SPECIFIC CAPITAL, LONG TERM IMPLICIT CONTRACTS,

AND TEMPORARY LAYOFFS

by

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The new "implicit contract theory," whose proponents claim explains temporary layoffs, has received considerable attention. This should not be surprising since contract theory's central tenet that labor market transactions can best be understood by examining the long term relationships that develop between employers and employees has a natural intuitive appeal. However, in this paper, we argue that there are a number of essential elements missing from the contract theoretic explanation for temporary layoffs. The term "temporary layoff" connotes an image of a firm, facing intertemporal fluctuations in demand, finding it optimal in successive periods of time to employ, temporarily lay off, and then recall the same worker. This description suggests that a distinguishing characteristic of a temporary layoff is that it is in fact temporary since the laid off worker is recalled. That the firm chooses to recall the laid off worker when demand recovers rather than hiring a new worker suggests that there is a value to the firm associated with a continuing worker-firm attachment. That the laid off worker is available for recall suggests that there is a value to the worker associated with a continuing worker-firm attachment. It is therefore troubling, given this characterization of a temporary layoff as an inherently intertemporal phenomenon involving considerations of the value of long term worker-firm attachments, that the prototype implicit contract model is a one period model in which labor is assumed to be homogeneous.[1]

The prototype implicit contract model (e.g., Azariadis (1975), Burdett and Mortensen (1980)) supposes that an individual firm is faced with the problem of deciding on the size of its labor pool for the coming period prior to its knowing the actual level of product demand that will occur in the period. Both workers and firms are presumed to be perfectly mobile ex ante (prior to the realization of product demand.) An individual firm is a perfect competitor in this ex ante labor market implying that the firm can contract for an unlimited number of workers by offering a contract that offers expected utility (expected income if workers are risk neutral) at least as good as is available elsewhere. Ex post (after workers have been contracted for and after product demand is realized), workers are assumed to be either perfectly immobile (e.g., Azariadis (1975)) or at least partially immobile (e.g., Baily (1977) and Holmstrom (1981)). The degree of ex post immobility is important because this determines the ex post opportunity cost of a worker's time. A worker is "temporarily" laid off in this context when ex post realized demand is sufficiently low to cause the ex post value of the marginal product to fall below the ex post opportunity cost of the worker's time.

A fundamental problem with this explanation for layoffs is the inconsistency between the development of long term attachments in the labor market and the combined assumptions of labor homogeneity and ex ante mobility for all workers prior to each contract period. The underlying basis for the existence of implicit contracts is the inherent immobility in the labor market caused by the costs of reallocating workers in the labor market at each moment in time. The reallocation costs are presumably due to such factors as the costs of information, hiring costs, moving costs, and the accumulation of firm specific human capital. In light of these factors underlying labor immobility, while it may be reasonable prior to any given period to characterize new entrants or reentrants to the labor force (who by definition have no attachment to any particular firm) as being both homogeneous and mobile, surely workers who have been attached to a specific firm in the previous contract period are not so mobile or homogeneous.

Another troubling characteristic of the contract theoretic explanation for layoffs is that the probability of layoffs in any given period depends on the variance of the price distribution associated with the price uncertainty in that period and not on intertemporal variations in demand. This is easily seen as no layoffs occur if product demand is certain but nevertheless varies over time. A peculiar consequence of this property is observed by considering a situation in which the ex ante distribution of product demand varies over time. In particular, if one considers a sequence of contract periods in which the firm's ex ante expected product demand is initially at trend level, falls below trend for one period, and then returns to trend level, then contract theory has nothing to say about the predicted layoff rates as a result of such intertemporal demand variability. In fact, if the within period variance of prices associated with the price uncertainty in each period is lower in the period of relatively low demand than it is in the periods with expected trend level demand, then the probability of layoffs may be higher in the periods of expected trend level demand than it is in the period of relatively low expected demand. This possibility of a positive correlation between the incidence of layoffs and product demand is certainly not consistent with empirical observation.[2]

Given these objections to the existing contract theoretic explanation for layoffs, this paper proposes an alternative contract model. In what follows, a model in which long term mutually advantageous attachments develop between workers and firms is formulated. The primary factors underlying the development of long term attachments are presumed to be the accumulation of firm specific skills, hiring costs, and information costs. Since long term attachments develop, a worker considering "attaching" himself to a particular firm is not only concerned with the expected utility available for the coming period (as is assumed in the existing contract models) but is concerned with the long term expected utility over many periods. Hence, long term implicit contracts are hypothesized. This specification allows us to analyze the interaction between the incentives for layoffs, the causes of long term attachments developing and the optimal intertemporal wage structure of the firm.

Much of the remainder of the paper is concerned with working out the properties of the model. The model is one in which the firm must make a number of interdependent decisions in each period of time that affect its current and future profits. The firm must decide on whether to permanently lay off and/or temporarily lay off any of the workers in its existing pool of experienced workers, it must decide on whether or not it should add to this pool by hiring and training new workers, and although the firm is presumed to be constrained to offer a long term contract with expected discounted income at least as good as is available elsewhere, the firm has flexibility with regard to how wages are distributed over time. Individual worker behavior plays an important role in the model as well. In particular, the worker's decision on whether or not to quit when temporarily laid off plays a fundamental role in the analysis.

The Model

Consider a model with the following assumptions:

(i) A worker's productivity is assumed to increase with firm specific experience. This increase in productivity is attributed to the natural accumulation of skills with experience and the simple hiring costs associated with processing a new worker with the firm. This type of worker heterogeneity is incorporated by assuming two classes of workers: senior workers who are assumed to have at least one period of experience with the firm in period i are denoted by L_i^e and new, entry level workers are denoted by L_i^n. As a first approximation, assume that the difference in productivity implies that new, entry level workers produce no output in their initial period of experience with the firm and experienced workers produce output according to a strictly concave production function given by:

(1) $$F(L_i^e) \quad \text{where} \quad F' > 0,\ F'' < 0.$$

Hours per worker are assumed to be fixed and normalized to one.

(ii) Since "experience" can only be acquired within the firm, the firm has only a limited number of experienced workers available in any given period. Defining m_i^e to be the number of available experienced workers in period i and R_i^e the number actually contracted for in period i, it must be true that $m_i^e \geq R_i^e$. The determinants of m_i^e and R_i^e are discussed below.

(iii) The firm is assumed to be a price taker for all periods. The type of ex ante-ex post uncertainty posited by the typical contract model is not considered here. That type of uncertainty requires that the firm make a decision with regard to the actual number of workers it will contract for in the current period prior to knowing the actual level of demand for the period. Alternatively, we suppose that the decisions made in the current period that take effect in the current period are made with certain knowledge of demand. For instance, the decisions on the number of experienced workers to retain, to temporarily layoff and the number of new workers to hire in the current period are made with certain knowledge of demand. However, the firm may be uncertain in the current period about future demand. Since this is a model in which it takes time for workers to acquire experience, this implies that the firm must, in general, make a decision on the size of the available experienced labor pool for a given period prior to knowing the actual level of demand for the period. Formally, the type of uncertainty presumed in this analysis implies that from the perspective of any given period i, the firm knows with certainty the price in period i, P_i, but P_{i+k} $(k>0)$ is a random variable.

(iv) The firm faces a competitive constraint for new workers (assumed to be risk neutral) which requires that the expected discounted income associated with contracting with the firm is at least as good as is available elsewhere. This assumption takes the form:

$$W_i^n + V_{i+1}^e \rho \geq V_i^n, \ \rho < 1 \tag{2}$$

where V_i^n is the market determined expected discounted income available elsewhere, W_i^n is the wage promised to the new worker in period i, V_{i+1}^e is the expected discounted income promised to the workers for the future (i.e., the present discounted value of income for all periods starting with period $i+1$) and ρ is the discount factor.[3] It is assumed that there are no problems of enforceability.

