ANTICIPATED PREEMPTION AND THE DETERMINATION OF INITIAL STRUCTURE IN A GROWING MARKET

by

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ABSTRACT

Studies of entry deterring strategies typically assume a specific initial market structure. Single product monopolies in markets for differentiable goods have been shown to have an incentive to preempt their markets by introducing new products before they would be introduced by entrants. Here we show that while preemption is optimal once a firm is committed to a market, the necessity of preemptive product introductions at later dates entails costs which may prevent the emergence of a single product monopoly initial structure; especially if the market is growing rapidly and competition does not reduce prices significantly.

I. Introduction

Much of the recent literature on entry deterrence and barriers to entry examines conditions under which firms already established in a market may or may not profitably commit themselves to reaction functions that would yield negative returns to entrants. Analyses deal most frequently with strategic investment in new capacity (e.g., see Dixit, 1979, 1980; Eaton and Lipsey, 1980, 1981; Spence, 1977, 1979), but an important subset of this literature focuses on what have come to be called preemptive strategies, strategies by which established firms introduce new variants of differentiated products or product innovations before the new products or innovations could profitably be introduced by new firms. Studies of the preemptive introduction of differentiated products have generally employed a spatial characterization of differentiation.[1] Eaton and Lipsey (1979) examine the situation of a monopolist in a spatial market and conclude that the returns to preemption always exceed the returns to entry., Wildman (1980) and Schmalensee (1978) make similar claims on behalf of established firms in a spatial oligopoly, although their analytical approaches differ.

Studies of preemption, and studies of entry deterrence in general, typically take some initial market structure as given and then examine the returns to entry deterring activities from that point on. The most common assumption is that the market is initially served by a single firm which is threatened by entry at some future date. When justified, the single firm assumption is usually supported by the observation that firms generally enter markets sequentially. Clearly, if conditions favor the employment of preemptive strategies by established firms, the organization of a market in equilibrium is largely predetermined by its initial structure. Thus, to fully explain equilibrium market structures one must study the determinants of the initial structures from which they evolve.

In this paper I identify and examine factors which influence the selection of initial structure in a growing spatial market in which firms, once established, benefit from the employment of preemptive strategies. The rules of the multi-firm pricing game, the rate of market growth, and the returns to Hotelling-type locational strategies are shown to be important determinants of initial structure.

The assumptions of the model are set out in Section 2. The returns to preemption for a single product monopolist are examined in Section 3 and the Eaton-Lipsey argument that such a firm will always preempt is restated. Conditions under which the initial market structure is likely to be a single product monopoly are examined in Section 4 and the consequences of relaxing the assumptions about the nature of sunk costs are discussed in Section 5. The conclusions of the paper are summarized in Section 6.

II. Assumptions

The assumptions about firm and market characteristics underlying this analysis are essentially those employed by Eaton and Lipsey (1979) in their study of entry deterrence in a growing spatial market. The only significant departure is that demand grows continuously in this model instead of increasing in a single discontinuous jump as assumed by Eaton and Lipsey.

We are studying entry into a growing market for a differentiable product. The range of potential product differentiation is represented by a line segment of two units length with endpoints of 0 and 2. Consumers are uniformly distributed across the product spectrum and, with the exception of their locational preferences, are assumed identical. At a given price a consumer values a nearby product more than one farther away. Thus, holding price constant, an individual's rate of consumption is a decreasing function of his distance from the product purchased.

In this model market growth refers to an increase in the density of consumers in the market. Growth in this sense is increasing demand for potential products, not increasing sales as in some analyses. Thus it is possible to have market growth when sales are zero, as is assumed to be the case prior to the entry of the first firm. Consumer density is assumed to increase exponentially at rate g until time τ, at which time growth ceases. Density remains at this terminal level from τ onward. It will prove to be convenient to let $\tau = 0$ and index dates prior to τ by negative numbers and dates after τ by positive numbers.

We begin by assuming that during some portion of the period prior to τ the market is served by a monopolist with a single product located at 1, the midpoint of the product spectrum. By τ, when growth has ceased, consumer density will be sufficient to support the introduction of two new products via the entry of new firms, one on each side of the original firm. Without loss of generality we can set consumer density at τ equal to $1 = e^{g\tau}$.

