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THE LIQUIDITY PREMIUM AND THE SOLIDITY PREMIUM

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The term structure of real interest rates is normally ascending. Given an ascending term structure, the implicit forward short-term rate of interest (for any specified future date) must be higher than the current short-term interest rate. Nevertheless, there is no evidence of any upward trend in real interest rates over time. It follows that implicit forward short-term rates must also be higher than the actual short-term rates that will be realized on average, in the future. This difference, between the implicit forward short-term rate and the mathematical expectation of the future short-term rate, is called the liquidity premium.

Thus two stylized facts: 1) an ascending term structure at any moment of time, and 2) historical stationarity of interest rates over time, imply a positive liquidity premium. Since historical stationarity might seem to need no special justification, explaining the liquidity premium reduces to explaining why the term structure is normally ascending. However, non-ascending term structures are not rare. As for the historical stationarity of interest rates, empirically there seems to be no strong ground on which to postulate such a degree of uniformity. I therefore do not adopt the stylized facts as my point of departure. Instead, my intention is to identify the forces explaining the liquidity premium as such, independent of the shape of the term structure and the level of interest rates. Doing so will illuminate not only the determination of the liquidity premium but of the term structure as well.

Hicks [1946] and Keynes [1930] argued that it is risk-aversion which causes the forward rate to be greater than the expected future rate. Possible future variations in interest rates will affect the values of long-term bonds more than short-term bonds. Consequently, they argued, a yield premium is needed to induce risk-averse investors to hold the "less liquid" (more risky) longer-term bonds.

The Keynes-Hicks view has been criticized, and properly so, for its over-emphasis on capital-value risk as opposed to income risk. Someone who wants only to lock in a more-distant-future flow of income could simply make a long-term investment commitment, and then be entirely unconcerned about possible interest-rate variations and consequent fluctuations in capital values at intermediate dates. For such an individual, a yield premium might be required to induce him to hold a shorter-term instrument.

Such a consideration led Modigliani and Sutch [1966] to the notion of the "preferred habitat". They argued that because of personal variations in individual motives to save or to consume at different dates (due, for example, to life-cycle considerations), different investors would typically be concerned about consumption risk at different dates. Thus, nothing of a high order of generality could be said about the sign of the liquidity premium.

While the Modigliani-Sutch criticism of Keynes and Hicks is well-founded, their terminology may incorrectly suggest that each investor is typically concerned only with income risk at some single "habitat" date, or even that an individual's temporal consumption plans are exogenous data rather than endogenous variables. In the models to be presented here individuals are concerned about income risks at every date (up to their planning horizons), and their temporal consumption plans are choice variables that depend upon relative prices and interest rates.

Once we acknowledge that individuals are risk averse with respect to income at all dates in their lifetimes, we must also accept that there is generally no single measure of the riskiness of an asset. For example, in comparing two pure discount, default-free bonds, one maturing at an earlier date and one maturing at a later date, the risk configuration that first comes to mind (recognizing throughout that each bond yields a riskless return as of its own maturity date) is: (a) the longer-term bond is risky as of the earlier date, since its liquidation value will vary with intervening interest-rate fluctuations, while (b) the shorter-term bond is risky as of the later date, since the investor's terminal return will vary with the rollover interest rate. The question "which maturity-strategy is more risky?" has no single unambiguous answer.

Suppose, on the other hand that the terminal value of the short-term bond rolled-over covaries against the endowed consumption risk at the later date; that is, short-term bonds provide insurance with respect to the later date. Then buying the short-term bond and rolling over is <u>even less risky</u> with respect to the later date than holding a riskless bond maturing at that date. Here we have a configuration where at the earlier date the long-term bond is risky and the short-term bond is riskless, while at the later date the long-term bond is merely riskless but the short-term strategy provides insurance. The short-term strategy is the less risky strategy for obtaining consumption <u>at both the earlier date and the later date</u>! Hence in equilibrium the short-term strategy must have a lower rate of return, on average, over both the short-haul and the long-haul, than the long-term strategy. This means, of course, that the liquidity premium is positive.

In what follows I will show that the liquidity premium will be positive if and only if the second risk-configuration described above prevails. I will also show there is a third configuration in which the long-term bonds are less risky and therefore have on average the lower rate of return. I will be employing a general-equilibrium model to analyze how individuals' time-and-state distributed endowments, preferences (for consumption and also for risk-bearing), beliefs (and the timing of information arrival affecting beliefs), and productive opportunities all contribute to the shaping of the term structure and to the differential rates of return to short-term and long-term bonds. I will first use a simplified world of pure exchange to highlight certain elements of the general theory of the liquidity premium. I will show, in particular, how the amount of and timing of anticipated information arrival affect the liquidity premium. I will then introduce intertemporal production. Here the interaction of production with information about the realized magnitude of representative endowments (e.g., good crop or bad crop) affects beliefs about consumption in a systematic way, and plays a critical role in determining the liquidity premium.

I. DEFINITIONS

We can cover all the essentials of the problem by dealing with a simple three-date model (dates 0, 1, and 2 years from the present). The current short-term real interest rate ${}_0r_1$ is defined in:

$$\frac{{}_0P_1}{{}_0P_0} \equiv \frac{1}{1 + {}_0r_1} \tag{1}$$

Here ${}_0P_1$ is the price quoted today (date-0) of a unit claim to real income to be received next year (at date-1). ${}_0P_0$ is the price today of a unit of real income today; I will ordinarily take current income of any date as the numeraire commodity for prices quoted at that date, so that ${}_0P_0 \equiv 1$. The interest rate denoted ${}_0r_1$ is then the rate quoted at date-0, today, for discounting one-year future claims into their current (present-value) equivalent.

Analogously, the current long-term (2-year) interest rate ${}_0R_2$ is defined in:

$$\frac{{}_0P_2}{{}_0P_0} \equiv \frac{1}{(1 + {}_0R_2)^2} \tag{2}$$

Here ${}_0P_2$ is the price quoted today of a claim to income to be received at date-2, and ${}_0R_2$ is the corresponding long-term interest rate for discounting such claims into their present-value equivalent. But the price ratio on the L.H.S. of equation (2) can also be written in another way that serves to define the forward short-term rate ${}_0r_2$:

$$\frac{{}_0P_2}{{}_0P_0} \equiv \frac{1}{(1 + {}_0R_2)^2} \equiv \frac{1}{(1 + {}_0r_1)(1 + {}_0r_2)} \tag{3}$$

The notation ${}_0r_2$ indicates that this interest rate, though short-term in that it discounts income claims from date-2 back only to the previous year (date-1), is a rate <u>quoted</u> (or else <u>implicit</u> in other quoted prices and interest rates) at date-0.

