

A NOTE ON THE THEORY OF LAYOFFS

By

John Haltiwanger
University of California, Los Angeles

Working Paper No. 232
February 1982
Department of Economics
University of California, Los Angeles

In a recent article, Baily (1977) considers a two period implicit contract model to explain optimal hours, employment reduction strategies for a firm in the face of a downturn in demand. In Baily's specification of the two period model, the firm knows the price that will occur in the first period but is uncertain about the price in the second period except that it is assumed that the price in the second period will be less than or equal to the price in the first period. In addition, it is assumed that the firm can employ no more workers in the second period than it contracted for in the first period. This paper considers two slight variations on these assumptions in order to demonstrate some peculiarities about the explanation for layoffs offered by implicit contract theory.

First, the assumption that the uncertain second period price must be less than or equal to the first period price is relaxed and it is simply assumed that the price in the second period is uncertain. In this modified version of the Baily model, it is demonstrated that hours reductions and layoffs may still result in the face of a realized upturn in demand. Such a possibility is obviously problematic since it is not clear that we ever empirically observe firms laying off workers in response to upturns in demand.

Second, the assumption that the price in the first period is known with certainty is relaxed. In this specification, it is assumed that the firm contracts for workers *ex ante* (before the price in either period is known) and can employ no more workers *ex post* in either period than it has contracted for *ex ante*. This later specification is closer to the specification of the more common one period models (e.g. Azariadis (1975), Burdett and Mortensen (1980)) but is a straightforward extension to two periods. This new specification allows us to identify two distinct types of layoffs that are possible in this

context. The first type is associated entirely with unexpected variations in demand and is common to all contract models. It is the type that gives rise to the possibility of layoffs in response to intertemporal upturns in demand. The second type is associated entirely with expected variations in demand and is essentially due to the contract being made for a sufficiently long period of time to allow for a change in the state of expectations to occur within the contract period. Layoffs of the second type have the property that they are only associated with intertemporal downturns in demand.

The Model

The model is essentially that of Baily (1977). The firm under consideration is assumed to be a price taker in both periods. In period zero, the price P_0 is known and the price in period one, $P_1(s)$ is unknown and is a random variable (where s indexes the states of the world). The firm announces a contingent two period contract ex ante (before the price in period one is known) which specifies, W_0 , the wage in period 0, L_0 , the initial employment, H_0 , the hours of work in period 0, and the contingent decision variables for period 1, $W_1(s)$, the contingent wage for period 1, $L_1(s)$, the contingent employment for period 1 and $H_1(s)$, the contingent hours for period 1. Following Baily, the constraint $L_1(s) \leq L_0$ is imposed. However in departing from Baily, it is no longer assumed that $0 \leq P_1(s) \leq P_0$ ($\forall s$) but rather that simply $P_1(s) \geq 0$ ($\forall s$).

Workers evaluate the expected two period income offer given by:

$$(1) \quad W_0 H_0 - D(H_0) + E \left[\frac{L_1(s) \left\{ W_1(s) H_1(s) - D(H_1(s)) \right\}}{L_0} + \left(1 - \frac{L_1(s)}{L_0} \right) Y \right] \rho = v$$

where $D(H) > 0$, $D'(H) > 0$ and $D''(H) > 0$ for $H > 0$. The terms $D(H_0)$ and $D(H_1(s))$ represent the disutility of work in periods 0 and 1, respectively. In what follows, the firm takes (1) as a constraint with V and Y fixed. V is the expected two period income available elsewhere and Y is the ex post opportunity cost of a worker's time. Although it is not necessary for a solution, following Baily it is assumed that $V > Y + Y\rho$ (where ρ is the discount factor).