The formulation of the constraint (2) with the decision variables W_i^n and V_{i+1}^e introduces a convenient analytical device. Instead of the firm specifying to new workers the initial wage along with an explicit specification of future wages and future permanent and temporary layoff probabilities, the firm simply promises that the expected discounted income in the future will equal or exceed V_{i+1}^e. In accordance with this, it is assumed that in each future period the firm will explicitly specify the wage and the probabilities of permanent and temporary layoffs for that particular period along with a promise of what expected discounted income will be in the future beyond that period. This "rolling" process works in the following manner. Suppose that period 0 is the initial period of the firm's existence implying that the firm has no available experienced workers in period 0 (i.e., $m_o^e = 0$). The firm hires L_o^n new workers in period 0 by promising them expected discounted income V_o^n. Since new workers in period 0 have an expected discounted income at least as good as is available elsewhere, no new workers quit in period 0 and hence $m_1^e = L_o^n$. In period 1, the firm faces the constraint that the expected discount income starting with period 1 must equal $V_1^e = (V_o^n - W_o^n)/\rho$. This constraint takes the form:

$$(3) \qquad \frac{R_1^e}{m_1^e}\left[\frac{L_1^e}{R_1^e}\ (W_1^e + V_2^e\rho) + (1 - \frac{L_1^e}{R_1^e})\ K_1\right] + (1 - \frac{R_1^e}{m_1^e})\ Y_1 \geq V_1^e$$

It is helpful to explain (3) in a piecemeal fashion. A worker who is in the available experienced labor pool in period 1 faces a number of possibilities. The worker may be employed, permanently laid off, temporarily laid off or may choose to quit. Permanent and temporary layoffs are distinguished in this analysis by specifying that a worker who is permanently laid off is permanently separated from the firm with the provision that the worker cannot be recalled. Alternatively, a worker who is temporarily laid off is assumed to be in the pool of available experienced workers for the following period (given that the worker does not quit). This distinction essentially represents a difference in commitment on the firm's part between telling a worker when laid off that the firm has no future plans that include the worker and telling a worker when laid off that the firm expects to be able to recall the worker within a reasonable period of time.

Since within the class of experienced workers all workers are assumed to be homogeneous, permanent layoffs are made randomly out of the experienced labor pool. This implies that the probability of being permanently laid off is given by $1 - (R_1^e/m_1^e)$ (that is, it is equal to one minus the probability of being recontracted for). Y_1 represents the expected discounted income available to a permanently laid off worker through the best use of the worker's time given that he will not be recalled. In what follows, the firm takes Y_1 to be exogenous where $Y_1 \leq V_1^n$ due to search and transfer costs. Hence, the term $(1 - (R_1^e/L_1^e))Y_1$ on the LHS of equation (3) represents the expected discounted income available elsewhere net of search costs weighted by the probability of being permanently laid off.

A worker who is not permanently laid off, may be temporarily laid off, employed or choose to quit. Temporary layoffs are made randomly out of the retained experienced labor pool, R_1^e. The probability of being temporarily laid off is given by $1 - (L_1^e/R_1^e)$. A worker who is temporarily laid off may either quit (in which case by definition he is not available for recall) or may remain attached to the firm. K_1 represents the expected discounted income available to a temporarily laid off worker through the best use of the worker's time taking into account the possibility of quitting. The determinants of K_1 and in particular the manner in which the firm can alter K_1 are discussed below. Given these definitions, the term $(R_1^e/m_1^e)\ (1 - (L_1^e/R_1^e))K_1$ on the LHS of (3) represents the expected discounted income of a temporarily laid off worker weighted by the probability of being temporarily laid off conditional on the probability of not being permanently laid off.

A worker who is actually employed earns W_1^e for period 1 and is in the pool of available experienced workers for period 2 with future expected discounted income given by V_2^e. In order to concentrate on the possibility of temporarily laid off workers quitting, it is assumed that employed workers do not quit. In order to justify this rather strict assumption, it is assumed that V_i^e is always chosen so that $V_i^e > Y_i$ (which implies a worker would always prefer to remain attached to the firm). A more general framework would explicitly analyze the possibility of employed workers quitting as well as the possibility of temporarily laid off workers quitting.

In total, then, the LHS of (3) represents the expected discounted income for each experienced worker starting with period 1 taking into account current wages, the probabilities of permanent layoffs, temporary layoffs and quits, and future expected income. The constraint (3) requires that this expected discounted income equal or exceed that previously promised to the workers.

In period 1, the firm may be hiring new workers as well as rehiring experienced workers. The contract constraint the firm faces for new workers in period 1 is (2) moved one period forward. Observe that this implies that the firm's promise made to both new workers and experienced workers in period 1 for future expected income starting in period 2 is the same. This is reasonable in this context because starting in period 2, workers hired in either period 0 or 1 are by assumption identical in terms of productivity. Hence, for any given period i, it is assumed that the firm faces the following two constraints for new workers and experienced workers, respectively:

$$(2)' \qquad W_i^n + V_{i+1}^e\rho > V_i^n \quad , \quad V_i^n > Y_i$$

$$(3)' \qquad \frac{R_i^e}{m_i^e}\left[\frac{L_i^e}{R_i^e}(W_i^e + V_{i+1}^e\rho) + \left(1 - \frac{L_i^e}{R_i^e}\right)K_i\right] + \left(1 - \frac{R_i^e}{m_i^e}\right)Y_i > V_i^e \quad ,$$

(V) The quit function of temporarily laid off workers, but not the actual number of quits, is taken by the firm as given. The number of quits will depend on the firm's wage policy. It seems reasonable to assume that quits by temporarily laid off workers are positively related to the net gain associated with quitting. Formally, denoting γ_i as the percentage of temporarily laid off workers that quit, γ_i is assumed to be positively related to G_i where $G_i = Y_i - (B_i + V_{i+1}{}^e\rho)$(where G_i is the difference between the expected discounted income associated with quitting and the expected discounted income associated with remaining attached to the firm.)[4] B_i is the expected income available to a temporarily laid off worker in period i given that the worker remains available for recall. In general, B_i can be thought to consist of unemployment benefits and the income equivalent of the value of the additional leisure a temporarily laid off worker acquires.[5] Observe that the expected discounted income associated with remaining attached to the firm is assumed to be $B_i + V_{i+1}{}^e\rho$. This represents an assumption that employed workers and temporarily laid off workers in period i are indiscriminately thrown back together in the attached experienced labor pool and treated equally in period $i + 1$. The key insight here is that firms can reduce the incentive for temporarily laid off workers to quit by specifying a wage structure that increases with job tenure.

Now one obvious possibility is that γ_i is a simple step function taking a value of 0 when $G_i < 0$ and a value of 1 when $G_i > 0$. Indeed, all of the results that follow are consistent with this specification for γ_i.[6] However, since it is easy to imagine reasons why γ_i would be a continuously differentiable function of G_i and since it is simpler analytically if γ_i is a continuously differentiable function of G_i, we assume that γ_i has the following properties:

(4) $\gamma_i = \gamma_i(G_i)$ where

(i) $\frac{\partial \gamma_i}{\partial G_i} = \gamma_i' > 0$

(ii) $\gamma_i' > 0$ for $G_i > 0$

(iii) $\gamma_i = \gamma_i' = 0$ for $G_i \leq 0$

Equation (4) indicates that γ_i is endogenous through V_{i+1}^e. Since by assumption, $V_i^e > Y_i$ ($\forall i$), it is assumed that $Y_i > B_i + Y_{i+1}\rho$ in order to insure that there is some possibility that $G_i > 0$.

Since all attached experienced workers are assumed to be homogeneous, the percentage of temporarily laid off workers who quit in period i, γ_i, gives the probability that any particular temporarily laid off worker will quit.[7] Hence, the expected discounted income of a worker temporarily laid off in period i, K_i, is given by:

$$K_i = \gamma_i Y_i + (1 - \gamma_i)(B_i + V_{i+1}^e \rho)$$

(vi) The available experienced labor pool in period i, m_i^e, consists of the experienced workers who were actually employed in period $i - 1$, the experienced workers who were temporarily laid off but did not quit in period $i - 1$, and the new, entry level workers hired in period $i - 1$. This implies $m_i^e = L_{i-1}^e + (1 - \gamma_{i-1})(R_{i-1}^e - L_{i-1}^e) + L_{i-1}^n$. Observe that this specification allows us to analyze the optimality of a seniority based hiring system (i.e., under what circumstances, if any, will the firm find it optimal to hire new workers at the same time it is either permanently or temporarily laying off experienced workers?)

Given the above assumptions, it is now possible to fully specify the firm's maximization problem. The firm in period t is concerned with maximizing the sum of expected discounted profits from period t onwards given by:

$$(5) \qquad P_tF(L_t^e) - W_t^eL_t^e - W_t^nL_t^n + E_t\sum_{k=t+1}^{\infty}[P_kF(L_k^e) - W_k^eL_k^e - W_k^nL_k^n]\rho^{k-t}$$

where E_t is the expectational operation conditional on information known at time t. Conceptually, the firm's maximization problem can be described in the following manner. The firm at the beginning of the current period t inherits a pool of experienced workers m_t^e and takes as given the labor cost parameters V_t^e, Y_{t+k} and B_{t+k} (for all $k > 0$). The firm knows P_t with certainty but is uncertain about P_{t+k} for $k > 0$. The firm must make decisions on the number of experienced workers to retain, to temporarily lay off and the number of new workers to hire in the current period. It must also decide on the intertemporal distribution of wages between the current period and the indefinite future. These are the binding decisions that the firm must make because these are decisions that take effect in the current period. It makes these binding decisions based on its knowledge of current demand and its expectations of future demand. The firm specific capital accumulation and the resulting long term implicit contracts necessitate the firm considering the impact of the current decisions on expected discounted future profits. Hence the intertemporal maximization problem. Formally, the firm maximizes (5) subject to the constraints (2)', (3)', $m_i^e > R_i^e$, $R_i^e > L_i^e$ and $V_i^e > Y_i(\forall_i)$. The firm chooses L_i^e, L_i^n, W_i^e, W_i^n, V_{i+1}^e and R_i^e $(\forall_i > t)$ contingent on the realization of product demand in period i.