As the economy moves historically through time, at date-1 an actual short-term rate ${}_1r_2$ will come into existence for discounting date-2 claims. This is the <u>future</u> short-term rate, defined in terms of the prices and interest rate quoted at date-1:

$$\frac{{}_1P_2}{{}_1P_1} \equiv \frac{1}{1 + {}_1r_2} \tag{4}$$

(Here, since date-1 income becomes the numeraire for all claims quoted at date-1, it must be that ${}_1P_1 \equiv 1$.) Viewed from date-0, however, this actual future short-term rate ${}_1r_2$ will be uncertain. This brings us, finally, to the formal definition of the liquidity premium L:

$$L \equiv {}_0r_2 - E({}_1\tilde{r}_2) \tag{5}$$

The liquidity premium is the excess, on average, of the known forward short-term rate of interest over the unknown future short-term rate.

The simple algebra of the "discount" concept is completely analogous. The current short-term real discount ${}_0d_1$ is defined in:

$$\frac{{}_0P_1}{{}_0P_0} \equiv 1 + {}_0d_1 \tag{6}$$

Interest and discount are thus related as:

$$1 + {}_0d_1 \equiv \frac{1}{1 + {}_0r_1} \tag{7}$$

Evidently, positive interest will correspond to negative discount.

The forward discount is defined in:

$$\frac{{}_0P_2}{{}_0P_1} \equiv 1 + {}_0d_2 \equiv \frac{1}{1 + {}_0r_2} \qquad (8)$$

And the actual future discount is defined in:

$$\frac{{}_1P_2}{{}_1P_1} \equiv 1 + {}_1d_2 \equiv \frac{1}{1 + {}_1r_2} \qquad (9)$$

This bring us to the formal definition of what will be our second risk measure, the solidity premium, S (McCulloch [1973]) and Bailey [1964]. S is the difference between the forward discount and the expected future discount. Just as L measures the riskiness of the short-term investment strategy as of the later date, S measures the riskiness of the long-term strategy at the earlier date:

$$S \equiv {}_0d_2 - E({}_1\tilde{d}_2) \qquad (10)$$

Of course, S can also be expressed in terms of interest rates, or in terms of price ratios:

$$S \equiv \frac{1}{1 + {}_0r_2} - E\left(\frac{1}{1 + {}_1\tilde{r}_2}\right) \equiv \frac{{}_0P_2}{{}_0P_1} - E\left(\frac{{}_1\tilde{P}_2}{{}_1\tilde{P}_1}\right) \qquad (11)$$

A similar easy development from (5) leads to an expression for L in terms of price ratios:

$$L \equiv \frac{{}_0P_1}{{}_0P_2} - E\left(\frac{{}_1\tilde{P}_1}{{}_1\tilde{P}_2}\right) \qquad (12)$$

Although there is a kind of inverse relationship, it will be evident that S is neither the reciprocal nor the negative of L.

The intuitive interpretation that makes these formalisms useful can be developed as follows. Think of the liquidity premium L as measuring the difference, as of the farther-future date (date-2), between the sure return (principal plus interest) on a long-term bond and the mean return on a short-term bond rolled-over at the uncertain future short-term rate. For, it follows by elementary manipulations that:

$$(1 + {}_0r_1)L \equiv (1 + {}_0R_2)^2 - (1 + {}_0r_1)\, E(1 + {}_1\tilde{r}_2) \qquad (13)$$

As for S, think of it as measuring the difference, as of the <u>nearer-future</u> date (date-1), between the sure return on a short-term bond and the mean return on a long-term bond liquidated at the uncertain future discount. For:

$$S(1+{}_0R_2)^2 \equiv \frac{(1+{}_0R_2)^2}{1+{}_0r_2} - E\left(\frac{(1+{}_0R_2)^2}{1+{}_1\tilde{r}_2}\right) \equiv (1+{}_0r_1) - (1+{}_0R_2)^2\, E(1+{}_1\tilde{d}_2) \qquad (14)$$

Thus, $L > 0$ implies an excess yield, on average, of long-term over short-term instruments <u>compared at date-2</u>. $S > 0$, on the other hand, indicates an excess yield, on average, of short-term over long-term instruments <u>compared at date-1</u>. Since the dates at which the comparisons are made differ, L and S need not have opposite signs.

II. RISK PREMIUMS AND THE RETURN TO LONG-TERM AND SHORT-TERM BOND STRATEGIES

This section analyzes the forces underlying the liquidity premium and solidity premium, using an explicit contingent-claim model of income uncertainty at near-future (date-1) and far-future (date-2) dates. The present date is assumed free of uncertainty; at date-0 each individual is supposed to have a specific known endowment $\bar{c}_0$ of the real income commodity ("corn"). But at date-1 his endowment will be the uncertain consumption vector $(\bar{c}_{11},\ldots,\bar{c}_{1E};\pi_{11},\ldots,\pi_{1E})$, where $e = 1,\ldots,E$ indicates the state of the world at the earlier date and π_{1e} represents the associated probability (assessed at date-0). Similarly, at date-2 the endowment will be $(\bar{c}_{21},\ldots,\bar{c}_{2S};\pi_{21},\ldots,\pi_{2S})$, where $s = 1,\ldots,S$ is the index for states of the world at the later date and π_{2s} is the associated probability in terms of beliefs at date-0. In this section pure exchange is assumed: there are no productive opportunities (e.g., storage) for physically transforming income of one date into income of any other date.