The firm's objective function is its two period expected profits given by:

$$(2) \quad P_0 G(H_0 L_0) - W_0 H_0 L_0 + E \left[P_1(s) G(H_1(s) L_1(s)) - W_1(s) H_1(s) L_1(s) - T(L_0 - L_1(s)) \right] \rho$$

The production function, with capital assumed fixed, is well behaved with the following properties (assumed by Baily):

$$(3a) \quad G'(HL) > 0, G''(HL) < 0 \text{ for all finite } H \text{ and } L$$

$$(3b) \quad G'(HL) \rightarrow \infty \text{ as } HL \rightarrow 0, G'(HL) \rightarrow 0 \text{ as } HL \rightarrow \infty$$

$$(3c) \quad G'(HL)H \text{ is bounded above for a fixed } L > 0.$$

$T(L_0 - L_1(s))$ represents the turnover costs associated with layoffs where T is a positive constant and is such that $T < Y$.

The firm maximizes (2) subject to (1) (associate this constraint with the Lagrange multiplier λ) and to the inequality constraint $L_1(s) \leq L_0$ (associate this constraint with the Kuhn-Tucker multiplier $\mu(s)$). After some work, the optimality conditions are:

$$(4) \quad P_0 G'(H_0 L_0) = D'(H_0)$$

$$(5) \quad P_1(s) G'(H_1(s) L_1(s)) = D'(H_1(s)) \quad (\forall s)$$

$$(6) \quad P_0 G'(L_0 H_0) H_0 - D(H_0) = V - (Y - T)\rho - E[\mu(s)]$$

$$(7) \quad P_1(s)G'(L_1(s)H_1(s))H_1(s) - D(H_1(s)) = Y - T + \frac{\mu(s)}{\rho} \quad (\forall s)$$

$$(8) \quad \mu(s) (L_0 - L_1(s)) = 0 \quad (\forall s)$$

and (1). The wages W_0 and $W_1(s)$ are determined only up to the expected income constraint (1). Equations (4) - (8) are a subsystem of equations that fully determine L_0 , $L_1(s)$, H_0 , $H_1(s)$ and $\mu(s)$. The following proposition demonstrates that layoffs may be optimal in response to a realized upturn in demand (the proofs of all propositions are contained in the appendix).

Proposition 1 There exists a distribution of $P_1(s)$ such that:

$$(i) \quad E \left[P_1(s) \right] > P_0$$

(ii) for at least one state of the world, s^* , where $P_1(s^*) > P_0$, it is the case that $L_1(s^*) < L_0$.

The reason that layoffs may be optimal following an upturn in demand is that layoffs in this context depend on the presumed within period uncertainty of demand as well as on intemporal variations in demand. In other words, it is the difference between the realized price and the expected price in period 1 that is causing the layoffs in proposition 1 rather than the differences between prices in periods 0 and 1.

It is important to emphasize that this aspect of the contract theoretic explanation for layoffs is not particular to the Baily specification. In the more common one period models, layoffs depend solely on the presumed uncertainty of demand in each period and not at all on intertemporal variations in demand. This uncertainty dependent explanations of layoffs has peculiar consequences in the one period contract theoretic context as well. Consider a sequence of contract periods in the one period contract context in which demand is initially expected to be relatively high, then relatively low and then relatively high again.

Under these conditions, it may be that the probability of layoffs in the periods of relatively high expected demand is higher than it is in the periods of relatively low expected demand.¹

A possible objective to proposition 1 is that if the assumption that $P_0 \geq P_1(s)$ ($\forall s$) is relaxed then it may be argued that the assumption that $L_0 \geq L_1(s)$ should be relaxed as well. In other words, if it is possible that the uncertain price of period 1 may be greater than the certain price of period 0, then perhaps one ought to stipulate that the firm in making its two period contract with workers, actually contracts for m workers prior to both periods and then may use up to m workers in either period, i.e., $m \geq L_0$ and $m \geq L_1(s)$. In what follows, this alternative specification is considered. However, the further modification of assuming that the price in period 0 is uncertain is made as well. Given these modifications, this specification becomes a straightforward extension of the one period contract model to two periods. This modified framework allows us to demonstrate that multiperiod contract models of the Baily-type generate two distinct types of layoffs.

