

ON THE ABSENCE OF SECRECY AND COMMITMENTS  
IN NON-CONFLICT SITUATIONS

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It is commonly observed that in pure non-conflict situations -- i.e., when all of the individuals in a group share the same basic preference orderings over all of their alternative social states -- individuals openly communicate. The team members neither behave secretively nor commit themselves to punish the informed decision of others in the group. And when the physical environment prevents some individuals from making decisions in full knowledge of the decisions of others, the individuals willingly submit to the directions of a more informed central coordinator, or "team captain."

These empirical regularities suggest that theoretical solutions to rational individual interactions in pure non-conflict situations are always best for the individuals, and therefore the group, when the various decision makers act under perfect information and without any commitments. The purpose of the first part of this paper is to prove this result. More specifically, it is to prove that in pure non-conflict situations, rational individual action under perfect information a la von Stackelberg and von Neumann-Morgenstern always leads to a joint solution, and the solution is always a joint optimum.

(It would be peculiar if such an obvious optimality result, the simple converse of Bellman's optimality principle, had not been proved elsewhere.

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The result, that if non-conflicting individuals sequentially maximize, then they will always reach their joint optimum, is implicit in numerous dynamical arguments. I just haven't been able to find the theorem either explicitly stated or proved elsewhere.)

Existence holds under quite general conditions and does not involve the indifference conundrums posed by Peleg and Yaari for perfect information games when later decision makers are indifferent between various solution points. (While Goldman has proven a fairly general existence result for these environments, we are still left without an algorithm for resolving the Peleg-Yaari indifference conundrums under generally conflicting preferences.)

The results can be used to test for conflict in a group. If the individuals are observed to keep secrets or establish committed reaction functions, the payoffs are not "team" payoffs. For example, the results can be used to determine if a single consumer -- viewed as a sequence of distinct decision makers -- represents a set of conflicting decision makers. A long chain of economic theorists (Stotz, Pollack, Peleg-Yaari, Hammond, Thaler-Shefrin and several other, no-less-sophisticated thinkers) have argued that our unconstrained future selves are likely to choose future consumption streams that are different than (or "inconsistent with") those that our current selves would most prefer and that the resulting conflict, an external diseconomy from future to present selves, leads current selves to constrain, with commitments or secrecy, the behavior of his future selves. However, the standard examples of consumption-commitments -- viz., joining Christmas clubs, avoiding vice-inducing situations, and hiring budget-enforcing agents -- are not unambiguous examples of such constraints.

Rather, a bit of introspection suggests that these are examples of constraints imposed on future selves that are less informed, impulse-buying, selves than the current, more thoughtful, self. The literature's inability to isolate clean examples of the "inconsistency" phenomenon speaks for the empirical rarity of intrapersonal externalities from informed future selves to present selves, at least among consumers of acceptable sanity.

While an absence of future consumption commitments by normal individuals in situations where they will be continually informed (and a corresponding absence of empirical examples in which normal consumers go out of their way to avoid receiving messages that might lead them to non-impulse purchases) implies an absence of intertemporal "inconsistencies," or externalities from future informed selves to current selves, it does not imply a complete absence of conflict. Current selves may still impose externalities on future selves, and -- without a reciprocal externality from future to current decision makers -- there would be no reason for a current decision maker to make commitments to protect himself from future decision makers. Part II of this paper shows that any perfect information, no-commitment solution under this limited form of conflict is best for the current decision maker. The result generalizes the theorem of Blackorby, Nissen, Primont, and Russell. Although the result enables us to interpret non-committing consumers as possibly possessing partial internal conflict, we have been unable to find any microbiological representation permitting this conflict that would not also lead to externalities from future to present decision makers. Once the individual is allowed to contain separate bodily components that resist current consumption, the body's relatively short-horizon components will gain by committing near-future consumption to favor themselves relative to the longer-horizon components.

Moreover, since all essential components of a human body live or die together, there is no biological reason -- given our Part I results -- for our bodies to have evolved into groups of conflicting components.

A more appropriate application of the second optimality result is to intergenerational allocations within a family in which parents -- for good biological reasons -- are sufficiently benevolent that they make lump-sum transfers to their offspring but the offspring have no such benevolence towards their parents (Fisher). Here, the later decision makers, the offspring, would choose less consumption for their parents but are not given the alternative because of the compensatory adjustments of the lump-sum transfer to redistributive efforts of the offspring (Becker). Highly benevolent parents are, of course, observed to keep secrets and establish punishment commitments, but only, I believe, when it is perceived to be for the less-informed child's own good. On the other hand, the commitments and secrecy of less benevolent parents -- those who cannot afford lump-sum transfers to their children -- are often of a redistributive nature and provide the basis of observed child labor, child abuse, compulsory education, and social security laws (Thompson-Ruhter).