Suppose at date-0 there are complete markets for claims to consumption contingent upon states of the world, at any date.[1] Then the current price of a claim to income at date-1 contingent upon state-e can be denoted ${}_0P_{1e}$, and similarly ${}_0P_{2s}$ is the current price of a claim to income at date-2 contingent upon state-s. Certainty claims to income as of any given date can be purchased by buying a full complement of the corresponding contingent claims. Thus:

$$\begin{aligned} {}_0P_1 &\equiv \sum_e {}_0P_{1e} \\ {}_0P_2 &\equiv \sum_s {}_0P_{2s} \end{aligned} \tag{15}$$

Once the passage of time reveals the state of the world at date-1, individuals will in general revise their beliefs about the probabilities of the date-2 states. (That is, the advent of state-e is not only an income-event but also generally an information-event.) The revised probabilities can be denoted $\pi_{s.e}$. Then the price of a certainty claim to date-2 consumption, quoted at date-1 after state-e has obtained, can be expressed as:

$$ {}_{1e}P_2 \equiv \sum_s {}_{1e}P_{2s.e} \tag{16} $$

We can define the future short-term interest rate ${}_{1e}r_2$, contingent upon state-e having been realized at date-1, in:

$$ \frac{{}_{1e}P_2}{{}_{1e}P_1} \equiv \frac{1}{1 + {}_{1e}r_2} \tag{17} $$

As usual, the denominator on the L.H.S. would be unity, since it is the price of the numeraire commodity current at date-1 after state-e obtains.

The expected future rate of interest, in terms of probability beliefs at date-0, can then be written:

$$ E(1 + {}_1r_2) \equiv E\left(\frac{{}_1P_1}{{}_1P_2}\right) \equiv \sum_e \pi_e \frac{{}_{1e}P_1}{\sum_s {}_{1e}P_{2.e}} \tag{18} $$

Recalling equation (12), the liquidity premium L can be expressed in terms of the underlying bundles of contingent claims as:

$$ L \equiv \frac{{}_0P_1}{{}_0P_2} - E\left(\frac{{}_1P_1}{{}_1P_2}\right) \equiv \frac{\sum_e {}_0P_{1e}}{\sum_s {}_0P_{2s}} - \sum_e \pi_e \frac{{}_{1e}P_1}{\sum_s {}_{1e}P_{2s.e}} \tag{19} $$

To press further, I shall have to say something more about the forces underlying the determination of prices. First of all, I will assume away any differences of beliefs in the economy: everyone agrees as to the probabilities π_e, π_s, and $\pi_{s.e}$. Let every individual make choices under uncertainty so as to maximize expected utility $U = E(V)$, where

$V(c_0,c_1,c_2)$ is his "cardinal," risk-averse, preference-scaling function for dated consumption income vectors. Individuals are also assumed to be identical in endowments. Taken together, the assumptions of representative tastes and endowments for all individuals guarantee that the prices emerging in our model are "sustaining" prices -- prices at which each agent merely holds his endowment and no trade takes place. In addition to the standard postulate of state-independence of the utility function, I will also be making the simplifying assumption of time-independence: that the V function is separable in the variables c_0, c_1, and c_2. While this limits the generality of our results, nonseparable tastes (i.e., allowing for possible intertemporal preference complementarities) would be a second-order effect that can only be accommodated by rather burdensome notation.[2]

If the social endowments of income are positive in every state, and if everyone assigns infinite negative utility to zero consumption at any date, an interior solution is guaranteed in which each individual holds positive amounts of every contingent claim c_{1e} and c_{2s}. Then in equilibrium at date-0 each individual's expected marginal utilities will be proportional to the prices:

$$\frac{v_0}{{}_0P_0} = \frac{\pi_e v_{2s}}{{}_0P_{1e}} = \frac{\pi_s v_{2s}}{{}_0P_{2s}} \tag{20}$$

where $v_0 \equiv \partial U/\partial c_0$, $\pi_{1e}v_{1e} \equiv \partial U/\partial c_{1e}$, and $\pi_{2s}v_{2s} \equiv \partial U/\partial c_{2s}$.[3] This equation reveals, therefore, the relation of prices to preferences and endowments (which together determine the marginal utilities) and to probability beliefs.

Substituting from (20) into (19) leads to:

$$L = \frac{\sum_e \pi_e v_{1e}}{\sum_s \pi_s v_{2s}} - \sum_s \pi_e \frac{v_{1e}}{\sum_s \pi_{s\cdot e} v_{2s}} \tag{21}$$

Or, in a more condensed notation:

$$L = \frac{E(v_1)}{E(v_2)} - E \frac{v_1}{\underset{e}{E}(v_2)} \tag{22}$$

Here $\underset{e}{E}$ indicates the expectation (of date-2 marginal utility) conditional upon state-e at date-1. (That is, calculated in terms of the $\pi_{s.e}$ probability beliefs.) Expectations symbolized simply as E are taken with respect to beliefs at date-0. Of course:

$$E(\underset{e}{E}(v_2)) = \sum_e \pi_e \underset{e}{E}(v_2) = E(v_2) \tag{23}$$

In parallel, we develop a similar expression for S. Starting from equation (11), we obtain in a similar fashion the analog of equation (22):

$$S = \frac{E(v_2)}{E(v_1)} - E \frac{\underset{e}{E}(v_2)}{v_1} \tag{24}$$

Certain simple relations must hold between the signs of L and S, following from Jensen's Inequality.[4] In particular, L and S may be both negative, but may not be both positive. $L > 0$ $(S > 0)$ implies $S < 0$ $(L < 0)$, $L = 0$ $(S = 0)$ implies $S < 0$ $(L < 0)$, but $L < 0$ $(S < 0)$ implies nothing about the sign of $S(L)$. Furthermore, only if both v_1 and $\underset{e}{E}(v_2)$ are non-random can both L and S be zero.[5]

Thus, the possible cases of interest are:

1) $L > 0$, $S < 0$

2) $L < 0$, $S > 0$

3) $L < 0$, $S < 0$

For the economic interpretation of these various cases, I now provide the promised demonstration that L and S are explicit measures of risk. L and S are not merely determined by the pattern of time-dated risks, but in fact can be rearranged to show that they can each be expressed as a covariance of an asset value with a dated marginal utility. Rearranging the expression for L in equation (22):[6]

$$L = \frac{1}{E(\underset{e}{E}(v_2))}\left[E(v_1) - E\left(\frac{v_1}{\underset{e}{E}(v_2)}\right) E(\underset{e}{E}(v_2))\right]$$

$$= \frac{1}{E(v_2)}\left\{E\left(\frac{v_1}{\underset{e}{E}(v_2)}\,\underset{e}{E}(v_2)\right) - E\left(\frac{v_1}{\underset{e}{E}(v_2)}\right) E(\underset{e}{E}(v_2))\right\}$$

which is just:

$$L = \frac{1}{E(v_2)}\,\mathrm{Cov}\left(\frac{v_1}{\underset{e}{E}(v_2)},\ \underset{e}{E}(v_2)\right) \tag{25}$$

