THE ECONOMICS OF BUYING PROBABILITIES:

PARALLEL AND INTENSIVE RESEARCH

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Ben T. Yu

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University of California, Los Angeles

Department of Economics

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This paper deals with alternative methods of buying probabilities in different market settings. It has general applications in many problems where the objective is to find something of positive value and the probability of finding it can be altered by alternative allocation of resources. For expositional simplicity, I will only concentrate on a specific problem, parallel and intensive research.

Parallel research refers to the simultaneous employment of several teams to solve an <u>identical</u> problem. Its effectiveness hinges on the statistical property that increasing the sample of observation drawn from a random distribution would increase the likelihood of reaching the tail end of the distribution. That is, parallel research is equivalent to more random sampling which would increase the chance of success.[1] (Nelson, 1961, Arditti and Levy 1980) The alternative to parallel research is intensive research, i.e., instead of spending x dollars on each of n projects, one can spend nx dollars on one project. Obviously, this will increase the chance of success of an individual project.[2]

The choice between parallel and intensive research has been inadequately considered in existing literatures. Although Nelson has suggested this possibility in his original formulation of the problem, he sees this as a trivial extension of his basic argument. Without formalizing, he argues that "a low rate of expenditure on several projects with a consequent improved choice is a far more effective way to buy time than a faster rate of expenditure on a single project right from the start." Recently, Arditti and Levy modelled parallel research as buying probability of success (rather than time). With zero rate of interest, their approach should not differ in substantive content from Nelson's original model. However, they have not considered alternative ways to buy the success probability, and their framework cannot be utilized to evaluate Nelson's proposition. Section I of this paper integrates the alternative of intensive research into Arditti and Levy's model and evaluates Nelson's proposition in terms of changes in the exogenous parameters affecting the success probability function. Several implications beyond those argued in Nelson's model can be shown from this more generalized model. The analyses also reveal the general principles in buying probabilities.

The second part of the paper examines how the choice of research methods (or buying probabilities) will be affected by perfect competition. This issue has also been ignored or confused in existing models of parallel research. Arditti and Levy's model implicitly assumes a single firm (or a single user of the innovation) organizing parallel research without the threat of competition. The set up of the models thus not only precludes the choice between parallel and intensive research, it precludes the important distinction between organized (co-operative) and unorganized (non-cooperative) parallel research.[3] The distinction is crucial: organized parallel research maximizes rent, but unorganized parallel research dissipates rent. The losers in organized research would still be compensated, but the losers in unorganized research receive nothing. As we will show in Section II, unorganized parallel research (i.e., free entry in buying probabilities) may be the unavoidable consequence of perfect competition. The number of teams doing parallel research in this environment is <u>more</u> than the number organized by a single firm if there is no threat of competition. Furthermore, perfect competition will reduce both organized parallel research and intensive research. In the limiting case, we show that organized parallel research would not survive at all. In other words, what may be an efficient method of inventing (buying probability) in a single firm environment may be seen as inefficient by the firm under some conditions of perfect competition.

I. Similar to the model of Arditti and Levy, I assume the expected present value of the gain to an innovation be R and the cost of development be A dollars per team (or per inventor). The probability of failure by a single team is $q(A)$, with $q' < 0$ and $q'' > 0$. If a single firm employs n teams with A dollars allocated to each team, the expected return to the innovation program is

$$\pi\,(n,A) = (1 - q^n(A))\,R - nA \tag{1}$$

To maximize the return, the firm chooses n and A to maximize $\pi(n,A)$. Assuming n can only take on integer value, the first order condition with respect to n is,

$$q^{n-1}(A)\;(1 - q(A))R = A \tag{2}$$

This condition is similar to the one in Arditti and Levy. It's interpretation is quite intuitive: the expected gain of employing an additional team is the expected gain of the marginal team, $(1 - q(A))R$, given that all the intramarginal teams fail, $q^{n-1}(A)$. Maximization requires the firm to employ additional parallel research teams until the expected marginal gain is equal to the marginal cost, A. The return maximizing number of teams, n*, will be called the extent of <u>organized</u> parallel research. <u>Unorganized</u> research in this model is zero by assumption.