## I

A convenient description of the first non-conflict situation has the utilities of each of the individuals in a group represented by monotone increasing functions of a common, continuous, real-valued function of individual actions,  $f(x_1, \dots, x_n)$ , with the action,  $x_i$ , of the  $i^{\text{th}}$  individual,  $i = 1, \dots, n$ , chosen from a compact set of feasible actions,  $X_i$ .

If the individuals in this situation independently choose their actions, each selecting an  $x_i$  that maximized  $f$  for given  $(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$ , the resulting, Cournot-type, solution set might obviously contain many local maxima that are not global maxima. There would be nothing to guarantee the achievement of a globally maximal value of  $f$ . The source of the problem is that there is no communication between the decision makers and therefore no resulting "coordination" of their activities.

To represent perfect communication, or "coordinated," noncooperative decision making, we assume "perfect information" in the von Stackelberg-von Neumann-Morgenstern sense, meaning that the individuals choose their actions in sequence, where individual 1 chooses first and communicates his action to the rest of the individuals before they move, individual 2 chooses next, similarly communicating his chosen actions to individuals 3, ...,  $n$ , before they move, etc. The  $n$ th individual will choose an action just as he did in the game without communication, choosing an  $x_n$  in  $X_n$  that maximizes  $f(x_1, \dots, x_{n-1}, x_n)$  for the previously given, chosen, values of  $x_1, \dots, x_{n-1}$ . We first show that an unambiguous solution to the above game always exists.

The existence of an optimal  $x_n$  for the last mover is assured by the compactness of  $X_n$  and the continuity of  $f$  (Weierstrass theorem). There may be several such maximizing values of  $x_n$ . We shall let  $x_n^*(x_1, \dots, x_{n-1})$  represent  $n$ 's

solution correspondence. Since  $x_n^*$  is going to be so picked, individual  $n-1$  will attempt to pick an  $x_{n-1}$  that maximizes, for given  $x_1, \dots, x_{n-2}$ , the function  $f(x_1, \dots, x_{n-2}, x_{n-1}, x_n^*(x_1, \dots, x_{n-2}, x_{n-1}))$ . Since the value of  $f$  for a given  $x_{n-1}$  is the same regardless of the value of  $x_n$  subsequently chosen from the non-empty image set of  $x_n^*(x_1, \dots, x_{n-1})$ , the actual choice by  $n$  from this set is a matter of indifference to  $n-1$  as well as to  $n$  and therefore does not affect the choice by  $n-1$ . Momentarily assuming the existence of a maximizing solution for individual  $n-1$ , an assumption validated in the next paragraph, the maximization yields another non-empty correspondence,  $x_{n-1}^*(x_1, \dots, x_{n-2})$ . Similarly, individual  $n-2$  attempts to pick, prior to the choices of  $n-1$  and  $n-2$ , an  $x_{n-2}$  that maximizes, for given  $x_1, \dots, x_{n-3}$ ,  $f(x_1, \dots, x_{n-3}, x_{n-2}, x_{n-1}^*(x_1, \dots, x_{n-3}, x_{n-2}), x_n^*(x_1, \dots, x_{n-3}, x_{n-2}, x_{n-1}^*(x_1, \dots, x_{n-3}, x_{n-2})))$ , etc. The solution set to this sequence of maximizations ( $x^*$ ), may, of course, contain several elements.

To prove that the set is non-empty, it is sufficient to prove that the response correspondences  $x_{n-1}^*(\cdot)$ ,  $\dots$ ,  $x_1^*(\cdot)$  are non-empty. Again using the Weierstrass theorem,  $x_{n-1}^*(\cdot)$  is non-empty if the domain of the objective function variables controlled by  $n-1$  (i.e.,  $(x_{n-1}, x_n^*(x_{n-1}))$ ) is compact. Since the domain of  $x_{n-1}$ ,  $X_{n-1}$ , is compact by assumption, we need only show that the range of  $x_n^*(x_{n-1})$ , or  $\bigcup_{x_{n-1} \in X_{n-1}} x_n^*(x_{n-1})$ , is compact. This is done in the following three steps. First, because  $(x_{n-1}, x_n^*(x_{n-1}))$  maximizes a continuous, real-valued objective function in  $(x_{n-1}, x_n)$  for a given  $x_{n-1}$ , we know that  $x_n^*(x_{n-1})$  is upper-semicontinuous (Berge). Second,  $x_n^*(x_{n-1})$  is closed for any given value of  $x_{n-1}$ . For suppose otherwise; then the set  $x_n^*(x_{n-1})$  would not contain all of its limit points.