Firms are assumed to possess perfect foresight, which means that the established monopolist and potential entrants always know what consumer density will be at any date in the future. In addition the monopolist is assumed to know the demand and cost conditions facing potential entrants. The monopolist can thus predict with certainty the timing of entry if it is allowed to occur.

Cost functions are identical for all firms. A firm introducing a new product incurs a once-and-for-all initial entry cost of F which is sunk and ties the firm to a specific location. Following the usage of Eaton and Lipsey

(1981) we call the asset purchased by the expenditure of F entry capital. For simplicity it is assumed that there are no other fixed costs and that vartiable costs are constant.

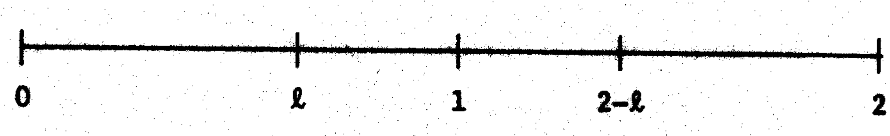
III. Second Round Entry and the Eaton-Lipsey Result

If the monopolist takes no action to prevent it, two new firms will enter the market, one on each side of the original firm, by some date no later than τ. We will refer to the new firms as second round entrants. The original monopolist is the first round entrant. Given the symmetry of their situations the second round entrants will locate equidistant from the original firm. This situation is depicted in Figure 1, with the original monopolist located at 1 and the two new firms at ℓ and $2-\ell$.

In spatial models of this type, where competitors are assumed to employ Hotelling-type locational strategies, for a given number of participants the equilibrium configuration of locations and prices is independent of density. This being the case, it is feasible and convenient to represent the value of revenue minus variable cost for one product in a particular locational configuration at any given time as density at that time multiplied by the value of revenue minus variable cost for the same product in the same configuration with consumer density equal to 1. For density of 1 define r_m to be revenue minus variable cost for a monopolist with a single product located at 1, r_e to be revenue minus variable cost for an entrant in the three firm configuration represented in Figure 1, and r_s to be revenue minus variable cost for the firm with the product located at 1 in the same three firm market.

Assume second round entry occurs T_e units of time prior to τ, or at time $-T_e$. The present value of a new product introduced by an entrant

FIGURE 1



(evaluated at $-T_e$) is given by (1).

(1) $$PV_e(T_e) = e^{-iT_e}[r_e\int_{-T_e}^{0} e^{(g-i)t}dt + \frac{r_e}{i} - Fe^{iT_e}],$$

where i is the discount rate assumed common to all firms. As long as entry is competitive, entry will occur at time $-T_e^*$, such that $PV_e(T_e^*) = 0$.

Now consider the situaion of the single product monopolist anticipating entry on both sides of its market at $-T_e^*$. If the monopolist were to commit itself to a reaction function, known to potential entrants, which would render entry unprofitable, entry would be deterred. One way for the monopolist to commit itself to such a reaction function would be to introduce its own new products in the same product space just prior to the date at which an entrant could enter and break even, that is, just prior to $-T_e^*$. Potential entrants, realizing that the monopolist has already incurred the sunk costs of entry for its new products and is therefore unlikely to withdraw them from the market, would not come in since attainable revenue is no longer sufficient to cover variable costs plus the fixed costs of entry.

Assume the monopolist does preempt and introduces new products just prior to $-T_e^*$ at ℓ and $2-\ell$ and replicates the pricing pattern that would have resulted had the new products been introduced through entry. Evaluated at $-T_e^*$, the present value of the monopolist in this situation is the present value of the two new products plus the present discounted value of r_s times consumer density from $-T_e^*$ on. That is, the present value of the monopoly would be the present value of the product at 1 if entry had been allowed plus the present value of the products at ℓ and $2-\ell$. Since the monopolist is assumed to have selected the locations and prices that would have occurred with entry, the new products have the same present value to the monopolist that they would for entrants, which is zero. Thus, if constrained to the same prices and locations that would prevail following entry, the monopolist is indiffferent about deterring entry.