The research intensity, A, can be similarly derived from the first order condition of $\pi(n,A)$ with respect to A, i.e.,

$$-q^{n-1}(A)\;q'R = 1 \tag{3}$$

The interpretation of (3) is also intuitive: the expected gain of increasing research intensity by a dollar to one team is the increase in the expected gain of that team, $-q'R$, given that other teams all fail, $q^{n-1}(A)$. Maximization requires this amount to be equal to one dollar.

Dividing equation (2) by (3), the return maximizing A can be shown to be independent of n and R, i.e.,

$$-\frac{(1 - q(A))}{q'} = A \tag{4}$$

The maximizing procedure is to solve for the research intensity per team from (4). The solution, A^*, will then be inserted in equation (2) to determine the extent of organized parallel research. A decrease in A^* is equivalent to a lowering of the marginal cost of parallel research, and ceteris parabus, the extent of organized parallel research should increase.

Graphing the solution of (4) in Figure 1 provides additional information on the nature of the problem. Treating the success probability, $1-q(A)$, and $-q'A$ as two separate functions of A, the return maximizing A^* can be derived from the intersection of these two functions. The success (failure) probability function monotonically approaches 1 (0) as A increases. $q'A$ is the product of the tangent of the failure function and its x-intercept. It has a graphical interpretation as given in figure one. If $q(A)$ is strictly declining, i.e., $q' > 0$ for all A, $q'A$ must first increase and then decline, and its value must always be less than one. Furthermore, one can use simple differentiation to show that the slope of $1-q(A)$ must always be greater than that of $-q'A$ by a factor of $q''A$. These considerations suggest that the intersection of the success probability function and $-q'A$ must occur at the corner, i.e., A^* equals zero.

The above solution, however, has no real meaning because with A^* equals to zero, the gain of research equals zero with perfect certainty. Furthermore, this situation is not likely to be of great importance. Consider a $q(A)$

function that has a fixed cost, x, (x can be small but finite), which must be spent before any results can be obtained. I.e.,

$$\tilde{q}(A) = \begin{cases} q(A-x) & \text{if } A \geq X \\ 1 & \text{if } A < X \end{cases} \tag{5}$$

