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MEASURING THE IMPACT OF EDUCATION ON PRODUCTIVITY

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Abstract

Measuring the Impact of Education on Productivity

In the U.S., growth in income has been accompanied by growth in education. Between 1929 and 1979 real gross national product more than tripled. During the same period annual expenditures on education rose from 3% of GNP to 7%. Thus in real dollars, Americans were spending ten times as much on education in 1979 than in 1929. In the same period the fraction of the prime-age population who were college graduates rose from 3.9% to 16.4%. The median number of school years completed rose from 8.4 in 1930 to 12.5 in 1979.[1] The economist views education as an investment and the concurrent growth of education and output has provided an impetus for establishing a causal link from education to productivity. The evaluation of this investment as a source of growth has been the task of a host of economic studies including the human capital literature and much of the growth accounting literature. The essential underpinning of this literature is simple: education is a factor of production. Its primary function may be allocative, as proposed by Nelson and Phelps (1966) and Welch (1970), or it may be physically productive. But, in either case, the basic inference drawn from statistics such as those presented is that education is a form of productive capital and the growth accountant's task is to measure its contribution to production.

Our purpose in this paper is to consider a variety of studies that attempt to evaluate the impact of education on aggregate production in the U.S. economy. We attempt to step back from the myriad of technical questions surrounding the complex growth accounting formulae used for this purpose, and consider these models in a simple, unadorned framework that hopefully will crystallize the essential assumptions underlying such formulae. We will raise

some basic questions concerning the application of growth accounting techniques to measuring the contribution of education. We contend that standard methods of growth accounting make sense for simple measurement of factor contributions where outputs are well-measured and when factor growth is exogenous. For education and other forms of producer capital which are legitimately viewed as intermediate products the standard techniques seem less desirable. We propose an alternative measure which we consider more amenable to measuring the contribution of an intermediate input such as education. This measure is derived using tools similar to those used to analyze consumer's surplus. A direct analogy with the consumer's case is given and the derivation of the alternative measure is based on this analogy. The theoretical and conceptual analysis of the measure of education's contribution to productivity is followed by a discussion of the empirical measures implemented by various authors.

In the U.S., growth in income has been accompanied by growth in education. Between 1929 and 1979 real gross national product more than tripled. During the same period annual expenditures on education rose from 3% of GNP to 7%. Thus in real dollars, Americans were spending ten times as much on education in 1979 than in 1929. In the same period the fraction of the prime-age population who were college graduates rose from 3.9% to 16.4%. The median number of school years completed rose from 8.4 in 1930 to 12.5 in 1979.[1] The economist views education as an investment and the concurrent growth of education and output has provided an impetus for establishing a causal link from education to productivity. The evaluation of this investment as a source of growth has been the task of a host of economic studies including the human capital literature and much of the growth accounting literature. The essential underpinning of this literature is simple: education is a factor of production. Its primary function may be allocative, as proposed by Nelson and Phelps (1966) and Welch (1970), or it may be physically productive. But, in either case, the basic inference drawn from statistics such as those presented is that education is a form of productive capital and the growth accountant's task is to measure its contribution to production.

Our purpose in this paper is to consider a variety of studies that attempt to evaluate the impact of education on aggregate production in the U.S. economy. We attempt to step back from the myriad of technical questions surrounding the complex growth accounting formulae used for this purpose, and consider these models in a simple, unadorned framework that hopefully will crystallize the essential assumptions underlying such formulae. We will raise some basic questions concerning the application of growth accounting techniques to measuring the contribution of education. We contend that standard methods of growth accounting make sense for simple measurement of factor contributions where outputs are well measured and when factor growth is exogenous. For education and other forms of producer capital which are legitimately viewed as intermediate products the standard techniques seem less desirable. We propose an alternative measure which we consider more amenable to measuring the contribution of an intermediate input such as education.

The paper proceeds as follows. In the next section we characterize the productive process using a simple model and discuss the theoretical problems with standard growth accounting measures. We also derive our alternative measure. In the second section we review a few of the empirical measures of education's contribution to productivity.

SECTION I

Theoretical Approaches to Measuring the Productivity of Education

In this section we develop a prototype of the basic growth accounting framework and analyze its potential usefulness in measuring the contribution of the growth in inputs to the growth in output. We concentrate on the characterization of education as a productive input and show that the standard procedures when applied to an intermediate input like education lead to some counter-intuitive conclusions that warrant consideration of an alternative approach.

Let us consider a very simple economy which produces one output Y, as a function of two homogeneous inputs, capital, K, and labor, N, according to the relationship.

$$Y = f(K,N).$$

The growth accounting question is a simple one. Suppose we observe growth in the amount of output produced. What pat of that growth can be attributed to growth in the inputs? The question we as economists must ask ourselves is whether it is possible to answer the preceding question in a meaningful manner. What assumptions must we make about the nature of the inputs and the production process to be able to make meaningful statements about the nature of the growth of production?

Suppose, first, that the amount of capital K was fixed and labor grew exogenously. Then, ignoring any possibility for technical advance all output growth is due to the growth in labor. If both output and labor are easily measurable quantities then the contribution of the increase in the input is easily measured. No complex formulae need to be called into play -- all growth is simply attributed to the growth in labor.

The situation becomes considerably more complicated when we allow for growth in both inputs, both conceptually and empirically. The basic problem is that it is difficult to identify the source of the growth in output if both inputs increase exogenously. There are numerous conceptual experiments we could perform to attribute fractions of output growth to each of the growing inputs. Denote the initial endowments of capital and labor as K^o and L^o and initial output as Y^o. Let the superscript 1 denote quantities in period 1, and thus $K^1 > K^o$, $L^1 > L^o$ and $Y^1 > Y^o$. First we could consider a sequential process, first incrementing the capital stock and then incrementing the labor stock. Thus the contribution of capital to the increased output is

$$S_K = f(K^1,L^o) - f(K^o,L^o)$$

and labor's contribution is

$$S_L = f(K^1,L^1) - f(K^1,L^o).$$

Clearly the order in which we sequence events will make a difference in such measurements. If we conceived of the incremental process as adding the $L^1 - L^o$ units of labor first, then labor's contribution would be

$$S_L^1 = f(K^o,L^1) - f(K^o,L^o),$$

and S_K^1 would be the residual growth in output. There is no guarantee $S_L = S_L^1$ or $S_K = S_K^1$. It depends on the algebraic form of the production function. In fact the only production function for which the measured contribution is independent of sequence is a linear production function. If $f(K,L)$ is linear, then each input has a constant marginal product independent of the level of the other inputs and thus the contribution of the exogenous growth in inputs is easily measurable. Such a functional form essentially assumes away the interesting part of the production process since it assumes all factors of production are perfect substitutes. If the production function is nonlinear (that is there exists factor complementarity), then even conceptually we cannot attribute part of the growth in output to the input and part of the growth in output to another. The extent of output growth depends on the extent of growth in both inputs. The product of the two are inextricably entwined because there is factor complementarity. There is no logical way to distinguish which input is "responsible" for a part of output -- since the inputs work together. The economist is thus faced with a difficult conundrum. In accounting for output growth, if the production function is linear there is no problem and if the production function is nonlinear there is no solution.

