ALTERNATIVE SOCIAL COMPOSITION FUNCTIONS AND THE VOLUNTARY PROVISION OF PUBLIC GOODS

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Revised

July 1982

Working paper #244 A

J. Hirshleifer July 1982

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Abstract

It has traditionally been assumed that the socially available amount X of a public good is the simple sum of the separate amounts x_i produced by the $i=1,\ldots,I$ members of the community. But there are many other possibilities of practical importance. Among them are: (i) Weakest-link rule, where the socially available amount is the minimum of the quantities individually provided, and (ii) Best-shot rule, where the socially available amount is the maximum of the individual quantities. The former tends to arise in linear situations, where each individual has a veto on the total to be provided (e.g., if each is responsible for one link of a chain); the latter tends to arise where there is a single prize of overwhelming importance for the community, with any individual's effort having a chance of securing the prize.

In comparison with the standard Summation formula of ordinary public-good theory, it is shown that underprovision of the public good tends to considerably moderated when the Weakest-link function is applicable, but aggravated when the Best-shot function is applicable. In time of disaster, where the survival of the community may depend upon each person's doing his duty, the conditions for applicability of the Weakest-link rule are approximated. This circumstance explains the historical observation that disaster conditions tend to elicit an extraordinary amount of unselfish behavior.

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Anarchia is a perfectly circular island, and each citizen owns a wedge-shaped slice (not all equal) from the center to the sea. Like Holland, Anarchia is protected by dikes from occasional storms that threaten to flood the land. But since Anarchia has no government, everyone makes his own decision as to how high a dike to build. While the height of each citizen's dike is perfectly visible to all, the customs of Anarchia forbid enforcement of any threat, inducement, or contract whereby some parties might influence the choices of others. In times of flood the sea will penetrate the sector belonging to whichever citizen has constructed the lowest dike, but the topography of Anarchia is such that no matter where the sea enters, damage will be suffered equally over the whole island. The economists of Anarchia have long realized that flood-protection for their island is a public good. Many centralized schemes for motivating individuals to build dikes of the socially optimal height have been discussed, but Anarchia's citizens find any such social planning intolerable. It so came about, however, that the United Nations generously paid for an analysis of the situation by the well-known international consulting firm of economists, Arthur "Dam" Little & Co. To everyone's surprise, the conclusion was that Anarchia's citizens have voluntarily invested in dikes (and therefore have provided themselves with the public good of flood-protection) to almost exactly — 98.17%, to be exact — the socially efficient amount.

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A. ALTERNATIVE SOCIAL COMPOSITION FUNCTIONS

Public goods, by definition, have a peculiar feature on the demand side: the amount produced, however determined, is equally available for concurrent consumption by all members of the community. Our standard models have assumed, however, that there is nothing at all special on the supply side, in the social technology for the production of public goods.

More specifically, it has always been assumed that the socially available amount X of a public good is nothing but the simple sum of the separate amounts x_i produced by the $i=1,\ldots,I$ members of the community. But many other possibilities come to mind. I shall speak of these alternative possibilities as social composition functions, i.e., as different possible ways of amalgamating individual productions into social availabilities of a public good.

I will concentrate on three especially simple cases:

SOCIAL COMPOSITION FUNCTIONS

(1)	$X = \sum_i x_i$	Summation
(2)	$X = \min_i(x_i)$	Weakest-link
(3)	$X = \max_i(x_i)$	Best-shot

Imagine a team markmanship contest. Suppose that the payoff in the form of glory goes as a public good to all the members of the winning team, collectively. Now, it is perfectly easy to imagine that the relevant social composition function, the formula for calculating a team's score, might be (among many other possibilities) any one of the three rules above: the team declared the winner might be the one with the best total score, or the highest minimum score (the best of the individual marksmen's worst shots), or the highest maximum score (the best single shot).

While in this illustration the alternative social composition functions represent contrived formulas for judging a social pastime, other examples can be instanced that correspond to "natural" environmental payoff structures. The Anarchia fable is an instance of the Weakest-link function. More generally the Weakest-link measure approximates a wide variety of "linear" situations where each member of a social group successively has a kind of veto power over the extent of collective achievement. E.g., if each member is responsible for one link of a chain. As for the Best-shot function, imagine a number of anti-missile batteries ringing a city, firing at a single incoming nuclear-armed ICBM, where destruction for all will be the consequence if the enemy device gets through the defensive ring. Then, for all practical purposes, the only relevant question is whether the single best defensive shot is good enough to destroy the incoming bogey. Or a logically similar situation: the supporters of two claimants to the throne might engage in battle, with all the combatants on each side instructed to aim exclusively to kill the rival pretender.

As a different kind of example, consider the "natural economy" of families. In modern evolutionary-genetic theory, the behavior of animals is ultimately determined by the need to maximize reproductive survival. For a diploid organism, survival of an offspring represents a public good for its two parents. Since parents share equally in the genetic heritage of their offspring, any genetic payoff to the father due to enhanced offspring survival is equally an achievement for the mother, and vice versa. If father and mother forage separately for food, nutritional success for offspring might essentially depend only upon the total of the calories they bring back to the nest (Summation), Or, perhaps mother and father forage for distinct nutritional elements needed in fixed proportions by the offspring (Weakest-link). Or, more fancifully, the distinct elements may be equally useful but incompatible, rather like pickles and ice cream, so that the offspring will feed only on the larger of the two loads brought home by the two parents (Best-shot).