However, for the monopolist the optimal locations for new products would not be at ℓ and $2-\ell$. Maximization of joint profits for three products dictates that the new products be located at 1/3 and 5/3, with all three priced the same so that each product serves 1/3 of the market. Entrants, on the other hand, would not locate at 1/3 and 5/3. Behaving as Hotelling-type competitors, they would locate closer to the product at 1 since they can capture a portion of its market without suffering a commensurate loss of sales on the outside segments of their own market areas. In addition, a monopolist controlling all three products could set the joint profit maximizing price.

Call the configuration of prices and locations that would obtain with entry the entry configuration and the optimal monopoly price and locational scheme described above the monopoly configuration. We have shown that, since the present value of a new product to a competitive entrant is zero, if the monopolist preempts employing the entry configuration its present value, calculated at $-T_e^*$, is the total of the discounted value of r_s times density from $-T_e^*$ on. The monopoly firm will have the same value if it allows entry to occur. Since the monopolist's profits are greater with the monopoly configuration than with the entry configuration, the present value of the monopoly must also be greater in this configuration. Thus, if allowed to employ the monopoly configuration, the present value of the monopolist must be greater if it preempts the market than if it allows entry to occur. New products have a positive value to the established monopolist while their value is zero to potential entrants. This gives us Eaton and Lipsey's principle result. The established firm will always find preemption profitable.

Note that while this result tells us that preemption always increases the net worth of the established firm, it does not tell us that the present discounted value of a preemptive monopolist, evaluated at the time of first round entry, must be positive. The analysis of the next section demonstrates that this does not have to be the case. In situations where a preemptive single product monopoly has a negative value other initial market structures will be observed.

IV. Analysis of First Round Entry

Define r_3 to be revenue minus variable cost when density is 1 for one of the single products controlled by a three product monpolist employing the monopoly configuration. The present value (evaluated at time of entry) for a monopolist that introduces a single product at the midpoint of the product spectrum on date $-T_m$, $T_m > T_e^*$, and preempts potential entrants by introducing two new products just prior to $-T_e^*$ is given by (2).

$$PV_m(T_m) = e^{-iT_m}\Big[r_m \int_{-T_m}^{-T_e^*} e^{(g-i)t}dt + 3r_3\int_{-T_e^*}^{0} e^{(g-i)t}dt + 3\,\frac{r_3}{i} - Fe^{iT_m} - 2Fe^{iT_e^*}\Big]. \tag{2}$$

The initial market structure will be a single product monopoly if there exists a $T_m > T_e^*$ for which $PV_m(T_m)$ is nonnegative.

By adding and subtracting $Fe^{iT_e^*}$, (2) may be rewritten as

$$PV_m(T_m) = e^{-iT_m}\{[r_m \int_{T_m}^{-T_e^*} e^{(g-i)t}dt - Fe^{iT_m} + Fe^{iT_e^*}] \qquad (2')$$

$$+ 3[r_3 \int_{T_e^*}^{0} e^{(g-i)t}dt + \frac{r_3}{i} - Fe^{iT_e^*}]\}.$$

Call the first term in square brackets to the right of the equals sign in (2') R_1 and the second term in square brackets R_2. $R_1e^{-iT_m}$ corresponds to what would be the present value (evaluated at $-T_m$) of a single product monopolist in the market from $-T_m$ to $-T_e^*$ paying a strictly prorated share of entry costs, if such a transaction were possible. $R_2e^{-iT_m}$ is the present worth, evaluated at $-T_m$, of a three plant monopoly established at $-T_e^*$.

Note that R_2 is identical to the expression in square brackets in (1) with T_e^* substituted for T_e and r_3 substituted for r_e. Since $PV(T_e^*) = 0$, $R_2 \gtrless 0$ as $r_3 \gtrless r_e$. If $r_3 > r_e$ first round entry by a single firm intent on following a deterrence strategy is assured. The present value of the 3 plant monopoly configuration from T_e^* on is positive, so competition for the market among prospective first round entrants will produce a $T_m > T_e^*$ for which $PV_m(T_m) = 0$. Given $r_3 > r_e$, the successful first round entrant will accept a loss on operations prior to $-T_e^*$ in exchange for the right to positive returns from $-T_e^*$ on. This is the standard result of competition for the market resulting in entry at an earlier than optimal date.

If $r_e > r_3$ the nature of first round entry is more difficult to determine. This makes R_2 negative and it is possible for the present value of "losses" from $-T_e^*$ on to exceed the value of "profits" during the earlier period ($|R_2| > R_1$). In this case first round entry by a single product monopolist would not be feasible.