After some work, the optimality conditions reduce to:[8]

(6) $$P_iF'(L_i^e) = B_i + \mu_i + \gamma_i(Y_i - B_i - E_i(\sum_{k=i+1}^{\infty} P_kF'(L_k^e)\rho^{k-i})) \qquad (\forall_i > t)$$

(7) $$P_iF'(L_i^e) + E_i(\sum_{k=i+1}^{\infty} P_kF'(L_k^e)\rho^{k-i}) = Y_i + \delta_i \qquad (\forall_i > t)$$

(8) $$z_i = E_i(\sum_{k=i+1}^{\infty} P_kF'(L_k^e)\rho^{k-i}) - V_i^n < 0,\ L_i^n < 0,\ L_i^n \cdot z_i = 0 \qquad (\forall_i > t)$$

(9) $$(R_i^e - L_i^e)\gamma_i'[B_i + E_i(\sum_{k=i+1}^{\infty} P_kF'(L_k^e)\rho^{k-i}) - Y_i] = 0 \qquad (\forall_i > t)$$

(10) $$\mu_i(R_i^e - L_i^e) = 0 \qquad (\forall_i > t)$$

(11) $$\delta_i(M_i^e - R_i^e) = 0 \qquad (\forall_i > t)$$

and (2)' and (3)' (associate the Kuhn-Tucker multiplier μ_i with the constraint $R_i^e > L_i^e$, and the Kuhn-Tucker multiplier δ_i with the constrant $m_i^e > R_i^e$). Equations (6)-(11) are an independent subsystem of the optimality conditions that fully determine L_i^e, L_i^n, R_i^e and V_{i+1}^e $(\forall_i > t)$. The contractual wages W_i^e and W_i^n are determined recursively by (2)' and (3)' respectively, given L_i^e, L_i^n, R_i^e and V_i^e.

Comparing (6)-(11) with the optimality conditions found in the existing contract models indicates substantial differences. In most contract models, there is a condition that implies that temporary layoffs are optimal in any given period only if the value of the marginal product for that period associated with fully employing all contractual workers falls below the opportunity cost of a worker's time for that period. If we altered our assumptions by specifying workers to be perfectly immobile after being contracted for, this is precisely the implication of condition (6) (since in this event $\gamma_i = 0$ and condition (6) would become $P_i F'(L_i^e) = B_i + \mu_i$.) However, since this specification allows for the possibility that workers are not perfectly immobile after being contracted for, condition (6) includes an additional term that takes into account the cost of potential quits by temporarily laid off workers.

Conditions (7) and (8) provide insights into the circumstances under which it is optimal to permanently lay off workers in the experienced labor pool and hire new workers, respectively. Condition (7) indicates that experienced workers should be permanently laid off only if the expected present discounted value of the marginal product in period i associated with retaining all of the available experienced workers falls below the expected discounted income a worker can expect to earn elsewhere (net of search costs). Condition (8) indicates that a new worker should be hired in period i only if the expected present discounted value of the marginal product associated with hiring a new worker is equal to or exceeds the total discounted cost of the new worker to the firm. Since $V_i^n > Y_i$, conditions (7) and (8) together imply that if any experienced workers are permanently laid off in period i, then no new workers are hired in period i. Hence, at least in one sense, seniority based hiring is optimal in this context.

Condition (9) provides insight into the optimal intertemporal distribution of wages. Specifically, observe that if there are no temporary layoffs in period i, then the LHS of condition (9) for period i is equal to zero. This implies that if there are no temporary layoffs in period i, the intertemporal distribution of compensation between W_i^e and V_{i+1}^e is indeterminate. However, when there are temporary layoffs in period i, condition (9) suggests that the intertemporal distribution of compensation between W_i^e and V_{i+1}^e becomes significant insofar as it affects the probability that the workers who are temporarily laid off in period i will quit.

The primary objective of this analysis is to determine the circumstances under which it is optimal for the firm to make temporary and/or permanent layoffs following a downturn in demand. The following proposition begins to identify some of the important factors (proofs of all propositions are contained in the appendix).

Proposition 1: The following are necessary conditions for temporary layoffs to be optimal in period i:

(i) $$E_i(\delta_{i+1}) > 0$$

(ii) $$B_i + E_i\left(\sum_{k=i+1}^{\infty} P_k F'(L_k^e)\rho^{k-i}\right) > P_i F'(R_i^e) + E_i\left(\sum_{k=i+1}^{\infty} P_k F'(L_k^e)\rho^{k-i}\right)$$

(iii) $$B_i + E_i\left(\sum_{k=i+1}^{\infty} P_k F'(L_k^e)\rho^{k-i}\right) > Y_i$$

(iv) either $\gamma_i' = \gamma_i = 0$ or $B_i + E_i\left(\sum_{k=i+1}^{\infty} P_k F'(L_k^e)\rho^{k-i}\right) = Y_i$ (or both)

Condition (i) of Proposition 1 indicates that temporary layoffs are optimal in period i only if there is a positive probability associated with the firm recalling all laid off workers in period $i + 1$. This suggested that temporary layoffs may be understood as a means of temporarily "storing" workers during temporary downturns in demand. That the firm only lays off a worker temporarily if there is a positive probability of recall suggests that what is relevant for temporary layoffs is not so much that current demand is lower than expected but rather that current demand is low relative to both past and expected future demand. In other words, uncertainty of product demand is not a necessary factor for explaining temporary layoffs in this context. Temporary layoffs are seen to be optimal in this context as a means of the firm holding inventories of experienced attached workers during intertemporal downturns in demand whether these downturns are anticipated or unanticipated. This represents a significant departure from the typical contract theoretic explanation for layoffs which relies heavily on demand being uncertain. This is not meant to deny that temporary layoffs may be explained as a result of the firm holding an inventory of workers against potential unexpected short run variations in demand (which is the basis of the standard contract theoretic explanation) but that temporary layoffs may also be explained as a means of the firm smoothing the variations in its attached, experienced labor pool by using temporary layoffs during intertemporal fluctuations in demand (whether these fluctuations are anticipated or not.)

Conditions (ii) and (iii) of Proposition 1 are best interpreted by considering the best use of a worker's time in terms of maximizing the expected discounted joint income to both the firm and the worker. The expected discounted joint income associated with the marginal worker being temporarily laid off in period i given that the worker does not quit is given by $B_i + E_i(\sum_{k=i+1}^{\infty} P_k F'(L_k^e)\rho^{k-i})$. The expected discounted joint income associated with the marginal worker being permanently laid off is given by Y_i. The expected discounted joint income associated with the marginal worker being employed (given that all retained workers are employed) is given by $P_i F'(R_i^e) + E_i(\sum_{k=i+1}^{\infty} P_k F'(L_k^e)\rho^{k-i})$. Hence, conditions (ii) and (iii) indicate that a worker should only be temporarily laid off if this represents the best use of the worker's time relative to either being employed or permanently laid off.

As with condition (i) of Proposition 1, conditions (ii) and (iii) suggest that temporary layoffs are more likely to be optimal when current demand is low relative to both past and expected future demand. In accord with this, condition (ii) implies that current demand must be sufficiently low to cause the value of the marginal product (given full employment of all retained workers) to fall below B_i, the opportunity cost in period i of the time of a temporarily laid off worker who remains available for recall.

The firm in making temporary layoffs as a means of "storing" experienced workers over temporary downturns in demand must be concerned with the possibility that a temporarily laid off worker may quit. The key insight here is that the firm can reduce the probability that a temporarily laid off worker will quit by specifying a wage structure that rises with tenure at the firm. Accordingly, condition (iv) indicates that a necessary condition for temporary layoffs to be optimal if $B_i + E_i(\sum_{k=i+1}^{\infty} P_k F'(L_k^e)\rho^{k-i}) > Y_i$ is that V_{i+1}^e must be sufficiently high to induce temporarily laid off workers not to quit. In other words, if the expected discounted joint income associated with the marginal worker being temporarily laid off but remaining attached exceeds the expected discounted joint income associated with the marginal worker quitting, then the firm finds it optimal to induce temporarily laid off workers not to quit. This implies that there will never be any suboptimal quits by those temporarily laid off.