Of course, all kinds of intermediate cases and combinations can be imagined. Social composition functions observed in practice may well involve strands of all three rules mentioned above, as well as of others not yet identified. Not the best shot or the highest total but the location of the top decile, or the total of the best three shots, or the average of the best and worst shots, or the variance or skewness (and so on in indefinite variety) might be the relevant scoring formula. And there may be trade-offs: an excellent best-shot might partially compensate for a mediocre total, etc. When we think of the variety of collective enterprises — families and nations and armies and firms — and the vast range of their associated activities, we can realize how descriptively incomplete has been our standard assumption that all social composition functions take the form of simple summation.

B. EFFICIENT AND EQUILIBRIUM SOLUTIONS — SUMMATION VERSUS WEAKEST-LINK

In this Section, I will examine Nash-Cournot equilibrium outcomes versus efficiency conditions for provision of public goods, comparing the standard textbook Summation situation with the Weakest-link composition function of our Anarchia fable.

In making the comparison it will be helpful first to construct a somewhat novel diagrammatic illustration[1] of the standard public-good solution — for a simple world of two individuals A and B. In Figure 1, x_A on the horizontal axis represents A's production of the public good while x_B on the vertical axis represents B's contribution. A's indifference curves $U_A^o, U'_A, U''_A, \ldots$ on these axes are drawn solid, while B's $U_B^o, U'_B, U''_B, \ldots$ are shown as dashed. The shapes of the U_A curves can be explained as follows. First of all, other things equal an increase in x_B always benefits A, hence any move North in the diagram gets A onto a higher indifference curve. Moving East, on the other hand, means that for given x_B individual A is producing a larger amount of the public good. He will derive some marginal-utility benefit from doing so, but will also incur a marginal cost. It is reasonable to suppose that as he increases x_A from zero at the vertical axis, utility at first rises but ultimately begins to fall. These conditions dictate the shapes of the U_A curves in the diagram. A corresponding argument explains the shapes of the U_B curves.

Evidently, for any given x_B, the optimum x_A will be found where the associated U_A curve is horizontal. Hence, under Nash-Cournot assumptions, a Reaction Curve R_A can be passed through all these points of horizontality; the curve R_A shows A's chosen output of the public good, given B's output. The negative slope of R_A reflects the fact that B's production of the public good substitutes for A's own production in A's utility function. Hence diminishing marginal utility for A's own production of the public good sets in earlier, the larger is x_B.

It may be helpful to introduce some symbolism at this point. Individual $i=A,B$ is maximizing $U_i(X,y_i)$, where $X \equiv x_A + x_B$, and y_i represents his consumption availability of an ordinary private good. His choice between x_i and y_i is constrained by some production function $Q_i(x_i,y_i) = 0$. Then the private optimality solution can be written in two alternative self-explanatory notations as:

