imprecision of this approximation, we can see in equations (33) and (34) that negative $\rho_{x,y}$ assures $L < 0$, $S < 0$ — Scenario #3 above. Scenarios #1 and 2 can only occur when $\rho_{x,y}$ is positive; that is, when realized marginal utility at date-1 is positively associated with expected marginal utility at date-2 (or realized date-1 consumption with expected date-2 consumption).

(However, as the equations indicate, we could have Scenario #3 even with $\rho_{x,y} > 0$.)

As between Scenarios #1 and #2, which of the two transpires depends upon the magnitude of the associated positive regression. Suppose occurrence of a good state at date-1 raises the probability of good states at date-2, but not by so much as to make the expected date-2 outcome better (in marginal-utility terms) than the realized date-1 outcome. Thus, $v_{1e} < E_e(v_2) < E(v_2)$. (And, of course, there would then be a similar "regression toward the mean" effect in the event of a bad state at date-1.) The coefficient of variation for date-2 expected marginal utility, $\sigma_y/E(y)$, then tends to be small in comparison with the cor-

responding date-1 statistic, $\sigma_x/E(x)$. This leads, in equations (33) and (34), to the pattern $L>0, S<0$ -- Scenario #1. Here the anticipated revision of beliefs regarding date-1 is, on average, more extensive than that regarding date-2. But if a good state at date-1 means, on average, an even better state at date-2 (and a poor date-1 state an even worse expected date-2 outcome), this "regression away from the mean" leads to Scenario #2 with $L<0, S>0$. In this case the anticipated revision of beliefs regarding date-2 is, on average, more extensive, even though we get conclusive information about date-1 and only partial information about date-2.

An interest-rate interpretation will also be helpful. If the magnitude of the positive regression is small ("regression toward the mean"), the contingent future short-term interest rate will be low in rich branches of the date-state consumption tree and high in poor branches of the tree. Here a good date-1 state implies a not-quite-as-good date-2 state on average, and therefore low r_2 -- while a poor date-1 state implies a high r_2 . Low returns when you are rich, and high returns when you are poor means that the short-term maturity strategy has the superior risk characteristics, so the long-terms must unambiguously yield more, on average, over both long and short horizons ($L>0, S<0$). But, with "regression away from the mean," the contingent future short-term rate is high when you are rich and low when you are poor; the short-term strategy has definitely inferior risk characteristics, and hence must always yield more, on average, again over both long and short horizons ($L<0, S>0$).

Finally, with negative $\rho_{x,y}$ a good [bad] date-1 state implies a bad [good] date-2 on average, and thus a low [high] contingent interest rate. So the short-term strategy is risk-enlarging at date-2 (implying $L<0$) while the long-term strategy is risk-enlarging at date-1 (implying $S<0$). (Recall, however, that the

combination $L < 0$, $S < 0$ may occur even if $\rho_{x,y}$ is positive so long as it is not very large.)

Note that the aspect of date-2 consumption that is important in determining the sign of the liquidity premium is not the variation in c_2 (date-2 consumption) or in v_2 (date-2 marginal utility), but the variation in $E_e(v_2)$ -- the date-1 revised expectation of date-2 marginal utility. The variation in v_2 constrains the variation in $E_e(v_2)$, of course; the variation in $E_e(v_2)$ can be as great as that of v_2 only if the information about date-2 arriving at date-1 is conclusive, and in all other cases must be smaller. The variation in v_2 does not matter, however, except to the extent that it is manifested in $E_e(v_2)$, and hence in interest rates, prior to date-2. Specifically, it follows from equations (25) and (26) that if no information is revealed at date-1 about date-2, (so that $E_e(v_2)$ is a constant), then $L=0$ and $S < 0$, even if there is much more consumption risk at date-2 than at date-1.

Part of what Hicks and Keynes had in mind as a source of the liquidity premium was that the far future was inherently more uncertain than the near future, a proposition few of us would dispute. But this is neither a necessary nor sufficient condition for $L > 0$. From equation (33) the needed conditions are: 1) that date-1 and date-2 consumption be positively correlated, and 2) that information arriving at date-1 is relatively more informative about date-1 consumption than date-2 consumption. The combined weight of these conditions is rather restrictive, so that in a world of pure exchange, there really could be no clear presumption that the liquidity premium is positive. As we shall see, it is the possibility of intertemporal productive transformation, in association with information anticipated to be received at date-1, that more powerfully forces $L > 0$.