Figure 1

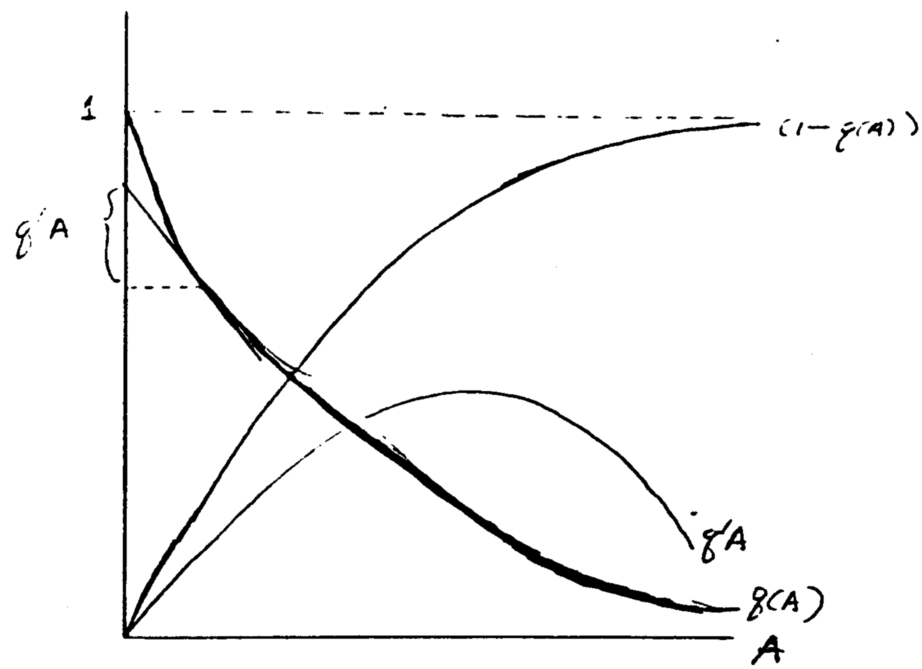
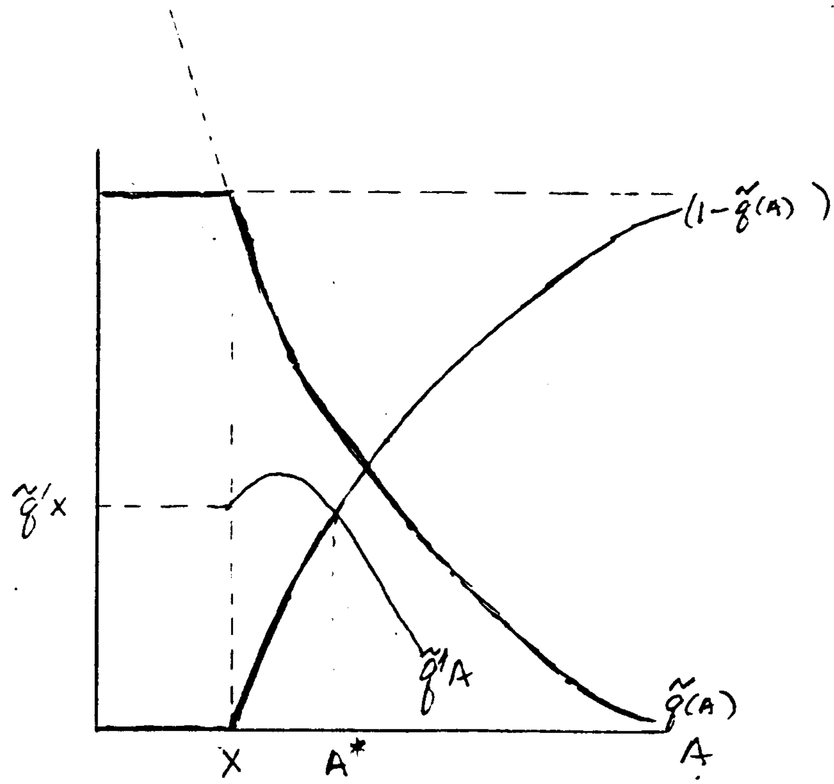


Figure 2



An example of (5) is

$$\tilde{q}(A) = \begin{cases} \dfrac{K}{h(A-x)+K} & \text{for } A \geq x;\ h, K \text{ are positive constants} \\ 1 & \text{for } A < x \end{cases} \qquad (6)$$

Since $-\tilde{q}'A$ is positive at $A = x$, there is usually an interior solution (as shown in Figure Two). For example, using the explicit function in (6), we can solve for A^* in (4),

$$A^* = x + \sqrt{\frac{Kx}{h}} \qquad (7)$$

That is, the return maximizing level of intensive research is a function of the fixed cost plus a term proportional to the size of the fixed cost.

Equation (4) and Figure Two provide a convenient framework to evaluate Nelson's proposition. If one interprets the early development of an innovation as research with high failure probability, i.e., an unward shift of $\tilde{q}(A)$, both the $(1-\tilde{q}(A))$ and $-\tilde{q}(A)\cdot A$ curves will in general shift and the change in the marginal cost of parallel research, A^*, is ambiguous. In addition, as pointed out by Arditti and Levy, an increase in failure probability may increase or decrease the marginal gain of parallel research.[4] Thus, the effect of higher failure probability on organized research is also ambiguous. In fact, if the explicit functional form of (6) is used, the extent of organized research can be shown to decrease as failure probability increases.[5]

The ambiguous effects on intensive research, A^*, can be further analyzed. The model here views A^* and parallel research n^* as alternative methods of buying success probability. The cost <u>to each team</u> (or inventor) of buying probability is the inverse of q', which decreases if a higher failure probability also simultaneously flattens the slope of $\tilde{q}(A)$. On the other hand, if the cost of buying probability remains the same, a higher failure probability may lower the conditional success probability of hiring <u>an extra team</u> (or inventor), inducing a substitution away from parallel research to intensive research.