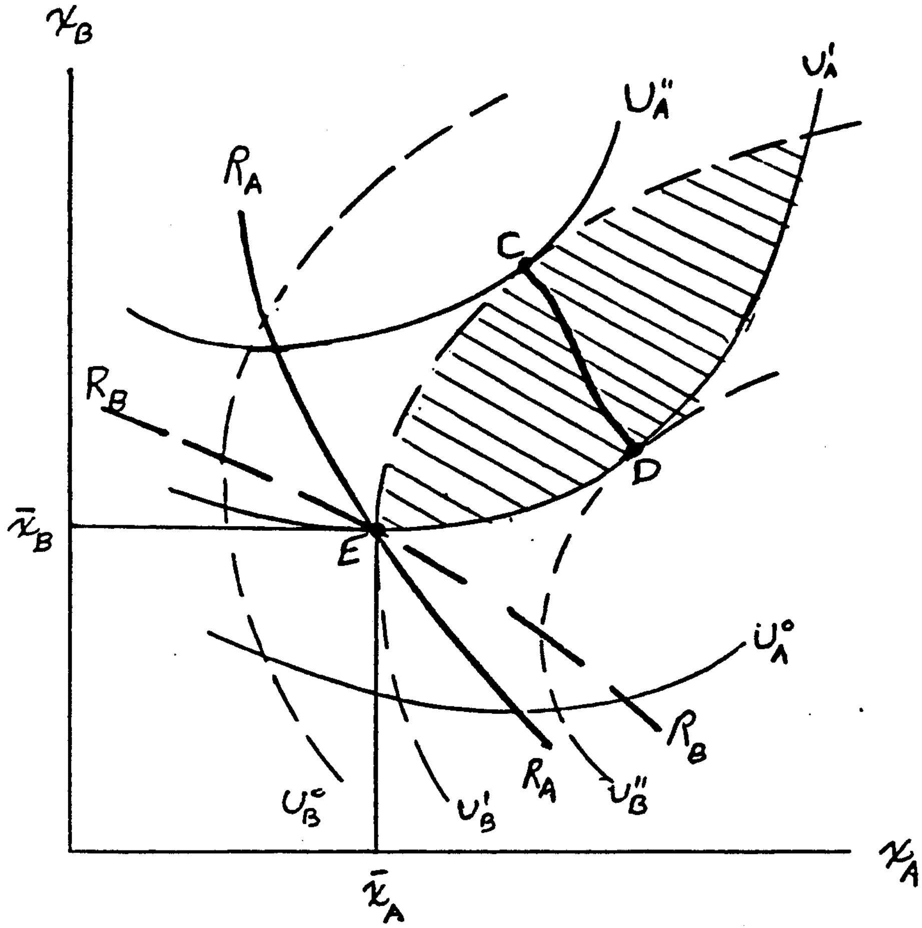

FIGURE 1: Efficient and equilibrium solutions, Summation Composition Function

$$\left.\frac{dy_i}{dX}\right|_{U_i} = \left.\frac{dy_i}{dx_i}\right|_{Q_i} \tag{1a}$$

or, $$MRS_i(X) = MC_i(x_i) \tag{1b}$$

Given our standard assumptions about the shapes of the Marginal Rate of Substitution (MRS) and Marginal Cost (MC) functions, a rise in x_B tends to lower $MRS_A(X)$ without affecting $MC_A(x_A)$ — assuming, as we usually do, that the individuals' production functions are not interdependent. A's indifference-curve slope on x_A,x_B axes in Figure 1 can be written:[2]

$$\left.\frac{dx_B}{dx_A}\right|_{U_A} \equiv -\frac{\partial U_A/\partial x_A}{\partial U_A/\partial x_B} \equiv -\frac{MRS_A(X) - MC_A(x_A)}{MRS_A(X)} \tag{2}$$

Since upon moving North in the diagram $MRS_A(X)$ tends to decrease and $MC_A(x_A)$ remain unchanged, the indifference curve slope becomes less negative or more positive. Thus, the indifference-curve minima shift to the left, explaining the negative slope of the R_A curve as asserted above.

For the other individual, we have.

$$\left.\frac{dx_B}{dx_A}\right|_{U_B} \equiv -\frac{\partial U_B/\partial x_A}{\partial U_B/\partial x_B} \equiv -\frac{MRS_B(X)}{MRS_B(X) - MC_B(x_B)} \tag{3}$$

A corresponding argument explains the negative slope of individual B's Reaction Curve R_B.

Evidently, the Nash-Cournot solution is at point $E = (\bar{x}_A,\bar{x}_B)$ in Figure 1, the intersection of the two Reaction Curves. (The equilibrium is stable if, as shown here, the R_A curve is absolutely steeper than the R_B curve.) Equally evidently, this equilibrium is not efficient. In fact, the shaded portion of the diagram pictures the familiar region of mutual advantage.

The curve CD is the portion of the contract curve (set of mutual indifference-curve tangencies) included within the region of mutual advantage, and the efficient solution must lie on this curve. The mutual-tangency condition can be written:

$$\frac{MRS_A(X) - MC_A(x_A)}{MRS_A(X)} = \frac{MRS_B(X)}{MRS_B(X) - MC_B(x_B)} \qquad (4)$$

Imposing the additional necessary condition for efficiency that $MC_A = MC_B$, this reduces to the familiar:

$$MC_A(x_A) = MC_B(x_B) = \sum_i MRS_i(X) \qquad (5)$$

That is, for efficiency in the production of a public good, each producer should set the Marginal Cost of his output equal to the sum of the individuals' separate Marginal Rates of Substitution. This condition will be met only by a subset of the points, or perhaps only by a unique point, within the range CD of the contract curve in Figure 1.

With this standard solution for the Summation rule of social composition as background, now consider the Weakest-link function of the Anarchia fable.

Figure 2 is constructed on the same principles as Figure 1. However, the 45° line where $x_A = x_B$ now plays an important role. In drawing individual A's indifference curves (solid) we clearly need only consider the region where $x_A \leq x_B$: A would never choose to produce more output than B does, since he would incur additional costs without generating any benefits for anyone under the social composition function $X = \min(x_A, x_B)$. And similarly, B's indifference curves (dashed) are shown only in the region where $x_B \leq x_A$.

Consider A's indifference curves $U_A^o, U_A', U_A'', \ldots$. Equation (2) continues to define their slopes. But within the interesting region Northwest of the 45° line, along any vertical line the slopes now will be the same -- since