Given this difficulty many economists have nevertheless proceeded to make some good and useful approximations to the measurement of productivity growth. Let us rewrite the production function in a more general form:

$$Y = f(\underset{\sim}{X},\tau)$$

where $\underset{\sim}{X}$ denotes an n-vector of inputs and τ summarizes the part of technology that is subject to change. Implicitly Y, $\underset{\sim}{X}$ and τ carry time subscripts. Even though growth is an inherently dynamic process and results from durable investments that take time, the points that we make can be simplified by using instantaneous relationships. Denoting $\frac{\partial z}{\partial t}$ by $\dot{z}$, the standard growth accounting formula decomposes growth in output into two components: that due to growth in inputs and that which occurs because of technical advance:

$$\frac{\dot{Y}}{Y} = \sum_i S_i \frac{\dot{X}_i}{X_i} + \frac{\tau \frac{\partial Y}{\partial \tau}}{Y} \frac{\dot{\tau}}{\tau} \tag{1}$$

where $S_i = \frac{X_i f_i}{Y}$, and $f_i = \frac{\partial Y}{\partial X_i}$. The production shares, S_i are not typically observed, but assuming cost minimizing behavior and exogenous factor prices we know that

$$S_i = SC_i \tag{2}$$

where C_i is the share of the i^{th} input in total cost, $C_i = \frac{P_i X_i}{\Sigma P_i K_i}$ and S is the scale elasticity which is simply equal to the ratio of average to marginal cost.[2] If we assume constant returns to scale, average cost equals marginal cost, and since the C_i are observed we have the following accounting definitions:

$$\text{Explained growth} = \sum_i C_i \frac{\dot{X}_i}{X_i} \tag{3}$$

$$\text{Unexplained growth} = \frac{\dot{Y}}{Y} - \sum_i C_i \frac{\dot{X}_i}{X_i} \ . \tag{4}$$

The contribution of the i^{th} factor is defined as $C_i \frac{\dot{X}_i}{X_i}$ and the residual is referred to as growth in total factor productivity. By evaluating the contribution of a growth in an individual input by using an indirect measurement of marginal product, the economist is taking a linear approximation to the production function as his basis for attributing output growth to the various factors. This technique of treating an insoluble nonlinear problem as linear, and thus approximating a solution is not an uncommon practice and seems a quite reasonable way of gauging the relative contributions of the exogenous growth of the various factors of production to output growth.

Unfortunately, the complications in analyzing productivity growth do not end with the introduction of several growing inputs. Not all inputs in the production process are exogenously determined. The amount of intermediate inputs available for production of the final product is endogenously determined and produced using factors that have real opportunity costs. Although standard techniques for growth accounting serve as a useful tool when some inputs are intermediate inputs, caution must be used in interpreting the results of such studies. Often intermediate input growth is treated as being exogenous and thus costless. If the growth of some inputs is endogenous and thus costly we must consider what would have been produced had the factors used to produce those inputs been used differently. In studying the contribution of education to growth in output these considerations are extremely important.

To distinguish education as an intermediate product we rearrange the simple model proposed above. Specifically let the production technology be described by

(5) $$Y = g(\underset{\sim}{X}_1, E)$$

(6) $$E = h(\underset{\sim}{X}_2)$$

and $\underset{\sim}{X} = \underset{\sim}{X}_1 + \underset{\sim}{X}_2$. The subscripts on $\underset{\sim}{X}$, the vector of inputs, indicate use in primary and intermediate production. We have omitted exogenous technology, τ, for the moment.

The economic problem is to allocate $\underset{\sim}{X}$ between primary and intermediate production. If we use as our allocation rule, the maximization of output Y, subject to the resource constraint $\underset{\sim}{X}$, the first order efficiency conditions are:

(7) $$g_i = h_i \frac{\partial g}{\partial E} .$$

The right hand side (RHS) of (7) measures the indirect marginal product of primary factors and the left hand side (LHS) measures the direct product. A factor is efficiently allocated when direct and indirect marginal products are equal. The RHS of (7) measures the value of factors diverted to production of E while the LHS measures the opportunity cost of those factors. Suppose, now, that $\underset{\sim}{X}$ grows exogenously, and we want to observe the contribution of education to observed output growth. If we view education, not as an intermediate input, but as a basic input, using equation (2) we find the "contribution" of educated labor is

$$\frac{\partial Y}{\partial E}\frac{dE}{d\underset{\sim}{X}} = \frac{\partial Y}{\partial E}\Sigma h_i \frac{dX_{i2}}{d\underset{\sim}{X}} . \tag{8}$$

This calculation ignores the cost of factors used to produce E. If we explicitly recognize opportunity costs we find that education's contribution to growth is zero. To see this consider the change in output associated with a change in the i^{th} input, dX_i:

$$\frac{dY}{dX_i} dX_i = g_i\, dX_{i1} + \frac{\partial Y}{\partial E} h_i\, dX_{i2} . \tag{9}$$

Using equation (7) this implies

$$\begin{aligned} \frac{dY}{dX_i} dX_i &= g_i\, dX_{i1} + g_i\, dX_{i2} \\ &= g_i (dX_{i1} + dX_{i2}) \\ &= g_i\, dX_i \end{aligned} \tag{10}$$

since, by definition, $dX_i = dX_{i1} + dX_{i2}$. Equation (10) demonstrates that the <u>net</u> marginal product of the factor X_{i2} diverted to the production of education is zero. By the definition of efficient production, the <u>marginal</u> contribution of education is zero. If it were not, basic resources could be rearranged to increase total output. Therefore, the use of marginal accounting to measure education's contribution to growth seems inappropriate, since at the margin the educational process makes no net contribution. Where, then, do growth accountants err in their calculations of a positive contribution of education, and how might we interpret or reconstruct their results?

Let us consider a simpler model that will help illustrate the problem with the calculations made by the growth accountants. Let Y denote per capita output, let N_1 denote the number of unskilled workers in the labor force and N_2 denote the number of skilled (educated) workers. Suppose $N_1 + N_2 = N$. Then we characterize our production relationship as:

(11a) $$Y = g(N_1, K_1, E)$$

(11b) $$E = h(N_2, K_2)$$

(11c) $$N_1 + N_2 = \bar{N}$$

(11d) $$K_1 + K_2 = \bar{K}$$

where $\bar{K}$ is the amount of a second primary input which we call capital, and is allocated between primary and intermediate production, K_1 and K_2 respectively. In equations 11a-d we have depicted a production process where the education process is factor absorbing. Assume that the number of educated laborers that provide services E is N_2. That is, no person works only in the education sector. Each worker is educated and then the worker's services are used for primary production. The educational process merely embodies capital in workers. We can apply the standard technique to equations 11 to get:

(12a) $$\dot{Y} = g_1\dot{N}_1 + g_2\dot{K}_1 + g_3\dot{E}$$

(12b) $$\dot{E} = h_1\dot{N}_2 + h_2\dot{K}_2$$

Using the standard growth accounting approach we would say that the contribution of education to total growth in output was $g_3\dot{E}$ and would calculate that contribution by measuring the wage return to the N_2 educated laborers. However, we cannot ignore equation (12b). Substituting (12a) into (12b) we get

$$\dot{Y} = g_1\dot{N}_1 + g_2\dot{K}_1 + g_3h_1\dot{N}_2 + g_3h_2\dot{K}_2 \tag{13}$$

Since firms are cost minimizers the marginal product of any "unrefined" unit of labor must be the same and the marginal product of capital must be the same across sectors. Thus

$$g_1 = g_3h_1 \tag{14a}$$

$$g_2 = g_3h_2. \tag{14b}$$

Substituting equations (14) into (13) we get

$$\dot{Y} = g_1(\dot{N}_1 + \dot{N}_2) + g_2(\dot{K}_1 + \dot{K}_2)$$

$$\dot{Y} = g_1\dot{\bar{N}} + g_2\dot{\bar{K}}.$$

We find that all growth in output is due to growth in the total amount of primary inputs. Because factors are always allocated so that marginal productivities in different uses are equal, there is <u>no marginal</u> contribution of education to the production process.[3] The accounting error comes in treating $g_3\dot{E}$ as the total contribution of educated laborers. In fact, $g_3\dot{E} = g_3h_1\dot{N}_2 + g_3h_2\dot{K}_2$. The marginal productivity of the educated laborers is the same as that of unskilled laborers. It appears higher to the growth accountant because the capital embodied in the laborer is not seen. The return to E is not just the return to the N_2 educated laborers that provide skilled services but also incorporates the return to the capital necessary to provide the workers with their skills. Because education is an intermediate good that is factor absorbing in its production, simply calculating the financial rewards to workers who are educated as a measure of their productivity ignores the economic rewards to the resources needed to produce the education. At the margin education contributes nothing because raw labor is allocated so that its marginal productivity is equal across sectors. If educated workers are receiving a net reward in excess of that received by the uneducated, the uneducated will become educated until the net rewards are equalized.

