

A Decomposition of the Harberger
Expression for Tax Incidence

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The now classic article by Harberger (1962) on the incidence of the corporate income tax introduced the two sector general equilibrium model to the field of public finance. Since then his analysis has been extended to various other branches of public finance and to labor economics.¹

In the Harberger model an imposition of a corporate income tax creates two distortions in Paretian conditions: the one between the marginal rates of technical substitution of the two sectors, and the other between the marginal rate of transformation and the marginal rate of substitution. The first may be called the factor price distortion and the second the commodity price distortion. If we imagine that the tax creates these two distortions in succession, we can decompose the tax impact upon the rate of return into the "factor price distortion (or ω)" effect and the "commodity price distortion (or π)" effect corresponding to the respective distortions. Mieszkowski (1967, pp. 253-254) was the first to suggest such a geometric interpretation for the Harberger expression.

Among the two effects, the π effect is a more indirect and a more subtle concept. Perhaps for this reason all of Harberger's predecessors had neglected the π effect in their estimation of the corporate tax incidence. Introduction of the demand side into the two sector general equilibrium model for the first time enabled Harberger to take into account the π effect in his estimation of the tax incidence. The π effect is a theoretically interesting concept, because its sign is explained by the celebrated theory of Stolper and Samuelson. But its empirical importance in the total tax incidence is unknown.

The aim of the present paper is two-fold. First, we precisely define the above geometric decomposition and derive its formal expression. Mieszkowski argued that the first and the second terms of the numerator in the Harberger expression correspond to the above geometric decomposition.² It will be shown that this correspondence does not hold generally. Second, by making use of our formal expression we will numerically measure the two terms of the decomposition by applying Harberger's estimates of the parameters for the U.S. economy. We will demonstrate that the ω effect roughly approximates the total impact of the tax on the rate of return. This suggests that the π effect is rather unimportant in explaining the total impact of a tax change upon the rate of return.

The first section summarizes the Harberger model and introduces notation used in this paper. The decomposition is geometrically defined in Section 2, and an explicit expression of the decomposition in terms of the utility and production functions and their derivatives is presented in Section 3. Numerical estimation of the two effects is given in Section 4. Harberger's original expression and Mieszkowski's two effects are discussed in Section 5. The final section summarizes our results.

1. THE MODEL

The Harberger model divides the economy up into two sectors, one corporate (producing good x) and the other non-corporate (producing good y), with each sector employing capital and labor. It is a generalization of the standard Heckscher-Ohlin International Trade Model expanded by explicitly introducing a demand sector.

Let K_x , L_x , and X denote the amounts of capital, labor, and the output level, respectively, of the x industry. Let r_x be the price of capital this industry faces. Define K_y , L_y , Y , and r_y , similarly for the industry y . Harberger assumes that the two industries face the same undistorted price of labor, and chooses the labor as the numeraire so that its price is equal to one. Thus the compensated labor demand function of x industry is written as

$$L_x = L_x(r_x, X) \quad (1)$$

In similar fashion, we can define the functions $K_x(r_x, X)$, $L_y(r_y, Y)$, and $K_y(r_y, Y)$. The tax we are considering is a tax on the capital used in the corporate sector, and hence we have

$$r_x = r + t \quad (2)$$

$$r_y = r, \quad (3)$$

where r is the market price of capital that capital owners face and t is the corporate tax rate.

Assuming that all the factors are fully employed, we have

$$K_x + K_y = \bar{K} \quad \text{and} \quad L_x + L_y = \bar{L}, \quad \text{where } \bar{K} \text{ and } \bar{L} \text{ denote the fixed total amount}$$

of capital and labor existing in the economy. Substituting (1) and similar equations for K_x , L_y , and K_y , and noting (2) and (3), we have

$$K_x(r + t, X) + K_y(r, Y) = \bar{K} \quad (4)$$

$$L_x(r + t, X) + L_y(r, Y) = \bar{L} \quad (5)$$

On the other hand, in the long run equilibrium the profit of each industry must be zero. Thus we have

$$(r + t) \cdot K_x(r + t, X) + L_x(r + t, X) = p_x \cdot X \quad (6)$$

$$r \cdot K_y(r, Y) + L_y(r, Y) = p_y \cdot Y \quad (7)$$

where p_x and p_y are the prices of x and y , respectively.

In the Harberger model, the government will spend the tax proceeds in the identical manner that the private sector would have spent the taxes, and the pattern of demand remains unchanged by the redistribution of income among consumers. Thus he specifies the demand for x as a function of relative commodity prices:

$$X = D(p_x/p_y). \quad (8)$$

This specification implicitly assumes that the income effect for the commodity x is zero.

The set of five equations (4)-(8) describes a complete model for the five variables P_x , P_y , X , Y , and r . The solutions for each of these can be expressed as a function of t . In particular, we write the solution functions for Y and r as

$$Y = Y^*(t) \quad \text{and} \quad (9)$$

$$r = r^*(t). \quad (10)$$

Our main concern is in characterizing the derivative of the last equation.

2. THE DIAGRAMMATIC DEFINITION OF THE DECOMPOSITION

Figure 1 is the box-diagram for this two-sector economy. Assume that the point N on the contract curve represents the initial equilibrium with no tax, and that the point T represents the new equilibrium after the corporate tax is imposed on x. When the corporate tax is imposed, the relative price of the capital that the x industry faces is higher than that the y industry faces. Thus, at T the isoquant curves are steeper for x than for y. On the other hand, the output level of y at T is drawn at the level higher than at N reflecting the consumer's response to the reduced relative price of the commodity y.