The maximization problem in this alternative specification becomes:

$$\max (9) \quad E_t \left[P_0(t)G(L_0(t)H_0(t)) - W_0(t)L_0(t)H_0(t) \right] \\ + E_s \left[P_1(s)G(L_1(s)H_1(s)) - W_1(s)L_1(s)H_1(s) \right] \rho$$

subject to:

$$(10) \quad E_t \left[\frac{L_0(t)}{m} (W_0(t)H_0(t) - D(H_0(t))) + \left(1 - \frac{L_0(t)}{m}\right)Y \right] \\ + E_s \left[\frac{L_1(s)}{m} (W_1(s)H_1(s) - D(H_1(s))) + \left(1 - \frac{L_1(s)}{m}\right)Y \right] \rho = v$$

$$(11) \quad L_0(t) \leq m \quad (\forall t)$$

$$(12) \quad L_1(s) \leq m \quad (\forall s)$$

where $P_0(s)$ and $P_1(t)$ are assumed to be independently distributed

(note that turnover costs T are suppressed for simplicity). In this specification, the optimality conditions reduce to:

$$(13) \quad P_0(t)G'(L_0(t)H_0(t)) = D'(H_0(t)) \quad (\forall t)$$

$$(14) \quad P_1(s)G'(L_1(s)H_1(s)) = D'(H_1(s)) \quad (\forall s)$$

$$(15) \quad P_0(t)G'(L_0(t)H_0(t))H_0(t) - D(H_0(t)) = Y + \mu_0(t) \quad (\forall t)$$

$$(16) \quad P_1(s)G'(L_1(s)H_1(s))H_1(s) - D(H_1(s)) = Y + \frac{\mu_1(s)}{\rho} \quad (\forall s)$$

$$(17) \quad E_t \left[\mu_0(t) \right] + E_s \left[\mu_1(s) \right] = V - Y(1 + \rho)$$

$$(18) \quad \mu_0(t) (m - L_0(t)) = 0 \quad (\forall t)$$

$$(19) \quad \mu_1(s) (m - L_1(s)) = 0 \quad (\forall s)$$

and (10). The contractual wages $W_0(t)$ and $W_1(s)$ are determined only up to the expected income constraint (10). The following propositions help to distinguish between uncertainty dependent layoffs and layoffs due to intertemporal downturns in demand.

Proposition 2 If $P_0(t)$ and $P_1(s)$ are identically distributed, then $L_0(t)$ and $L_1(s)$ are identically distributed and $H_0(t)$ and $H_1(s)$ are identically distributed.

Proposition 3 In a world of price certainty, if $P_0 = P_1$, then full employment is optimal for both periods. However, in a world of price certainty, there exists a critical price for period 1, P_1^c such that:

$$(i) \quad P_1^c < P_0$$

$$(ii) \quad \text{if } P_1 < P_1^c, \text{ then } L_0 = m > L_1$$

$$(iii) \quad \text{if } P_1 \geq P_1^c, \text{ then } m = L_1$$

Proposition 2 demonstrates that if there are no intertemporal changes in the state of expectations (i.e., P_0 and P_1 are identically distributed) then there is no reason to expect more layoffs in one period than the next. However, more layoffs may occur in one period than another due to the randomness of demand. The first part of proposition 3 provides further evidence that layoffs are possible when P_0 and P_1 are identically distributed only when demand is random. The second part of proposition 3 establishes that layoffs may be optimal in multiperiod contract models even in the absence of price uncertainty. This is significant in and of itself since, as aforementioned, price certainty implies a full employment contract in the one period contract theoretic context. Proposition 3 further demonstrates that layoffs that occur in the absence of price uncertainty are only associated with intertemporal downturns in demand.

In comparing and contrasting the incentives underlying the two types of layoffs identified by propositions 2 and 3, observe that the uncertainty dependent layoffs are essentially due to the stipulation that the firm must make a binding decision with regard to the maximum number of workers it can employ prior to it knowing the actual price. The firm has the incentive to "hedge its bet" by contracting for more workers than it intends to employ in the lowest possible states of demand. Thus, it is the possibility of unexpected variations in demand that is critical rather than intertemporal variations in demand. Alternatively, the layoffs identified in proposition 3 as being associated with intertemporal downturns in demand are essentially due to the stipulation that the contract is made for an extended period of time over which intertemporal variations in demand are expected. The firm, in this situation, has the incentive to contract for more workers than it intends to utilize in the period of lowest demand. Thus, in this case, intertemporal variations of demand are a critical factor.