The marginal accounting method, correctly applied, will lead us to conclude that the net contribution of education is zero. Clearly this is not the case. Although the net contribution of the last worker educated is zero, the inframarginal educated workers have a positive contribution to output. The correct way to evaluate the contribution of education is to measure its inframarginal contribution to production. Although such measures are common in applied welfare economics, they are most often presented for consumer goods. To provide motivation for our proposed measure of education's contribution to growth, let us consider an analogy from consumer goods. Suppose we increase a consumer's income by $1000, holding commodity prices constant, and we observe that the consumer's expenditure on food increase by $200. We might be tempted to conclude that the contribution of this additional food to his increased welfare is $200, but in doing so we would ignore the alternatives on which the $200 could have otherwise been spent. The correct measure of the contribution of food to his welfare is his utility given he can spend his $1000 as he pleases minus his utility if he is constrained to spend the additional $1,000 on anything but food. This measurement is his increase in welfare because he can spend money on the available food. The economist is able, in theory, to make two observations in this case. First, the economist can account for an individual's expenditures: "Out of the additional $1000, $200 was spent on food." Secondly the economist can calculate how much this expenditure increased the consumer's welfare: "The welfare contribution of the additional $200 in food is the excess of his utility over that which would have attained had the consumer been unable to buy food." These are two very different statements. Both are interesting observations, but the first has only descriptive import and the second has normative import. Before completing the analogy by presenting the production equivalent to this example, let us re-enforce our point with another simple illustration. Suppose that food were an inferior good. Our consumer, when given the extra $1000 income, would decrease his expenditure on food, say, by $50. We could hardly argue that food reduced the consumer's welfare! The accounting observation would show that expenditures on food had decreased. The correct welfare measurement of the contribution of food would measure the consumer's utility given his additional $1,000 spent as desired minus the consumer's utility if constrained not to change his expenditures on food. Clearly the welfare measure would show a positive value to food purchases.

The consumer example is fully analogous to the production case. Consider an exogenous increase in primary factors of production, such that marginal rates of substitution among factors remain constant if resources are efficiently allocated. This change is equivalent in the consumer example to increasing income holding commodity prices constant. Output will increase. The growth accountant's measure of the increase in output due to the increase in education is the opportunity cost of factors diverted toward education. It is a measure of how much of the growth in endowments is "spent" on education. To claim that this is the contribution of education to increased welfare ignores the alternative ways those basic resources could have been allocated. The correct measure of the contribution of education to output is the amount of output that actually is produced minus the output level that would have been chosen had none of the additional resources been allocated to the education sector. The growth accountant's observation tells us how much of our resources growth we devoted to the education sector. Although this is interesting as a descriptive measure, the normative measure of education's contribution must take into account what would have been had the education option been unavailable. Again we have two measures. The first measures how inputs were actually allocated to produce the additional output -- in particular how many resources were devoted to education. The second measure tells us what contribution the availability of the education alternative made to output growth. It measures what welfare is as compared to what it might have been had we not had the education sector.

The theoretical distinction between the growth accountant's measure and the surplus measure we propose should be clear. The question now becomes how one might implement the second measure empirically. Let us consider the consumer example again. To derive this measure we first have to trace the value of food as income changes, that is the shadow price of food. Let Z denote quantity of food, I denote income and P denote the actual price while $\tilde{P}$ denotes the shadow price. Then the movement of the shadow price is described by

$$\frac{dZ}{dI} = \frac{\partial Z}{\partial I} + \frac{\partial Z}{\partial P}\frac{\partial \tilde{P}}{\partial I} = 0. \tag{15}$$

The first term on the RHS of (15) is the ordinary income effect. The second term has two parts. The first is a pure substitution effect and the other is the induced rate of increase in food's shadow price. The induced price increase is just enough to hold the net change in Z at zero given the change in income. In other words, equation (15) implicitly describes the marginal value of food at the initial level of consumption of Z as income changes. To a second order approximation the consumer's surplus is

$$\frac{1}{2}\Delta\tilde{P}\Delta Z \tag{16}$$

where

(17) $$\Delta\tilde{P} = \frac{\partial\tilde{P}}{\partial I}\Delta I = \frac{-\frac{\partial Z}{\partial I}}{\frac{\partial Z}{\partial P}}\Delta I \qquad \text{(from 15)}$$

and

(18) $$\Delta Z = \frac{\partial Z}{\partial I}\Delta I.$$

Substituting (17) and (18) into (16) and rearranging terms we find that the measure of consumer's surplus is

(19) $$-\frac{C}{2}\frac{\varepsilon^2}{\eta}\frac{(\Delta I)^2}{I}$$

where C is food's expenditure share, ε is the income elasticity of food, and η is the utility constant own price elasticity.[4] We have estimates or observations on all the components of our measure and thus it can be empirically implemented.

The exact same analysis can be done for the production side of the economy. The analogous formula (see Appendix A) for education's net productivity is

(20) $$-\frac{S}{2}\frac{\xi^2}{\sigma_{ii}}\frac{(\Delta Y)^2}{Y}$$

where S is the scale elasticity described in equation (3), ξ is the elasticity of demand for education with respect to aggregate output (holding marginal rates of factor substitution constant), σ_{ii} is the Allen-Uzawa[5] own substitution elasticity, and Y denotes the value of aggregate output.[6]

We are not familiar with estimates in the literature of any of the requisite parameters needed to evaluate the net contribution of education to productivity. If the production process were Cobb-Douglas with parameter β_i as a coefficient on education, and subject to constant returns to scale, then $S = \xi = 1$ and $\sigma_{ii} = \frac{-(1-\beta_i)}{\beta_i}$. If $\frac{\Delta Y}{Y}$ were equal to 1 and $\beta_i = .25$, then education's net contribution would be approximately one-sixth of the increase in output. If $\frac{\Delta Y}{Y}$ were equal to 0.1, then education would have only contributed $\frac{1}{60}$ of the growth.[7]

In the next section we examine a few of the empirical attempts to measure education's contribution to growth. None of them uses either of the concepts presented in their extreme form, but they are usually closer to the first method than the second.

Section II:

Methods of Empirical Measurement

Much detailed empirical work has been aimed at measuring the sources of growth in the economies of the United States and other developed countries. Clearly education must have played a role in this productivity growth, but as demonstrated in Section 1, the standard approach for measuring the extent of that role does not properly measure the opportunity cost of producing educated workers. In this Section, we describe several methods of estimating the contribution of education, and show their potential inadequacy in explaining growth due to education using two simple examples.