It is easy to show that for given values of r_m, r_e and r_3 with $r_3 - r_e$ negative and finite there must exist a value for g above which the sum of R_1 and R_2 is always negative. Substitute T_e^* for T_e in (1) and call the term inside the square brackets in (1) R_e. Define $R_d = R_2 - R_e$.

$$R_d = (r_3 - r_e) \left[\int_{-T_e^*}^{o} e^{(g-i)} dt + \frac{1}{i} \right].$$

Since $PV_e(T^*) = 0$, $R_e = 0$ and the value of R_d is the same as the value of R_2. For $r_3 - r_e$ negative and finite R_2 can be no greater than $(r_3 - r_e)/i$, which is also negative and finite.

By setting g sufficiently high consumer density, and thereby, potential sales, on any given date prior to τ can be made as small as desired. Thus, given $T_m - T_e^*$ finite,[2] R_1 can be driven arbitrarily close to zero by increasing g. Therefore there must exist a value of g above which $|R_2| > R_1$ and $PV_m < 0$ always.

Projecting intuitively from this result to situations with lower values of g, one would predict that, for fixed value of r_m, r_e, r_3, i and F, increasing g would reduce the rents available to a first round monopolist or, alternatively, if the values of these parameters were unknown, that the probability of observing first round entry by a single product monopolist would be a decreasing function of g. Unfortunately I was unable to derive a mathematical expression that revealed anything about the nature of this relationship. Therefore its properties were explored with a computer model for various combinations of values for the parameters listed in the first sentence of this paragraph.

Setting PV_m equal to zero and solving for the largest possible value of T_m gives the earliest date at which first round entry could occur. Denote this value of T_m by T_m^*. If entry is competitive on the first round and single product monopoly is a feasible initial structure then first round entry will take place at $-T_m^*$.[3] Since firms compete for rents through earlier than optimal entry, large monopoly rents will be reflected in high values for T_m^*. Thus the effect of changes in g on the rents available to a first round monopoly entrant may be studied by examining the variation in T_m^* as g is changed. This was done for numerous combinations of r_m, r_e, r_3, i and F. In every case T_m^* was revealed to be a decreasing function of g.[4] T_e^* showed the same pattern and the difference between the two declined as g increased. The results for one set of parameter values are reported in Table 1. Notice that for a growth rate slightly over half the discount rate a single product monopoly is untenable as an initial structure since $PV_m < 0$. This result is typical of our findings for the other combinations of parameter values tested. Intuition appears to have been a reliable guide in this case. Single product monopoly initial structures are less likely to be observed in markets in which demand is increasing rapidly.

Factors which increase r_e relative to r_3 clearly make the market less attractive to potential single product monopolists in the first round. Locational considerations by themselves favor second round entrants. If price was set by an external authority and not allowed to vary in response to entry, firms entering in the second round would profit from occupying positions closer to the original firm than would be dictated by joint maximization. This would raise r_e above r_3. However, it is possible for the effect of second round entrants' locational advantage on the relative values of r_3 and r_e to be more than offset by postentry price competition. So the rules of the postentry pricing game may be critical in determining initial structure.

The relative values of r_3 and r_e are examined for two different pricing games in the appendices under the assumptions that individual demand

TABLE 1

Effect of Growth Rate on Entry Dates

g	T_m^*	T_e^*	$T_m^* - T_e^*$
0.01	120.4	13.3	107.1
0.02	65.6	9.0	56.6
0.03	46.2	7.2	39.0
0.04	35.1	6.2	28.9
0.05	26.6	5.5	21.1
0.06		$PV_m < 0 \ \forall \ T_m > T^*_e$	

*T_m^* and T_e^* calculated for $F = 5.0$, $r_m = 1.49$, $r_e = 0.53$, $r_3 = 0.51$.

functions are linear and marginal costs and transportation costs are constant. In Appendix 1 it is shown that, given these assumptions, if entrants come in committed to a policy of always setting a price (mill price) identical to the price of the established firm, then r_e will always exceed r_3. On the other hand, if the post entry game is Nash in prices r_e may be either greater than or less than r_3, depending on transport costs (in this analysis the utility loss per unit distance attributable to consuming a product not at one's preferred location) and the parameters of the demand function. This is demonstrated in Appendix 2. Subject to limits on parameters imposed by the constraint that a profit maximizing single product monopolist sell to at least one-third of all consumers,[5] steep demand functions in combination with high transport costs produce $r_e > r_3$. Within a limited range this relationship may be maintained by trading off steeper demand functions for lower transport costs or vice versa. However, with flat demand functions and low transport costs r_e is always less than r_3.