The critical role of the firm's ability to influence a temporarily laid off worker's quit decision by altering the intertemporal distribution of wages can be better understood by briefly considering an alternative specification for γ_i. Suppose that the quit function γ_i is such that the firm can potentially only lower γ_i to some positive minimum. Under this alternative specification, the following would be a necessary condition for temporary layoffs to be optimal in period i:

$$(12) \qquad P_i F'(R_i^e) < B_i - \gamma_{min}(B_i + E_i(\sum_{k=i+1}^{\infty} P_k F'(L_k^e)\rho^{k-i}) - Y_i)$$

Condition (12) reflects the possibility that if $\gamma_{min} > 0$, the firm may suffer a reduction in the available pool of experienced workers if it temporarily lays off any workers. A not surprising implication of this is that the higher γ_{min}, the less likely the firm will find it optimal to use temporary layoffs during temporary downturns in demand.[9]

Our analysis, not surprisingly, suggests that distinguishing between temporary and permanent separations is important because the circumstances that generate temporary layoffs will be quite different from the circumstances that yield permanent layoffs. In spite of this apparent importance, existing analyses have either blurred, neglected, or assumed away the distinction between temporary and permanent layoffs. For example, the recent work of Baily (1977), Holmstrom (1981) and Card (1982) blurs the distinction by suggesting that the probability of "layoffs" at a given firm should simultaneously be an increasing function of the valuation of leisure by workers, the level of imperfectly experienced rated unemployment benefits and the expected income from alternative permanent employment opportunities for workers. To illustrate the problems with this blurred distinction, observe that in the present context that condition (ii) of Proposition 1 implies that $B_i > 0$ is a necessary condition for temporary layoffs to be optimal. This implies that if $B_i = 0$, then regardless of a worker's alternative permanent employment opportunities, temporary layoffs will not be optimal. In other words, an improvement in alternative permanent employment opportunities while perhaps increasing the probability of a permanent layoffs may not change the probability of temporary layoffs. In fact, one might argue that an improvement in alternative permanent employment opportunities for workers may actually lower the probability of temporary layoffs. This is because an improvement in alternative employment opportunities for workers will, other considerations apart, increase the likelihood that a temporarily laid off worker will quit and thereby will increase the costs associated with temporary layoffs.

The firm in this analysis has an incentive to maintain a pool of experienced workers consistent with long run trend demand. The problem the firm faces when suffering a downturn in demand in the current period is deciding on whether the downturn is permanent or transitory. In view of this signal extraction problem, it is helpful to consider the firm's optimal strategy in response to the limiting cases of known permanent and known transitory fluctuations in demand. The following proposition helps to characterize the firm's response to known permanent changes in product demand.

<u>Proposition 2:</u> If beginning in period t the firm expects that the

exogenous product demand and labor cost variables that it will face will be both intertemporally constant and non-stochastic (i.e., $P_t = P_{t+k}$, $V_t^n = V_{t+k}^n$, $Y_t = Y_{t+k}$, and $B_t = B_{t+k}$ $(\forall k > 0)$), then:

(i) $L_t^e + L_t^n = L_{t+k}^e \quad (\forall k > 0)$

(ii) $R_{t+k}^e = L_{t+k}^e \quad (\forall k > 0)$

(iii) $L_{t+k}^n = 0 \quad (\forall k > 0)$

(iv) if $m_t^e > L^{max}$, then $L_t^e = L_{t+k}^e = L^{max}$ and $L_{t+k}^n = 0 \quad (\forall k > 0)$

(v) if $m_t^e < L^{min}$, then $L_t^e + L_t^n = L_{t+k}^e = L^{min}$, $L_t^n > 0$, and $L_{t+k}^n = 0 \quad (\forall k > 0)$

(vi) if $L^{max} > m_t^e > L^{min}$, then $L_t^e = L_{t+k}^e = m_t^e$, and $L_{t+k}^n = 0$ $(\forall k > 0)$.

where L^{max} and L^{min} are defined by:

$$P_t F'(L^{max}) = Y_t(1-\rho), \; P_t F'(L^{min}) = V_t^n(\frac{1}{\rho} - 1)$$

Proposition 2 establishes that if beginning in period t the firm expects constant exogenous product demand and labor costs for the indefinite future, then the firm will immediately "jump" to the optimal stationary state experienced labor pool and will fully employ this labor pool in all periods. This implies that if the initial experienced labor pool in period t is larger than the optimum, the firm will permanently lay off the excess in period t. Similarly, if the initial labor pool in period t is smaller than the optimum, the firm will hire the requisite number of new workers in period t in order to have the optimum number of experienced workers for periods t+1 onwards.

Taken together, Propositions 1 and 2 suggest that the circumstances under which temporary and permanent layoffs are optimal are quite different. The following simple special case of the model helps identify additional distinguishing characteristics.

Special Case: A Two Period Model

Consider the firm in period t choosing the optimal wage-employment strategy for the present and the future. Suppose that the firm expects that the labor costs will be constant for all periods and that it expects product demand to be both intertemporally constant and non-stochastic starting in period t+1. By Proposition 2 this implies that the firm is in a stationary state from period t+1 onwards. Under these circumstances, the model reduces to essentially two periods: one period representing the current period (period t), and one period representing the indefinite future (period t+1).

Formally, this special case can be characterized by letting V^n, Y and B represent the intertemporally constant labor cost parameters, P_t current demand and P_{t+1} the demand for the indefinite future. In this special case, by Proposition 3, $L_{t+1}^e + L_{t+1}^n = L_{t+1+k}^e = R_{t+1+k}^e = m_{t+1+k}^e$, $R_{t+1}^e = L_{t+1}^e$, and $L_{t+1+k}^n = 0$ for all $k > 0$. Moreover, (7) and (8) together imply $L_{t+1}^n = 0$. Therefore, since $L_{t+1}^n = L_{t+1+k}^n = 0$ and $L_{t+1+k}^e = L_{t+1}^e = R_{t+1}^e = R_{t+1+k}^e = m_{t+1+k}^e$ for all $k > 0$, the model does collapse to essentially two periods. Since $R_{t+k}^e = L_{t+k}^e$ for all $k > 0$, the LHS of condition (9) is equal to zero for all periods $t + k$ $(k > 0)$, implying that the intertemporal distribution of wages from $t+1$ onwards is indeterminate. This allows us, without loss of generality, to assume that the wage paid in each period commencing with period $t+1$ is the same. By (3)', this implies $W_{t+1}{}^{e} = V_{t+1}{}^{e}(1 - \rho)$.

The optimality conditions in this collapsed model reduce to:

$$P_t F'(L_t{}^e) = (1 - \gamma_t)B + \gamma_t(Y - P_{t+1}F'(L_{t+1}{}^e)\frac{\rho}{1-\rho}) + \mu_t \tag{13}$$

$$P_t F'(L_t{}^e) + P_{t+1}F'(L_{t+1}{}^e)\frac{\rho}{1-\rho} = Y + \delta_t \tag{14}$$

$$z_t = P_{t+1}F'(L_{t+1}{}^e)\frac{\rho}{1-\rho} - V^n < 0, \quad z_t L_t{}^n = 0 \tag{15}$$

$$(R_t{}^e - L_t{}^e)\gamma_t'[B + P_{t+1}F'(L_{t+1}{}^e)\frac{\rho}{1-\rho} - Y] = 0 \tag{16}$$

$$P_{t+1}F'(L_{t+1}{}^e)\frac{1}{1-\rho} = Y + \delta_{t+1} \tag{17}$$

$$L_{t+1}{}^e = R_{t+1}{}^e, \quad L_{t+1}{}^n = 0 \tag{18}$$

$$\mu_t(R_t{}^e - L_t{}^e) = 0 \tag{19}$$

$$\delta_t(m_t{}^e - R_t{}^e) = 0 \tag{20}$$

$$\delta_{t+1}(m_{t+1}{}^e - R_{t+1}{}^e) = 0 \tag{21}$$

Equations (13) - (21) represent an independent subsystem of equations that fully determine $L_t{}^e$, $L_t{}^n$, $L_{t+1}{}^e$, $L_{t+1}{}^n$, $R_t{}^e$, $R_{t+1}{}^e$, $V_{t+1}{}^e$, μ_t, δ_t and

$\dot{o}_{t+1}$ for given m_t^e, P_t, P_{t+1}, Y, B, V^n and V_t^e.

Since this two period model is a special case of the more general model presented above, Propositions 1 and 2 hold in this context as well. However, the following propositions demonstrate that this simplified framework allows us to identify additional factors in determining the optimality of temporary and permanent layoffs.