And similarly:

$$S = \frac{1}{E(v_1)}\,\mathrm{Cov}\left(\frac{\underset{e}{E}(v_2)}{v_1},\ v_1\right) \tag{26}$$

The first covariable in (25), $\frac{v_1}{\underset{e}{E}(v_2)}$, can be seen from equations (17), (18), and (20) to equal $1 + {}_{1e}r_2$, one plus the interest rate contingent on the occurrence of state-e at date-1. Similarly, its reciprocal, $\frac{\underset{e}{E}(v_2)}{v_1}$, the first covariable in equation (26), is one plus the contingent discount.

From equations (25) and (26) we can derive three propositions:

PROPOSITION 1: $L > 0$ if and only if the covariance between future interest rates and date-2 conditional expected marginal utility is positive, or roughly speaking, if and only if the covariance between interest rates and date-2 consumption is negative. This is the same thing as saying that $L > 0$ when the value of a rolled-over short-term bond covaries against consumption at date-2, which makes the strategy of holding short-term bonds a form of insurance with respect to date-2. Recall that $L > 0$ implies $S < 0$, that is, that the covariance between the value of the long-term bond liquidated at date-1 and date-1 consumption is positive. The short-term bond strategy is thus the less risky strategy for obtaining consumption at both the earlier and the later date, and hence has a lower rate of return over both the short-haul and the long-haul. This is Scenario #1.

PROPOSITION 2: $S > 0$ if and only if the covariance between future discounts and date-1 marginal utility is positive, or, roughly speaking, if the covariance between future discounts and date-1 consumption is negative. Again, recall that $S > 0$ implies $L < 0$, or that the covariance between interest rates and date-2 consumption is positive. Thus, this scenario #2 occurs when it is the long-term bond that is the less risky instrument for obtaining consumption at both dates. Hence the long-term bond has a lower rate of return than the series of short-term bonds, on average, over both the short haul and the long haul.

PROPOSITION 3: $L < 0$ and $S < 0$ if and only if the covariance between interest rates and date-2 conditional expected marginal utility is negative and the covariance between discounts and date-1 marginal utility is also negative. Roughly speaking, this case occurs when the long-bond liquidity covaries with date-1 consumption and the short-term bond rolled-over covaries with date-2 consumption. This seemingly paradoxical case thus corresponds to the most easily imagined risk-configuration — that in which each bond is the least risky strategy for obtaining consumption at its own maturity date (Scenario #3).[7]

III. THE STOCHASTIC PROPERTIES OF CONSUMPTION AND THE RELATIVE RATES OF RETURN TO LONG-TERM AND SHORT-TERM BOND STRATEGIES

We have seen that it is not the mere presence of risk that shapes the term premiums on bonds, but the pattern in which the resolution of time-dated risks is visited upon prices. Two aspects of the stochastic properties of consumption describe this pattern and determine the signs and relative magnitudes of L and S: (1) the serial correlation of dated marginal utilities or -- speaking more loosely -- the serial correlation of consumption over time, and (2) the relative magnitude of two coefficients of variation, the first of date-1 marginal utility and the second of date-2 marginal utility.

We shall see in the next section how intertemporal productive transformations tend to induce positive serial correlation. In this section we shall see how arrival of information about date-2, either due to the mere realization of some state at date-1 or due to the arrival of some side-message, may affect the serial correlation either way. As for the coefficients of variation of the dated marginal utilities, these are reflections of the power of the date-1 information as pertaining to date-1 and to date-2. Since our knowledge about the date-1 state will be complete at date-1, the extent of the information gained is essentially measured by our date-0 uncertainty about the date-1 state — the coefficient of variation of v_1. But we may also at date-1 receive information about date-2. The coefficient of variation of $\underset{e}{E}(v_2)$ measures how much we expect (as of date-0) to revise (as of date-1) our initial beliefs about the date-2 state. Thus, both the relative *size* of the revisions (indicated by the coefficients of variation) and also the direction of *dependence* of the revisions (indicated by the serial correlation) affect L and S.

Rearranging (25) and (26), and simplifying notation by letting $x \equiv v_1$ and $y \equiv \underset{e}{E}(v_2)$:

$$L = \frac{1}{E(y)} \text{Cov } (y, \frac{x}{y})$$
$$S = \frac{1}{E(x)} \text{Cov } (x, \frac{y}{x}) \qquad (27)$$

These expressions can be rearranged to obtain:[8]

$$L = E(x)\, \sigma_{1/y} \left[\frac{\sigma_y}{E(y)} \rho_{y,1/y} - \frac{\sigma_x}{E(x)} \rho_{x,1/y}\right] \qquad (28)$$

$$S = E(y)\, \sigma_{1/x} \left[\frac{\sigma_x}{E(x)} \rho_{x,1/x} - \frac{\sigma_y}{E(y)} \rho_{y,1/x}\right] \qquad (29)$$

Here σ_x and σ_y are respectively the standard deviation of date-1 marginal utility and the standard deviation of conditional expected date-2 marginal utility; $\sigma_{1/x}$ and $\sigma_{1/y}$ are the standard deviations of their reciprocals; and ρ is the coefficient of correlation.

The sign of $\rho_{x,y}$ does not conclusively determine the sign of $\rho_{x,1/y}$, but the two will fail to take on opposite signs only in rather bizarre cases where one variable is highly skewed. For purposes of exposition, therefore, I shall carry on as though the signs of $\rho_{x,1/y}$ and $\rho_{y,1/x}$ are always the opposite of $\rho_{x,y}$. (As for $\rho_{y,1/y}$, the correlation of any variable with its own reciprocal is always negative.)