A third possibility would be for an established monopolist to commit itself to a postentry pricing strategy of always setting a price low enough to ensure $r_3 > r_e$. If first round entrants can credibly make commitments of this type then the initial structure will always be a single product monopoly. The point, of course, is not to argue for the plausibility of any particular set of pricing rules, but to demonstrate that with commonly employed assumptions about price competition conditions both favorable and unfavorable to the emergence of a single product monopoly initial structure may be generated.

Up to now we have assumed that a single product monopolist will locate its product at the midpoint of the product spectrum. Relaxing this assumption has little affect on the conclusions reached above. While it is possible that in some circumstances first round single product monopolies may be viable at an off-center location but not at the midpoint, the basic argument developed above is still valid. A second round entrant would still locate too close to the first firm to permit joint profit maximization and unless price competition is sufficiently strong it would also realize a higher net revenue on sales. Given this relationship, if the market is growing fast enough a single product monopolist would have a negative value as a first round entrant.

The existence of a single product monopolist is frequently used as a starting point in analyses of entry deterrence. The basic point developed in this section is that the viability of this type of firm as the first round entrant in a growing market cannot be taken for granted. Even with modest growth one should not be surprised to observe either simultaneous first round entry by two or more firms or first round entry by a multiproduct firm.

V. First Round Entry with Short-Lived Entry Capital

The results developed above depend critically upon the assumption of infinitely durable, or at least very long-lived, entry capital. If the durability of entry capital was a choice variable to the firm, then the structural evolution of a growing market would proceed differently, at least in those situations in which the present value of a first round single product monopoly would be negative.

Consider the opposite case in which there are no cost advantages or disadvantages associated with units of entry capital of differing durability -- the discounted cost of a sequence of investments in short-lived entry capital being equal to the cost of a single unit of entry capital with the same durability. In growing markets where infinitely durable entry capital would make a first round single product monopoly infeasible one might now see first round entry by single product monopolists that enter the market with the intention of leaving later in response to second round entry.[6]

VI. Conclusions

Studies of entry deterring strategies typically assume an initial structure and examine tactics available to established firms. Here we have shown that for a spatial model in which firms are blessed with perfect foresight and deterrence is always the optimal strategy that knowledge of the deterrence strategy to be employed also provides us with information about the viability of different initial market structures. Initial structure is strongly influenced by the rate of growth of potential demand, the characteristics of consumer demand functions, and the rules by which firms compete in prices. Single product monopoly initial structures should be observed less frequently in rapidly growing markets than in markets growing slowly while postentry price competition makes their appearance more likely. The effects of consumer demand characteristics on initial structure are important and vary according to the pricing strategies employed if entry does occur.

APPENDIX 1

In addition to the assumptions stated in Section 2 we now restrict the analysis to situations with linear demand curves and constant marginal cost. Given these restrictions we can show that if entrants are able to force an established firm into the role of price leader by committing themselves to a policy of always charging the same price (mill price) as the established firm, then r_e will always be larger than r_3.

Let demand for a consumer located distance α from the product he buys be given by

$$\text{(A1.1)} \quad q(\alpha,P) = a - b(P+c\alpha),$$

where P is price and a,b and c are positive constants. c is the transport cost which is interpreted as utility loss per unit distance in nongeographic space. Using the same two unit linear product space, the initial single product monopolist is at 1 and the two entrants are located distance β from the initial product on each side $(\beta = 1- \ell)$. Since entrants are committed to charging the same price as the original firm, the initial product will be sold to all consumers out to distance $\beta/2$ from its location on both sides. For convenience we assume that marginal cost is zero. Thus, if entry occurs, the original firm will set price to maximize revenue, where revenue is given by density times r_s and r_s is