We have spoken so far only of bonds, i.e., of certainty claims to income, either at date-1 or at date-2. But more generally, for any asset representing

a bundle of contingent claims, its date-0 price will vary so that its expected yield to any date will be determined by the covariance of its future value with consumption at that date. Specifically, for any bundle of contingent claims, if we were to take the difference between its forward price (in terms of date-2 consumption as the numeraire) and its expected future price (same numeraire) we would obtain an expression analogous to that for L in equation (12). Substitution for expressions parallel to (22), and rearrangement to obtain the parallel to (25), readily show the expected change in price over the interval from date-0 to date-2 to be determined by the covariance of the value of this asset with consumption at date-2. The same difference between forward and expected future price of any bundle in terms of date-1 consumption yields an expression analogous to that for S in equation (11). Again, substitution and rearrangement yield analogs to (24) and (26), which show the expected price change to date-1 to be determined by the covariance of the value of the asset with consumption at date-1.

What we really have here is a multi-period generalization of the single-period Sharpe-Lintner-Mossin capital asset pricing model (CAPM)--in which the question "How risky is this asset?" has possibly as many answers as there are dates. In the CAPM, the factor determining an asset's expected yield is its covariance with the market portfolio, where the value of the market portfolio is simply the realization of date-1 (end-of-period) consumption. In the model here, or rather its rearrangement to allow for risky bundles as well as bonds, the covariance of the asset values with the realization of consumption at each date determines the expected yield to that date. The covariance of the value of any asset with the market portfolio (the present value of all future contingent claims) is in general not the same as its covariance with consumption at any given date. This is fun, but for now, back to the liquidity premium.

III. THE INFLUENCE OF INTERTEMPORAL PRODUCTION

The foregoing provided a relatively complete analysis of the determinants of L and S under pure exchange, and makes possible an economical treatment here of the consequences of introducing intertemporal productive transformations. I am building here upon a model introduced in more primitive form by Hirshleifer [1972]. Hirshleifer believed that when costless storage is possible, a unit claim to the inherently more flexible date-1 crop becomes (other things equal) more desirable than a corresponding claim upon the date-2 crop, tending to bring about an ascending term structure -- and, he thought, a positive liquidity premium. While it is true that the forward-only nature of intertemporal production always enriches the future at the expense of the present and hence tends to make the term structure of interest rates ascending, I will show that it does not always imply $L > 0$.

I will discuss here two illuminating polar cases. In each, I distinguish between the endowment and the actual consumption at each date. With the opportunity to invest, not all of the current crop need be consumed; a later date's crop may include returns from investment of part of the crop of an earlier date. I assume here that investment exhibits diminishing returns, and that the marginal return to investment depends only on the amount invested, not on the endowments at either date. I also assume that production goes forward only; the current crop may be invested for consumption later, but there is no mechanism for transforming the future endowment into consumption today.

In our first case, which leads to $L > 0$, the state realized at date-1 determines the endowment at that date but does not modify the probabilities of the possible endowments at date-2. It does, however, affect the amount individuals choose to invest at date-1. This case isolates the interaction of revised beliefs about the

date-1 crop with their implied investment decisions in providing information about date-2 consumption.

In the second polar case, which leads to $S > 0$, there is only one possible crop at date-1, but different possible messages about what the endowment might be at date-2. Here it is the message about date-1 which determines the optimal investment at date-1. This case isolates the interaction of revised beliefs about date-2 with their implied investment decisions in providing information about date-1 consumption.

For the first case (beliefs about the date-2 endowment unaffected by the outcome of date-1) it is evident that the better the date-1 outcome, the more will be invested at date-1. Then we can easily establish that the covariance between $E_e(v_2)$ and contingent interest rates is positive. The greater the investment at date-1, the higher consumption at date-2, and so the lower is conditional expected marginal utility at the later date. Further, since investment exhibits diminishing returns, the higher the investment, the lower is the marginal rate of return and hence the contingent interest rate. So in those (relatively well-endowed) date-1 states where investment takes place, $Cov[E_e(v_2), {}_1r_2]$ is positive. For those date-1 outcomes so poor that no investment is undertaken, the value of $E_e(v_2)$ is invariant. The contingent interest rates in these corner-solution states will be higher than over the average of all states, and the invariant $E_e(v_2)$ will also be higher than its average over all states. Therefore, counting the corner-solution states as well, the covariance between contingent interest rate and $E_e(v_2)$ is positive. The conclusion is clear: the liquidity premium is

positive. (A limiting case of $L=0$ results if all states at date-1 are so poor that investment never takes place.) Of course, $L>0$ implies $S\leq 0$, but since there is variation in v_1 , the strict inequality $S<0$ holds.