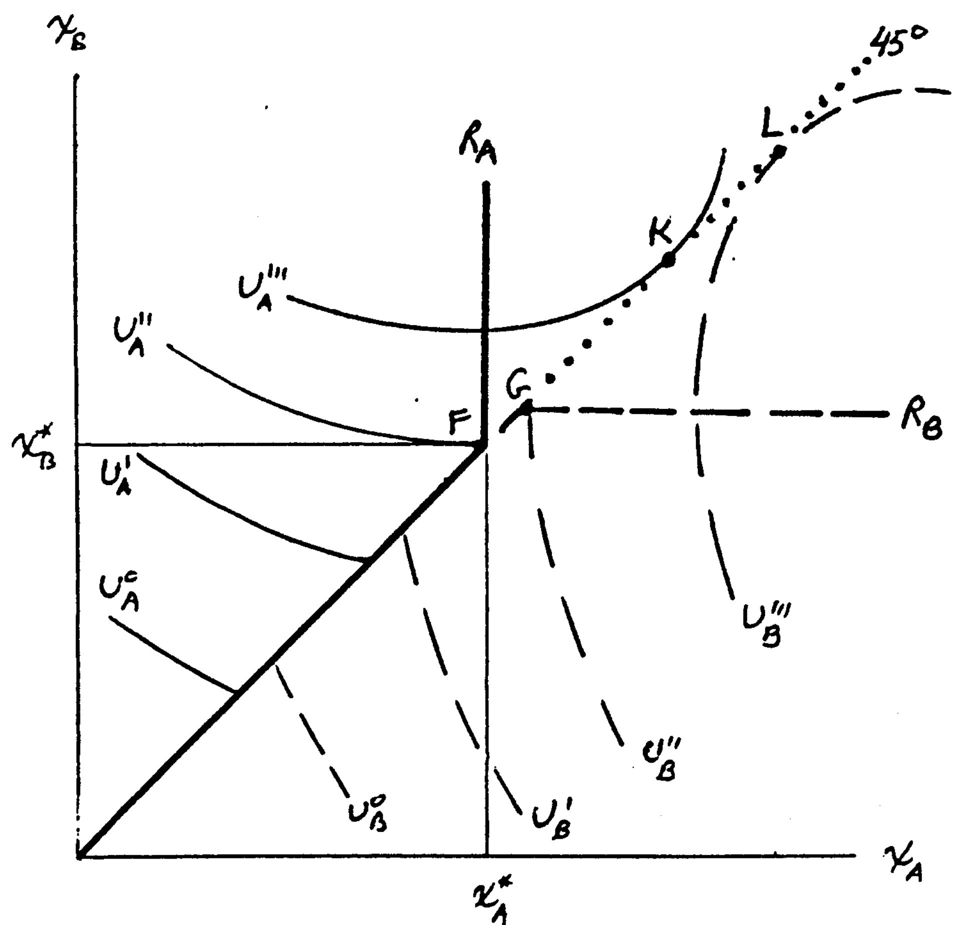

FIGURE 2: Efficient and equilibrium solutions, Weakest-link Composition Function

$MC_A(x_A)$ depends only on x_A, while $MRS_A(X)$ remains unchanged when $X = \min(x_A, x_B)$. For low levels of utility, A's indifference curves will have negative slope where they contact the 45° line, but eventually a point F will be reached where the contact occurs at a horizontal indifference-curve slope. Between points F and K in the diagram, contact occurs along a positive-sloped portion of the indifference curve. And, finally, above K, indifference curves will not touch the 45° line at all. Correspondingly for individual B: below point G the indifference curves have negative slope where they contact the 45° line; above point G, they have positive slope; while above point L they do not touch the 45° line at all.

As for the Reaction Curves, it will be evident that R_A must lie along the 45° line until point F is reached, after which it becomes vertical. A similar analysis for individual B indicates that R_B must also at first overlie the 45° line. But eventually, at the point G where a vertical-sloped indifference curve contacts the 45° line, the R_B curve becomes horizontal.

Under the strict logic of the Nash-Cournot equilibrium concept, the outcome may equally well fall anywhere along the entire range where R_A and R_B both overlie the 45° line, that is, anywhere in the range OF. (Note that F corresponds to the preferred X on the part of the individual <u>least desirous</u> of the public good, relative to its private Marginal Cost to him.) I want to argue, however, that in the spirit of "rational expectations" we would expect to find the final equilibrium at point F, the upper limit of this range, where $X = x^*$. While all the potential equilibria along OF are possible, there is a gain to all parties the nearer the outcome is to point F. Apart from the possibility of learning via repeated play of the game, it is not difficult to imagine a dynamic that would lead directly to this solution. Specifically, if the players moved in sequence, the first player (A, let us say) would "rationally" pick output $x_A = x^*$ in the confident belief that player B would then follow his lead. If B were the first player, knowing A's situation he would also choose point F.[3] So long as the choices are visible,[4] the economic logic leading to the solution at F seems compelling.[5] It should be noted, also, that this outcome is not vulnerable to the Prisoners' Dilemma paradox. In the Prisoners' Dilemma, once having achieved a mutually profitable outcome it pays each party to "defect". Here defection is not an issue — the cooperative outcome at F is perfectly stable, if it can only be achieved in the first place.

The (modified) Nash-Cournot equilibrium, then, can be said to be where all parties produce output $X = x^*$ such that:

$$\begin{aligned} MC_i(x_i^*) &= MRS_i(x_i^*) \qquad \text{for some } i \in I \\ \text{and} \quad MC_j(x_j^*) &\leq MRS_j(x_j^*) \qquad \text{for all } j \neq i. \end{aligned} \tag{6}$$

Note: Individual i is the one who first hits the equality in the upper equation.