Our concern is in the change in r caused by the shift of equilibrium from N to T. The change in r can be measured by comparing the slopes of the y-isoquant at N and T. To compare them, it is convenient to decompose the movement NT into two parts. Let A be the intersection of the efficiency locus and the y-isoquant at the output level of the new equilibrium. Then the movement NT may be conceptually decomposed into the two components NA and AT.

We call the change in r associated with the movement from N to A the commodity price distortion (or simply π) effect, and that accompanied by the movement from A to T the factor price distortion

(or simply ω) effect. The π effect is equal to the impact upon r of an excise tax on x that would increase the consumption of y by the same amount as the corporate tax t would. Thus, the π effect may be considered as caused by the commodity price distortion created by the corporate tax. On the other hand, the ω effect measures the effect upon r of the factor price distortion created by the tax when the consumption of y is hypothetically kept constant.

The sign of the ω effect is unequivocally negative as the y isoquant is convex to its origin. The sign of the π effect, on the other hand, can be determined by the following proposition:

The Stolper-Samuelson Lemma³

In the standard two-sector model, an increase in the output of an industry along the contract curve increases (reduces) the relative price of the factor which the industry uses more (less) intensively.

Thus, the π effect will be negative or positive according as the x sector is more or less capital intensive. Putting these observations together, we can conclude that the total effect is unequivocally negative when the x industry is the more capital intensive sector, as in the case of Figure 1, while it cannot be signed a priori when the x industry is the less capital intensive sector.

The geometric decomposition above enabled us to exploit the familiar Stolper-Samuelson Lemma in analyzing the tax impact upon r . Our task now is to obtain formal expressions of this decomposition so that we can determine their numerical magnitudes.

3. THE FORMAL DEFINITION OF THE DECOMPOSITION AND AN EXPLICIT EXPRESSION

Equations (4) and (5) can be combined to eliminate X yielding:⁴

$$G(Y,r,t) = 0 \quad (11)$$

From (11), r can be solved for as a function of Y and t:⁵

$$r = R(Y,t). \quad (12)$$

Equation (11) represents the relationship that variables Y, r, and t have to satisfy if the economy is in full employment, while the y industry is minimizing cost under the prevailing factor prices, and the x industry is doing so under the tax distorted factor prices. Equation (12), therefore, gives the slope of the y-isoquant in the box diagram when t and Y are known. When t = 0, this equation identifies the relationship between Y and r on the contract curve.

Let us examine the level of r when the corporate tax is t and the output of the y industry is the one corresponding to A in Figure 1, which is $Y^*(t)$ as defined in (9). Move along the y-isoquant curve passing through A until the angle between the isoquants of the two industries corresponds to t. Then the slope of the isoquant of the y industry at that point is given by $R(Y^*(t),t)$. This must be identical to the r given by (10); hence we have the identity

$$r^*(t) = R(Y^*(t),t) \quad \text{for all } t.$$

Differentiating this with respect to t we have:

$$\frac{dr^*}{dt} = \frac{\partial R}{\partial Y} \cdot \frac{dY^*}{dt} + \frac{\partial R}{\partial t} \quad (13)$$

[the total effect] [the π effect] [the ω effect]

The derivative $\frac{dY^*}{dt}$ in (13) gives the output change due to the shift from N to T, and hence the accompanying change in r is represented by $\frac{\partial R}{\partial Y} \cdot \frac{dY^*}{dt}$. On the other hand, the movement along the y -isoquant from A to T corresponds to $\partial R/\partial t$.⁶ The first and the second terms of RHS of (13), therefore, give the π and ω effects, respectively, that are discussed in the previous section. We will use these two terms as formal definitions for the respective effects.

We are now in a position to derive the explicit expressions of the π and ω effects in terms of the commodity and factor demand functions, rather than in terms of the functions R and Y^* . Following Harberger, we assume that

$$p_x = p_y = r = 1 \quad (14)$$

and

$$t = 0 \quad (15)$$

hold at the initial equilibrium.⁷ (Note that only the price of labor is assumed to be identically equal to 1.)

Given (14) and (15), the explicit expressions of the π and ω effects in terms of the commodity and factor demand functions is obtained by total differentiation of the system (4) - (8) which yields:⁸

$$\frac{dr^*}{dt} = \frac{-(k_x - k_y)(K'_y k_x + K'_x k_y)}{(K'_x + K'_y)\{(k_x - k_y)^2 + (K'_x + K'_y)/D'\}} + \frac{-K'_x}{K'_x + K'_y} \quad (16)$$

[the total effect]

[the π effect]

[the ω effect]

where

$$K'_x = \frac{\partial K_x}{\partial r_x}, \quad K'_y = \frac{\partial K_y}{\partial r_y}$$

and k_x and l_x are unit input demand functions that are defined by

$$k_x(r_x+t) \equiv K_x(r_x+t, X)/X \quad , \quad l_x(r_x) \equiv L_x(r_x, X)/X \quad (17)$$

Note that K'_x and K'_y are Hicksian substitution terms and are always negative. Note also that the functions k_x and l_x do not contain X and Y because constant returns are assumed.