Call one of these excluded limit points  $z$ . Since  $X_n$  is closed,  $z \in X_n$ . And since  $z$  is not in  $x_n^*(x_{n-1})$ ,  $f(x_1, \dots, x_{n-1}, z) < f(x_1, \dots, x_{n-1}, x_n^*(x_{n-1}))$ . From these facts, it would follow that  $\lim_{x_n^v \rightarrow z} f(x_1, \dots, x_{n-1}, x_n^v(x_{n-1})) > f(x_1, \dots, x_{n-1}, z)$ , which contradicts the continuity of  $f$ . So  $x_n^*(x_{n-1})$  is an upper-semicontinuous function with a closed image for any given  $x_{n-1}$ . We can now complete the proof by applying the result of Nikaido (Lemma 4.5) stating that such a function defined over a compact set produces a total image set, our  $\bigcup_{x_{n-1} \in X_n} x_n^*(x_{n-1})$ , which is compact. So  $x_{n-1}^*(x_1, \dots, x_{n-2})$  is non-empty. The same procedure can be repeated to show that  $x_{n-2}^*(x_1, \dots, x_{n-2})$  is non-empty, etc.

This completes our existence proof. We are now prepared to discuss optimality.

In general, that is when conflict may be present, perfect information solutions are not generally jointly efficient. Standard prisoner's dilemma games illustrate this simple fact. But we are dealing here with a non-conflict situation, where the possible payoffs do not permit the redistributive opportunities presented in a standard prisoner's dilemma game.<sup>1</sup>

We now prove that a perfect information solution will always achieve a joint optimum in the above, non-conflict situation:

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<sup>1</sup>An additional, well-known difficulty with perfect information solutions is that when a later mover is indifferent between several possible actions, prior movers -- not knowing which among the later mover's indifferent actions will actually be selected -- do not really know what to do. This difficulty also disappears in non-conflict situations because, as we have already indicated, when prior movers always share the indifference of later ones, the particular actions of later movers within their solution correspondences have no effect on the utilities or decisions of prior movers.



Suppose  $x^*$  were not a global maximum point. Then there would be an  $x^0$ , say a global maximum point, such that  $f(x^0) > f(x^*)$ . Had individual  $n$  been presented with  $x_1^0, \dots, x_{n-1}^0$ , he would have picked  $x_n^0$  (i.e.,  $x_n^*(x_1^0, \dots, x_{n-1}^0) = x_n^0$ ); and  $x^0$  would have resulted instead of  $x^*$ . So  $n$  was not presented with  $(x_1^0, \dots, x_{n-1}^0)$ . It also follows that if individual  $n-1$  had been presented with  $x_1^0, \dots, x_{n-2}^0$ , he would have picked  $x_{n-1}^0$ ; for  $x_{n-1}^*(x_1^0, \dots, x_{n-2}^0) = x_{n-1}^0$  and  $f(x^0) > f(x^*)$ . So  $n-1$  was not presented with  $x_1^0, \dots, x_{n-2}^0$ . Similarly,  $n-2$  was not presented with  $x_1^0, \dots, x_{n-3}^0$ , etc. up to individual 1. But individual 1 has no excuse. He must have not maximized his utility. For, according to the above sequence, wherein  $x^* \neq x^0$  implies  $x_1^* \neq x_1^0$ , if he had picked  $x_1^* = x_1^0$ , then  $x^*$  would have equalled  $x^0$  and his utility would have been higher. So the supposition that  $x_1^* \neq x_1^0$  contradicts the assumption of individually rational choice. The solution must be a global maximum.

Jim Mirrlees has suggested an alternative, more direct, optimality proof, which can be paraphrased as follows: Pick any  $x$ . Then restrict  $x_n$  so as to maximize  $f$  for the given  $x_1, \dots, x_{n-1}$ . This  $f$  defines a particular value of the function,  $f_{n-1}(x_1, \dots, x_{n-1})$ . Then pick an  $x_{n-1}$  that maximizes the latter function, thus yielding an  $f$  that defines  $f_{n-2}(x_1, \dots, x_{n-2})$ , etc. By definition,  $f_1 \geq f_2(x_1) \geq \dots \geq f_{n-1}(x_1, \dots, x_{n-1}) \geq f(x_1, \dots, x_n)$ . Since  $f_1$  depends on no variables, it is unique and therefore the same regardless of what value of  $x$  we initially chose. In particular, if  $x = x^0$ ,  $f_1 = f_2 = \dots = f_{n-1} = f(x^0) = \max_x f(x_1, \dots, x_n)$ . So  $f_1$  is our maximum and theorem is proved.