As for the efficient solutions, inspection of Figure 2 will make it clear that these all lie along the 45° line in the range between points K and L — which corresponds to the portion of the contract curve within the region of mutual advantage. Along the 45° line below K, both parties move onto higher indifference curves the closer they get to K. Similarly, above point L there is a mutual gain as the parties move down toward L. Along the segment KL, however, neither can gain without some loss being imposed upon the other party. Within KL, a subset or possibly a unique point will also meet the production condition:

$$\sum_i MC_i(x_i) = \sum_i MRS_i(x_i) \quad (7)$$

C. THE EXTENT OF VOLUNTARY PROVISION -- DISCUSSION

Is there reason to believe that, under the Weakest-link composition function, voluntary provision of the public good will more nearly approximate the efficient amount?

The first point to note from Figures 1 and 2 is that, for both the Summation and Weakest-link functions, under-provision is to be anticipated. Nor is it immediately obvious that the extent of under-provision should be systematically greater in either case -- so long as we stick to the 2-party interaction pictured in the diagram. I want, however, to examine the systematic effect of two sorts of variation: (1) When we allow the parties to have distinctly unequal "weights" within the social total. (2) When we allow the number of independent interacting parties to increase.

Effect of unequal weight: For the standard Summation case, it is a well-known result that there tends to be "exploitation of the great by the small."[6] That is, larger members of social groups more than carry their share, at least comparatively speaking.. As a corollary, the more unequal the degree of greatness, the more the total provision will be. Despite the force of this argument, however, inequality does not have very great impact upon under-provision unless the disparity of weight becomes very large indeed.

Under the Weakest-link function, inequality of weight has essentially no effect upon the outcome. The reason is that, in both equations (6) and (7), a large-size individual i can be expected to have both high MC_i and high MRS_i, for given x_i. In Anarchia, suppose one person has a large wedge-shaped slice and another only a small slice of the island. For the former, the larger perimeter makes it more costly to add a marginal inch to the height of his sector of the dike -- but correspondingly, the larger area of his sector raises the benefit to him of another inch added to dikes all around.

Effect of larger numbers: In the standard Summation situation, it is well-known that at the Nash-Cournot equilibrium: (1) total production X of the public good rises as the population size I grows, but (2) the absolute and relative under-provision also increases with I.[7] (These results follow if neither the public good nor the private good is inferior.) Intuitively, think of a typical individual A in Figure 1 where B represents "everyone else". When others provide more of the public good, individual A now being in effect wealthier will want to consume more X, as well as more of the private good. Then his Reaction Curve R_A will have negative slope (x_A falls as x_B rises) but its absolute slope will be greater than unity ($x_A + x_B$ rises as x_B rises.) As X rises with increasing population size I, the typical individual A finds himself almost saturated with the public good X, and hence relatively unwilling to sacrifice consumption of the private good to generate even more X. We can say that A's weight in the social total relative to "everyone else" diminishes; being small, he will "exploit the great" by cutting x_A back almost one-for-one as x_B rises (the slope of his Reaction Curve approaches -1). But in the efficiency condition of equation (5), the rising $\sum_i MRS_i$ on the right-hand-side as I increases dictates that each individual i should increase his output x_i -- whereas, we have just seen, our typical individual A will want to decrease his output. So the degree of under-provision will rise sharply as numbers increase.

The result for the Weakest-link social composition function is quite different. The degree of under-provision may increase, but if so only weakly as I increases. Equation (7) says that the efficient social output X will tend to be unchanged as I increases, since the summations on both left-hand-side and right-hand-side of the equation rise more or less equally. Equation (6) indicates that the actual output will surely be unchanged, except when one of the new entrants is less desirous of the public good (relative to his private Marginal Cost) than the least desirous old member of the population. If the populations, old and new, were quite uniform there would be no change at all, but with heterogeneous populations there would remain some tendency for increased underprovision as I rises. Intuitively, think of each individual as supplying a link in a linear chain, whose strength is the public good enjoyed by all. If all individuals had identical Marginal Cost MC and Marginal Rate of Substitution MRS, adding another link would not affect either the equilibrium strength or the efficient strength of the chain. However, as a second approximation, a new entrant relatively undesirous of strength would tend to reduce the equilibrium amount provided relative to the efficient amount.

D. BEST-SHOT COMPOSITION FUNCTION

I will discuss the third social composition function, the Best-shot formula, only briefly. Recall that here the social total X of the public good represents the largest of the individual contributions x_i.

In the 2-party case, if we suppose as a first approximation that the individuals are identical, each would surely want to escape the burden — leaving the other to bear the cost. But if the parties diverge sufficiently in desire for the public good (relative to its private cost), the one most desirous will very likely end up being the provider. It is difficult to set up a plausible dynamic leading to a definite equilibrium under visible conditions,[8] however.