4. INTERPRETATION AND ESTIMATION OF THE TWO EFFECTS

Since K'_x , K'_y , and D' are negative, equation (16) shows that the sign of the π effect is determined by that of $k_x - k_y$. Since we have

$$k_x - k_y = \left(\frac{L_y}{K_y} - \frac{L_x}{K_x} \right) k_x k_y \quad ,$$

this implies that the sign of the π effect is determined by the relative factor intensity of the two industries. This is in agreement with the Stolper-Samuelson Lemma. We also observe that, other things being equal, the higher the absolute value of D' is, the greater is the absolute value of the π effect. This is natural because a higher absolute value of D' tends to lengthen the distance between N and A in Figure 1.

Equation (16) shows that the ω effect is an increasing function of K'_y/K'_x , a measure of the relative curvature of the isoquants. We can geometrically demonstrate that the magnitude of the ω effect depends on the relative curvature of the isoquants. The ω effect can be thought of as the change in r necessary to achieve the proper differential between the slopes of the x and y isoquants (the slope of the x isoquants must equal the slope of y isoquant plus t in equilibrium). Figure 2,

which reproduces a part of Figure 1, shows why increasing the absolute value of K'_x keeping K'_y fixed reduces the ω effect. The point T^0 is a post-tax equilibrium. But if the absolute value of K'_x goes up, a typical isoquant will look like JJ, which is less bent than II. This will result in a new equilibrium to be T^1 with a lower value of r .

Let us now numerically compute the value of each term in (16) by applying Harberger's estimate of parameters for the U.S. economy. His estimates of the relevant parameters are:⁹

$$k_x = \frac{1}{6}; \quad k_y = \frac{1}{2};$$

$$\frac{K'_x}{X} = -\frac{5}{36}; \quad \frac{K'_y}{Y} = -\frac{1}{24}; \quad \text{and} \quad \frac{D'}{X} = -\frac{1}{7}$$

Substituting these values into the terms of (16) we get:

$$\text{the } \pi \text{ effect} = 4/39$$

$$\text{the } \omega \text{ effect} = -30/39$$

$$\text{the total effect} = -2/3.$$

In this particular case, therefore, the ω effect explains 115% of the total effect. This suggests a fairly good approximating formula in this particular case.

$$\frac{\partial r^*}{\partial t} = -\frac{K'_x}{K'_x + K'_y}.$$

This formula is useful insofar as it allows us to approximate the total effect with estimates K'_x and K'_y , without requiring estimates of any parameters of product demands. The fact that the ω effect dominates is

of interest because one of the primary intentions of the original Harberger analysis was to incorporate general equilibrium considerations by explicitly taking account of product demand. Yet, using Harberger's own numbers, we find that the factor price distortion effect which is independent of commodity demand parameters dominates.

5 A NOTE ON THE LITERATURE

A. The Harberger Expression

What is the relationship between the RHS of (16) and the Harberger's expression for dr^*/dt ? By simplifying (16), we get

$$\frac{dr^*}{dt} = - \frac{D' \cdot k_x \cdot (k_x - k_y) + K'_x}{D' \cdot (k_x - k_y)^2 + K'_x + K'_y} \quad (18)$$

Despite its simple appearance, equation (18) is essentially the same as Harberger's expression for dr^*/dt :¹⁰

$$\frac{dr^*}{dt} = \frac{E \cdot k_x \cdot \left(\frac{K_x}{K_y} - \frac{L_x}{L_y}\right) + S_x \left(\frac{l_x K_x}{K_y} + \frac{k_x L_x}{L_y}\right)}{E \cdot (k_y - k_x) \left(\frac{K_x}{K_y} - \frac{L_x}{L_y}\right) - S_y - S_x \cdot \left(\frac{l_x K_x}{K_y} + \frac{k_x L_x}{L_y}\right)}, \quad (19)$$

where

$$S_x \equiv \frac{d\left(\frac{K_x}{L_x}\right)}{dr} \cdot \frac{r}{\left(\frac{K_x}{L_x}\right)}, \quad S_y \equiv \frac{d\left(\frac{K_y}{L_y}\right)}{dr} \cdot \frac{r}{\frac{K_y}{L_y}}, \quad \text{and} \quad E \equiv \frac{D'}{X}. \quad (20)$$

In fact, (19) is obtained by multiplying both the numerator and the denominator of (18) by $\frac{L_y + K_y}{L_y K_y}$.¹¹

B. Mieszkowski's Output and Factor Substitution Effects

Mieszkowski (1967, pp. 253-254) called the first and the second terms in the numerator of (19) the output and the factor substitution effects.¹² He suggested that they can be given geometric interpretations similar to our π and ω effects.

This, however, does not hold in general. Figure 3 illustrates this point. Here elasticity of substitution in the y industry is assumed infinite and the straight lines Y^0 and Y^t represent its isoquant curves. Hence the straight line passing through N and A is the efficiency locus of this economy. If the corporate income tax brings the economy from N to T, this diagram immediately reveals that the ω effect is zero, since the slopes of the y-isoquant is zero both at A and T. Mieszkowski's pure factor substitution effect in this case as measured by the second term in the numerator of (19), however, is negative.