Maxim Engers has pointed out that the optimality theorem leads to an alternative, greatly simplified, existence proof: Since the converse of this

theorem, Bellman's Optimality Principle, also holds, a solution is equivalent to a maximum. Therefore, since a maximum in our model obviously exists, so does a solution. Unfortunately, this elegant proof does not extend to the generalized problem of Section II while our more cumbersome, direct proof does.

## II

As noted in the Introduction, the absence of incentives to hide information or make future commitments implied in the above, pure, non-conflict situation continues to hold under a weakening of the conditions on preferences. In particular, it continues to hold as long as each future decision makers have the same preferences over future actions as the immediately preceding decision maker. In this case, the successive objective functions are described as follows:

$$\begin{aligned} f_1(x) &= U_1(x_1, f_2(x)) \\ f_2(x) &= U_2(x_1, x_2, f_3(x)) \\ &\vdots \\ f_{n-1}(x) &= U_{n-1}(x_1, \dots, x_{n-1}, f_n(x)) \\ f_n(x) &= U_n(x_1, \dots, x_n) \end{aligned}$$

where  $f'_{i+1} > f''_{i+1} \rightarrow U_i(x_1, \dots, x_i, f'_{i+1}) > U_i(x_1, \dots, x_i, f''_{i+1})$ . Thus, for any given  $x_1, \dots, x_i$ , the  $i^{\text{th}}$  individual's objective function is a monotone increasing function of the  $i + 1^{\text{st}}$  individual's function.

A perfect information solution,  $x_1^*$ , is, as above, an  $x$  such that:  $x_n$  is picked so as to maximize  $f_n(x)$  given  $x_1, \dots, x_{n-1}$ ;  $x_{n-1}$  is picked so as to maximize  $f_{n-1}(x)$  given  $x_1, \dots, x_{n-2}$ ; and the dependence of  $x_n$  or  $x_{n-1}$ , etc. The existence of a solution holds under the same additional conditions on preferences, and through the same argument, as in the direct proof of Section I; the exercise will not be repeated. What we wish to show is that no decision maker has an incentive to influence the decisions of later decision makers, i.e., that all subsequent decision makers will choose a sequence of actions that maximizes the utility of a current decision maker. (From this it

follows that a current decision maker has no incentive to commit the decisions of, or withhold information from, future decision makers.) More precisely, we wish to show that  $x^*$  maximizes  $f_1(x_1^*, \dots, x_{i-1}^*, x_i, x_{i+1}, \dots, x_n)$  over all  $x_i, x_{i+1}, \dots, x_n$  in  $\prod_{j=i+1}^n (X_j)$ .

The result holds trivially for  $i = n$  as it follows from the mere definition of  $x^*$ . To show it for  $i = n - 1$ , first recall that our specification on the forms of the successive objective functions implies that any  $x_n$  that maximizes  $f_n$  given  $x_1, \dots, x_{n-1}$  will also maximize  $f_{n-1}$  given  $x_1, \dots, x_{n-1}$ . Therefore, the result holds for  $n - 1$  as long as any pair of actions resulting from  $n - 1$ 's first rationally picking an  $x_{n-1}$  in anticipation of his later choice and then picking an  $x_n$  that maximizes his same, initial utility function given  $x_{n-1}$  -- call the pair  $x'_{n-1}, x'_n$  -- also maximizes his utility,  $f_{n-1}(x_1^*, \dots, x_{n-2}^*, x_{n-1}, x_n)$ , over all  $x_{n-1}, x_n$  in  $X_{n-1} \times X_n$ . Theorem 1, that rational, perfectly informed, sequential choice under a common utility function achieves a maximum of that function, tells us that  $x'_{n-1}, x'_n$  does indeed unconditionally maximize  $f_{n-1}(x_1^*, \dots, x_{n-2}^*, x_{n-1}, x_n)$ . Building on this, we can show in the same way that  $x'_{n-2}, x'_{n-1}, x'_n$  unconditionally maximizes  $f_{n-2}(x_1^*, \dots, x_{n-3}^*, x_{n-2}, x_{n-1}, x_n)$ , and so on until we arrive at individual 1, at which point our theorem is proved.

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