The efficient solution has two parts: choosing the low-cost supplier, and determining the quantity he produces. We can say:

For some individual k such that $TC_k(x_k) \leq TC_j(x_k)$ for all $j \neq k$:

$$MC_k(x_k) = \sum_i MRS_i(x_k)$$
$$\text{and} \quad x_j = 0, \text{ for } j \neq k. \tag{8}$$

The output should be produced by the individual whose Total Cost TC_k is lowest, and the amount provided should be such that his Marginal Cost MC_k equals the sum of all the Marginal Rates of Substitution MRS_i. (There may be more than one productive arrangement meeting these conditions.) But it is immediately clear that the actual provision will not get anywhere near this. Even if the most efficient producer were to become the single generator of the public good for the entire community, he would clearly produce only to the point where Marginal Cost equalled his <u>individual</u> Marginal Rate of Substitution:

$$MC_k(x_k) = MRS_k(x_k) \tag{9}$$

As numbers increase, the amount provided might rise slightly whenever a new entrant turns out to be the new low-cost provider. Nevertheless, it is clear that as I grows the Best-shot function implies drastically and increasingly unsatisfactory outcomes.

E. SUMMARY AND REMARKS

In the provision of public goods, our standard textbook assumption -- that the amount X socially available is simply the <u>sum</u> of the private amounts x_i individually produced -- is only one of a number of possible important social composition functions. Two other social composition functions were selected for comparative investigation: under the Weakest-link rule the socially available quantity corresponds to the <u>minimum</u> of the individual x_i, while under the Best-shot function the social availability X corresponds to the <u>maximum</u> of the x_i. It was shown that each of the three functions applies to important types of social phenomena. Furthermore, other public-good situations correspond to mixtures of composition functions or to more complicated functions than were discussed here. Consequently, exclusive concentration upon the Summation formula has led to a seriously distorted view of the private provision of public goods.

Each of the three social composition functions leads to a distinct pattern of provision of X, and in particular of underprovision of the public good as population size I increases. Without pretending to any degree of rigor, overall results are roughly summarized in Figure 3. The bold curves indicate the general trend of the efficient social total of X as population size I grows, while the faint curves indicate the equilibrium provision. The solid pair of curves indicate the working of the Summation function, the dashed pair stand for the Weakest-link function, and the dotted pair the Best-shot function. At I = 1, the public good is of course merely a private good; all curves coincide along the X-axis where I - 1 = 0.

When the social composition function for public goods represents the standard textbook Summation situation (solid curves), efficient

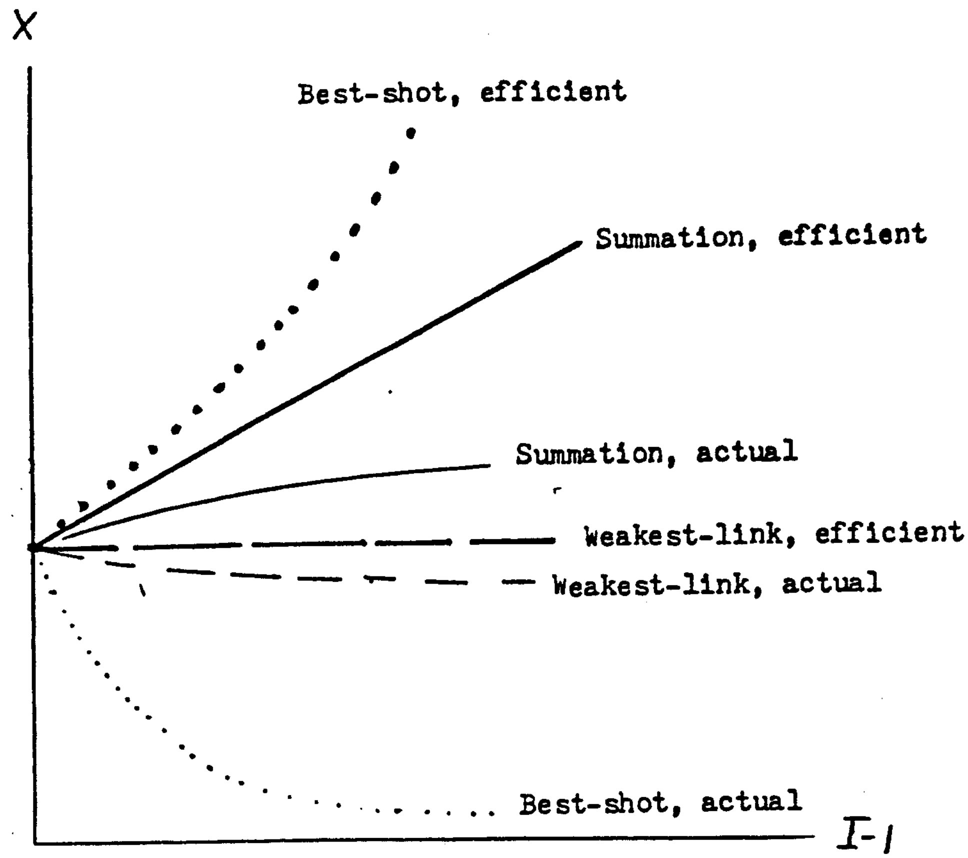

FIGURE 3: Trend of efficient and actual solutions as population grows

provision rises rapidly with I but actual provision only very slowly, hence absolute and relative underprovision of the public good tend to increase strongly with numbers. When the social composition function follows the Weakest-link formula (dashed curves) -- i.e., when the socially available amount X is the minimum of the amounts privately produced -- the efficient amount tends to remain unchanged as I grows while the actual equilibrium provision may fall slowly as I increases. Underprovision is mild, particularly when the population is homogeneous. (In fabled Anarchia, underprovision was alleged to amount to just 1.83%, a figure selected for dramatic rather than veridical illustration.) Finally, when the social composition function follows the Best-shot rule (dotted curves) -- i.e., when the socially available amount X is the maximum of the quantities privately produced -- efficient provision rises very sharply with I, while actual provision falls rapidly to a rather low level and then remains substantially constant. Clearly, underprovision is the most severe in this case.