The work by Griliches and Jorgenson (1967) is exemplary of the "pure" growth accounting approach to measuring the effect of education on productivity.[8] They begin with the basic growth accounting equation as we have characterized it in equation (2):

$$\frac{\dot{Y}}{Y} = \sum_i S_i \frac{\dot{X}_i}{X_i} + \frac{\tau}{Y} \frac{\partial Y}{\partial \tau} \frac{\dot{\tau}}{\tau} .$$

One of their inputs is labor, and they construct the index of labor services as

$$\frac{\dot{L}}{L} = \Sigma S_i \frac{\dot{L}_i}{L_i} \tag{21}$$

where the L_i represent hours of labor input of education type i. They separate the rate of growth of labor input into three components: the rate of growth of the labor force, $\frac{\dot{N}}{N}$; the rate of growth of hours per man $\frac{\dot{H}}{H}$; and the change in the proportional distribution of labor among the educational types. Letting e_i denote the proportion of workers of education type i,

$$\frac{\dot{L}}{L} = \frac{\dot{H}}{H} + \frac{\dot{N}}{N} + \Sigma S_i \frac{\dot{e}_i}{e_i} . \tag{22}$$

The last term on the RHS is computed by summing the share weighted change in proportions. They break labor into eight educational groups and using the last term of (22) compute what they call the annual percentage change in labor-input per man hour. This index varies from 0.62% during the period from 1948-52 to 1.2% from 1957-59. The average annual rate of change from 1940 to 1965 was 0.74%. The GJ study of productivity is certainly a pioneering work and it is improper to criticize it for not computing a fine enough index of change in labor input. They admit that the classification of labor should be made by age, sex, occupation, industry, among other components but such detailed data was not available to them. Thus their only breakdown of labor is by educational level, and the value of each additional hour of labor in any educational group i is measured at its "value of marginal product" as reflected by cost share. They treat additional education as if it had no opportunity cost or alternatively as if the determination of the education level were exogenous. Such an index is an adequate way of measuring changes in the labor force due to exogenous demographic shifts in composition but ignores the whole notion of opportunity cost of factors whose allocations are determined endogenously. In essence, the GJ study represents the accountant's breakdown of growth into its various components, without regard to what growth would have been had resources been arranged differently. It is a descriptive measure of how we spent the increase in primary inputs and is not a measure of the opportunity costs of those inputs. The valuation of such factors at true value of marginal product, that is gross VMP less marginal cost, is meaningless since at the margin the product of education just equals the opportunity cost.

The recent analyses by Chinloy (1980), Denison and Jorgenson et al. are similar in nature to the GJ study, but some attempt is made to make use of the notion of the opportunity cost of an educated worker's time. We characterize Chinloy's work since he constructs a more complete index of labor productivity than do the other authors, but they are all very much in the same spirit.

The Chinloy model follows the basic growth accounting model as introduced in the first section. Changing notation slightly, let superscripts denote time period and subscripts denote educational group. We observe N^t laborers in period t, N_1^t of them being uneducated and N_2^t of them educated. The wages paid to each uneducated laborer in period t is w_1^t and educated laborers receive w_2^t for their services. Consider only two periods, $t=0,1$. Then the growth in total labor force (Chinloy uses hours, we will use number of workers) is defined by:

$$(23) \qquad h = \ell n(N^1) - \ell n(N^0) = \ell n(\frac{N^1}{N^0}) .$$

Chinloy compares this growth in pure units of input to an index in the change in labor productivity derived from an assumed translog production function. Specifically let V_1 be the average share of the total labor bill received by uneducated workers and $V_2 = 1 - V_1$ be the average share received by educated workers. Then,

$$(24) \qquad V_1 = \frac{1}{2}(\frac{w_1^o N_1^o}{w_1^o N_1^o + w_2^o N_2^o} + \frac{w_1^1 N_1^1}{w_1^1 N_1^1 + w_2^1 N_2^1}) .$$

These shares are used to take a weighted average of indices in the growth of the size of each segment of the labor force. Specifically, let

$$\Delta\ell n\ h_1 = \ell n(\frac{N_1^1}{N_1^o})$$

(25)

$$\Delta\ell n\ h_2 = \ell n(\frac{N_2^1}{N_2^o})$$

The index of the growth rate for labor productivity is

$$d = V_1\ \Delta\ell n\ h_1 + V_2\ \Delta\ell n\ h_2.$$

The growth rate for quality change is

$$q = d - h.$$

In this setting, q represents the contribution of education to productivity growth. Chinloy's index of quality change, q, implicitly includes a measure of cost. The index h is a measure of what output would have been had growth in the labor force maintained the initial proportions between educated and uneducated labor. It is in some sense a measure of what would have been had labor not "reallocated" itself. Thus the excess of what was over what would have been, Chinloy calls quality change. Chinloy's index of quality change, q, implicitly includes a measure of opportunity cost of the education process. The index h, is a measure of what the growth in output would have been had the initial proportions of educated and uneducated of labor been maintained. Specifically, if

$$\frac{N^1}{N^o} = \frac{N_1^1}{N_1^o} = \frac{N_2^1}{N_2^o},$$

Then

$$d = V_1 \ \ell n(\frac{N_1^1}{N_o^1}) + V_2 \ (\frac{N_2^1}{N_o^2})$$

$$= V_1 \ \ell n(\frac{N^1}{N^o}) + (1-V_1) \ \ell n(\frac{N^1}{N^o})$$

$$= \ell n(\frac{N^1}{N^o})$$

$$= h$$

which implies $$q = d - h$$

$$= 0.$$

Chinloy would conclude there has been no quality change in the composition of the labor force. This conclusion is in some sense correct. The average level of education of the populace has not changed, and the proportion of output rewarded to educated workers has not changed. It seems the entire growth of output is due to population growth. It would be incorrect however to say that the increased level of education contributed nothing to increased output. Consider, for example, the simple production process depicted in Figure 1. Initially there are N^o laborers available and N_2^o are educated, N_1^o are uneducated. Output is initially Y^o. Suppose the total workforce expands from N^o to N^1, output expands from Y^o to Y^1 and the proportion of educated workers remains constant. As shown, the Chinloy formulation would show a zero contribution to the change in the educational level of the population. However, suppose the education alternative had not been available for the $N^1 - N^o$ new workers. Clearly output would not have increased to Y^o, but to $Y* < Y^o$ as depicted in Figure 2. The contribution of education at the margin is zero, but there is an inframarginal contribution that is positive.