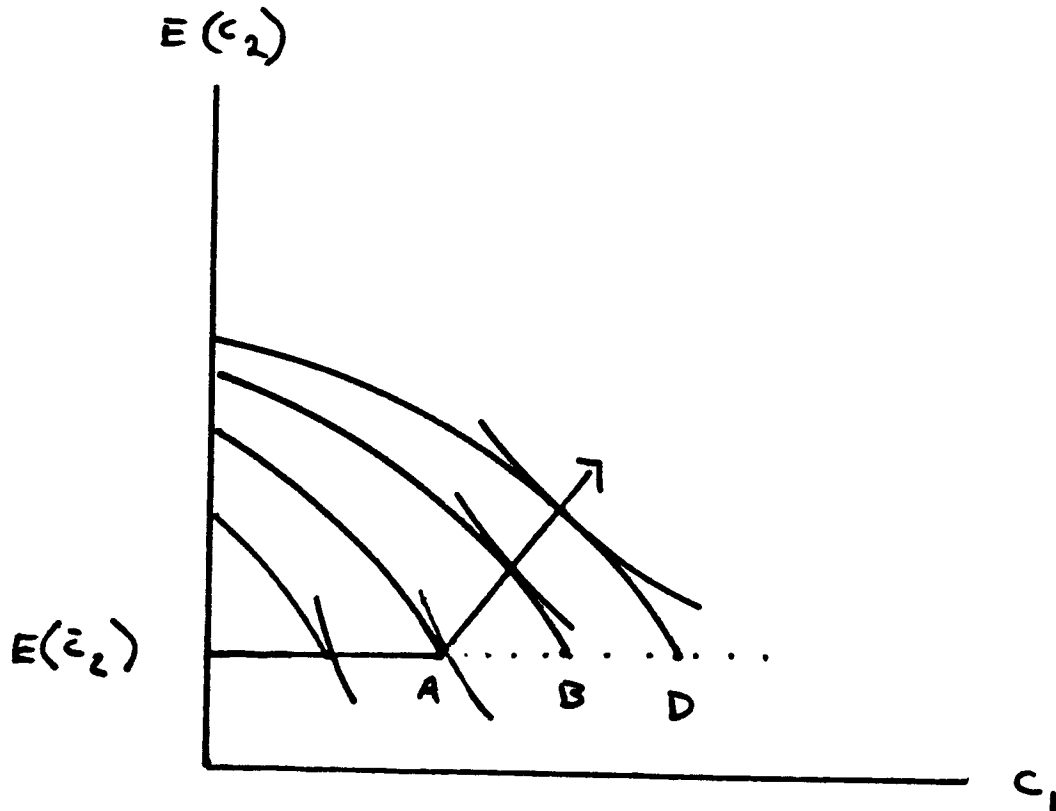


Figure 1

Figure 1 illustrates, for a representative individual situation, how a negative covariance between contingent interest rates and $E(c_2)$ arises. The latter is of course inversely related to $E(v_2)$ because the news changes only the mean of the c_2 distribution. As the date-1 endowment increases from very low levels, the expected date-2 endowment $E(\bar{c}_2)$ being constant throughout, no investment takes place until the critical point A is reached. Beyond A, from endowment points like B and D, the scale of investment increases. The interpretation, given diminishing

returns to investment, is a negative association between the achieved $E(c_2)$ and the equilibrium contingent interest rate ${}_1r_2$.

In terms of the interpretation associated with equations (33) and (34), our first case reveals that intertemporal production makes consumption positively serially correlated even if the endowments are independent. A rich date-1 crop and hence a richer-than-average date-1 consumption now tends to be followed by enriched date-2 consumption, and a poor date-1 crop by a less enriched date-2 consumption. Moreover, the low interest rates occur in the rich branches of the tree of outcomes, and the high ones in the poor branches.