I will conclude with a possibly significant application of the ideas proposed here.[9]

Observers of human behavior in disaster situations have repeatedly been struck by the degree of cooperation and self-sacrifice that these tragic events typically elicit. Refugees are often sheltered gratis in private homes, while food, blankets, medical services and the like flow copiously into the stricken area. An interesting observation: after the Alaskan earthquake of 1964, suppliers of essential goods and transport services actually reduced rather than raised their prices.[10] Furthermore, this is not merely a matter of assistance coming from outside: the victims themselves often display a remarkable degree of restraint and mutual cooperation. In the Alaskan case, for example, the purchasers of low-priced essential goods refrained from hoarding and took no more than an equitable quantity. Reports of a similar tenor have been made about the Halifax explosion of 1917,[11] the New York power blackout of 1965,[12] and more generally about the bombing disasters of World War II.[13] The sociologist C.E. Fritz (1961) has contended that shared communal disaster is a unifying experience without equal.

More specifically, the strong feeling of community identification, and the consequent unselfish cooperative efforts in repair and relief activity, are characteristic of the immediate postimpact period. The motivation for mutual assistance appears to erode after some days or weeks, and jealousies over relief distribution and the like typically lead to widespread recrimination as a more normal society is restored. And similarly for the "counter-disaster syndrome," the tendency of outsiders to rush assistance to the disaster zone. Some time later a reaction tends to set in, frequently leading to bad relations between victim and support populations.

Before seeing the evidence, it would have been plausible to conjecture that the weakening of normal control mechanisms in the immediate disaster period would allow antisocial elements to violate laws and customs, and that even normally law-abiding individuals would attach much higher priority than before to the selfish necessities of personal survival as against serving community interests. Certainly, popular novelistic treatments typically paint a lurid picture of disaster aftermaths. And indeed there is some divergence of opinion and evidence on this score. The sociologist Sorokin (1942) maintained that disaster releases a variety of extreme behaviors both good and bad. And incidents of looting in the Chicago blizzard of February 1967 counterbalance the good-fellowship observed during the 1965 New York blackout. Or to cite military experience: front-line troops who have heroically withstood an enemy may, as at Waterloo, suddenly dissolve into a frantic, pushing mob. It is this juxtaposition or alternation of good and bad behavior that needs explaining.

Dacy and Kunreuther (1969) argued that disaster increases "community feeling." De Alessi (1975) pointed out that we need not call upon a shift of the utility function; the evidence is consistent simply with a movement on a given utility map. In particular, potential donors with a positive "taste" for charity now find new groups of impoverished targets for benevolence.

Without necessarily disagreeing with these explanations, so far as they go, I want to emphasize the public-good aspect of the problem. The alliance we call society, normally not in danger of collapse, is threatened in time of disaster. In these circumstances alliance-supportive activities, cooperativeness and self-sacrifice, become an important public good. But, I also want to argue, a public good in large part describable in terms of the Weakest-link social composition function.

In our Anarchia example, the public need for flood protection was met by a technological solution -- construction of dikes. Here we are speaking of behavioral solutions. While the underlying principles remain the same, the greater volatility of behavior allows variation over time in response to the perceived magnitude of the threat. In normal periods when the threat is small, the dikes already in place are sufficient and there is no need for exceptional behavior. In effect, the "permanent" component of individuals' adjustment to the long-term probability distribution of threats provides more than enough protection. But as the specific threat grows and appears to be overwhelming the existing permanent defenses, the Weakest-link formula elicits a "transitory" additional adjustment: members of the community turn out to plug the leaks, to add sandbags on top, to dig out alternative water channels, etc.

While not intrinsic to the logic of the Weakest-link rule, behavioral response to disaster may be intensified by a critical-mass aspect of the social alliance. Instead of the loss rising more or less in proportion to the magnitude of the non-countered threat, an all-or-nothing situation may be involved. In terms of the dike metaphor: if the dike is not breached, little or no loss will be suffered, but once breached even by a little bit the whole structure may give way. In terms of human cooperative behavior, a critical-mass situation (see Schelling [1978]) implies two stable equilibria: survival of the alliance intact, or its complete collapse.

Thus, consistent with the evidence, the goodness of behavior to be expected in disaster will vary in response to the magnitude of the threat. In normal times people behave in a conventionally cooperative way because individually they find it profitable to do so: while there is some slippage around the edges, on the whole the social control mechanisms deter evildoing. As the threat grows, eventually the balance may hang by a hair — so that any single person can reason that his own behavior might be the social alliance's weakest link. It is in these circumstances that the Weakest-link rule elicits extraordinary heroism and self-sacrifice even from normally selfish people. But once the balance seems to be swinging the other way, so that even heroism can no longer rescue the situation — or, at least, once many people believe that others might so perceive matters — social collapse is likely to be swift.