In fact we can demonstrate that Mieszkowski's two effects coincide with the relative magnitude of the π and ω effects only in exceptional cases. Consider the situation where neither the factor substitution effect nor the ω effect is equal to zero. Let α be the ratio of the output effect to the factor substitution effect, and β be the ratio of the π effect to ω effect. Thus we have

$$\alpha \equiv \frac{\text{the output effect}}{\text{the f. subst. effect}} = \frac{D' \cdot k_x \cdot (k_x - k_y)}{K'_x}, \quad (21)$$

while from the definitions of the π and ω effects, we get

$$\beta \equiv \frac{\text{the } \pi \text{ effect}}{\text{the } \omega \text{ effect}} = \frac{D' - (k_x - k_y) \cdot (K'_y k_x + K'_x k_y)}{K'_x \{D' (k_x - k_y)^2 + (K'_x + K'_y)\}}. \quad (22)$$

It follows that¹³

$$\alpha = \beta + \frac{(\beta + 1)(k_x - k_y)^2}{(K'_x + K'_y)/D'} \quad (23)$$

Since the denominator of the second term of the RHS is positive, we have

$$\alpha = \beta \text{ if and only if } \beta + 1 = 0 \text{ or } k_x = k_y. \quad (24)$$

It may be noted that, $\beta + 1 = 0$ is equivalent to $\partial r^*/\partial t = 0$. Thus

(24) implies that the ratio of the output effect to the factor substitution effect is equal to the ratio of the π effect to the ω effect, if the tax change happens to have no impact on r or if the factor intensities of the two sectors are identical. It implies, however, that the two ratios are unequal otherwise.

In retrospect, it is obvious that the factor substitution effect cannot adequately represent the ω effect, since the former does not contain S_y (or K'_y) while the latter is determined by an interaction of S_x and S_y (or K'_x and K'_y).

6 CONCLUDING REMARKS

In the present paper we have decomposed Harberger's tax incidence formula into two components: the π effect which is due to the distortion in commodity prices created by the tax and the ω effect which is due to the distortion in factor prices created by the tax. The numerical measurement of these terms based on Harberger's estimates of the parameters showed the ω effect, which has the simple expression of

$$= \frac{K'_x}{K'_x + K'_y}$$

explains 115% of the total effect of the corporate tax upon the rate of return. We have also shown that the relative magnitude of Mieszkowski's output and factor substitution effects does not represent that of the π and ω effects.

APPENDIX A: A CLASS OF EQUIVALENT GEOMETRIC INTERPRETATIONS
OF (13) AND (16)

The geometric decomposition via A given in Section 2. is obviously not the only possible decomposition of the movement from N to T in Figure 1. In footnote 4 we have given another decomposition via A'. We call the former decomposition Type A and the latter Type A', or the transpose of Type A. Table 1 lists these and a few other obvious decompositions of the movement from N to T.

The points that appear in the list are illustrated in Figure 4. B and C are the points on the contract curve with the post-tax output ratio of x and y, and with the post-tax output level of x, respectively. To define points B' and C', the concept of the tax-distorted contract curve is useful. It is the locus of the points where the angles of the isoquants of the two industries correspond to the tax rate under consideration. The dotted line in Figure 4 illustrates this curve. B' and C' are the points on this tax-distorted contract curve with the pre-tax output ratio of x and y, and with the pre-tax output level of x, respectively.

It is clear that in terms of the tax impact on r the decompositions listed in Table 1 are generally different for finite tax changes. It can readily be proved, however, that they are all identical for infinitesimal tax changes.¹⁴ Therefore, (13) and (16) formally represent all of the types in Table 1 in terms of the impact on r of infinitesimal tax changes.

In Section 6.B. we demonstrated that Mieszkowski's formal

definitions of the substitution and output effects cannot be given geometric interpretations of either Type A or Type A'. Our finding in the present Appendix that (13) and (16) represent all of the geometric decompositions listed in Table 1 shows that Mieszkowski's two effects cannot be given any of these geometric interpretations.¹⁵

Table 1

Types	1 st Step	2 nd Step
A	NA	AT
A'	NA'	A'T
B	NB	BT
B'	NB'	B'T
C	NC	CT
C'	NC'	C'T

APPENDIX B: DERIVATION OF (16)

The ω effect

The ω effect is represented by the partial derivative of R with respect to t. In view of the definition of R, we have for all (Y,t):

$$G(Y, R(Y, t), t) \equiv 0. \quad (A1)$$

Hence we get

$$\frac{\partial R}{\partial t} = - \frac{\partial G}{\partial t} / \frac{\partial G}{\partial r} \quad (A2)$$

The function G was defined in footnote 4 as (A3)

$$G(Y, r, t) \equiv \ell_x(r+t) K_y(r, Y) - k_x(r+t) L_y(r, Y) - \ell_x(r+t) \bar{K} + k_x(r+t) \bar{L}$$

In order to find the derivatives of G, it is useful to explicitly state a few relationships among ℓ_x , k_x , ℓ_y , and k_y , L_x , K_x , L_y and K_y and their derivatives with respect to r. In view of (17) we have

$$K'_x = k'_x \cdot X \quad \text{and} \quad L'_x = \ell'_x \cdot X \quad (A4)$$

On the other hand, a Hicksian rule on the substitution terms yields $L'_x + (r+t)K'_x = 0$ and $L'_y + rK'_y = 0$. Thus, in view of (14) and (15), we find that

$$L'_x + k'_x = 0 \quad \text{and} \quad L'_y + K'_y = 0 \quad (A5)$$

hold at the equilibrium. In view of (14) and (15), we find from (6), (7), and the definitions of ℓ_x , K_x , ℓ_y and k_y that

$$\ell_x + k_x = 1 \quad \text{and} \quad \ell_y + k_y = 1 \quad (A6)$$

also hold at the initial equilibrium.