Proposition 3: In the two period model, the following are necessary conditions for permanent layoffs to be optimal in period t:

(i) $P_tF'(m_t^e) < Y(1 - \rho)$

(ii) $(P_t + P_{t+1}(\rho/(1-\rho)))F'(m_t^e) < Y$

Moreover, condition (i) combined with the condition that $P_t > P_{t+1}$ constitute a set of sufficient conditions for permanent layoffs to be optimal in period t.

Proposition 4: In the two period model, the following constitute a set of necessary and sufficient conditions for temporary layoffs to be optimal in period t:

(i) $P_t < P_{t+1}$

(ii) $P_tF'(R_t^e) < B$

Proposition 5: In the two period model, if

(i) $P_t < P_{t+1}$, $P_tF'(m_t^e) < B$

and

(ii) $(P_t + P_{t+1}(\rho/(1-\rho)))F'(m_t^e) > Y$

Then temporary layoffs are optimal in period t, permanent layoffs are not optimal in period t, and W_{t+1}^e must be such that $W_{t+1}^e > ((Y - B)(1 - \rho))/\rho$ (which implies $\gamma_t = \gamma'_t = 0$.)

Taken together, Propositions 3, 4 and 5 provide a comprehensive description of the optimal employment strategy for a firm in the face of a downturn in demand. Consider a situation in which demand is relatively low in the current period (i.e., period t). How is relatively low demand defined? A sufficient condition for current demand to be considered relatively low is $P_tF'(m_t^e) < Y(1 - \rho)$. This is a reasonable characterization of relative low current demand because if past demand and future demand are no greater than current demand and $P_tF'(m_t^e) < Y(1 - \rho)$, then the optimality condition (7) indicates that it would not have been optimal to have accumulated m_t^e experienced workers. This "definition" of relatively low demand is of interest because Propositions 3 and 4 identify $P_tF'(m_t^e) < Y(1 - \rho)$ as a necessary condition for either permanent or temporary layoffs to be optimal in period t (given $B < Y(1 - \rho)$).

How does the firm respond to this period of relatively low demand? If the downturn is perceived to be permanent (i.e., $P_t > P_{t+1}$), then Proposition 3 indicates that permanent layoffs may be optimal. To understand this, consider the firm's marginal decision on whether or not to permanently lay off the m_t^e-th worker in the situation in which $P_t = P_{t+1}$ and $P_tF'(m_t^e) < Y(1 - \rho)$ (observe that $P_t = P_{t+1}$ implies by Proposition 4 that temporary layoffs are not optimal in period t). In this situation, if the m_t^e-th worker is permanently laid off, the firm "loses" the present discounted value of the marginal product of the m_t^e-th worker given by $P_tF'(m_t^e)/(1 - \rho)$, the firm "gains" V_t^e through the reduction in the wage bill associated with having to pay one less worker, and the firm "loses" $V_t^e - Y$ which is the increase in the wage bill associated with compensating workers for a marginally higher probability of permanent layoffs (this compensation is equal to the difference between the expected discounted income of a worker who is retained by the firm and the expected discounted income of a worker who is permanently laid off). Adding up the marginal gains and losses, it should be clear that if $P_tF'(m_t^e) < Y(1 - \rho)$, then the marginal gains of permanently laying off the m_t^e-th worker outweigh the marginl losses. Hence, if $P_t = P_{t+1}$ and $P_tF'(m_t^e) < Y(1 - \rho)$, permanent layoffs are optimal.

Focusing on the costs of compensating workers for a lower probability of being employed helps explain the result that in the face of a permanent downturn in demand it is always optimal for the firm to reduce the employed work force through permanent layoffs rather than through temporary layoffs. The reason is that, for a given permanent reduction in the optimal employed work force, the cost of compensating workers for what would be an increase in the probability of temporary layoffs in every period exceeds the cost of compensating workers for a one time increase in the probabiity of permanent layoffs in period t (given $B < Y(1 - \rho)$.) Of course, to the extent that the firm is uncertain with regard to whether the downfall in demand is temporary or permanent, the firm may be reluctant to permanently lay off experienced workers. This implies, other considerations apart, that when this type of signal extraction problem exists the firm may be more likely to use temporary layoffs rather than permanent layoffs (at least in the initial periods of the downturn).

Alternatively, suppose that current demand is sufficiently low to cause $P_tF'(m_t^e) < Y(1 - \rho)$ but that this relatively low level of demand is expected to be temporary (i.e., $P_t < P_{t+1}$). For purposes of exposition, it is helpful to consider a temporary downturn in demand that is "purely" temporary. By a "purely" temporary downturn in demand we mean a situation in which current demand is low relative to both past and future levels of demand but future demand is consistent with past levels of demand (so that the future derived demand for contractual workers equals or exceeds the current available pool of experienced workers). In essence, this is a situation in which future demand makes permanent layoffs clearly suboptimal but current demand is such that temporary layoffs may be optimal. In particular consider a situation in which $(P_t + P_{t+1}(\rho/(1-\rho)))F'(m_t^e) > Y$. Requiring $(P_t + P_{t+1}(\rho/(1-\rho)))F'(m_t^e) > Y$ guarantees that future demand is sufficiently high to make permanent layoffs suboptimal. In this instance, under what circumstances is it optimal for the firm to temporarily lay off workers? Since permanent layoffs are not optimal in this situation, the relevant consideration is whether the firm should temporarily lay off the m_t^e-th worker. If the firm temporarily lays off the m_t^e-th worker, the firm loses the value of the worker's marginal product for period t, $P_tF'(m_t^e)$. The firm may also lose the future discounted value of the worker's marginal product if the worker quits when laid off but by Proposition 5 this is a situaion in which the firm has an incentive to intertemporally distribute wages in such a way as to induce workers temporarily laid off in period t not to quit. In laying off the m_t^e-th worker, the firm gains W_t^e through the reduction in the wage bill associated with having to pay one less worker in period t, and the firm loses $(W_t^e + V_{t+1}^e\rho) - (B + V_{t+1}^e\rho) = W_t^e - B$ which is the increase in the wage bill associated with the firm compensating workers for a marginally higher probability of temporary layoffs. Adding up the marginal gains and losses, if $P_tF'(m_t^e) < B$, then temporarily layoffs are optimal in this situation.

Summarizing the results from the two period model, permanent layoffs are seen to be optimal only in response to sufficiently severe permanent downturns in demand that cause the present discounted value of the marginal product of a worker to fall below the expected discounted income available to a worker from alternative permanent employment. Temporary layoffs, on the other hand, are optimal only in response to temporary downturns in demand (note that $P_{t+1} > P_t$ is a necessary condition for temporary layoffs to be optimal in period t). A worker is temporarily laid off when current demand is sufficiently low to make the value of the worker's current marginal product to fall below the expected income (taking into account the income equivalent of the value of leisure) of the worker in the current period (given that the worker remains available for recall) but future demand is sufficiently high to make it optimal to retain the worker. It is worth emphasizing that the temporary layoffs depicted in this two period framework are not dependent on demand being stochastic but rather depend critically on intertemporal fluctuations in demand. This is important because it suggests temporary layoffs may be explained even without agents making expectational errors. The key is that the adjustment costs associated with the firm varying its attached pool of experienced, trained workers make it optimal for the firm to smooth its variation in its attached labor pool relative to intertemporal variations in demand by using temporary layoffs as a means of storing workers during temporary downturns in demand. Of course, the firm only finds this to be an optimal strategy given that it can induce temporarily laid off workers not to quit by specifying a wage structure that rises with job tenure.

Concluding Remarks

This paper develops a theory of the firm's demand for labor in an intertemporal context in which the firm faces a variety of adjustment costs for varying its labor force. The adjustment costs are attributable to the natural accumulation of firm specific skills by workers, hiring costs, and the costs imposed upon a firm through having to pay higher wages on average to workers when employment at that firm is unstable. The presence of these adjustment costs imply the development of mutually advantageous long term attachments in the labor market. These long term attachments are modeled in this analysis by hypothesizing that the firm makes fully enforceable long term implicit contracts with the workers. The long term implicit contracts are such that a worker in joining the firm is promised an expected discounted income that is at least as good as is available elsewhere.

The derived optimal employment strategy calls for the firm to maintain a pool of experienced workers consistent with the long run trend level of demand. Temporary layoffs are shown to be optimal in response to temporary downturns in demand of sufficient severity given that the firm is able to induce temporarily laid off workers not to quit by specifying a wage structure that rises with experience. Permanent layoffs, on the other hand, are shown to be optimal only in response to sufficiently severe permanent downturns in the long run trend level of demand. Both temporary and permanent layoffs are "efficient" in this context given the adjustment costs discussed above, the search technology specified and the assumed intertemporal variance of demand. That is, a worker is only permanently laid off if the present discounted value of the worker to the firm falls below the present discounted value of the worker's expected income from alternative employment net of search costs. Analagously, a worker is only temporarily laid off in a given period only if the value of the worker to the firm for that period falls below the opportunity cost of the worker's time for that period (not including the possibility of alternative permanent employment since temporarily laid off workers are induced not to quit). Hence, layoffs are only made when this represents the best use of the worker's time.