Subject to the imprecision of this approximation, we have now three additional propositions:

PROPOSITION 4: Negative serial correlation in the marginal utility of consumption implies $L < 0$ and $S < 0$. Roughly speaking, negative serial correlation in consumption insures that neither bond will command an obvious premium over the other — we have Scenario #3 corresponding to Proposition

#3. (Strictly speaking, as the equations indicate, we could have Scenario #3 even with $\rho(x,y) > 0$, but only if it is small and if one distribution or both are highly skewed.)

PROPOSITION 5: Positive serial correlation in the marginal utility of consumption implies either $L > 0$ or $S > 0$. Roughly speaking, positive serial correlation in consumption implies that either one bond or the other will command an obvious premium -- that we will have either Scenario #1 or #2, corresponding to Propositions 1 and 2.

PROPOSITION 6: Given positive serial correlation in the marginal utility of consumption, $\sigma_x/E(x) > \sigma_y/E(y)$ implies $S > 0$, and $\sigma_y/E(y) > \sigma_x/E(x)$ implies $L > 0$.

Proposition 6 tells us that as between Scenarios #1 and #2, which of the two prevails depends upon the <u>magnitude</u> of the associated positive regression. Suppose occurrence of a good state at date-1 raises the probability of good states at date-2, but not by so much as to make the expected date-2 outcome better (in marginal-utility terms) than the realized date-1 outcome. Thus, $v_{1e} < \underset{e}{E}(v_2) < E(v_2)$. (And, of course, there would then be a similar "regression toward the mean" effect in the event of a bad state at date-1.) The coefficient of variation for date-2 expected marginal utility, $\sigma_y/E(y)$, then tends to be small in comparison with the corresponding date-1 statistic, $\sigma_x/E(x)$. This leads, in equations (28) and (29), to the pattern $L > 0$, $S < 0$ — Scenario #1. Here the anticipated revision of beliefs regarding date-1 is, on average, more extensive than that regarding date-2. But if a good state at date-1 means, on average, an even better outcome at date-2 (and a poor date-1 state an even worse expected date-2 outcome), this "regression away from the mean" leads to Scenario #2 with $L < 0$, $S > 0$. In this case the anticipated revision of beliefs regarding date-2 is, on average, more extensive, even though we get conclusive information about date-1 and only partial information about date-2.

An interest-rate interpretation will also be helpful. If the magnitude of the positive regression is small ("regression toward the mean"), the contingent future short-term interest rate will be low in rich branches of the date-state consumption tree and high in poor branches of the tree. Here a good date-1 state implies a not-quite-as-good date-2 state on average, and therefore low $_1r_2$ — while a poor date-1 state implies a high $_1r_2$. Low rollover yields when you are rich, and high yields when you are poor means that the short-term maturity strategy has the superior risk characteristics, so the long-terms must unambiguously yield more, on average, over both long and short horizong $(L > 0,\ S < 0)$. But, with "regression away from the mean," the contingent future short-term rate is high when you are rich and low when you are poor; the short-term strategy has definitely inferior risk characteristics, and hence must always yield more, on average, again over both long and short horizons $(L < 0,\ S > 0)$.

Finally, with negative $\rho_{x,y}$ a good [bad] date-1 state implies a bad [good] date-2 on average, and thus a low [high] contingent interest rate. So the short-term strategy is risk-enlarging at date-2 (implying $L < 0$) while the long-term strategy is risk-enlarging at date-1 (implying $S < 0$). (Recall, however, that the combination $L < 0$, $S < 0$ may occur even if $\rho_{x,y}$ is positive so long as it is not very large.)

Note that the aspect of date-2 consumption that is important in determining the sign of the liquidity premium is not the variation in c_2

(date-2 consumption) or in v_2 (date-2 marginal utility), but the variation in $\underset{e}{E}(v_2)$ -- the date-1 revised expectation of date-2 marginal utility. The variation in v_2 constrains the variation in $\underset{e}{E}(v_2)$, of course; the variation in $\underset{e}{E}(v_2)$ can be as great as that of v_2 only if the information about date-2 arriving at date-1 is conclusive, and in all other cases must be smaller. The variation in v_2 does not matter, however, except to the extent that it is manifested in $\underset{e}{E}(v_2)$, and hence in interest rates, prior to date-2. Specifically, it follows from equations (25) and (26) that if no information is revealed at date-1 about date-2, (so that $\underset{e}{E}(v_2)$ is a constant), then $L = 0$ and $S < 0$, <u>even if there is much more consumption risk at date-2 than at date-1</u>.

Part of what Hicks and Keynes had in mind as a source of the liquidity premium was that the far future was inherently more uncertain than the near future, a proposition few of us would dispute. But this is neither a necessary nor sufficient condition for $L > 0$. From Equation (28) the needed conditions are: 1) that date-1 and date-2 consumption be positively correlated, and 2) that information arriving at date-1 is relatively more informative about date-1 consumption than date-2 consumption. The combined weight of these conditions is rather restrictive, so that in a world of pure exchange, there really could be no clear presumption that the liquidity premium is positive. As we shall see, it is the possibility of intertemporal productive transformation, in association with information anticipated to be received at date-1, that more powerfully forces $L > 0$.

I have spoken so far only of bonds, i.e., of certainty claims to income, either at date-1 or at date-2. But more generally, for any asset representing a bundle of contingent claims, its date-0 price will vary so that its expected yield to any date will be determined by the covariance of its future value with consumption at that date. What we really have here is a multi-period generalization of the single-period Sharpe-Lintner-Mossin capital asset pricing model (CAPM) -- in which the question "How risky is this asset?" has possible as many answers as there are dates.[9] In the CAPM, the factor determining an asset's expected yield is its covariance with the market portfolio, where the value of the market portfolio is simply the realization of date-1 (end-of-period) consumption. In the model here, or rather its rearrangement to allow for risky bundles as well as bonds, the covariance of the asset values with the realization of consumption at each date determines the expected yield to that date.