The two opposing effects can sometimes be separated. Consider a case where nature only allows increases in success probability within certain limits. That is, even if no research effort is expended, the success probability may still be positive; and even if an infinite research effort is expended, the success probability may still be less than one. We can describe the failure probability of such a function as

$$\tilde{\tilde{q}}(A) = d + q(A-x) \quad \text{if } A \geq x,\ 0 < d < 1,\ \lim_{A \to \infty} q(A-x) = u$$
$$= d \qquad\qquad \text{if } A < x,\ 0 < d < 1 \tag{8}$$

This function and the success probability function have the shapes as shown in Figure Three. The intersection of the derived $-\tilde{\tilde{q}}'A$ and $(1-\tilde{\tilde{q}})$ occurs at A^*. Now consider a <u>vertical</u> shift of the failure probability by a positive Δd, i.e., the future becomes uniformly more uncertain, but the cost of buying probability, $1/\tilde{\tilde{q}}'$, remains the same. This will shift the success probability curve uniformly downward without changing the shape of the $-\tilde{\tilde{q}}'A$ curve. As shown in Figure Four, the new return maximizing A^* must be higher.

The economics of parallel research and the two effects of intensive research described above have many interesting real world implications. Consider a research institute (or a University) looking for an inventor (or a faculty member) with R expected value of research potential. The institute can improve the probability of getting the right person in two ways: (1) on-the-job training, i.e., spending resources on improving the success probability of an individual inventor. This can be accomplished by either granting a longer employment contract or more heavily subsidizing the inventor

Figure 3

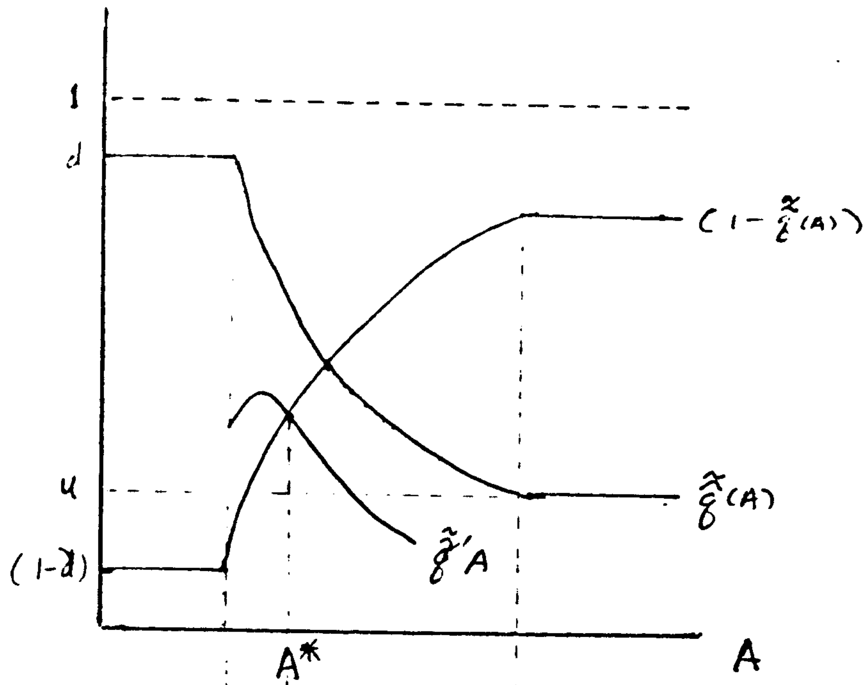
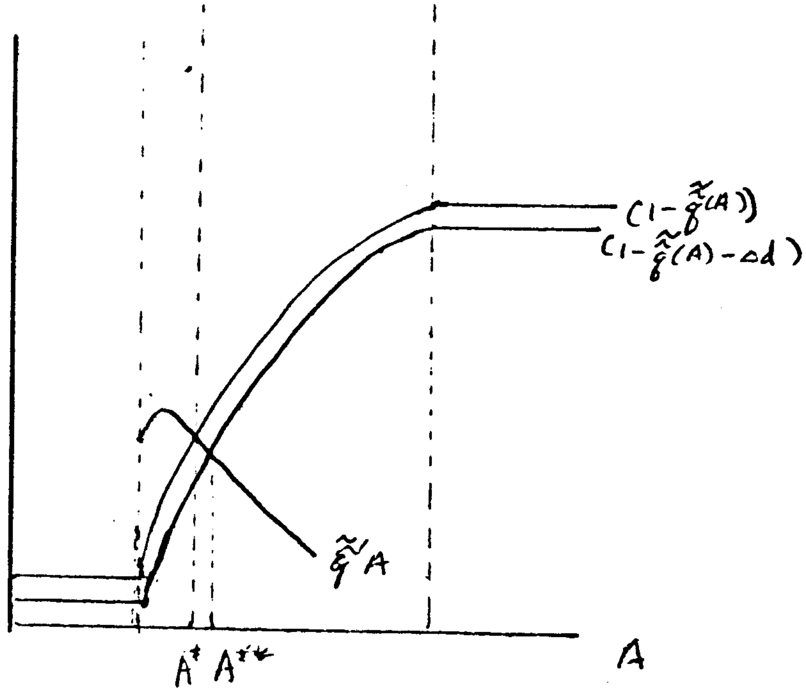