Differentiating (A3), and then applying (4), (5), (A4) through (A6), and the definitions of l_x , k_x , l_y , and k_y , we get

$$\begin{aligned}
 \frac{\partial G}{\partial t} &= l'_x K_y - k'_x L_y - l'_x \bar{K} + k'_x \bar{L} \\
 &= l'_x K_x + k'_x L_x \\
 &= K'_x (k_x + l_x) \\
 &= K'_x
 \end{aligned} \tag{A7}$$

$$\begin{aligned}
 \frac{\partial G}{\partial r} &= \frac{\partial G}{\partial t} + l_x K'_y - k_x L'_y \\
 &= K'_x + (l_x + k_x) K'_y \\
 &= K'_x + K'_y
 \end{aligned} \tag{A8}$$

Thus, in view of (A4), we have

$$\frac{\partial R}{\partial t} = - \frac{K'_x}{K'_x + K'_y} \tag{A9}$$

The π effect

As is seen from (13), the π effect is the product of $\partial R/\partial Y$ and $\partial Y^*/\partial t$. In view of (A1), we have

$$\frac{\partial R}{\partial Y} = - \frac{\partial G}{\partial Y} / \frac{\partial G}{\partial r} .$$

Since

$$\begin{aligned} \frac{\partial G}{\partial Y} &= \ell_x k_y - k_x \ell_y \\ &= (1 - k_x) k_y - k_x (1 - k_y) \\ &= k_y - k_x \end{aligned} \tag{A10}$$

this and (A8) yield

$$\frac{\partial R}{\partial Y} = \frac{k_y - k_x}{K'_x + K'_y} \tag{A11}$$

We now derive an expression for dY^*/dt . Rather than directly differentiate the complete model, we will first simplify it into the model with two equations and two unknowns, Y and r .

From (4), (5), (6), (7), and the definitions of $k_x(r+t)$, we get

$$p_x X + p_y Y = \bar{L} + r\bar{K} + t k_x(r+t) \cdot X \tag{A12}$$

This states that the National Product ($p_x X + p_y Y$) is equal to the Personal Income ($\bar{L} + r\bar{K}$) plus the tax revenue ($t \cdot k_x(r+t) \cdot X$). Substituting (8) for X in (A12), we have

$$\{p_x - t \cdot k_x(r+t)\} D\left(\frac{p_x}{p_y}\right) + p_y Y - \bar{L} - r\bar{K} = 0 \tag{A13}$$

This equation implicitly gives the demand for y when all the prices and the tax rate are given.

Define the unit cost functions, c_x and c_y , by

$$c_x(r+t) \equiv l_x(r+t) + (r+t) \cdot k_x(r+t)$$

$$c_y(r) \equiv l_y(r) + rk_y(r).$$

Then (6) and (7) may be rewritten as

$$p_x = c_x(r+t) \tag{A14}$$

$$p_y = c_y(r) \tag{A15}$$

Substituting the last two equations for p_x and p_y in (A13), we obtain

$$F(Y,r,t) = 0, \tag{A16}$$

where

$$F(Y,r,t) \equiv \{c_x(r+t) - tk_x(r+t)\} D\left(\frac{c_x(r+t)}{c_y(r)}\right) + c_y(r)Y - \bar{L} - r\bar{K} \tag{A17}$$

Equation (A16) and equation (11), i.e.,

$$G(Y,r,t) = 0, \tag{A18}$$

together give solutions for Y and r for a given t , and this reduced model enables us to analyze the properties of $Y^*(t)$. Totally differentiate (A16) and (A18) and solve for dY^*/dt by applying Cramer's rule, and we obtain

$$\frac{dY^*}{dt} = - \left(\frac{\partial F}{\partial t} \frac{\partial G}{\partial r} - \frac{\partial F}{\partial r} \frac{\partial G}{\partial t} \right) / \left(\frac{\partial F}{\partial Y} \frac{\partial G}{\partial r} - \frac{\partial F}{\partial r} \frac{\partial G}{\partial Y} \right) \tag{A19}$$

In order to derive the expressions for the derivatives of F , the

Shephard-Samuelson Lemma for c_x and c_y is useful:

$$c'_x(r+t) = k'_x(r+t) \quad \text{and} \quad c'_y(r) = k'_y(r) \quad (\text{A20})$$

Also note that in view of (14), (15), (A14) and (A15), the following holds at the initial equilibrium.

$$c_x = c_y = 1 \quad (\text{A21})$$

Differentiating (A17) and applying (14), (15), (17), (A20) and (A21), we have

$$\frac{\partial F}{\partial t} = D' \cdot k_x, \quad \frac{\partial F}{\partial r} = D' \cdot (k_x - k_y), \quad \text{and} \quad \frac{\partial F}{\partial Y} = 1.$$

Substituting these, (A7), (A8) and (A10) in (A19) we obtain

$$\begin{aligned} \frac{dY^*}{dt} &= - \frac{D' (K'_y k_x + K'_x k_y)}{D' \cdot (k_x - k_y)^2 + (K'_x + K'_y)} \\ &= - \frac{K'_y k_x + K'_x k_y}{(k_x - k_y)^2 + (K'_x + K'_y) / D'} \end{aligned}$$

Substituting this, (A9), and (A11) in (13), we obtain (16).