IV. THE INFLUENCE OF INTERTEMPORAL PRODUCTION

The foregoing analysis of the determinants of L and S under pure exchange makes possible a condensed treatment of the consequences of introducing intertemporal productive transformations. Hirshleifer [1972] argued that, when intertemporal production in the form of costless storage is possible, a unit claim to the inherently more flexible date-1 crop becomes (other things equal) more desirable than a corresponding claim upon the date-2 crop, tending to bring about an ascending term structure — and, he thought, a positive liquidity premium. While it is true that the forward-only nature of intertemporal production always enriches the future at the expense of the present and hence tends to make the term structure of interest rates ascending, I will show that it does not always imply $L > 0$.

I will discuss here two illuminating polar cases. In each, I distinguish between the endowment and the actual consumption at each date. I assume here that investment exhibits constant or diminishing returns, and that the marginal return to investment depends only on the amount invested, not on the endowments at either date. I also assume that production goes forward only; part of the current crop may be invested for consumption later, but there is no mechanism for transforming the future endowment into consumption today.

In our first case, which leads to $L > 0$, the state realized at date-1 determines the endowment at that date but does not modify the probabilities of the possible endowments at date-2. It does, however, affect the amount individuals choose to invest at date-1. Here revised beliefs about the date-1 crop imply investment decisions that give us additional information about date-2 consumption.

In the second polar case, which leads to $S > 0$, there is only one possible crop at date-1, but different possible messages about what the endowment might be at date-2. Here it is the message about date-2 that determines the optimal investment at date-1. Here revised beliefs about date-2 imply investment decisions that give us additional information about date-1 consumption. Below I provide a numerical example of each polar case (with production taking the form of simple constant-returns storage).

For the first case (beliefs about the date-2 endowment unaffected by the outcome of date-1) it is evident that the better the date-1 outcome, the more will be invested at date-1. Then given a diminishing-returns production function we can easily establish that the covariance between $\underset{e}{E}(v_2)$ and contingent interest rates is positive. The greater the investment at date-1, the higher consumption at date-2, and so the lower is conditional expected marginal utility at the later date. Further, since investment exhibits diminishing returns, the higher the investment, the lower is the marginal rate of return and hence the contingent interest rate. So in those (relatively well-endowed) date-1 states where investment takes place, $Cov[\underset{e}{E}(v_2), {}_1r_2]$ is positive. For those date-1 outcomes so poor that no investment is undertaken, the value of $\underset{e}{E}(v_2)$ is invariant. The contingent interest rates in these corner-solution states will be higher than over the average of all states, and the invariant $\underset{e}{E}(v_2)$ will also be higher than its average over all states. Therefore, counting the corner-solution states as well, the covariance between contingent interest rate and $\underset{e}{E}(v_2)$ is positive. The conclusion is clear: the liquidity premium is positive. (A limiting case of $L = 0$ results if all states at date-1 are so poor that investment never takes place.) Of course, $L > 0$ implies $S < 0$.

TABLE 1 -- NUMERICAL EXAMPLES

	FIRST CASE (information concerns earlier date)			SECOND CASE (information concerns later date)		

PART A: DATED ENDOWMENT CONFIGURATIONS

t:	0	1	2	0	1	2
	90	150 or 75	150 or 75	90	100	150 or 75

The good states (150) and bad states (75) are assumed equally probable at each date. Here date-1 information conclusively determines the date-1 state but does not modify probability beliefs about date-2.

The good and bad states at date-2 are equally probable. Information arriving at date-1 will conclusively announce the state at date-2.

PART B: CONTINGENT CONSUMPTION CHOICES

(based on $U=\ln(c_0,c_1,c_2)$ and production in form of costless storage at date-1)

0	1	2	0	1	2
		175 or 100			
	125 (store 25)			100 (store 0)	150
90	or		90	or	or
	75 (store 0)			87.5 (store 12.5)	87.5
		150 or 75			

PART C: CONTINGENT-CLAIM PRICES AT DATE-0

	0	1	2	0	1	2
			.12			
		.36			.45	.3
			.225			
	1			1		
			.15			
		.6			.51	.51
			.3			
Totals		.96	.795		.96	.81

These contingent-claim prices, effective at date-0, take into account the possibilities for storage at date-1 (storage at date-0 is assumed not possible.)

PART D: INTEREST RATES AND TERM PREMIUMS

First case	Second case
${}_0r_1 = .04$	${}_0r_1 = .04$
${}_0R_2 = .12$	${}_0R_2 = .11$
${}_0r_2 = .21$	${}_0r_2 = .185$
${}_1r_{2g} = .05$	${}_1r_{1g} = .50$
${}_1r_{2b} = .33$	${}_1r_{(1)} = 0$
$E({}_1r_2) = .185$	$E({}_1r_1) = .25$
$L = .025$	$L = -.065$
$S = -.025$	$S = +.067$

Figure 1

Figure 1 illustrates, for a representative individual situation, how a negative covariance between contingent interest rates and $E(c_2)$ arises. The latter is of course inversely related to $\underset{e}{E}(v_2)$ because the news changes only the mean of the c_2 distribution. As the date-1 endowment increases from very low levels, the expected date-2 endowment $E(\bar{c}_2)$ being constant throughout, no investment takes place until the critical point A is reached. Beyond A, from endowment points like B and D, the scale of investment increases. The interpretation, given diminishing returns to investment, is a negative association between the achieved $E(c_2)$ and the equilibrium contingent interest rate ${}_1r_2$.

In terms of equations (28) and (29), our first case reveals that <u>intertemporal production makes consumption positively serially correlated even if the endowments are independent</u> (see also Table 1). A rich date-1 crop and hence a richer-than-average date-1 consumption now tend to be followed by enriched date-2 consumption, and a poor date-1 crop by a less enriched date-2 consumption. Moreover, the low interest rates occur in the <u>rich</u> branches of the tree of outcomes, and the high ones in the poor branches.

This result, that intertemporal production implies $L > 0$, is very robust provided we stick to the condition that beliefs about the date-2 endowment are unaffected. Notably, the positive liquidity premium does not depend upon the (perhaps more stylish than factual) stylized facts. $L > 0$ obtains regardless of the distributions of c_1 and c_2 (and c_0), i.e., whether or not the term structure is ascending (the first stylized fact). Further, $L > 0$ in no way depends upon a stationary trend in interest rates (the second of the stylized facts). The result will also hold for a paradigm with any number of dates, and for comparison of any maturities so long as all the dated endowments are statistically independent, and also for both linear and diminishing-returns production functions.[10]

Now consider the second polar case: here the crop at date-1 is non-stochastic, but information about date-2 will arrive at date-1 in the form of a side-message. This means that the date-1 investment decision depends only upon the news about date-2. The worse the news, the more will be set aside at date-1 (invested) for consumption at date-2. Note that uncertainty regarding date-1 <u>consumption</u> emerges even though there is certainty about the date-1 endowment. In effect, some of the risk at date-2 is shifted <u>back</u> to date-1.