Figure 4.



with research facilities. (2) Pyramidical elimination, i.e., employing an inventory of inventors for a short time period with smaller resources, picking the best inventor at the end of a specified period.

The choice between on-the-job training and pyramidical elimination in finding the right inventor is exactly analogous to the choice between parallel and intensive research in finding a useful innovation. Both are problems of buying probabilities. Setting aside the influence of competition which we will analyze in the next section, the analyses in this section yield the following implications:

(a) The higher the standard of research potential required by an institute is (i.e., a higher R), the larger will be the number of parallel researchers employed by the institute. However, resources for on-the-job training should be indifferent to changes in standards, R (see equation [4]). Casual observations of a few respectable universities in the nation support this contention. The tenure to non-tenure ratio is higher than the average among such universities, but their employment contracts appear similar to those in other universities.

(b) The more difficult it is to train an inventor, the shorter the employment contract and the less amount of subsidy will be given to the inventor. In terms of our model, a greater difficulty in training can be represented by a higher cost of buying probability along the intensive margin, i.e., a decrease in q'. A shift of the success (or failure) probability function with this property implies that more resources will be required in order to buy a given probability level, and less probability should be purchased. In general, a change in the slope of the success (fialure) probability function would also simultaneously change the level of such probability function, and implication (c) below should be jointly considered with the effects described here.

(c) The more difficult it is to find the right inventor, or stated differently, the smaller the expected percentage of inventors having research characteristics as required by the Institute, the longer the employment contract and the larger a subsidy an individual inventor would receive. This is the implication of the example given in equation (8). A greater difficulty of finding the right person means that for any given resources spent on an inventor, the probability of finding the right person is less. This is exactly equivalent to an upward vertical shift of the failure function. Holding the training cost, q', constant, the marginal gain of hiring parallel researchers should go down relative to that of intensive research and more of the latter will be conducted.

II. The analyses in the previous section must be modified under perfect competition. In an environment where there are N inventors, a firm may have contracted with n inventors (or teams) to do parallel research, but here remains N-n unorganized parallel inventors (or teams). If N is large, the chance of one of them succeeding seems almost certain, and the incentive to innovate should diminish. On the other hand, a competitive firm has the incentive to erode profitable gain of research by either rushing or spending more to innovate. The seemingly conflicting incentives can be analyzed by drawing a distinction between organized and unorganized parallel research. It will be demonstrated below that the disincentive to innovate should reduce the extent of organized parallel research; and if inventors are identical, such method of research would be totally displaced by unorganized parallel research. The incentive to rush to innovate cannot be demonstrated in the existing timeless model. However, the incentive to erode rent can still be demonstrated in terms of an increase in unorganized parallel research.