In comparing and contrasting the explanation of layoffs offered by this analysis with that of the "typical" implicit contract model, there are some stark differences and some interesting similarities. A fundamental difference in approach and results stems from our emphasis on temporary layoffs as inherently intertemporal phenomena involving considerations of the value of continuing long term worker-firm attachements. Our making endogenous the recall decision by firms, the quit decision by workers who have been temporarily laid off, and explicitly modeling the intertemporal firm specific training acquisition highlights the difference in approach. The differences in results can be characterized as being analogous to the different motives for holding inventories of goods. One reason that firms end up holding inventories of goods is that they must make some binding productive capacity decisions for a given period prior to their knowing the actual realization of demand for that period. This motive for holding inventories of goods is essentially the motive for holding inventories of workers in the existng contract models. This motive depends criticially on demand being stochastic. Alternatively, one can argue that the motive for firms to hold inventories of goods is due to the fact that demand fluctuates intertemporally and there are adjustment costs associated with varying the productive capacity of the firm. Thus, this motive suggests that inventories allow firms to smooth variations in productive capacity relative to intertemporal variations in demand. Analogously, the analysis in this paper suggests that there are adjustment costs associated with varying the firm's pool of experienced, trained workers. This implies a motive for the firm to smooth the variations in its pool of experienced workers relative to intertemporal variations in demand by making temporary layoffs during temporary downturns in demand. It is interesting to note that the motive to smooth the variation in the pool of experienced, trained workers has long been recognized in the firm specific human capital literature. However, there the implication associated with this smoothing motive is that firms should be reluctant to layoff experienced, trained workers during temporary downturns in demand for fear of losing the trained workers permanently. While not disputing the firm's concern with the possibility that a temporarily laid off worker might quit, our analysis suggests that a firm may induce a temporarily laid off worker not to quit by specifying a wage structure that rises with tenure at the firm. Given this ability to induce temporarily laid off workers not to quit, temporary layoffs become an optimal strategy.

An obvious limitation of the analysis in this paper is that we do not consider either the problem of the risk of default in our long term implicit contracts or the problem of what the long term contract would look like if there exists asymmetry in the information known by workers and firms. These are important problems and not surprisingly are beginning to receive considerable attention in the literature.[10] However, it would seem that a necessary first step before considering the problems of default and asymmetric information is the proper characterization of the circumstances under which temporary and permanent layoffs will occur in the absence of these problems. The analysis in this paper is intended to help provide this characterization.

FOOTNOTES

[1]Notable exceptions are the multiperiod models of Baily (1974), Baily (1977) and Holmstrom (1981). However, they maintain the assumption of worker homogeneity, their long term contracts are two period contracts in which layoffs only occur in the second period, and their resulting explanation for layoffs is very similar to that offered by the one period models.

[2]For further discussion of this point see Haltiwanger (1981).

[3]Since V_{i+1}^e does not "take effect" until period $i+1$, it is possible that V_{i+1}^e could be chosen to be contingent on the realized level of demand for period $t+1$. However, since in this model both the firm under consideration and the workers are assumed to be risk neutral, both the firm and the workers are indifferent between specifying in period i a non-contingent V_{i+1}^e and a contingent V_{i+1}^e as long as $E_i(V_{i+1}^e)$ remains the same. Hence, we assume V_{i+1}^e is not contingent on the realization of demand in period $i+1$.

[4]A more inclusive quit function in a search framework would include an explicit modeling of search costs and search intensity as in Mortensen (1978) and Burdett and Mortensen (1980). However, for analytical simplicity, we simply assume that γ_i is positively related to G_i.

[5]It is assumed that the government financed unemployment benefits are financed by a tax system that is not experienced rated.

[6]I am grateful to Finis Welch for pointing this out.

[7]If γ_i is stochastic, then it is actually the ex ante expected γ_i that gives the probability that a temporarily laid off worker will quit. However, in what follows, we impose a type of certainty equivalence by assuming that the expected percentage of quits is equal to the actual ex post

percentage of quits.

[8]Observe that neither the constant $V_i^e > Y_i$, nor the Kuhn-Tucker multiplier associated with this constraint appear in the optimality conditions (6) - (11). This is because this constraint is never binding (i.e., the multiplier associated with this constant is always equal to zero).

[9]Observe that if $\gamma_{min} = 1$, then "temporary layoffs" effectively become "permanent layoffs" as far as the firm is concerned because in both cases those laid off are unavailable for recall.

[10]For a good survey of the recent work in this area see Hall and Lazear (1982).

APPENDIX

Proof of Proposition 1:

(i) If $R_i^e > L_i^e$, then by (10), $\mu_i = 0$. Combining (6) and (7) when $\mu_i = 0$ yields:

$$(A1) \qquad (1 - \gamma_i)\,(Y_i - B_i - Y_{i+1}\rho - E_i(\delta_{i+1}\rho)) + \delta_i = 0.$$

Hence, $B_i + Y_{i+1}\rho + E_i(\delta_{i+1})\rho - Y_i \geq 0$ and since by assumption $B_i + Y_{i+1}\rho < Y_i$, this implies $E_i(\delta_{i+1}) > 0$.

(ii) If $R_i^e > L_i^e$, then by (A1) and (9), either $\gamma_i' = \gamma_i = 0$ or $B_i + E_i(\sum_{k=i+1}^{\infty} P_k F'(L_k^e)\rho^{k-i}) = Y_i$. In either event, this implies with condition (6) that $P_i F'(R_i^e) < B_i$.

(iii) Follows directly from (A1) and (7).

(iv) Proved in the proof for condition (ii). Q.E.D.

Proof of Proposition 2: We prove (i) - (vi) by considering the following three exhaustive characterizations of the initial condition for m_t: (I) $m_t^e > L^{max}$; (II) $m_t^e < L^{min}$; and (III) $L^{min} \leq m_t^e \leq L^{max}$. Note that the problem is now non-stochastic.

(I) $m_t^e > L^{max}$. Suppose $R_t^e > L^{max}$. If $R_t^e > L^{max}$, then $P_t F'(R_t^e) < Y_t(1-\rho)$. This implies $\delta_t - \delta_{t+1}\rho < 0$ by (7) if $R_t^e = L_t^e$. If $R_t^e > L_t^e$, then $P_t F'(L_t^e) = B_t < Y_t(1-\rho)$ so that $\delta_t - \delta_{t+1}\rho < 0$ even if $R_t^e > L_t^e$. Hence, $\delta_t - \delta_{t+1}\rho < 0$ if $R_t^e > L^{max}$. Since $\delta_{t+1} > 0$, $m_{t+1}^e = R_{t+1}^e$. Now $m_{t+1}^e = \gamma_t L_t^e + (1 - \gamma_t)R_t^e + L_t^n$. Can $L_t^n > 0$? No, since $L_t^n > 0$, implies $\gamma_t(R_t^e - L_t^e) = 0$ by the arguments given in Proposition 1 implying $m_{t+1}^e = R_t^e + L_t^n$. This would then imply that

$R_{t+1}^e > L^{max}$ and by the above argument $\delta_{t+2}\rho > \delta_{t+1}$. But when $L_t^n > 0$, $\delta_{t+1}\rho = V^n - Y\rho$. This would imply $\delta_{t+2}\rho > V^n - Y\rho$ which by (7) and (8) yields a contradiction. Hence, $L_t^n = 0$.

With $L_t^n = 0$, $m_{t+1}^e = R_{t+1}^e = \gamma_t L_t^e + (1 - \gamma_t)R_t^e$. Two possibilities exist, either $R_t^e = L_t^e$ or $R_t^e > L_t^e$. If $R_t^e = L_t^e$, then $R_{t+1}^e > L^{max}$ and by the arguments above $\delta_{t+2}\rho > \delta_{t+1}$. Otherwise, if $R_t^e > L_t^e$, $P_t F'(L_t^e) = B_t$. Now $R_{t+1}^e > L_t^e$. If $R_{t+1}^e > L_{t+1}^e$, then $P_t F'(L_{t+1}^e) = B_t < Y_t(1 - \rho)$ implying $\delta_{t+2}\rho > \delta_{t+1}$. If $R_{t+1}^e = L_{t+1}^e$, then since $R_{t+1}^e > L_t^e$, $P_t F'(L_{t+1}^e) < B_t$ again implying $\delta_{t+2}\rho > \delta_{t+1}$.