Under the conditions of the second case, it is easy to see that the covariance between v_1 and $\frac{\underset{e}{E}(v_2)}{v_1}$ (one plus the contingent discount) will be positive, and hence by equation (26), the solidity premium must be positive. If the date-1 news about date-2 is very bad, investment is large and the contingent interest rate is very low. Correspondingly, since the date-1 endowment does not vary, date-1 consumption is very low. So, when investment occurs, lower interest rates are associated with lower date-1 consumption (implying high v_1). As for corner-solution states in which the date-2 news is so good that no investment is undertaken, in those states v_1 must be lower than average while interest rates are higher than average. Thus, overall, the covariance between date-1 marginal utility v_1 and the interest rate is negative. But ${}_1r_2$ is inversely associated with $\frac{\underset{e}{E}(v_2)}{v_1}$. So by equation (26) it must be that S is positive, which implies $L < 0$. A limiting case of $S = 0$ results if date-2 is always so well endowed that investment never takes place.)

As in our first case, the availability of intertemporal productive opportunities makes consumption serially correlated. But here the high interest rates come in the <u>rich</u> branches of the tree of possible outcomes, and the low ones (along with large investment!) in the <u>poor</u> ones. Put another way, in our first case higher investment (implying lower interest rates) takes place <u>from richer states</u> at date-1, while in our second case, higher investment takes place <u>toward poorer states</u> at date-2.

Figure 2 illustrates, for a representative individual, how a negative covariance between interest rates and v_1 arises. Along the horizontal axis c_1, date-1 consumption, is inversely related to v_1, the marginal utility of the date-1 outcome. On the vertical axis $\hat{c}_2$ represents the certainty equivalent of expected date-2 consumption. Since c_2 is still a random variable as of date-1, $\hat{c}_2$ is the analog of c_1, and is inversely related to $\underset{e}{E}(v_2)$. The date-1 endowment, $\bar{c}_1$, being constant throughout, as $\hat{c}_2$ increases investment undertaken at date-1 from endowment points like E will decline until the critical point F is reached. Beyond F no investment at all takes place, but the slopes of the indifference curves along the vertical ray from F are ever-increasing. The interpretation is that, given diminishing returns to investment, there is a negative association between the consumed c_1 and the rate of interest.

Our second case illustrates that, from a sufficiently abstract point of view, even the availability of intertemporal production does not necessarily tend to make for a positive liquidity premium. Nevertheless, I want to argue that the general presumption in favor of a positive liquidity premium is not so wrong. The reason is that, apart from unusual situations, the information that individuals can anticipate receiving will typically do more to resolve uncertainty about near-future than about far-future events. The implication is that our first polar case above (in which nothing was learned about the date-2 endowment) is likely to be a closer approximation of reality than our second polar case (in which nothing was learned about the date-1 endowment). It is this first case, we saw, that led to a systematically positive liquidity premium.

Figure 2

V. SUMMARY AND CONCLUSIONS

Why does anybody care about the sign of the liquidity premium? Because the term structure of interest rates alone does not reveal how risky it would be to provide for consumption at a given date by buying bonds that mature earlier (with the intention of rolling over) or by buying bonds that mature later (with the intention of liquidation). The liquidity premium, L, measures the riskiness (with respect to consumption at a terminal date) of rolling over a series of short-term bonds. S, the solidity premium, reveals the riskiness (as of an intermediate date) of the strategy of holding a longer-term bond and liquidating it.

The concern of Keynes and Hicks was for the investor who might, if he bought long-term bonds, have to liquidate them at an unfavorable price if he came upon a rainy day before they matured. In terms of the analysis here, this corresponds to $S < 0$ — which means that the value of the long-term bond at an intermediate date (before its maturity) covaries <u>with</u> income at that date. When $S < 0$, the long-term bonds will be risk-enlarging at intermediate dates, i.e., they will indeed have their lowest values on rainy days and their highest values on sunny days. But $S < 0$ does not imply $L > 0$. This means that Keynes' and Hicks' investor could still be vulnerable even if he buys the short-term bond, as he may have to roll it over at an unfavorable rate. Only if $L > 0$ is the strategy of buying short-term bonds the less risky strategy for both the near future and the far future.

Hicks' and Keynes' investor faces risk at two different future dates. Since consumption incomes at these two different dates are, in the Fisherian tradition, regarded as two distinct commodities, we need two measures of riskiness — one for each risky good, to fully describe the risk he faces.

For comparison of two bonds of different maturities, three basic scenarios can emerge:

#1: Positive L (which implies $S \leq 0$) occurs only if the short-term bond is the less risky instrument with respect to both the earlier and the later future dates. This can happen only if the short-term bond provides insurance at the later date, implying (through Jensen's Inequality) that the long-term bond is risky with respect to the earlier date.

#2: Positive S (which implies $L \leq 0$) occurs only if the long-term bond is the less risky instrument with respect to both dates, that is, it provides insurance with respect to the earlier date's endowed consumption risk.

#3: Negative L and negative S occurs if both bonds are risky assets with respect to income at dates other than their own maturity. The expected yield on the longer-term bond at the shorter horizon is thus on average higher than the yield on the short-term bond; similarly the expected yield on the short-term rolled-over must be higher than the yield on a long-term bond.

Which scenario emerges can be explained by i) the serial correlation of consumption, and ii) the relative sizes of the coefficients of variation of (a) marginal utility at the earlier date and (b) the conditional expectation (as of the earlier date) of marginal utility at the later date. The latter can be thought of as a measure of the relative informativeness of the earlier date's news regarding both the earlier date's and the later date's consumption.