Following the standard Nash assumption, a firm maximizes the return to his innovation, treating resource allocation of his competitors as given. The firm's probability of winning thus not only depends on his own effort, but also depends on the effort and the success probability of his competitors. Furthermore, unlike the case of a single firm setting where simultaneous successes by more than one inventor does not affect the return function of the firm, a competitive firm cannot get the full share of the innovative return if both him and his competitors simultaneously succeed. The assumption in this paper is that if i inventors simultaneously succeed, the probability of one winning is 1/i. The considerations suggest the following specification for the return function of firm j,

$$\tilde{\pi}_j(n,A) = \sum_{i=o}^{N-n} \left[q^{N-(n+i)} \, (1-q)^i (1-q_j^n(A)) \, \frac{R}{i+1} \right] - nA \qquad (9)$$

q is distinguished from $q_j(A)$ in that the former is the failure probability of the competitors as seen by firm j, the latter is the failure probability of firm j. Firm j can only influence $q_j(A)$ but not q.

The marginal contribution of parallel researcher in firm j is the increase in the expected gain of innovation given that all outsiders fail. It is immaterial to address the question of whether the competitors also form their own organized research or not. From firm j's viewpoint, the probability that all outsiders fail and one of firm j's parallel research team succeeds is the same regardless of whether the outsiders are organized or not. Also, when the firm decides the extent of parallel research, intensive research can be assumed to be constant and identical for all inventors, i.e., $q = q_j(A)$ for all j. Thus, assuming n takes on integer values only, the first order condition of (9) with respect to n can be written as[6]

$$\sum_{i=0}^{N-(n+1)} \left[q^{N-(n+1+i)} (1-q)^{i+1} \frac{R}{i+1} \right] - \frac{(1-q)^{N-n}(1-q^{n})R}{N-n+1} = A \quad , \quad (10)$$

If N is relatively large compared to n, the second term on the left hand side becomes zero, and (10) becomes

$$\sum_{i=0}^{N-(n+1)} q^{N-(n+1+i)} (1-q)^{i+1} \frac{R}{i+1} = A \quad (11)$$

To determine the extent of intensive research, one should also maximize (9) with respect to A. Since the first order condition must be determined jointly with (11) the appropriate return function to maximize is $\tilde{\pi}_j(n+1,A)$. This gives[7]

$$\sum_{i=0}^{N-(n+1)} -q_j^n(A) q_j'(A) q^{N-(n+1+i)} (1-q)^i \frac{R}{i+1} = 1 \quad (12)$$

Dividing (11) by (12), we have a condition similar to (4),

i.e.,
$$- \frac{(1-q_j(A))}{q_j^n(A) \; q_j'(A)} = A \quad (13)$$

Solving n* from (13), we have

$$n^* = \frac{\ln \left[\frac{1-q_j(A)}{-q_j' \cdot A} \right]}{\ln q_j(A)} \quad (14)$$

$\ln q_j(A) < 0$ by assumption. If organized parallel research is continued under this situation, $\ln \left[\frac{1-q_j(A)}{-q_j' \cdot A} \right]$ must also be less than zero. This implies $1 - q_j(A) < -q_j' A$. Since we know from previous section that $-q_j' A$ must intersect $(1-q_j(A))$ from above, the equilibrium A^* implied by (13) must be less than that under a single firm setting if n* is positive. This counter intuitive result arises because of our simplified timeless model. With time included, one ought to obtain the standard result of rushing to innovate (see Barzel, 1968)

Furthermore, substituting (13) into (11), we have

$$\sum_{i=0}^{N-(n+1)} -q^{N-(1+i)} q_j'(A)(1-q)^i \frac{R}{i+1} = 1 \qquad (15)$$