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¹See McLure (1975) for a survey.

²The geometric interpretation of the two terms was further discussed by McLure (1975, p. 140) and Atkinson and Stiglitz (1980, pp. 173-183) along the line of Mieszkowski (1967).

³Stolper and Samuelson (1941, pp. 68-69).

⁴To be explicit G is defined as

$$G(Y, r, t) \equiv l_x(r+t) \cdot K_y(r, Y) - k_x(r+t)L_y(r, Y) - l_x(r+t)\bar{K} + k_x(r+t)\bar{L},$$

where the functions l_x and k_x are defined in (17) below.

⁵Equation (12) cannot generally be solved for in that explicit form. However, under the conditions of the implicit function theorem, such a relationship will exist in the neighborhood of y^* and t^* .

⁶There are obviously other ways to decompose the movement from N to T . For example, let A' be a point on the y -isoquant of the pre-tax output level where discrepancy between the angles of the x -isoquant and the y -isoquant correspond to the given tax rate. In terms of the point A' the shift from N to T can be decomposed into the two movements: one from N to A' and the other from A' to T .

Comparing this decomposition with the one given in the text, it is true that, for a finite tax change, movements NA' and AT represent different tax effects on r . But the tax effects upon r are identical for an infinitesimal change in t , and $\partial H/\partial t$ in (13) formally represents both movements. Similarly, $\frac{\partial H}{\partial Y} \cdot \frac{dY^*}{dt}$ represents both the tax impact on r of NA and $A'T$ for an infinitesimal change.

The reason why equation (13) has these different geometric interpretations is exactly the same as the reason why the Hicks-Slutsky decomposition has different geometric interpretations depending upon whether the substitution term is taken to reflect a shift along the indifference curve for the old or new utility level. (Compare Figures 1 and 2 in Mosak (1942), for example.)

We will discuss other alternative geometric interpretations of (13) in Appendix A.

⁷It should be noted that assuming that there are no initial distortions is critical for the results that follow. If initial distortions are not zero then the correspondence between the sign of the "Stolper-Samuelson" effect (our Π effect) and the sign of the difference between physical factor intensities may not hold (see Jones (1971) and Neary (1978)). Moreover, to justify the assumption in equation (8) that income effects may be ignored it is necessary to assume that there are no initial distortions and that the tax under consideration is infinitesimal.

⁸See Appendix B for derivation.

⁹See Set II in Section VII of Harberger (1962). In our (17), the values of K'_x/X and K'_y/X are derived from his estimates:

$$S_x = S_y = -1, \ell_x = \frac{5}{6}, \frac{K_y}{K_x} = \frac{1}{2}, \text{ and } \frac{L_y}{L_x} = \frac{1}{10},$$

where (20) of the present paper gives the definitions of S_x and S_y .

It is also the case that:

$$\frac{K'_x}{X} = S_x \ell_x k_x = -1 \cdot \frac{5}{6} \cdot \frac{1}{6} = -\frac{5}{36} \text{ and}$$

$$\frac{K'_y}{X} = S_y \ell_x k_y \frac{L_y}{L_x} = -1 \cdot \frac{5}{6} \cdot \frac{1}{2} \cdot \frac{1}{10} = -\frac{1}{24}.$$

¹⁰Equation (7.12) of Harberger (1962).

¹¹Detailed derivation available upon request.

¹²See Atkinson and Stiglitz (1980, pp. 173-183) for an extensive discussion of these two effects.

¹³It follows from (21) and (22) that

$$\beta + 1 = (\alpha + 1) \frac{K'_x + K'_y}{D'(k_x - k_y)^2 + K'_x + k'_y}.$$

This immediately yields (23). Note that this equation implies that the signs of $\beta + 1$ and $\alpha + 1$ are equal.

¹⁴This conclusion should not be surprising since this discussion of equivalent geometric interpretations of (13) and (16) is analogous to the well-known discussions of equivalent geometric interpretations of the Slutsky decomposition. See J. Mosak (1942), for example.

¹⁵Even without the conclusion that (13) and (16) represent types A, A', B, B', C and C', it is easy to show that Mieszkowski's formal decomposition is neither a type B nor a type C. To see this, decompose the RHS of (10) in proportion to Mieszkowski's two effects to get

$$\frac{dr^*}{dt} = \frac{-D' \cdot k_x \cdot (k_x - k_y)}{D' \cdot (k_x - k_y)^2 + K'_x + K'_y} + \frac{-K'_x}{D' \cdot (k_x - k_y)^2 + K'_x + K'_y}$$

[the output term] [the f. subst. term]

Call the first and the second terms the output and the factor substitution terms, respectively. Applying Harberger's estimates of the parameters values listed in Section 4 we find that the output term is equal to 4/99 and the factor substitution term is -70/99. The magnitude of the output term is less than half of our π . Since our π effect represents the movement from N to A in Figure 4, this implies that Mieszkowski's output term must represent a movement from N to some point on the NA portion of the contract curve, if it can be given a geometric interpretation of a movement from N to a point on the contract curve.