Following similar arguments, it can be shown that $R_t^e > L^{max}$ implies $\delta_{t+k}\rho > \delta_{t+k-1}$ $(\forall k > 0)$. This implies $\delta_{t+k} > \delta_{t+1}(1/\rho)$. Since $\delta_{t+1} > 0$, this implies $\delta_{t+k} \to \infty$ as $k \to \infty$. However, this yields a contradiction because (7) and (8) together imply $\delta_{t+k} < (V^n/\rho) - Y$ $(\forall k)$. Hence, if $m_t^e > L^{max}$, then $R_t^e < L^{max}$.

Suppose $R_t^e < L^{max}$. This implies $R_t^e < m_t^e$ and hence $\delta_t = 0$. However, $R_t^e < L^{max}$ by (7) implies $P_t F'(L_t^e) = Y_t(1 - \rho) + \delta_t - \delta_{t+1}\rho > Y_t(1 - \rho)$. Yet this implies $\delta_t > 0$, a contradiction. Hence, if $m_t^e > L^{max}$, then $R_t^e = L^{max}$.

Given that $R_t^e = L^{max}$, suppose $L_t^e < R_t^e$. This would imply $P_t F'(L_t^e) = B_t > Y_t(1 - \rho)$ which is a contradiction.

Given that $R_t^e = L_t^e = L^{max}$, suppose $L_t^n > 0$. This implies by (7) and (8) that $\delta_{t+1}\rho = V_t^n - Y_t\rho$ and thus $m_{t+1}^e = R_t^e + L_t^n = R_{t+1}^e$. If $R_{t+1}^e > L_{t+1}^e$, then $P_t F'(L_{t+1}^e) = B_t = Y_t(1 - \rho) + \delta_{t+1} - \delta_{t+2}\rho$. This implies $\delta_{t+2}\rho > \delta_{t+1} = (V_t^n/\rho) - Y_t$ which is impossible. If $R_{t+1}^e = L_{t+1}^e$, then $P_t F'(R_t^e + L_t^n) < Y_t(1 - \rho) = Y_t(1 - \rho) + \delta_{t+1} - \delta_{t+2}\rho$ which again implies $\delta_{t+2}\rho > \delta_{t+1} = (V_t^n/\rho) - Y_t$ which is impossible. Hence, $L_t^n = 0$.

In total, then, if $m_t^e > L^{max}$, it must be the case that $R_t^e = L_t^e = L^{max}$ and $L_t^n = 0$.

(II) $m_t^e < L^{min}$. Since $m_t^e \geq R_t^e \geq L_t^e$, by (7), $Y_t(1 - \rho) + \delta_t - \delta_{t+1}\rho > V_t^n(1/\rho - 1)$ which implies $\delta_t > 0$. Moreover, since $V_t^n(1/\rho - 1) > B_t$, this implies by (6) that $R_t^e = L_t^e$ and $\mu_t > 0$.

Suppose $L_t^n = 0$. This implies by (7) that $P_tF'(L_{t+1}^e) = Y_t(1 - \rho) + \delta_{t+1} - \delta_{t+2}\rho > V_t^n(1/\rho - 1)$. If $L_{t+1}^n > 0$, then this implies $\delta_{t+1} > V_t^n(1/\rho) - 1)$ which is a contradiction. Otherwise, if $L_{t+1}^n = 0$, then this implies by (7) and (8) that $\delta_{t+2} > V_t^n(1/\rho - 1) - Y_t(1 - \rho) + \delta_{t+3}\rho$ and hence $\delta_{t+1} > [V_t^n(1/\rho - 1) - Y_t(1 - \rho)]\ [1 + \rho] + \delta_{t+3}\rho^2$. Following this line of argument either $L_{t+k}^n > 0$ for some k which will yield a contradiction or $\delta_{t+1} > [V_t^n(1/\rho - 1) - Y_t(1 - \rho)](1 + \rho + \rho^2 + \rho^3 + \ldots) = V_t^n(1/\rho) - Y_t$ which again yields a contradiction. Hence, $L_t^n > 0$.

Suppose $L_t^n + m_t^e < L^{min}$. This implies by (7) that $(V_t^n/\rho) - Y_t\rho - \delta_{t+2}\rho > V_t^n(1/\rho - 1)$. Hence, $V_t^n - Y_t\rho > \delta_{t+2}\rho$. This implies $L_{t+1}^n = 0$. However, by the above arguments for any period i in which $m_i^e < L^{min}$, $L_i^n > 0$. Hence, we have a contradiction.

Suppose $L_t^n + m_t^e > L^{min}$. There are two possibilities. Either $R_{t+1}^e = L_{t+1}^e$ which by (7) implies $P_tF'(L_{t+1}^e) = (V_t^n/\rho) - Y_t\rho - \delta_{t+2}\rho < V_t^n(1/\rho - 1)$ and hence $\delta_{t+2}\rho > V_t^n - Y_t\rho$ which is a contradiction. Alternatively, if $R_{t+1}^e > L_{t+1}^e$, then $P_tF'(L_{t+1}^e) = B_t = (V_t^n/\rho) - Y_t\rho - \delta_{t+2}\rho$. If $L_{t+1}^e > L^{min}$, then $\delta_{t+2}\rho > V_t^n - Y_t\rho$ which is a contradiction. Otherwise, if $L_{t+1}^e \leq L^{min}$, this implies $B_t \geq V_t^n(1/\rho - 1)$ which again is a contradiction. Hence $L_t^n + m_t^e = L^{min}$.

In total, then, when $m_t^e < L^{min}$, $m_t^e = R_t^e = L_t^e$ and $m_t^e + L_t^n = L^{min}$.

(III) $L^{min} \leq m_t^e \leq L^{max}$. Suppose $R_t^e < m_t^e$. This implies $\delta_t = 0$ and by (7) $P_tF'(L_t^e) = Y_t(1 - \rho) - \delta_{t+1}\rho$. However, since $L_t^e \leq R_t^e < m_t^e \leq L^{max}$, $P_tF'(L_t^e) > Y_t(1 - \rho)$ which is a contradiction. Hence, $R_t^e = m_t^e$.

Suppose $R_t^e > L_t^e$. This implies $P_tF'(L_t^e) = B_t$. However, since $L^{max} \geq m_t^e = R_t^e > L_t^e$, $P_tF'(L_t^e) > Y_t(1 - \rho) > B_t$ which is a contradiction.

Given that $m_t^e = R_t^e = L_t^e$, suppose $L_t^n > 0$. This implies $\delta_{t+1} = V_t^n(1/\rho) - Y_t$ which implies by (7) that $P_tF'(L_{t+1}^e) = (V_t^n/\rho) - Y_t - \delta_{t+2}\rho$. Since $m_t^e \geq L^{min}$, and $R_{t+1}^e = m_{t+1}^e = m_t^e + L_t^n$, this implies that $P_tF'(L_{t+1}^e) < V_t^n(1/\rho - 1)$. Hence, $\delta_{t+2}\rho > V_t^n - Y_t\rho$ which is a contradiction. Q.E.D.

<u>Proof of Proposition 3:</u>

(i) Suppose $P_tF'(m_t^e) \geq Y_t(1 - \rho)$ and $R_t^e < m_t^e$. By (14) and (17) this implies $\delta_t - \delta_{t+1}\rho > 0$ which in turn implies $R_t^e = m_t^e$. Hence, we have a contradiction.

(ii) By (14), if $R_t^e < m_t^e$, then $\delta_t = 0$. Since $\delta_t = 0$, by (14) and (15) $L_t^n = 0$. Since $L_t^n = 0$, $m_t^e \geq L_{t+1}^e$. Taken together $R_t^e < m_t^e$, $m_t^e \geq L_{t+1}^e$ and (14) imply (ii).

<u>Sufficiency</u>: Suppose $P_tF'(m_t^e) < Y(1 - \rho)$ and $P_t \geq P_{t+1}$ but $m_t^e = R_t^e$. Using the proof of condition (ii) of Proposition (4), $P_t \geq P_{t+1}$ implies $R_t^e = L_t^e$. Now, if $m_{t+1}^e = R_t^e > R_{t+1}^e = L_{t+1}^e$, then $\delta_{t+1} = 0$ which with (14) and (17) yields a contradiction. Otherwise, if $m_{t+1}^e = R_{t+1}^e$, then by (17) $P_{t+1}F'(m_t^e) = Y(1 - \rho) + \delta_{t+1}(1 - \rho)$ which again yields a contradiction given that $P_t \geq P_{t+1}$.