(15) resembles the first order condition of the maximization problem of an independent inventor competing for the industry return of innovation. For this problem, the objective function is $\tilde{\pi}(1,A)$, and the inventor only adjusts one variable, A, treating the q of other inventors as given. The first order condition is

$$\sum_{i=0}^{N-1} -q^{N-(1+i)} q'(A)(1-q)^i \frac{R}{i+1} = 1 \qquad (16)$$

Comparision between (15) and (16) suggests that as long as $n \geq 1$, the left hand side of (15) will always be less than the left hand side of (16). This can be interpreted as saying that the gain per dollar spent on organized research will always be less than the gain per dollar spent on unorganized parallel research. To maximize the gain per dollar, the firm would choose the lowest n, i.e., n=1 implying no parallel research. Stated differently, an inventor will find his marginal gain of research to be higher when he works independently than when he organizes with others. Thus, organized parallel research cannot be a stable equilibrium under competition.

Parallel research seems prevalent in the real world, however. At least two possibilities would account for this. First, the preconditions of the model, namely, inventors have identical costs may not be true. If inventors have differing abilities, the q' in equation (16) and (15) will not be identical, and there may still be organized parallel research among the superior inventors. Second, competition here is assumed to be "within the market." The alternative is potential competition, i.e., competition for the market. If transaction cost permits prior contracting so that potential competition becomes realistic (see Yu 1981), organized parallel research may be used even under competition.

Finally, we wish to show that unorganized parallel research under competition will be more than organized parallel research in a single firm setting. The optimal team size, n^*, in the latter situation has already been described by (2). To draw a comparision, the equilibrium number of unorganized parallel research under competition, N^*, can be derived from the zero profit condition due to entry i.e., $\tilde{\pi}(1,A) = 0$. Writing out the summation, we have

$$q^{N-1}(1-q(A))R + q^{N-2}(1-q)(1-q(A)) \frac{R}{2} \ldots\ldots (1-q)^{N-1}(1-q(A)) \frac{R}{N} = A \quad (17)$$

Inventors are identical, i.e., $q=q(A)$, at equilibrium. Thus, the first term in (17) is the same as the left hand side of (2). This term can be interpreted as the social gain of employing one additional research inventor (or team). The rest of the terms in (17) i.e., $\sum_{i=1}^{N-1} q^{N-(1+i)}(1-q)^{i}(1-q(A)) \frac{R}{i+1}$ only represent a redistribution of gain from one successful inventor to another. But this was seen by each competing inventor as his own private gain. The marginal gain of having one extra inventor is thus higher under perfect competition as described by (17) than that under single firm setting as described by (2). Since the marginal costs, A, are identical in the two situations, $N^* > n^*$.

Concluding Remarks

This paper views parallel and intensive research as alternative ways to improve the chance of getting useful innovations. In a single firm setting, it has been shown that intensive research (R & D allocated to each inventor) will not be directly affected by either the expected value of the innovation or parallel research (the number of inventors). Parallel research, however, should increase as expected value of the innovation increases. Changes in the failure probability affects intensive research (and thus the marginal cost of parallel research) in two seperable ways: it directly affects the cost of buying probability along the intensive margin, and it changes the gain of parallel research thus affecting intensive research indirectly.

Since both the marginal gain and cost of parallel research are affected, comparative static result of failure probability on parallel research is ambiguous. Nevertheless, our analyses can provide several implications regarding the structure of organization and inventors' employment contracts among research institutes.

The second part of the paper examines parallel and intensive research under perfect competition. The distinction between organized and unorganized parallel research is drawn. It is shown that although unorganized parallel research will be more under perfect competition, both organized parallel and intensive research will be less in this environment. Furthermore, under some specific conditions, organized parallel research may be completely displaced by perfect competition. The preconditions leading to this result, however, may be infrequent in many real world situations.

The analyses in this paper can be applied potentially in a variety of situations beside research activities. Whenever there is uncertainty in finding something of positive expected value and if the uncertainty can be reduced by spending more resources in searching, our analyses should apply. For example, consider natural resource exploration (drilling a deeper oil well as opposed to drilling more oil wells), or advertising (advertising intensively on one brand as opposed to introducing numerous brands), or investment in human capital (specializing in one profession as opposed to general training). Each can be modelled as a problem of buying probabilities. While there may always be some phenomena specific to a problem uncaptured in the model displayed here, we believe the model can provide a convenient base by which additional specific features can be built on.