This result, that intertemporal production implies $L > 0$, is very robust provided we stick to the condition that beliefs about the date-2 endowment are unaffected. Notably, the positive liquidity premium does not depend upon the (somewhat dubious) stylized facts. $L > 0$ obtains regardless of the distributions of c_1 and c_2 (and of course, c_0), i.e., whether or not the term structure is ascending (the first stylized fact). Further, $L > 0$ in no way depends upon a stationary trend in interest rates (the second of the stylized facts). The result will also hold for a paradigm with any number of dates, and for comparison of any maturities so long as all the dated endowments are statistically independent, and also for linear as well as diminishing returns production functions.⁹

Now consider the second case: here the crop at date-1 is non-stochastic, but information about date-2 will arrive at date-1 in the form of a "side-message". This means that the date-1 investment decision depends only upon the news about date-2. The worse the news, the more will be set aside at date-1 (invested) for consumption at date-2. Note that uncertainty regarding date-1 consumption emerges even though there is no uncertainty about the date-1 endowment. In effect, some of the risk at date-2 is shifted back to date-1.

Under the conditions of the second case, it is easy to see that the covariance between v_1 and $\frac{E(v_2)}{v_1}$ (one plus the contingent discount) will be positive, and hence by equation (26), the solidity premium must be positive. If the date-1 news about date-2 is very bad, investment is large and the contingent interest rate is very low. Correspondingly, since the date-1 endowment does not vary, date-1 consumption is very low. So, when investment occurs, lower interest rates are associated with lower date-1 consumption (implying high v_1). As for corner-solution states in which the date-2 news is so good that no investment is undertaken, in those states v_1 must be lower than average while interest rates are higher than average. Thus, overall, the covariance between date-1 marginal utility v_1 and the interest rate is negative. But r_2 is inversely associated with $\frac{E(v_2)}{v_1}$. So by equation (26) it must be that S is positive, which implies $L \leq 0$. (Again, variation in $E(v_2)$ gives us the strict inequality $L < 0$, and a limiting case of $S=0$ results if date-2 is always so well endowed that investment never takes place.)

As in our first case, the availability of intertemporal productive opportunities makes consumption serially correlated. News of a bad date-2 endowment induces investment, causing consumption at date-1 to be lower than otherwise, while news of a good date-2 state endowment encourages consumption of the date-1 crop. But here, the high interest rates come in the rich branches of the tree of possible outcomes, and the low ones (with large investment!) in the poor ones. Put another way, in our first case higher investment (implying lower interest rates) takes place from richer states at date-1, while in our second case, higher investment takes place toward poorer states at date-2.

We see, therefore, that from a sufficiently abstract point of view, even the availability of intertemporal production does not necessarily tend to make for a

positive liquidity premium. Nevertheless, I want to argue that the general presumption in favor of a positive liquidity premium is not so wrong. The reason is that, apart from unusual situations, the information that individuals can anticipate receiving will typically do more to resolve uncertainty about near-future than about far-future events. The implication is that our first polar case above (in which nothing was learned about the date-2 endowment) is likely to be a closer approximation of reality than our second polar case (in which nothing was learned about the date-1 endowment). It is this first case, we saw, that led to a systematically positive liquidity premium.