FIGURE 1

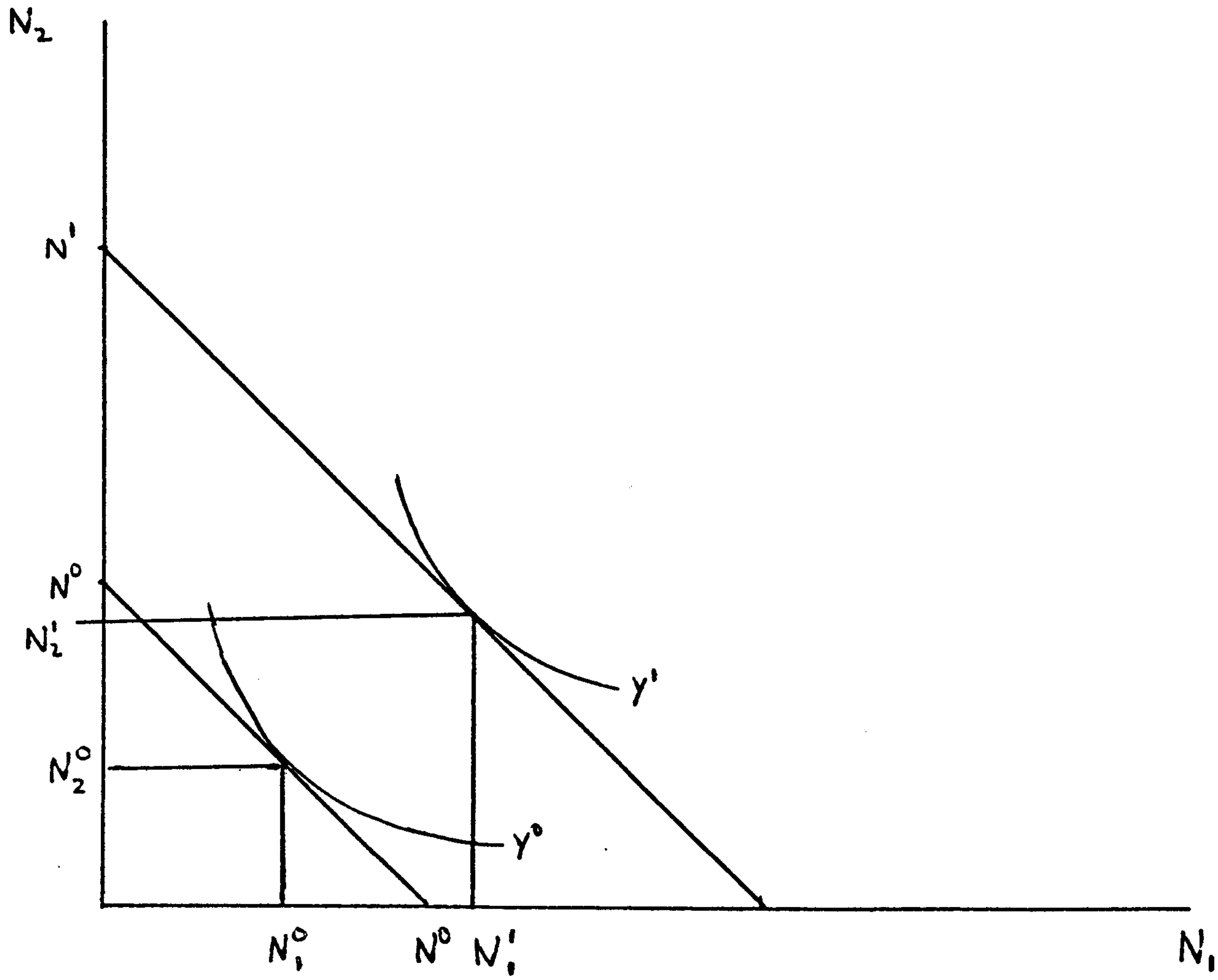
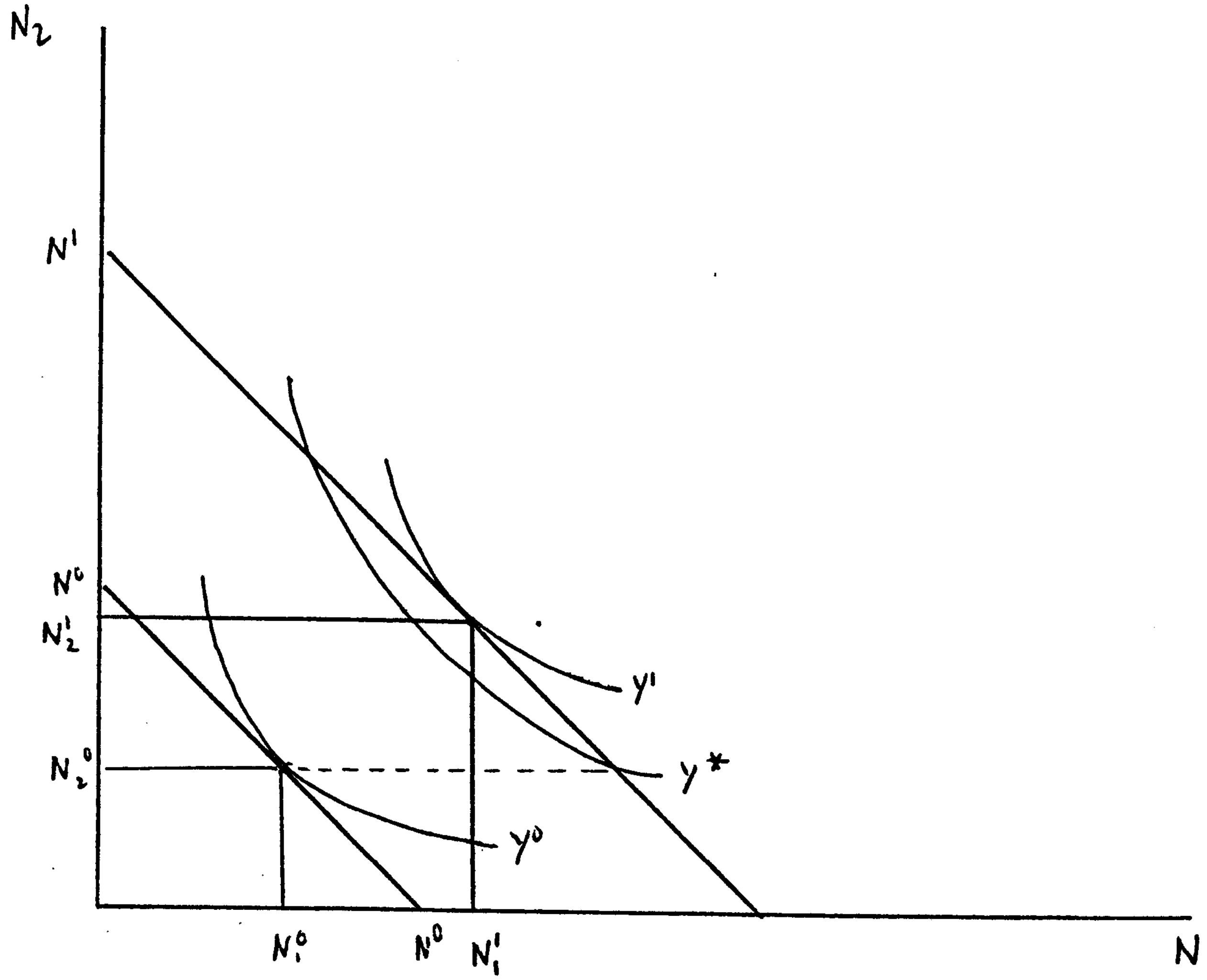


FIGURE 2



Denison (1974, for example) explicitly forms an index of labor services based on the norm of an eight-grade educated worker. Using the ratio of wages of higher educated workers to eighth grade workers he forms an index that computes base productivity of labor as if all workers were eighth grade educated, (correcting for the correlation of ability and education) and attributes the excess of actual returns to labor over baseline returns to labor as the contribution of education to the productive process. In essence, Denison recognizes that (part of) the opportunity cost of educating a worker is his foregone productivity as an uneducated worker. However, Denison, like Chinloy, uses a marginalist approach to the productivity accounting. The "returns" to education that he measures are returns to factors used in the production of education that he does not include in his measure of opportunity cost. Referring back to Section 1, Denison and Chinloy implicitly have a model like that represented in equations 11a-d in mind, but the returns they measure are returns to the capital "imbued" in the workers who are educated. The increased wages rewarded to educated workers are a return to a costly investment, and at the margin the net value of that investment is zero. Denison and Chinloy, through comparing actual productivity to some baseline expected productivity attempt to measure the opportunity cost of the educated laborer, but they do not measure the opportunity cost of other resources used in the educational process.

The Denison and Chinloy studies are an intermediate stage between a purely descriptive characterization of the sources of growth and an evaluation of the contribution of intermediate production processes using an opportunity cost measure. Both authors partially compute the opportunity cost of an educated laborer. Since the calculation is a marginal one, if they had calculated the cost fully, the contribution of the educational process would have been zero. The return to education that they measure is a return to the capital used in producing educated labor, but does not incorporate the opportunity cost of using that capital. The relevant normative measure used to evaluate the contribution of education to the productive process would be the excess of the actual return to capital used in the educational process above the return to that capital had it not been allocated to educational production. At the margin this contribution is zero, but there is a productive surplus to the inframarginal units.