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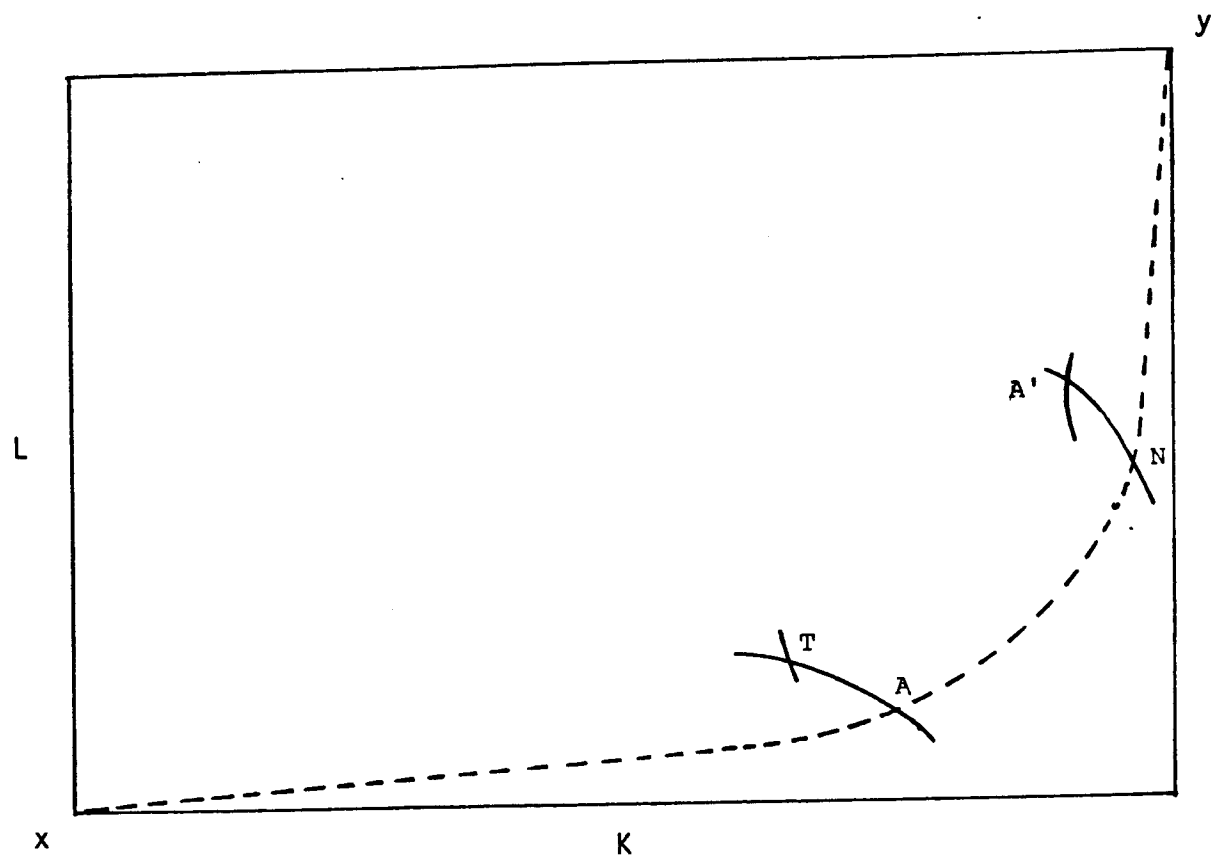