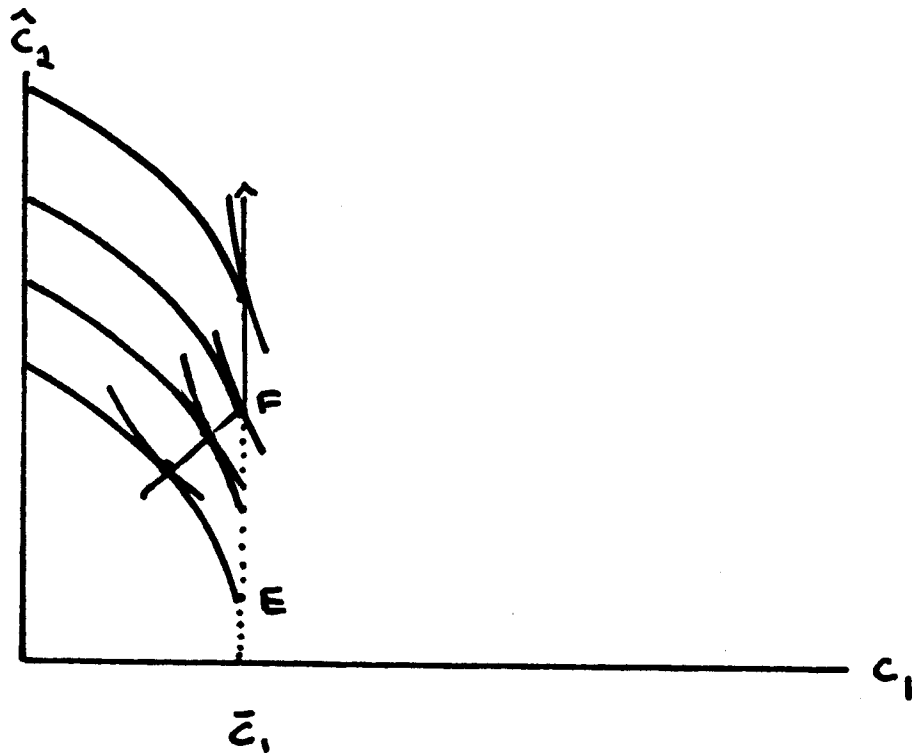


Figure 2

Figure 2 illustrates, for a representative individual, how a negative covariance between interest rates and v_1 arises. Along the horizontal axis is c_1 , date-1 consumption, which is inversely related to v_1 , the marginal utility of the date-1 outcome. On the vertical axis \hat{c}_2 represents the certainty equivalent of expected date-2 consumption. Since c_2 is still a random variable as of date-1, \hat{c}_2 is the analog of c_1 , and is inversely related to $E_e(v_2)$. The date-1 endowment, \bar{c}_1 , being constant throughout, as \hat{c}_2 increases investment undertaken at date-1 from endowment points like E will decline until the critical point F is reached. Beyond F no investment at all takes place, but the slopes of the indifference curves along the vertical ray from F are ever-increasing. The interpretation is that, given diminishing returns to investment, there is a negative association between the consumed c_1 and the rate of interest.

IV. SUMMARY AND CONCLUSION

Why does anybody care about the sign of the liquidity premium? Because the term structure of interest rates alone displays only the progression of relative values of riskless claims to income at increasingly remote future dates; it does not reveal how risky it would be to provide for consumption at a given date by buying bonds that mature earlier (with the intention of rolling-over) or by buying bonds that mature later (with the intention of liquidation). We tend to assume that the least risky strategy for any date's consumption is simply to hold a bond maturing at that date, but this is in error. If you want to know about the riskiness of various maturity strategies for different dates, you want to know about the liquidity premium and the solidity premium.

The concern of Hicks and Keynes was for the investor who might, if he bought long-term bonds, have to liquidate them at an unfavorable price if he came upon a rainy day before they matured. In terms of the analysis here, this corresponds to $S < 0$ -- which means that the value of the long-term bond at the intermediate date (the date prior to its maturity in a three-date paradigm) covaries with income at that date. When $S < 0$, the long-term bonds will be risk-enlarging at intermediate dates, i.e., they will indeed have their lowest (general equilibrium) values on rainy days together with their highest values on sunny days. But $S < 0$ does not necessarily imply $L > 0$. This means that Hicks' and Keynes' investor could still be vulnerable even if he buys the short-term bonds, as he may have to roll them over at an unfavorable rate. Only if $L > 0$ is the strategy of buying short-term bonds the less risky strategy for both the near future and the far future.

For a comparison of two bonds of different maturities, three basic scenarios can emerge:

Scenario #1: Positive L (which implies, by Jensen's Inequality, $S \leq 0$)

The forward rate of interest is greater than the expected future rate of interest, and the longer-term bond has the higher average yield over both the shorter horizon and the longer horizon. This occurs when the value of the shorter-term bond rolled-over covaries against income at the later date, and hence provides insurance against income-risk at that date.

Scenario #2: Positive S (which implies, by Jensen's Inequality, $L \leq 0$)

The forward discount is greater than the expected future discount, so that the longer-term bond has the lower average yield over both the longer and shorter horizons. This case occurs when the value of the longer-term bond covaries against income at the earlier date and hence provides insurance against the consumption risk present there.