At first glance Schultz (1961) takes a completely different tack in evaluating the educational process. He explicitly recognizes that education is an intermediate good that is costly to produce. The productive value of education in the economy is the yield from a stock of education. His method is to evaluate that stock in terms of resource cost and extrapolate the contribution of education to total product by imputing a value of the flow of services from that stock. Schultz begins by evaluating the stock of education in two years. Let us denote the resource cost of that stock as V^i, $i = 0,1$. Let Y_i continue to denote output in period i. The growth in the labor force over time is

$$\Delta N = \frac{N^1 - N^o}{N^o}.$$

To keep the per capita value of the stock of education constant, the value of the stock in the latter period should have been $(1 + \Delta N)V^o$, and thus the difference of actual from constant per capita value is

$$V^1 - (1 + \Delta N)V^o.$$

Of course the actual increase in the total value of the stock of education is simply $V^1 - V^o$. Assuming that education is purely an investment good, and is not made for consumption purposes, then the return to a dollar of capital invested in education should be the market rate of return, r. Thus the annual contribution of the additional stock is

$$r(V^1 - V^o).$$

Schultz assumes that over time the proportional contribution of labor to total product is a constant, s_N. Therefore, the proportion of labor's share of income growth due to increasing the total stock of education is

$$\frac{r(V^1 - V^o)}{s_N(y^1 - y^o)} .$$

The proportion of labor's share of output due to the increase in education per person is

$$\frac{r(V^1 - (1+\Delta N)V^o)}{s_N(Y^1 - Y^o)}$$

and the contribution of the increased stock of education to the increase in total output is simply

$$\frac{r(V^1 - (1+\Delta N)V^o)}{(Y^1 - Y^o)} . \tag{26}$$

To implement these formulae, Schultz makes some involved calculations regarding the costs of education, and assumes that labor's share of output is a constant, $s_N = 0.75$. His most critical assumption is that the correct rate of return is the individual, internal rate of return to an investment in schooling such as those calculated by Becker. At the margin, that rate of return must be chosen so that the present value of the earnings stream of the educated individual just equals the present value of the opportunity cost of that education, otherwise more individuals will become educated until the marginal return to an education is zero. Using a market rate of return to value the return to the inframarginal units of education is wrong for the same reasons we have outlined in the first section. At the margin the net return is zero, but for inframarginal units the net contribution is positive.

Although Schultz's method is cosmetically different from the mainstream growth accountants' techniques, in essence he is making the same measurement. The application of the internal rate of return to the value of the stock of education simply translates the contribution of education into a flow measure such as those used by the other authors. His comparison measurement of contribution represented in equation (26) is basically the same as that of Denison and Chinloy. rV^1 is simply the output that resulted from the actual capital outlay on education and $r(1+\Delta N)V^o$ is that output which would have been produced had the per capita "amount" of education remained constant. This does not measure what would have been produced had the $V^1 - (1+\Delta N)V^o$ dollars worth of capital been used in the next best alternative productive use. Thus like Chinloy and Denison, Schultz measures what growth would have been had we had no resources to devote to education in excess of those which would have kept per capita education the same (i.e., all new labor comes in as "eighth" graders). Instead he should measure what output would have been had we devoted the additional resources spent on increasing the educational level of the population to other production processes.

To illustrate the inability of the marginal accountants' approach to evaluating productivity to explain education's share in that productivity we present two simple numerical examples. First consider the following non-CRS production function:[9]

$$Y = \frac{(N_2-4)}{(N_1-8)^2}$$

where $N_2 > 4$ and $N_1 < 8$, where N_2 represents the number of educated laborers and N_1 represents the number of uneducated laborers. Suppose that in period 0 $N_1 + N_2 \leq 9$, and in period 1, $N_1 + N_2 \leq 10$, that is the labor force grew by one unit. The output maximizing labor allocations and value of marginal products are

$$N_1^o = 2 \quad VMP_1^o = 0.027$$

$$N_2^o = 7 \quad VMP_2^o = 1$$

and

$$N_1^1 = 4 \quad VMP_1^1 = 0.0625$$

$$N_2^1 = 6 \quad VMP_2^1 = 0.25$$

In period 0, $Y^o = 0.0833$ and later $Y^1 = 0.125$. Clearly, in this example, educated labor is an inferior good, since as total labor available increases, output increases, but the amount of educated labor decreases. Using base year weights, the basic growth accounting formula would be:

$$\frac{y^1 - y^o}{y^o} = s_1^o\left(\frac{N_1^1 - N_1^o}{N_1^o}\right) + s_2^o\left(\frac{N_2^1 - N_2^o}{N_1^o}\right) + \tau$$

$$\frac{y^1 - y^o}{y^o} = .01\ \left(\frac{4-2}{2}\right) + .99\ \left(\frac{6-7}{7}\right) + \tau$$

$$= 0.13 + \tau^{10}$$

The "growth" attributed to the educated workforce would be negative, and in fact, due to the non-CRS nature of the production function most of the growth would be attributed to the residual term which in fact reflects a change in scale. Using marginal growth accounting would lead us to conclude education contributed -13% to the 50% growth in output! The measure we proposed in equation (20) in section 1 yields a different answer. Our measure demonstrates that education contributes 18% to productivity growth in this simple example.[11] Clearly education has had a positive contribution, but the fact that in this example educated workers were an inferior input shows plainly how simple application of marginal growth accounting can err.

We have presented the previous example not with realism in mind, but instead as a polar case -- one in which the surplus measure of the value of education gives a reasonable answer and the growth accounting measures does not. The second example we present uses a more standard production technology and is designed to show how different measures of the contribution of education to productivity can vary in a non-pathological setting. Specifically, growth accountants will overestimate the contribution of education because they ignore the opportunity cost of the resources used.

Let the production process be characterized by a two-step process in which a certain amount of capital is "embodied" in a number of workers we will call educated and these educated workers enter and help in the production of the final product. Specifically, let K_1 denote the number of units of capital devoted to production of the final product and K_2 denote the number of units of capital devoted to the education process. Then our economy is characterized by the relationships

$$E = g(N_2,K_2)$$

$$Y = f(N_1,K_1,E)$$

$$N_1 + N_2 \leq N$$

$$K_1 + K_2 \leq K.$$

For illustration's sake assume g is a constant elasticity of substitution production function and f is a Cobb-Douglas production function:

$$E = (\gamma_N N_2^{-\rho} + \gamma_K K_2^{-\rho})^{-\frac{1}{\rho}}$$

$$Y = N_1^{\beta_N} K_1^{\beta_K} E^{\beta_E}$$

where $\gamma_N + \gamma_K = 1$ and $\beta_N + \beta_K + \beta_E = 1$. For the following example we assume the production technology remains fixed over time, as does the available number of units of capital K. In Table 1, we present the parameter values used in the production functions and the resulting allocation of resources in each of two periods. In the first period we assume $N = 1$ and in the second period $N = 2$. As N grows the marginal productivity of labor decreases in both educated and uneducated forms, because the capital stock remains fixed. We assume the education process is not "labor-using," that is all N_2 workers are employed in producing Y and they each receive equal fractions of the total payment to educated labor.

Let us first apply the Chinloy method of computing the quality of the labor force to this example. Using the formulae presented earlier, the following indices can be computed using the information in Table 1:

$V_1 = 0.125$

$V_2 = 0.875$

$\Delta \ell n\ N_1 = 0.8353$

$\Delta \ell n\ N_2 = 0.6187$

$d = 0.6457$

$h = 0.6931$

$q = -0.0474$

Thus Chinloy would conclude that the quality of the labor force has decreased. We get this "decrease in quality" because we are increasing the

Table 1

Production Function Example

Values of Parameters

$\gamma_N = 0.3$

$\gamma_K = 0.7$

$\rho = 0.5$

$\beta_N = 0.1$

$\beta_K = 0.2$

$\beta_E = 0.7$

$K = 1$

Period 1 $N = 1$

$N_1 = 0.3195$ $N_2 = 0.6805$

$K_1 = 0.2911$ $K_2 = 0.7089$

$Y = 0.5431$

$VMP_N = 0.1700$

$VMP_K = 0.3731$

$VMP_E = 0.5430$

Total payments to uneducated labor = 0.05431

Total payments to capital in productive sector = 0.1086

Total payments to educated labor = 0.3801

Payment per educated worker = 0.5587

Period 2 $N = 2$

$N_1 = 0.7364$ $N_2 = 1.2636$

$K_1 = 0.2746$ $K_2 = 0.7254$

$Y = 0.6649$

$VMP_N = 0.09029$

$VMP_K = 0.4843$

$VMP_E = 0.5517$

Total payments to uneducated labor = 0.06649

Total payments to capital in productive sector = 0.1330

Total payments to educated labor = 0.4654

Payments per educated laborer = 0.3683

size of the labor force holding the capital stock fixed, thus decreasing the marginal product of the labor force. One might be led to conclude from the Chinloy accounting method that had education growth "kept pace" with population growth output would be higher. In fact this is not true, since the allocations presented for period 2 maximize output subject to the resource constraints. We will grant that the proportion of educated workers has decreased from 68.05% to 63.18%, but this does not imply increased productivity had the proportion remained constant.