Figure I

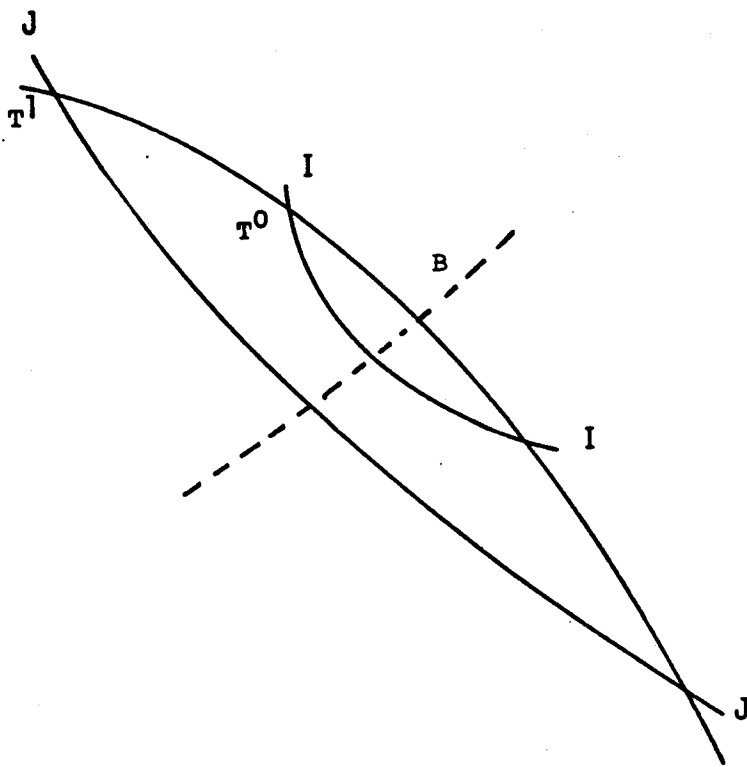


Figure 2

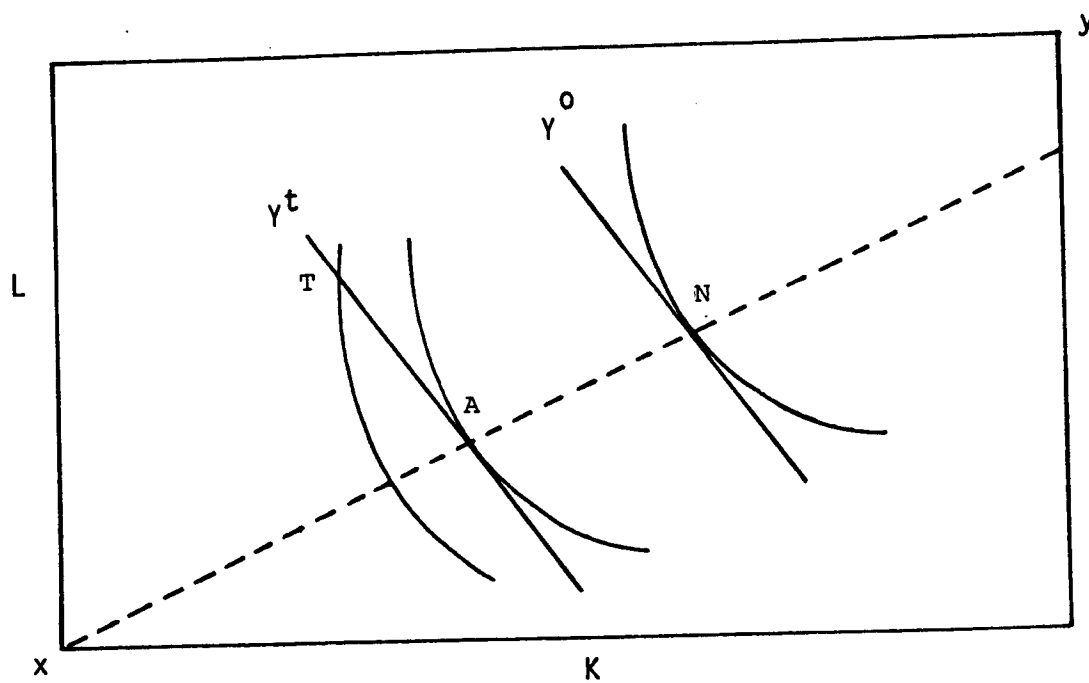


Figure 3

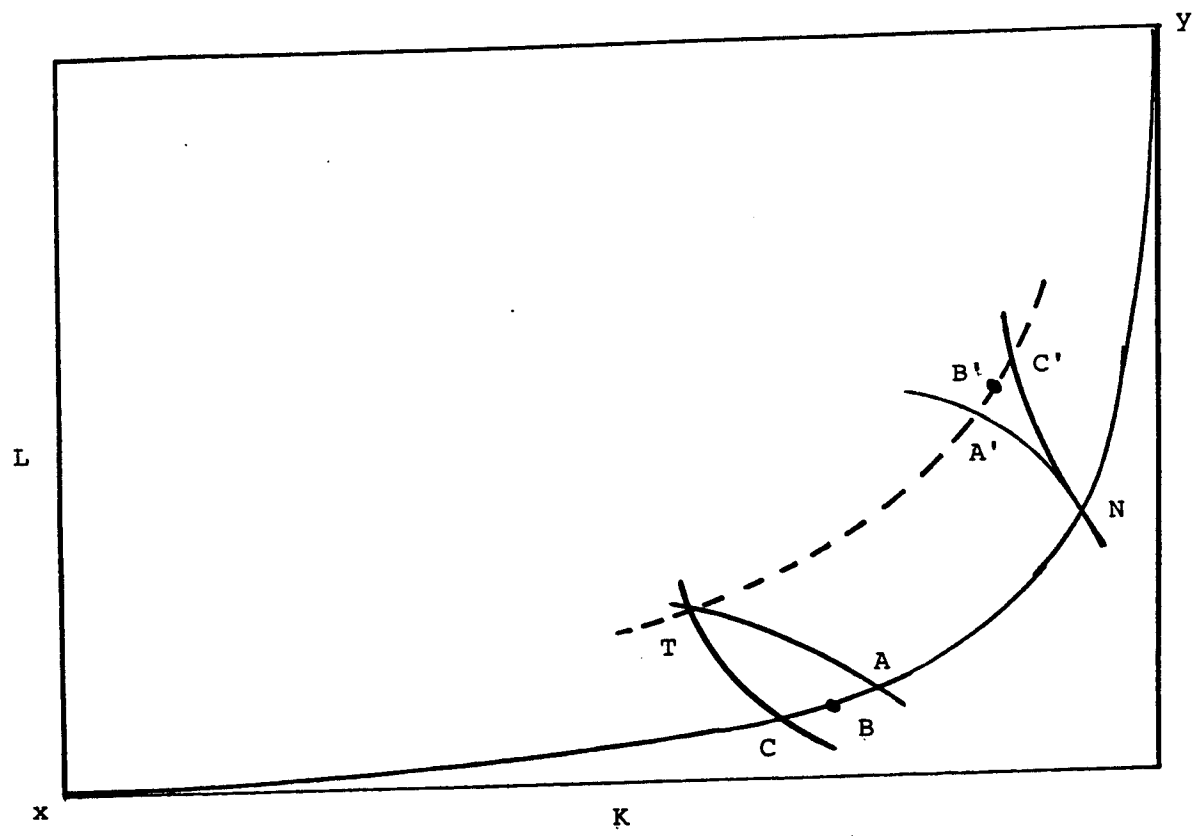


Figure 4