To sum up, this analysis suggests that the layoffs explained by contract theory do not, in general, depend upon intertemporal variations in demand but rather on the presumed uncertainty of demand within each period. A consequence of this dependence on the uncertainty of demand is that a positive correlation between the incidence of layoffs and actual demand is possible in this context. An exception to the uncertainty dependent explanation for layoffs can be found in the multiperiod contract context. If the contract is made for an extended period of time over which demand is expected to vary, then layoffs may be explained as the result of the firm finding it optimal to contract for more workers than it intends to utilize in the periods of lowest demand.

Footnotes

¹ A trivial proof of this is to consider three consecutive periods, the first and third of which have relatively high but uncertain demand while the second has relatively low but certain demand. Since in the one period contract models, layoffs are not optimal if prices are certain, no layoffs occur in the second period. Layoffs may however, occur in the first and third periods when demand is uncertain. It should also be obvious that layoffs may occur in the first and the third periods even when the realized prices in the first and the third periods are higher than the second period price. Hence, a positive correlation between the incidence of layoffs and actual demand is possible in the one period contract theoretic context as well.

References

Azariadis, C., 1975, "Implicit Contracts and Underemployment Equilibria,"
Journal of Political Economy, December, 1183-1202.

Baily, M.N., 1977, "On the Theory of Layoffs and Unemployment," Econometrica,
July, 1043-1063.

Burdett, K. and D. Mortensen, 1980, "Search, Layoffs, and Labor Market Equilibrium,"
Journal of Political Economy, August, 652-672.

Appendix

Proof of Proposition 1: Let there be n possible states of the world indexed in such a way that $P_1(s_1) < \dots < P_1(s_n)$. For each of these states of the world associate $q(s_i)$ with the probability of $P_1(s_i)$ occurring. Since all that is necessary is to find one distribution that satisfies (i) and (ii), consider an extreme case where $P_1(s_i) > P_0$ ($\forall s_i$). Given this notation, a proof by contradiction is used. Suppose that for any arbitrary distribution of $P_1(s)$ where $P_1(s) > P_0$ ($\forall s$), that $L_1(s) = L_0$ ($\forall s$). Since $L_1(s) = L_0$ ($\forall s$), the optimality conditions (4) - (8) become:

$$(A1) \quad P_0 G'(H_0 L_0) = D'(H_0)$$

$$(A2) \quad P_1(s) G'(L_0 H_1(s)) = D'(H_1(s)) \quad (\forall s)$$

$$(A3) \quad P_0 G'(L_0 H_0) H_0 - D(H_0) = V - (Y - T)\rho - E \left[\mu(s) \right]$$

$$(A4) \quad P_1(s) G'(L_0 H_1(s)) H_1(s) - D(H_1(s)) = Y - T + \frac{\mu(s)}{\rho} \quad (\forall s)$$

Combining (A3) and (A4) yields:

$$(A5) \quad P_0 G'(H_0 L_0) H_0 - D(H_0) + E \left[P_1 G'(H_1(s) L_0) H_1(s) - D(H_1(s)) \right] \rho = V$$

Combining (A2) and (A4) yields:

$$(A6) \quad D'(H_1(s)) H_1(s) - D(H_1(s)) = P_1(s) G'(H_1(s) L_0) H_1(s) - D(H_1(s)) \\ = Y - T + \frac{\mu(s)}{\rho} \quad (\forall s)$$

Consider an infinite sequence of distributions of $P_1(s)$ that satisfy

$P_1(s) > P_0$ ($\forall s$) where the distributions are indexed by a superscript j and are such that:

- (i) $P^1(s_i) = P^2(s_i) = \dots = P^j(s_i) = \dots$ for $i = 1, \dots, n - 1$, ($\forall j$)
- (ii) $P^1(s_n) < P^2(s_n) < \dots < P^j(s_n) < \dots$ ($\forall j$)

$$(iii) \quad q^1(s_i) = q^2(s_i) = \dots = q^j(s_i) = \dots \quad (\forall i) \quad (\forall j)$$