As an alternative to Chinloy's approach to the quality of the labor force, we can consider the standard growth accounting approach as used by authors such as Griliches and Jorgenson (1967). The basic accounting equation for this simple model would be

$$\frac{\dot{Y}}{Y} = s_{K_1} \frac{\dot{K}_1}{K_1} + s_{N_1} \frac{\dot{N}_1}{N} + s_E \frac{\dot{E}}{E} .$$

In theory one would like to measure the units of education E, but these are not observed. Only the number of educated workers N_2 is observed. Using base year percentages (i.e., $\frac{\dot{Y}}{Y} \simeq \frac{\Delta Y}{Y_o}$) to make the required calculations from Table 1 we get:

$$\frac{\dot{Y}}{Y} = .2243$$

$$\text{explained growth} = .7189$$

$$\tau = -.4946.$$ [12]

Clearly, ignoring the means by which educated labor is generated, the growth accountants would make a serious mistake in evaluating the progress of the economy. Because the education process is capital using, and a growing labor force is being applied to a fixed stock of capital the marginal product of labor decreases substantially, however, the technology of the economy has not changed at all. The constraint of a fixed resource being used as input to the fabrication of an intermediate input is being interpreted as technological "regression". Perhaps we are being too harsh on the growth accountants. Suppose that they recognize that part of the capital stock is used in the educational process, and can measure its value to the economy as a separate input. Then the growth accounting equation would be

$$\frac{\dot{Y}}{Y} = s_{K_1}\frac{\dot{K}_1}{K_1} + s_{N_1}\frac{\dot{N}_1}{N_1} + s_{N_2}\frac{\dot{N}_2}{N_2} + s_{K_2}\frac{\dot{K}_2}{K_2} + \tau$$

where s_{N_2} and s_{K_2} would be the share in value of inputs of labor and capital used in the education process. Somehow, the returns to the labor input would have to be separated from the returns to the capital input. Using this last accounting identity we get

$$\frac{\dot{Y}}{Y} = .2243$$

$$\text{explained growth} = .2956$$

$$\tau = -.0713 \; .$$

This accounting is clearly more reasonable, although still misinterprets fixed resource constraints as decreased technical progress.

The Schultz formulation of the marginalist growth accounting in this simple example can also be computed. Using the Figures in Table 1, we can compute:

$$V^o = VMP_K^o \times K_2^o = (0.3731)(0.7089) = 0.2644$$

$$V^1 = VMP_K^1 \times K_2^1 = (0.4843)(0.7254) = 0.3513.$$

Furthermore,

$$\Delta N = \frac{2-1}{1} = 1$$

and thus the value of the stock in the latter period, had per capita education remained the same, should have been 0.5288. The difference of actual from constant per capita educational stock is -0.1775.

Since the capital stock in this example has no intertemporal nature, the return to capital is simply its valuation in the market. That is a dollars worth of capital returns a dollar per year and the capital is regenerated each year, thus $r = 1$. Rather than assigning a constant share to labor output we can compute that the total rewards to labor in period zero are 0.05431 + 0.3801 = 0.4344 and in the period one are 0.06649 + 0.4654 = 0.5319. Thus the contribution of education to the rewards to labor is:

$$\frac{V^1 - V^o}{s_n^1 Y^1 - s_n^o Y^o} = \frac{0.3513 - 0.2644}{0.5319 - 0.4344} = 0.8914.$$

The contribution of the increased stock of education to total output is

$$\frac{V^1 - V^o}{y^1 - y^o} = \frac{0.3513 - 0.2644}{0.6649 - 0.5431} = 0.7135.$$

Schultz' method also demonstrates the proportion of growth due to the change in per capita education. Since per capita education has decreased, we are calculating the extent of growth that would have occurred had per capita education remained constant. In particular, labor's share would have grown by 182% more than it did (-0.1775/(0.5319 - 0.4344)) and total output would have grown by 145% more than it did (-0.1775/(0.6649 - 0.5431)).

Finally, we can compute the contribution of education to growth using the surplus measure in equation 20. Since the production function is Cobb-Douglas, this equation simplifies to

$$\left|\frac{1}{2}\,\frac{\beta_E}{1-\beta_E}\,\frac{\Delta Y}{Y}\right|\,\Delta Y$$

which is in our example equal to

$$(0.2616)\ \Delta Y.$$

That is, the growth in education is responsible for 26% of the growth in total productivity. Clearly we have at our disposal the true parameters and the true functional form from which to make our calculation. Although this is a luxury not afforded the actual investigator, it illustrates the potential variation in the two different approaches to measurement. Table 2 summarizes the various measures in this simple example.

We conclude this paper by summarizing our basic point. When dealing with an intermediate input marginal growth accounting errs by treating an increase in the amount of the input and as if it were exogenously bestowed on the economy. The opportunity cost of the resources used to purchase that input are ignored. In fact, at the margin, if production is efficient, the net marginal contribution of an intermediate input is zero. The contribution of the input is an inframarginal contribution that the standard accounting framework cannot capture. The growth accounting framework is useful for calculating the part of actual output growth attributable to growth in certain inputs. It accounts for where out growth came from, but does not allow us to measure the value of the processes generating that growth. We propose instead a measure borrowed from the literature on consumers surplus that captures the returns to the inframarginal units of the intermediate input. The essential question is what would output have been if the option to educate new workers was not available? Our proposed measure is an attempt to answer that question. Clearly, the next order of business for those concerned with the value of education is to compute the necessary empirical measures to implement this formula.

Table 2

Summary of Measurements of Growth in Producitivty Using Examples in Table I

Chinloy:	Quality of Labor Force decreased by -0.0474
Standard Growth Accounting:	Actual growth = 22.49%
	Explained growth = 71.89%
	Residual = -49.46%
	Educated labor's share of explained growth = 83.5%
Sophisticated Growth Accounting:	Actual growth = 22.43%
	Explained growth = 29.56%
	Residual = -7.13%
	Educated labor's share of explained growth = 42.01%
Schultz:	Contribution of increased stock of education = 71.35%
Plant-Welch:	Contribution of increased stock of education = 26.2%

FOOTNOTES

[1]These statistics are taken from the Handbook of Labor Statistics (1979), Table 188 and the Digest of Educational Statistics (1980), Tables 11 and 18. The educational expenditure figures reported in the Digest do not include any estimates of the foregone earnings of potential workers who are being educated.