FOOTNOTES

[*] I thank Christoper Hall, Benjamin Eden, and John Riley for helpful discussions.

[1] Success can be defined in terms of a low cost of a production method, a high cost of inventing, or the winning of a patent.

[2] Conceptually, parallel research precludes inter-team exchange of information until some preliminary results are obtained. Intensive research, on the other hand, assumes continuous intra-team exchange of information in the course of research. The cost of exchanging information and diseconomies in scale in organizing information may be some reasons why intensive research is not always preferred.

[3] Nelson (p. 362 and 363) have ignored this distinction. He treated organized parallel research as identical to market unorganized research under competition.

[4] Recall the expected marginal gain of parallel research depends on the probability that all intramarginal inventors fail and the marginal inventor succeed. A change in failure probability increases the expected gain of the first component but decreases the expected gain of the second component. See Arditti and Levy, p. 1090.

[5] Substituting (7) into (6) in the text, we have

$$q(A^*) = \frac{1}{\sqrt{\frac{hx}{K}} + 1}$$

Substituting q(A*) and A* into (2), we can solve for the optimal team size, n*.

$$n^* = \frac{\ln A^* - \ln\left[(1-q(A^*))R\right]}{\ln[q(A^*)]} + 1$$

$$= \frac{\ln\left[\frac{\left(x+\sqrt{\frac{Kx}{h}}\right)^2}{xR}\right]}{\ln q(A^*)} + 1$$

Taking the derivative of n* with respect to h, we have,

$$\frac{dn^*}{dn} = -[\ln q(A^*)]^{-1}\left[\frac{\sqrt{\frac{Kx}{h}}}{\left(x+\sqrt{\frac{Kx}{h}}\right)h}\right] - \ln\left[\frac{\left(x+\sqrt{\frac{Kx}{h}}\right)^2}{xR}\right][\ln q(A^*)]^{-2}\,\frac{1}{q(A^*)}\,\frac{dq(A^*)}{dh}$$

$\ln q(A^*) < 0$: and a meaningful problem requiring $n^* > 1$ implies

$\ln\left[\frac{\left(x+\sqrt{\frac{Kx}{h}}\right)^2}{xR}\right] < 0$; $\frac{dq(A^*)}{d\,h} > 0$ from (5). Thus $\frac{dn^*}{dh} > 0$.

6 $\tilde{\pi}_j(n + 1, A)$

$$= q^{N-(n+1)}(1-q_j^{n+1})R + q^{N-(n+2)}(1-q)(1-q_j^{n+1})\frac{R}{2} \cdots\cdots$$

$$\cdots\cdots + (1-q)^{N-(n+1)}(1-q_j^{n+1})\frac{R}{N-n} - (n+1)A$$

$\tilde{\pi}_j(n,A)$

$$= q^{N-n}(1-q_j^n)R + q^{N-(n+1)}(1-q)(1-q_j^n)\frac{R}{2} \cdots\cdots$$

$$\cdots\cdots + (1-q)^{N-n}(1-q_j^n)\frac{R}{N-n+1} - nA$$

$\therefore \tilde{\pi}_j(n+1,A) - \tilde{\pi}_j(n,A)$

$$= q^{N-(n+1)}\left[1-q_j^{n+1} - q + q_j^{n+1}\right] R + q^{N-(n+2)}(1-q)\left[1-q_j^{n+1} - q + q_j^{n+1}\right] \frac{R}{2}$$

$$\cdots\cdots (1-q)^{N-n}(1-q_j^n)\frac{R}{N-n+1} - A$$

Setting this expression equals to zero yield (10).

7 $q_j(A) = q$ only at A^*. Taking the derivative of $\tilde{\pi}(n+1,A)$ with respect to A, q is treated as constant.

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