Observe that as the distribution changes from $P_1^j(s)$ to $P_1^{j+1}(s)$, either H_0^j , $H_1^j(s)$ for at least one s and/or L_0^j must change in order for (A5) to be satisfied. Suppose $L_0^j = L_0^{j+1}$. From (A1), this implies $H_0^j = H_0^{j+1}$. From (A2), this implies $H_1^j(s_i) = H_1^{j+1}(s_i)$ for $i = 1, \dots, n-1$ and $H_1^j(s_n) < H_1^{j+1}(s_n)$. From (A6), this implies:

$$\begin{aligned} P_1^j(s_i)G'(H_1^j(s_i)L_0^j)H_1^j(s_i) - D(H_1^j(s_i)) \\ = P_1^{j+1}(s_i)G'(H_1^{j+1}(s_i)L_0^{j+1})H_1^{j+1}(s_i) - D(H_1^{j+1}(s_i)) \end{aligned}$$

for $i = 1, \dots, n-1$

and:

$$\begin{aligned} P_1^j(s_n)G'(H_1^j(s_n)L_0^j)H_1^j(s_n) - D(H_1^j(s_n)) \\ < P_1^{j+1}(s_n)G'(H_1^{j+1}(s_n)L_0^{j+1})H_1^{j+1}(s_n) - D(H_1^{j+1}(s_n)) \end{aligned}$$

Hence, if $L_0^j = L_0^{j+1}$, then the LHS of (A5) would increase as the distribution of $P_1(s)$ changes from $P_1^j(s)$ to $P_1^{j+1}(s)$, but since the RHS of (A5) is constant, (A5) would be violated. Hence, $L_0^j \neq L_0^{j+1}$. By similar reasoning, it is possible to show that $L_0^j < L_0^{j+1}$. From (A2), this implies $H_1^j(s_i) > H_1^{j+1}(s_i)$ for $i = 1, \dots, n-1$. Since $D'(H)H - D(H)$ is monotonically increasing in H , from (A6) there will be a critical distribution $P_1^c(s)$ where $H_1^c(s_i)$ for $i = 1, \dots, n-1$ is such that:

$$(A7) \quad D'(H_1^c(s_i))H_1^c(s_i) - D(H_1^c(s_i)) = Y - T$$

This $H_1^c(s_i)$ is Baily's "minimum work week" H_1^{\min} .

Now consider a distribution $P_1^k(s)$ where $k > c$. By previous arguments $L_0^k > L_0^c$. However, $H_1^k(s_i) = H_1^{\min} = H_1^c(s_i)$ for $i = 1, \dots, n-1$. Hence, if $L_0^k = L_1^k(s)$ ($\forall s$) then conditions (5) and (7) will be violated. Therefore, for any distribution $P_1^k(s)$ where $k > c$, $L_1^k(s) < L_0^k$ for some state of the world s . Q.E.D.

Proof of Proposition 2: To specify that $P_0(t)$ and $P_1(s)$ are identically distributed, let there be n possible states of the world for both $P_0(t)$ and $P_1(s)$ and indexed in such a way that $P_0(t_i) = P_1(s_i)$ for $i = 1, \dots, n$. In addition, define $q_0(t_i)$ and $q_1(s_i)$ the probabilities associated with $P_0(t_i)$ and $P_1(s_i)$ occurring, respectively. In order for $P_0(t)$ and $P_1(s)$ to be identically distributed, $q_0(t_i) = q_1(s_i)$ for $i = 1, \dots, n$. Given this notation, a proof by contradiction is used. Suppose $P_0(t)$ and $P_1(s)$ are identically distributed and either $L_0(t_i) \neq L_1(s_i)$ or $H_0(t_i) \neq H_1(s_i)$ for some i . There are six possibilities:

- (i) $m = L_0(t_i) > L_1(s_i)$
- (ii) $m = L_1(s_i) > L_0(t_i)$
- (iii) $m > L_0(t_i) > L_1(s_i)$
- (iv) $m > L_1(s_i) > L_0(t_i)$
- (v) $H_0(t_i) > H_1(s_i)$
- (vi) $H_1(s_i) > H_0(t_i)$