2
$$s_i = \frac{x_i f_i}{y}$$

$$= \frac{p_i x_i}{\Sigma p_i x_i} \cdot \frac{f_i}{y} \cdot \frac{\Sigma p_i x_i}{p_i}$$

$$= C_i \left[\frac{\Sigma p_i x_i}{y}\right] \frac{f_i}{p_i}$$

$$= C_i \text{ (average cost) } \left(\frac{1}{\text{marginal cost}}\right).$$

[3]This model can be reformulated in per capita terms if the production function is linear homogeneous. The result is that per capita growth in output is completely explained by the per capita growth in non-labor inputs. Letting lower case letters denote per capita growth the result is derived as follows:

$$\dot{y} = g_1 \dot{n}_1 + g_2 \dot{k}_1 + g_3 \dot{e}$$

$$\dot{e} = h_1 \dot{n}_2 + h_2 \dot{k}_2$$

and using 14(a) and 14(b) in the text we get

$$\dot{y} = g_1(\dot{n}_1 + \dot{n}_2) + g_2(\dot{k}_1 + \dot{k}_2)$$

but

$$\dot{n}_1 + \dot{n}_2 = 0$$

since all units are in per capita terms. Thus

$$\dot{y} = g_2 \dot{k}.$$

4 $$\frac{1}{2}\Delta\tilde{P}\Delta Z = \frac{1}{2}\left[\frac{-\frac{\partial Z}{\partial I}}{\frac{\partial Z}{\partial P}}\right]\Delta I \frac{\partial Z}{\partial I}\Delta I$$

$$= -\frac{1}{2}(\Delta I)^2 \cdot (\frac{\partial Z}{\partial I})^2 \frac{\partial P}{\partial Z}$$

$$= -\frac{1}{2}\frac{(\Delta I)^2}{I}(\frac{\partial Z}{\partial I})^2 \frac{I^2}{Z^2}(\frac{\partial P}{\partial Z}\frac{Z}{P})(\frac{PZ}{I})$$

$$= -\frac{1}{2}\frac{(\Delta I^2)}{I}\frac{\varepsilon^2}{\eta} C$$

[5]For an explanation of the Allen-Uzawa substitution elasticity see Layard and Walters (1978).

[6]Note that $\eta_{ii} = C_i \sigma_{ii}$. Also $\Sigma S_i \dot{\xi}_i = 1$ and since $S = S_i C_i$, we conclude $\Sigma C_i \xi_i = S^{-1}$. The derivation of (20) is given in the Appendix.

[7]Suppose without loss of generality that

$$Y = K^{1-\beta} E^{\beta}$$

and consider the producer to be minimizing costs: $p_K K + p_E E$ subject to an output constraint.

(i) The scale elasticity is derived by introducing a scale parameter λ into the production function:

$$Y = (\lambda K)^{1-\beta}(\lambda E)^{\beta}$$
$$= \lambda K^{1-\beta} E^{\beta}$$

and computing

$$S = \frac{\lambda}{Y}\frac{\partial Y}{\partial \lambda} = \frac{\lambda}{Y}(K^{1-\beta}\ E^{\beta})$$

$$= \frac{\lambda K^{1-\beta}\ E^{\beta}}{\lambda K^{1-\beta}\ E^{\beta}}$$

$$= 1.$$

(ii) The demand function for education is

$$E = Y\left[\frac{P_E}{P_k}\left(\frac{1-\beta}{\beta}\right)\right]^{\beta-1}.$$

The elasticity of demand with respect to output holding the price ratio constant is

$$\frac{Y}{E}\frac{\partial E}{\partial Y} = \frac{Y}{E}\left[\frac{P_E}{P_k}\left(\frac{1-\beta}{\beta}\right)\right]^{\beta-1}$$

$$= \left[\frac{P_E}{P_k}\left(\frac{1-\beta}{\beta}\right)\right]^{1-\beta}\left[\frac{P_E}{P_k}\left(\frac{1-\beta}{\beta}\right)\right]^{\beta-1}$$

$$= 1.$$

(iii) The Allen-Uzawa own elasticity of substitution is equal to (see Layard and Walters (1978))

$$\sigma_{ii} = \frac{\varepsilon_{ii}}{\nu_i}$$

where ε_{ii} = output constant elasticity of demand

ν_i = share of input i in total costs

and for input E

$$\varepsilon_{ii} = \frac{P_E}{E}\frac{\partial E}{\partial P_E}$$

$$= -(1-\beta)$$

and

$$\nu_i = \beta$$

so

$$\sigma_{ii} = \frac{-(1-\beta)}{\beta}.$$

The computations in the text then follow directly.

[8]In all fairness to Griliches and Jorgenson, later work done is much more sophisticated. We present their early model as a prototype and recognize that this was a seminal paper. See the references for more recent papers. These later papers are best characterized as being like Chinloy (1980).

[9]This production function was found on page 199 of Ferguson (1969). We thank Michael Darby for pointing us toward this monograph.

10 $$S_1^o = \frac{N^o . VMP_1^o}{N_1^o . VMP_1^o + N_2^o . VMP_2^o} = \frac{.054}{7.054} \doteq 0.01$$

$$S_2^o = 1 - S_1^o = 0.09.$$

[11]For this production function:

$$S = \frac{N_2}{N_2-4} + \frac{2N_1}{N_1-8}$$

$$\xi = \frac{N_2-4}{N_2}$$

$$\sigma_{ii} = [\frac{-N_2-4}{N_2}] \cdot \text{(cost share)}.$$

These formula used with base year values in equation (20) lead to a contribution of education which is 17.99% of the growth in output.

[12]Explained growth $= s_{k_1} \frac{\Delta \dot{K}_1}{K_{10}} + s_{n_1} \frac{\Delta N_1}{N_{10}} + s_e \frac{\Delta N_2}{N_{20}}$

$$= .2(-.05668) + (.1)(1.3049) + (.7)(.8569)$$

$$= .7189$$

Note that due to the production technology the v's are constant over time.

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APPENDIX

Derivation of Equation (20)

The measure of the contribution of the i^{th} input (education) is simply

$$(1) \qquad \frac{1}{2}\,\Delta f_i \Delta x_i$$

where f_i denotes $\frac{\partial f}{\partial x_i}$. By first order conditions of cost minimization

$$(2) \qquad \Delta f_i = \frac{\Delta p_i}{\lambda}$$

where λ is the marginal cost of production. The constraint that defines the shadow price is

$$\frac{dx_i}{dy} = \frac{\partial x_i}{\partial y} + \frac{\partial x_i}{\partial p_i}\,\frac{\partial p_i}{\partial y} = 0.$$

Therefore

$$(3) \qquad \Delta p_i = \frac{\partial p_i}{\partial y}\,\Delta y = \frac{\frac{-\partial x_i}{\partial y}}{\frac{\partial x_i}{\partial p_i}}\,\Delta y.$$

Also

$$(4) \qquad \Delta x_i = \frac{\partial x_i}{\partial y}\,\Delta y.$$

Substituting (2), (3) and (4) into (1) we get

$$(5) \qquad -\frac{1}{2\lambda}\left(\frac{\partial x_i^2}{\partial y}\right)(\Delta y)^2\,\frac{\partial p_i}{\partial x_i}\ .$$

Rearranging terms we get:

$$(6) \qquad \frac{-1}{2\lambda}\left(\frac{\partial x_i}{\partial y}\frac{y}{x}\right)^2\left(\frac{\partial p_i}{\partial x_i}\frac{x_i}{p_i}\right)\frac{x_i^2}{y^2}\frac{p_i}{x_i}(\Delta y)^2$$

which equals

$$\frac{-1}{2\lambda}\frac{\xi_i^2}{\eta_{ii}}\frac{p_i x_i}{y}\frac{(\Delta y)^2}{y} \quad .$$

But $\frac{p_i}{\lambda} = f_i$

and $\eta_{ii} = \frac{f_i x_i}{\Sigma f_j x_j}\sigma_{ii}$

So

$$(7) \qquad \frac{-1}{2}\frac{\xi}{\sigma_{ii}}\frac{\Sigma f_j x_j}{y}\frac{(\Delta y)^2}{y} \quad .$$

But $\Sigma f_i x_i$ = marginal cost = Sy so (7) becomes

$$(8) \qquad \frac{-1}{2}\frac{\xi}{\sigma_{ii}} S \frac{(\Delta y)^2}{y} \quad ,$$

which is the desired formula.