$$\text{(A1.2)} \quad r_s = 2P \int_0^{\beta/2} (a-bP-bc\alpha)d\alpha.$$

Setting the derivative of r_s with respect to P equal to zero and solving for P we have

$$(A1.3) \quad P = \frac{a}{2b} - \frac{c\beta}{8}.$$

Assume for the moment that entrants locate at 1/3 and 5/3 ($\beta = 2/3$), duplicating the monopoly configuration of locations. The original firm, knowing the entrants will match its price, would set the joint profit maximizing price, the same price that would maximize revenue for a three product monopolist. So $r_s = r_3$ if $\beta = 2/3$. We want to show that entrants can do better by locating elsewhere. Since total revenue is maximized and $r_e = r_3$ with the monopoly configuration of locations and prices, if r_e is increased by a change in an entrant's location then $r_e > r_3$. We will prove this to be the case.

For an entrant revenue is density times r_e, with r_e given by

$$(A1.4) \quad r_e = P \int_0^{\beta/2} (a-bP-bc\alpha)d\alpha + P \int_0^{1-\beta} (a-bP-bc\alpha)d\alpha.$$

Substituting from (A1.3) for p in the expression for r_e, setting $\beta = 2/3$, and taking the derivative with respect to β yields, after some manipulation,

$$(A1.5) \quad \left.\frac{dr_e}{d\beta}\right|_{\beta=2/3} = ac/12 - bc/96 - a/8b.$$

If (A1.5) has a value other than zero, then entrants will choose a $\beta \neq 2/3$ and $r_e > r_3$, a factor which, by itself, is unfavorable to the emergence of a single product monopoly initial structure.

Implicit in the analysis developed in the text is the assumption that for some period in the market's history prior to τ demand was sufficient to support a single product at the center of the spectrum (in the sense that net revenue is at least as great as the opportunity cost of entry capital) but no others. This assumption implies that the customers of a profit maximizing single product monopoly will be spread over more than a third of the product spectrum. We now show that for the linear demand model developed in this appendix, this assumption restricts the relative values of the demand parameters to a range for which (A1.5) will always be negative.

With only one product in the market, for a given price, the distance from the product's location to its most distant purchaser is the value of α for which $q(P,\alpha)$ is zero. Denote this value of α by γ and call the price set by a single product monopolist P_1. The firm's product is purchased by all consumers located in a region of size 2γ centered on its product's location on the attribute spectrum. As long as firms are unable to discriminate among consumers on the basis of location, γ and P_1 are determined jointly according to (A1.6). Thus either P_1 or γ can be treated as the choice variable of a single produce monopolist.

(A1.6) $P_1 = \frac{a}{b} - c\gamma.$

Revenue for a single product monopoly, r_1, is given by

(A1.7) $r_1 = 2P_1 \int_0^{\gamma} (a-bP_1-bc\alpha)d\alpha.$

Substituting from (A1.6) and taking the derivative with respect to γ we have

(A1.8) $\frac{dr_1}{d\gamma} = 2ac\gamma - 3bc^2\gamma^2.$

If the customers of a profit maximizing single product monopoly are distributed over more than a third of the product spectrum, (A1.8) must be positive for $\gamma = 1/3$. In other words, the firm must still be able to profit from expanding the range over which its product is sold if its currently most distant buyers are no more than 1/3 units away. (A1.8) positive for $\gamma = 1/3$ requires $2a > bc$, in which case (A1.5) is negative. Entrants will choose locations less than 2/3 from the center of the product spectrum and $r_e > r_3$.

APPENDIX 2

We employ the same linear specification of individual demand functions here as in Appendix 1. $q(P,\alpha)$ is the demand of a consumer located α from the product he buys when the price is P. The functional form employed is given by (A2.1), which is identical to (A1.1). Consumers are uniformly distributed across the two unit characteristic spectrum. We also retain the assumption of Appendix 1 that marginal cost is constant and zero.

(A2.1) $q(\alpha,P) = a - b(P+c\alpha)$.

Again we are concerned with the relative values of r_3 and r_e, this time when the postentry game is Nash in prices. Initially there is a single firm with a product located at the midpoint (at 1) of the two unit spectrum. Two new firms introduce products on each side of the original product. Once established, products are immobile and the competitive game is Betrand. Entrants know the rules of the pricing game and select their locations to maximize profits.