Scenario #3: Negative L and negative S

Both bonds are risky assets with respect to income at dates other than their own maturity. The expected yield on the longer-term bond at the shorter horizon must therefore in equilibrium be higher than the yield on the shorter-term bond; and similarly the expected yield on the short-term rolled-over must be higher than the yield on a long-term bond.

When each of these scenarios emerges can be explained in terms of two surprisingly simple aspects of the pattern of risk through time: 1) the serial correlation of consumption, and 2) the relative size of the coefficients of variation of marginal utility at the earlier date and of the conditional expectation (as of the earlier date) of marginal utility at the later date. The latter can be thought of as a measure of the relative informativeness of the earlier date's news regarding both the earlier date's and the later date's consumption.

If the serial correlation in consumption is zero or negative, both L and S must be negative. Then the least risky instrument for any date's consumption is simply a riskless bond maturing at that date. But with positive serial correlation, one or the other of the two maturities becomes the less risky instrument for obtaining income at both dates.

Suppose the news anticipated arriving at date-1 is such that a good outcome at the earlier date implies, on average, a good, but not as good, outcome for the later date (and a bad outcome a bad, but not as bad outcome). This means, in effect, that the news is more powerful regarding the earlier than the later date. Under these conditions, the contingent future short-term rate of interest will be low in the good branches of the tree of possible outcomes, and high in the poor branches. Then the liquidity premium is positive. Should the news regarding the later date be the more belief-revising (so that a good state at the earlier date implies, on average, an even better state at the next, etc.), the high interest rates come in the rich branches, the low in the poor, and the solidity premium is positive.

The positive liquidity premium observed by Keynes and Hicks is not a result of mere risk-aversion. But, from the above, it can be shown to be a consequence of risk-aversion plus:

- (1) the positive serial correlation of consumption which arises from inter-temporal productive transformations, and
- (2) the predilection of Nature to give us more information about the near future than about the far future.

Aside from $L > 0$ there are many other phenomena this theory would lead us to expect.

A short catalog includes the following:

- (1) Positive L requires the serial correlation in consumption to be positive.
- (2) Positive L occurs when low interest rates are associated with prosperity and high interest rates with bad times.
- (3) Positive L occurs when the rolled-over value of short-term bonds covaries against consumption at the date past the short-term bond's maturity.
- (4) Positive L occurs when the liquidated value of a long-term bond covaries with consumption at dates before its maturity, although this is not a sufficient condition for positive L .

Perhaps the stylized facts are more stylish than factual, even with regard to the sign of the liquidity premium. If so, we could continue:

- (5) Negative L and S occur when the value of any bond covaries against consumption at any date other than its maturity.
- (6) Negative L and S occur when the covariance between interest rates and consumption is zero or negative.

Throughout this paper, I have dealt only with real interest rates.

A fuller analysis would clearly call for an integration of the real with the nominal interest rates, for two main reasons: (1) it is the nominal interest rate that is directly observable, and (2) the forces leading to divergences between real and nominal interest rates are associated with risks imposed on individuals that our present theory can make no allowance for. I hope to report on the results of my own (Woodward [1980]) efforts in this direction at a later date.

FOOTNOTES AND REFERENCES

¹ Specifically, we are assuming date-0 markets in E distinct date-1 claims and in S distinct date-2 claims--E+S markets. While more extensive regimes could be defined, for example trading in all E·S claims contingent upon both a particular state at date-1 and a particular state at date-2, our assumption suffices for achieving preferred consumption vectors, that is, no Pareto-preferred improvements are made available by opening more markets. (After the realization of a particular state at date-1, re-trading of the S date-2 claims will in general take place.)

² In an economy of representative individuals who are risk-neutral (V is linear in all arguments), marginal utilities do not vary with income and hence prices and interest rates do not vary with income. If interest rates are not random, there is nothing to explain. For a more detailed discussion, see LeRoy (1978).

³ For a discussion of this point in the context of the theory of speculation see Salant [1976] and Hirshleifer [1976].