<u>Proof of Proposition 4:</u> <u>Necessary</u> (i) If $R_t^e > L_t^e$, then $P_tF'(L_t^e) = B$. Hence, by (14), $P_{t+1}F'(L_{t+1}^e)\frac{\rho}{1-\rho} = Y - B + \delta_t$. Since

$(Y - B)(1/\rho - 1) > B$ and $L_{t+1}^e = R_{t+1}^e = R_t^e + L_t^n > L_t^e$ this implies that $P_{t+1} > P_t$. (ii) Since $P_tF'(L_t^e) = B$ when $R_t^e > L_t^e$.

Sufficiency: We prove sufficiency by showing that when $P_{t+1} > P_t$ and $P_tF'(R_t^e) < B$, a no temporary layoff strategy in period t (i.e., $R_t^e = L_t^e$) does not yield the highest feasible total discounted profits. Since $P_t > P_{t+1}$, by (14) and (17), $\delta_{t+1} > 0$. Hence, $R_{t+1}^e = (1 - \gamma_t)(R_t^e - L_t^e) + \gamma_t L_t^e + L_t^n$. This implies that for any given R_t^e, the total discounted wage bill if $R_t^e = L_t^e$ is equal to:

$$W_t^e R_t^e + V_{t+1}^e \rho(R_t^e + L_t^n) + W_t^n L_t^n$$

which by the constraints (2)' and (3)' is equal to:

$$m_t^e V_t^e + (R_t^e - m_t^e)Y + V^n L_t^n \tag{A2}$$

Hence the total discounted profits under this no layoff strategy are given by:

$$\pi(\text{no layoff}) = P_tF(R_t^e) + P_{t+1}F(R_t^e + L_t^n)\frac{\rho}{1-\rho} - m_t^e V_t^e + (m_t^e - R_t^e)Y - V^n L_t^n \tag{A3}$$

An alternative feasible strategy would be to retain the same number of workers in period t (so that R_t^e remains the same) but to temporarily lay off some of the R_t^e workers. It is also feasible for the firm following such a layoff strategy to make V_{t+1}^e sufficiently high so that $\gamma_t = 0$. Assuming that V_{t+1}^e is chosen so that $\gamma_t = 0$, $R_{t+1}^e = R_t^e + L_t^n$. Hence, the total discounted wage bill associated with the layoff strategy is given by (using the constants (2)' and (3)')

$$\text{(A4)} \qquad m_t^e V_t^e + (R_t^e - m_t^e)Y + (L_t^{e*} - R_t^e)B + V^n L_t^n$$

where $L_t^{e*} < R_t^e$. Note that by (15), L_t^n is the same regardless of whether $R_t^e > L_t^e$. Given (A4), the total discounted profits under the layoff strategy where $L_t^e = L_t^{e*} < R_t^e$ is given by:

$$\text{(A5)} \qquad \pi(\text{layoffs}) = P_t F(L_t^{e*}) + P_t F(R_t^e + L_t^n) \frac{\rho}{1-\rho} - m_t^e V_t^e$$

$$+ (m_t^e - R_t^e)Y + (R_t^e - L_t^{e*})B - V^n L_t^n$$

Taken together (A4) and (A5) imply that:

$$\text{(A6)} \qquad \pi(\text{no layoffs}) - \pi(\text{layoffs}) = P_t(F(R_t^e) - F(L_t^{e*})) - B(R_t^e - L_t^{e*})$$

Since by assumption $P_t F'(R_t^e) < B$ and $L_t^{e*} < R_t^e$, by the concavity of the production function, (A6) reveals that the no temporary layoff strategy yields strictly lower total discounted profits than any temporary layoff strategy with the same number of workers retained. Hence, temporary layoffs must be optimal.

Proof of Proposition 5: Condition (ii) by Proposition 3 implies that permanent layoffs are not optimal. Condition (i) by Proposition 4 implies that temporary layoffs are optimal. Conditions (i) and (ii) together imply $B + P_{t+1}F'(m_t^e) \frac{\rho}{1-\rho} > Y$ which by (16) implies that $\gamma_t' = \gamma_t = 0$.

REFERENCES

Akerlof, G. and H. Miyazaki, 1980, "The Implicit Contract Theory of Unemployment Meets the Wage Bill Argument," Review of Economic Studies, January , 321-338.

Azariadis, C., 1975, "Implicit Contracts and Underemployment Equilibria," Journal of Political Economy, December, 1183-1202.

Baily, M. N., 1974, "Wages and Employment Under Uncertain Demand," Review of Economic Studies, January, 37-50.

Baily, M. N., 1974, "On the Theory of Layoffs and Unemployment," Econometrica, July, 1043-1063.

Baily, M. N., 1980, Comments on "Employment Fluctuations and Wage Rigidity," Brookings Papers on Economic Activity, No. 1, 125-133.

Becker, G. S., 1962, "Investment in Human Capital: A Theoretical Analysis," Journal of Political Economy, October, 9-49.

Burdett, K. and D. Mortensen, 1980, "Search, Layoffs, and Labor Market Equilibrium," Journal of Political Economy 88, August, 652-672.

Card, D., 1981, "Indexation in Long Term Labor Contracts," Working Paper.

Carmichael, L., 1980, "Firm Specific Human Capital and Seniority Rules," presented at the Conference on the Economics of Compensation, N.B.E.R.

Doeringer, P., and M. Piore, 1971, Internal Labor Markets and Manpower Analysis, Heath Lexington Books, Lexington.

Ehrenberg, R., 1971, "Heterogeneous Labor, the Internal Labor Market, and the Dynamics of the Employment-Hours Decision," Journal of Economic Theory, February, 85-104.

Feldstein, M., 1976, "Temporary Layoffs in the Theory of Unemployment," Journal of Political Economy, June, 937-957.

Gordon, D., 1974, "A Neo-Classical Theory of Keynesian Unemployment," Economic Inquiry, December, 431-459.

Grossman, H., 1981, "Incomplete Information, Risk Shifting and Employment Fluctuations," Review of Economic Studies, April 189-199.

Grossman, H., 1979, "Employment Fluctuations and the Mitigation of Risk," Economic Inquiry, July, 344-359.

Hall, R. and E. Lazear, 1982, "The Excess Sensitivity of Layoffs and Quits to Demand," National Bureau of Economic Research Working Paper.

Haltiwanger, J., 1981, "Implicit Contract Theory: A Reexamination and Reformulation," unpublished Ph.D. dissertation, The Johns Hopkins University.

Hashimoto, M., 1979, "Bonus Payments, On-The-Job Training and Lifetime Employment in Japan," Journal of Political Economy, October, 1086-1104.

Holmstrom, B., 1981, "Equilibrium Long Term Contracts," Quarterly Journal of Economics.

Kuritani, M., 1973, "Theory of Training, Earnings, and Employment: An Application to Japan," Ph.D. dissertation, Columbia University.

Loewenstein, M., 1979, "Worker Heterogeneity, Hours Restrections, and Temporary Layoffs," Discussion Paper No. 378, The Center for Mathematical Studies in Economics and Management Science, Northwestern University.

Medoff, J., 1979, "Layoffs and Alternatives under Trade Unions in U.S. Manufacturing," American Economic Review, June, 380-395.

Mortensen, D., 1978, "On the Theory of Layoffs," Discussion Paper No. 322, The Center for Mathematical Studies in Economics and Management Science, Northwestern University.

Mortensen, D., "Specific Capital and Labor Turnover," Bell Journal of

Economic, Autumn, 572-586.

Oi, W., 1962, "Labor as a Quasi-fixed Factor," Journal of Political Economy, December, 538-555.

Okun, A., 1975, "Inflation: Its Mechanics and Welfare Costs," Brookings Papers on Economic Activity, No. 2, 351-90.

Okun, A., 1981, Prices and Quantities: A Macroeconomic Analysis, Brookings Institution, Washington, D.C.

Poole, W., 1976, "Rational Expectations in the Macro Model," Brookings Papers on Economic Activity, No. 2, 463-505.

Rothchild, M., 1973, "Models of Market Organization with Imperfect Information: A Survey," Journal of Political Economy, December 1283-1308.

Salop, S. C., 1973, "Wage Differentials in a Dynamic Theory of the Firm," Journal of Economic Theory, August, 321-344.

Sargent, T., 1979, Macroeconomic Theory, Chapter 8, Academic Press, New York.

Solow, R., 1968, "Short-run Adjustment of Employment to Output," Value, Capital and Growth. Papers in honour of Sir John Hicks, Edinberg University Press, Aldine Publishing Co., Chicago.

Solow, R., 1980, "On Theories of Unemployment," American Economic Review, March, 1-11.

Williamson, O. E., M. L. Wachter and J. E. Harris, 1975, "Understanding the Employment Relation: The Analysis of Idiosyncratic Exchange," The Bell Journal of Economics, Vol. 6, No. 2, Autumn.

U.S. Bureau of Labor Statistics, 1972, "Layoff, Recall, and Worksharing Procedures," Bulletin 1425-13, Washington.