If all products sell at the same price, as in the example developed in Appendix 1, consumers always buy the nearest product, resulting in an even division of the space between adjacent products. If products can be sold at different prices, as in the analysis developed here, the boundary between regions served by neighboring products is the point at which delivered prices (mill price plus transport cost) for the two products are equivalent.

Let P_s be the price of the original product at 1 and P_e be the price of a product introduced by an entrant β units from the first product. Then the boundary between the market areas served by the two products will be ξ

units from the first product, where ξ is given by (A2.2).

$$\text{(A2.2)} \quad \xi = \frac{P_e - P_s}{2c} + \frac{\beta}{2}.$$

Since the analysis is symmetric with respect to entrants on each side of the first firm, we simplify the analysis further by assuming that entrants behave simultaneously as well as symmetrically. This saves us the complications of having to explicitly consider the behavior of both entrants.

For given values of P_s and P_e revenue for the product at the center is density times r_s and revenue for an entrant β units from the midpoint is density times r_e. r_s and r_e are given by (A2.3) and (A2.4) respectively.

$$\text{(A2.3)} \quad r_s = 2P_s \int_0^{\xi} (a - bP_s - bc\alpha)d\alpha.$$

$$\text{(A2.4)} \quad r_e = P_e \int_0^{\beta-\xi} (a - bP_e - bc\alpha)d\alpha + P_e \int_0^{1-\beta} (a - bP_e - bc\alpha)d\alpha.$$

Substituting from (A2.2) for ξ and setting the partials of r_s with respect to P_s and r_e with respect to P_e equal to zero gives implicit reaction functions for the original firm, (A2.5), and an entrant, (A2.6), respectively.

$$\text{(A2.5)} \quad \partial r_s / \partial P_s = A_s P_s^2 + B_s P_s + C_s = 0.$$

$$\text{(A2.6)} \quad \partial r_e / \partial P_e = A_e P_e^2 + B_e P_e + C_e = 0.$$

Where $A_s = 9b/4c$,

$B_s = -(\partial a + bP_e)/c - b\beta/2$,

$$C_s = \beta(4a-bc\beta)/4 + P_e(a/c-b\beta/2) - bP_e^2/4c,$$

$$A_e = 9b/8c,$$

$$B_e = b(3\beta/2-2) - a/c - bP_s/2c, \text{ and}$$

$$C_e = a - bc/2 - a\beta/2 + bc\beta(1-5\beta/8) + P_s(a/2c-b\beta/4-bP_s/8c)$$

Note that both reaction functions, and thus P_s and P_e, are functions of β. Substituting from (A2.2) for ξ and solving (A2.5) and (A2.6) simultaneously for the two prices enables entrants to calculate r_e for a given β. Location (β) is then selected to maximize r_e.

The complexity of the quadratic forms of the reaction functions prevented a formal solution of the entrants' objective function. Instead a computer program was developed to run the Bertrand game to conclusion for given values of β, a, b, and c.[7] Holding the values of demand parameters constant, the game was rerun for different values of β from 0 to 2/3, changing the value of β by 0.02 on each iteration. Thus we were able to identify the value of β that maximized r_e within ±0.02. We will refer to the estimate of an entrant's optimal β as β^*. This procedure was repeated while systematically varying b and c with a held constant at zero subject to the constraint that $bc < 2$.[8] Calculated values of β and r_e are the bottom and top numbers, respectively, in the triplets of observations shown in Table A2.1. The middle number is r_3 for the same set of demand parameter values.

$$r_3 = 2P_3 \int_0^{1/3} (a-bP_3-b(\alpha)d\alpha,$$

with P_3 given by

$$P_3 = a/2b - c/12,$$

which is just a rearrangement of the first order condition with respect to price for a three product monopolist employing the monopoly configuration of locations.

Looking at Table A2.1, one sees that a rough diagonal running from the lower left corner to the upper right corner separates the table into two sections which differ according to the relative values of r_3 and r_e. In the upper left section $r_3 > r_e$, while in the lower right section $r_3 < r_e$. With linear demand functions and a Bertrand pricing game, low transport costs and flat demand functions yield $r_3 > r_e$ and ensure single product monopoly as the initial structure. By increasing transport costs and/or making consumers more price sensitive (demand functions steeper) one can bring about a reversal of this inequality, which, if the market is growing fast enough, will generate a multiproduct initial structure.