Possibility (i) implies $\mu_1(s_i) = 0$ by (19). This in turn implies $H_0(t_i) \geq H_1(s_i)$ since combining (13) - (16) yields:

$$(A8) \quad D'(H_0(t_i))H_0(t_i) - D(H_0(t_i)) = Y + \mu_0(t_i) \\ \geq Y = D'(H_1(s_i))H_1(s_i) - D(H_1(s_i))$$

and $D'(H)H - D(H)$ is monotonically increasing in H . Since $P_0(t_i) = P_1(s_i)$ and $H_0(t_i) \geq H_1(s_i)$, by (13) and (14) this means $L_0(t_i) \leq L_1(s_i)$. This rules out possibility (i) and possibility (ii) can be ruled out by similar reasoning.

Possibilities (iii) and (iv) imply $H_0(t_i) = H_1(s_i)$ since in both cases:

$$(A9) \quad D'(H_0(t_i))H_0(t_i) - D(H_0(t_i)) = Y = D'(H_1(s_i))H_1(s_i) - D(H_1(s_i))$$

However, if $H_0(t_i) = H_1(s_i)$, then neither (iii) nor (iv) are possible since

(13) and (14) would be violated. Since (i) - (iv) are ruled out,

$L_0(t_i) = L_1(s_i)$. This implies (v) and (vi) are not possible since

$P_0(t_i) = P_1(s_i)$, $L_0(t_i) = L_1(s_i)$, and conditions (13) and (14) imply

$H_0(t_i) = H_1(s_i)$.

Q.E.D.

Proof of Proposition 3: (First part) Suppose not, i.e., $P_0 = P_1$ and either $m > L_0$ or $m > L_1$. Combining (15) - (17) in a world of price certainty yields:

$$(A9) \quad P_0 G'(L_0 H_0) H_0 - D(H_0) + (P_1 G'(L_1 H_1) H_1 - D(H_1)) \rho = V$$

Since $P_0 = P_1$, by proposition 2, $L_0 = L_1$. Hence $m > L_0 = L_1$. However, with $m > L_0 = L_1$, (15), (16) and (A9) imply $Y(1 + \rho) = V$ which is a contradiction.

(Second part) First, it is established that there exists a $P_1^c > 0$ such that if $P_1 < P_1^c$, then $m > L_1$. Suppose that this is not the case, i.e., $m = L_1$ for all $P_1 > 0$. The boundedness of $G'(HL)H$ implies that $P_1 G'(HL)H$ can be made arbitrarily close to zero by choosing P_1 sufficiently small. Since $m = L_1$, (16) becomes:

$$(A10) \quad P_1 G'(mH_1) H_1 - D(H_1) = Y + \frac{\mu_1}{\rho}$$

Since $Y + \frac{\mu_1}{\rho} > 0$ and $D(H_1) > 0$, a P_1^c can be found such that if $P_1 < P_1^c$

(A10) will be violated. The value of P_1^c can be derived from the following set of four equations in four unknowns, P_1^c , H_1 , H_0 and m :

$$(A11) \quad P_1^c G'(H_1 m) H_1 - D(H_1) = Y$$

$$(A12) \quad P_0 G'(H_0 m) H_0 - D(H_0) = V - Y\rho$$

$$(A13) \quad P_1^c G'(H_1 m) = D'(H_1)$$

$$(A14) \quad P_0 G'(H_0 m) = D'(H_0)$$

Given this P_1^c , note that if $P_1 \geq P_1^c$, then $m = L_1$ since otherwise (14) and (16) would be violated. Also, if $P_1 < P_1^c$, then $L_0 = m$ since in this event $\mu_1 = 0$ and $\mu_0 = V - Y(1 + \rho) > 0$. Combining (A11) - (A14) yields:

$$D'(H_0)H_0 - D(H_0) = V - Y\rho > Y = D'(H_1)H_1 - D(H_1)$$

implying $H_0 > H_1$ when $P_1 = P_1^c$. Since $H_0 > H_1$ when $P_1 = P_1^c$, (A13) and (A14)

imply $P_1^c < P_0$. Hence, there exists a P_1^c that satisfies (i) - (iii). Q.E.D.