If the serial correlation in consumption is zero or negative, both L and S must be negative. Then the least risky instrument for any date's consumption is simply a riskless bond maturing at that date. But with positive serial correlation of consumption, one or the other of the two maturities becomes the less risky instrument for obtaining income at both dates.

Should the correlation be positive, if the news arriving at the earlier date is on average more powerful regarding the earlier date than the later one (that is, good news about the earlier date implies good, but not as good news for the later, and similarly for bad news) the low interest rates will be in the richer branches of the tree of possible outcomes and the high interest rates in the poorer ones. Then the liquidity premium will be positive. Should the news regarding the later date be the more belief-revising (good news for the earlier date means great news for the later, etc.) the high interest rates come in the rich branches, the low in the poor, and the solidity premium is positive.

The positive liquidity premium observed by Keynes and Hicks is not a result of mere risk-aversion. But, from the above, it can be shown as to follow from risk-aversion plus:

1) the positive serial correlation of consumption which arises from the forward-only nature of production, and
2) the predilection of Nature to give us more information about the near future than about the far future.

FOOTNOTES

[1]Specifically, we are assuming date-0 markets in E distinct date-1 claims and in S distinct date-2 claims -- $E + S$ markets. While more extensive regimes could be defined, for example trading in all $E \cdot S$ claims contingent upon *both* a particular state at date-1 and a particular state at date-2, our assumption suffices for achieving preferred consumption vectors, that is, no Pareto-preferred improvements are made available by opening more markets. (After the realization of a particular state at date-1, *re-trading* of the S date-2 claims will in general take place.)

[2]For a discussion of this point in the context of the theory of speculation see Salant [1976] and Hirshleifer [1976].

[3]Equation (20) follows directly from maximization of $U = E(V)$ subject to the following budget constraint at date-0 (where the overbars indicate endowed quantities):

$$ {}_0P_0c_0 + \sum_e({}_0P_{1e}c_{1e}) + \sum_s({}_0P_{2s}c_{2s}) = {}_0P_0\bar{c}_0 + \sum_e({}_0P_{1e}\bar{c}_{1e}) + \sum_s({}_0P_{2s}\bar{c}_{2s}) $$

[4]Jensen's Inequality: $E(1/x) > 1/E(x)$; equality holds only for non-random x.

[5]If L and S were defined in terms of continuously compounded interest rates instead of discrete exchange ratios, then necessarily $L = -S$ so that L and S could not both be negative. But, as the case of L and S both negative has an interesting economic meaning, something is lost in going to the continuous-interest form. Moreover, from an expository point of view the interpretation of L and S in terms of covariances of bond values with consumption is much easier with the exchange-ratio definition. For commodities distinguished *only* by *date*, either the continuum or the discrete-

period versions of time passage may be useful. But the discrete-period formalization lends itself more naturally to generalizations involving distinct commodities -- for example comparing the forward wheat-corn price ratio to the expectation of the future wheat-corn price ratio (Woodward [1980]) -- since the trade of corn for wheat is necessarily discrete. For a discussion of the term structure using continuous compounding, see Cox, Ross, and Ingersoll [1979].

[6]The development that follows is due to McCulloch [1973].

[7]As for the $L > 0$, $S > 0$ case, this cannot occur. $L > 0$ implies that the short-term strategy provides insurance at date-2, which means the high contingent interest rates must be associated with small expected consumption at date-2. $S > 0$ implies that long-term bonds provide insurance at date-1, which means that high interest rates (discounts) must be associated with large consumption endowments at date-1. But, if therealized state at date-1 turns out to be rich, while date-2 is likely to be poor, the contingent interest rate would have to be low, not high.

[8]Let $x = v_1$ and $y = \underset{e}{E}(v_2)$

$$L = \frac{1}{E(y)} \text{Cov } (y, \frac{x}{y})$$

$$\text{Cov } (y, x/y) = E(x) - E(y)E(x/y)$$

$$\text{Cov } (x, 1/y) = E(\frac{x}{y}) - E(x)E(1/y)$$

$$\begin{aligned} \text{Cov } (y, x/y) &= E(x) - E(y) \; [\text{Cov}(x,1/y) + E(x)E(1/y)] \\ &= E(x) - E(y)E(x)E(1/y) - E(y) \text{ Cov}(x,1/y) \\ &= E(x) \text{ Cov}(y,1/y) - E(y) \text{ Cov}(x,1/y) \end{aligned}$$

$$\begin{aligned} \frac{\text{Cov}(y,x/y)}{E(x)E(y)} &= \frac{\text{Cov}(y,1/y)}{E(y)} - \frac{\text{Cov}(x,1/y)}{E(x)} \\ &= \frac{\sigma_y \sigma_{1/y} \rho_{y,1/y}}{E(y)} - \frac{\sigma_x \sigma_{1/y} \rho_{x,1/y}}{E(x)} \end{aligned}$$

$$= \sigma_{1/y} \left[\frac{\sigma_y}{E(y)}, \rho_{y,1/y} - \frac{\sigma_x}{E(x)} \rho_{x,1/y}\right]$$

$$L = E(x)\, \sigma_{1/y} \left[\frac{\sigma_y}{E(y)} \rho_{y,1/y} - \frac{\sigma_x}{E(x)} \rho_{x,1/y}\right]$$

$$S = E(y)\, \sigma_{1/x} \left[\frac{\sigma_x}{E(x)} \rho_{x,1/x} - \frac{\sigma_y}{E(y)} \rho_{y,1/x}\right]$$

[9]Since the first draft of this paper, an explicit multiperiod CAPM in terms of dated consumptions has been provided by Breeden [1979].

[10]The liquidity premium is also positive with any linear production function. This can be shown by partitioning the covariance into storage (production) and no-storage (no-production) states, and showing that each partition is positive. The liquidity premium is also positive with a linear production function and independent or positively correlated endowments in infinite-time, two-state Markov models of the sort used by R.E. Lucas [1978] and by Stephen F. LeRoy and C.J. LaCivita [1980]. It is easy to show that if there is merely persistence in the Markov transition probabilities, $L > 0$, and that with a linear production function added, plus either independence or persistence, L is still greater than zero. However, if the transition probabilities already display persistence, the introduction of storage may not increase the size of L.

[11]I am indebted to John Riley for a classroom example which provided the basis for this idea.

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