⁴ Equation (20) follows directly from maximization of $U = E(V)$ subject to the following budget constraint at date-0 (where the overbars indicate endowed quantities):

$${}^0P_0 c_0 + \sum_e ({}^0P_{1e} c_{1e}) + \sum_s ({}^0P_{2s} c_{2s}) = {}^0P_0 \bar{c}_0 + \sum_e ({}^0P_{1e} \bar{c}_{1e}) + \sum_s ({}^0P_{2s} \bar{c}_{2s})$$

⁵ If L and S were defined in terms of continuously compounded interest rates instead of discrete exchange ratios, then necessarily $S = -L$ and L and S could not both be negative. But, as the case of L and S both negative has an interesting economic meaning, something is lost in going to the continuous-interest form. Moreover, from an expository point of view the interpretation of L and S in terms of covariances of bond values with consumption is much easier with the exchange-ratio definition. For commodities distinguished only by date, either the continuum or the discrete-period versions of time passage may be useful. But the discrete-period formalization lends itself more naturally to generalizations involving distinct commodities -- for example comparing the forward wheat-corn price ratio to the expectation of the future wheat-corn price ratio (Woodward [1980]) -- since the trade of corn for wheat is necessarily discrete. For a discussion of the term structure using continuous compounding, see Cox, Ross, and Ingersoll [1979].

⁶ If $L \geq 0$, then $\frac{O^P_1}{O^P_2} \geq E\left(\frac{1^P_1}{1^P_2}\right)$ from equation (12). Taking reciprocals:

$$\frac{O^P_2}{O^P_1} \leq \frac{1}{E\left(\frac{1^P_1}{1^P_2}\right)}$$

Assuming the price ratio is not a degenerate random variable, we can use the strong form of Jensen's Inequality so that:

$$\frac{1}{E\left(\frac{1^P_1}{1^P_2}\right)} < E\left(\frac{1^P_2}{1^P_1}\right)$$

Thus: $\frac{O^P_2}{O^P_1} < E\left(\frac{1^P_2}{1^P_1}\right)$, or $S < 0$ from equation (11).

A similar development shows that $S \geq 0$ implies $L < 0$.

⁷ The development that follows is due to McCulloch [1973].

8 Let $x = v_1$ and $y = E(v_2)$

$$L = \frac{1}{E(y)} \text{Cov} \left(y, \frac{x}{y} \right)$$

$$\text{Cov} (y, x/y) = E(x) - E(y)E(x/y)$$

$$\text{Cov} (x, 1/y) = E\left(\frac{x}{y}\right) - E(x)E(1/y)$$

$$\begin{aligned} \text{Cov} (y, x/y) &= E(x) - E(y) [\text{Cov}(x, 1/y) + E(x)E(1/y)] \\ &= E(x) - E(y)E(x)E(1/y) - E(y) \text{Cov} (x, 1/y) \\ &= E(x) \text{Cov} (y, 1/y) - E(y) \text{Cov} (x, 1/y) \end{aligned}$$

$$\begin{aligned} \frac{\text{Cov}(y, x/y)}{E(x)E(y)} &= \frac{\text{Cov}(y, 1/y)}{E(y)} - \frac{\text{Cov}(x, 1/y)}{E(x)} \\ &= \frac{\sigma_y \sigma_{1/y} \rho_{y, 1/y}}{E(y)} - \frac{\sigma_x \sigma_{1/y} \rho_{x, 1/y}}{E(x)} \\ &= \sigma_{1/y} \left[\frac{\sigma_y}{E(y)} \rho_{y, 1/y} - \frac{\sigma_x}{E(x)} \rho_{x, 1/y} \right] \end{aligned}$$

$$L = E(x) \sigma_{1/y} \left[\frac{\sigma_y}{E(y)} \rho_{y, 1/y} - \frac{\sigma_x}{E(x)} \rho_{x, 1/y} \right]$$

$$S = E(y) \sigma_{1/x} \left[\frac{\sigma_x}{E(x)} \rho_{x, 1/x} - \frac{\sigma_y}{E(y)} \rho_{y, 1/x} \right]$$

9 The liquidity premium is also positive with any linear production function. This can be shown by partitioning the covariance into storage (production) and no-storage (no-production) states, and showing that each partition is positive. The liquidity premium is also positive with a linear production function and independent or positively correlated endowments in infinite-time, two-state Markov models of the sort used by R.E. Lucas [1978] and by Stephen F. LeRoy and C.J. LaCivita [1980]. It is easy to show that if there is merely persistence in the Markov transition probabilities, $L > 0$, and that with a linear production function added, plus either independence or persistence, L is still greater than zero. However, if the transition probabilities already display persistence, the introduction of storage may not increase the size of L .

10 I am indebted to John Riley for a classroom example which provided the basis for this idea.

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