TABLE A2.1

r_e, r_3 and $\beta*$ as Functions of Demand Parameters

c \ b	0 – 0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17
0 1 3.0	$r_3 > r_e$	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
4.0	.	1.44 1.86 0.39	1.32 1.63 0.43	1.23 1.45 0.47	1.14 1.30 0.49	1.06 1.17 0.51	0.99 1.06 0.53	0.92 0.97 0.53	0.87 0.91 0.55	0.81 0.82 0.55	0.76 0.76 0.57
5.0	.	1.53 1.81 0.47	1.40 1.58 0.49	1.28 1.39 0.51	1.17 1.24 0.53	1.08 1.11 0.55	1.00 1.01 0.55	0.92 0.92 0.57	0.86 0.84 0.57	0.80 0.77 0.57	0.74 0.71 0.57
6.0	.	1.59 1.75 0.51	1.43 1.52 0.53	1.30 1.34 0.55	1.18 1.19 0.55	1.08 1.06 0.57	0.98 0.95 0.59	0.90 0.86 0.57	0.83 0.78 0.59	0.76 0.71 0.59	0.71 0.65 0.59
7.0	.	1.62 1.70 0.53	1.44 1.47 0.55	1.29 1.28 0.57	1.16 1.13 0.59	1.05 1.00 0.57	0.95 0.90 0.59	0.87 0.80 0.59	0.79 0.73 0.59	0.72 0.66 0.59	0.66 0.60 0.59
8.0	.	1.62 1.64 0.55	1.43 1.41 0.57	1.27 1.23 0.57	1.14 1.07 0.59	1.02 0.95 0.59	0.92 0.84 0.59	0.83 0.75 0.59	0.75 0.67 0.59	0.68 0.60 0.59	0.61 0.54 0.61
9.0	.	1.61 1.59 0.57	1.41 1.36 0.57	1.24 1.17 0.59	1.10 1.02 0.59	0.98 0.89 0.59	0.87 0.79 0.59	0.78 0.70 0.59	0.70 0.62 0.61	0.63 0.55 0.61	0.56 0.49 0.61

*The value of a was set equal to 1 for all calculations.

**The first number in each group of three is r_e calculated at $\beta*$, followed by r_3, followed by $\beta*$.

***$\beta*$'s are within ±0.02 of value of β that maximizes r_e for given values of a, b and c.

FOOTNOTES

[1]Omori and Yarrow (1982) differ in this respect. Their representation of product differentiation is similar to that introduced by Spence (1976) in his study of monopolistic competition. This approach is more general in some respects than the spatial analogue, but it is not useful for examining that aspect of competition captured by locational strategies in spatial models. Still, their conclusions about the value of preemptive product introductions to an established monopoly are the same as those reached by Eaton and Lipsey (1979) in their spatial analysis of the problem.

[2]The value of T_m is constrained by the requirement that $PV_m(T_m) > 0$.

[3]PV_m has two roots in T_m. The largest, T_m^*, is the date of first round entry if first round entry is competitive. The smaller root gives the latest possible date of entry for which a single product monopoly would have a nonnegative value.

[4]Restrictions on the values of r_m, r_e and r_3 were $r_m > r_e > r_3$ and $r_m < 3r_3$. The last inequality guarantees that sales are not diminished if a monopolist introduces additional products. In practice we found it necessary to set r_e close to r_3 and r_m fairly close to $3r_3$ to produce values of T_m for which PV_m was nonnegative.

[5]If the range of potential product variation is large relative to the area served by a single product only multiproduct initial structures are plausible and discussion of the viability of a single product monopoly is meaningless. This constraint rules out situations of this type.

[6]This will not necessarily be the case since in rapidly growing markets net revenue on sales for a single product monopolist may never grow large

enough prior to the date of second round entry to cover the opportunity cost of entry capital. That is, $r_m e^{-gT_e^*} < iF$.

[7]Using the quadratic formula to solve for the P's, only real roots of the form

$$P = \frac{-B-(B^2-4AC)^{1/2}}{\partial A}$$

gave economically meaningful reaction functions.

[8]The reason for imposing this constraint on the range of values taken by the demand parameters is explained in detail in Appendix 1.

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