FOOTNOTES

[1]Similar diagrammatics are employed, for a somewhat different purpose, in my "Natural Economy Versus Political Economy" (1978).

[2]More explicitly: $$\frac{dx_B}{dx_A}\bigg|_{U_A} \equiv - \frac{\frac{\partial U}{\partial X}\frac{dX}{dx_A} + \frac{\partial U}{\partial y_A}\frac{dy_A}{dx_A}}{\frac{\partial U}{\partial X}\frac{dX}{dx_B}}$$

$$\equiv - \left(1 + \frac{dy_A}{dx_A}\frac{\partial U/\partial y_A}{\partial U/\partial X}\right)$$

$$\equiv - (1 - MC_A/MRS_A)$$

$$\equiv - \frac{MRS_A - MC_A}{MRS_A}$$

[3]Lacking this knowledge, B might tentatively choose point G, but then would dismantle or liquidate his "excessive" output after A's choice of point F.

[4]If the choices are not visible (e.g., if no citizen of Anarchia could observe the height of other citizens' dikes), the problem becomes more complex. Presumably, some kind of probabilistic mixed strategy will become privately optimal. (A logically parallel example, though for a much simpler binary-strategy rather than continuous-strategy interaction, is the "Battle of the Sexes" game analyzed by Luce and Raiffa [1957], pp. 90-94.) Such a situation will probably lead to a greater shortfall of output in comparison with the efficient solution. Of course, under the Summation Rule as well, invisibility may lead to a greater shortfall of output.

[5]A somewhat related issue is discussed in Luce and Raiffa (1957), pp. 106-107. In a game with multiple Nash equilibria, those equilibria with

payoffs inferior for both players to the payoffs of some other strategy pair are termed "jointly inadmissible." In our situation the equilibria along the 45° line short of point F are all dominated by point F, and so are jointly inadmissible. On the other hand, as is evident from the Figure 2 diagram, point F itself is jointly inadmissible in view of the possibility of outcomes even higher up on the 45° line. Thus, like the Prisoners' Dilemma, the game pictured here does not have what Luce and Raiffa call a solution in the strict sense. But the equilibrium at F, like the "uncooperative" solution for the Prisoners' Dilemma, seems entirely stable.

[6]Olson (1965), p. 29.

[7]See Chamberlin (1974 and 1976).

[8]As under the Weakest-link rule, we would expect invisible interactions to lead to a choice of a mixed strategy by each party.

[9]These ideas derive in part from my "Disaster and Recovery: A Historical Survey" (1963), "Disaster Behavior: Altruism or Alliance?" (1975 [1967]), and from the discussions in De Alessi (1975) and Douty (1972).

[10]Dacy and Kunreuther (1969). To underline the anomalous element for conventional theory, truck rates to Alaska were lowered only for those commodities that could not be conveniently shipped by boat -- the competitive mode of transport!

[11]Prince (1920).

[12]Rosenthal and Gelb (1965).

[13]Iklé (1958).

REFERENCES

Chamberlin, John, "Provision of Collective Goods as a Function of Group Size," Am. Pol. Sci. Rev., v. 68 (June 1974).

__________, "A Diagrammatic Exposition of the Logic of Collection Action," Public Choice, v. 26 (Summer 1976).

Dacy, D.C. and Kunreuther, H.R., The Economics of Natural Disasters: Implications for Federal Policy (New York, 1969).

De Alessi, Louis, "Toward an Analysis of Postdisaster Cooperation," Am. Econ. Rev., v. 65 (March 1975).

Douty, Christopher M., "Disaster and Charity: Some Aspects of Cooperative Economic Behavior," Am. Econ. Rev., v. 62 (Sept. 1972).

Fritz, C.E., "Disaster," in R.K. Merton and R.A. Nisbet, Contemporary Social Problems (Harcourt, 1961).

Hirshleifer, J., "Natural Economy Versus Political Economy," J. Social & Biol. Structures, v. 1 (October 1978).

__________, "Disaster and Recovery: A Historical Survey," RAND Corp. Memorandum RM-3079-PR (April 1963).

__________, "Disaster Behavior: Altruism or Alliance?" UCLA Economics Dept. Discussion Paper No. 59 (May 1975 [March 1967]).

Iklé, Fred C., The Social Impact of Bomb Destruction (Univ. of Oklahoma Press, 1958).

Luce, R. Duncan and Raiffa, Howard, Games and Decisions (John Wiley & Sons, 1957).

Olson, Mancur, The Logic of Collective Action (Harvard Univ. Press, 1965.)

Prince, S.H., Catastrophe and Social Change (Columbia Univ. Press, 1920).

Rosenthal, A.M. and Gelb, Arthur, The Night the Lights Went Out (Signet, 1965).

Schelling, Thomas C., Micromotives and Macrobehavior (W.W. Norton, 1978).

Sorokin, P.A., Man and Society in Calamity (Dutton, 1942).