

THE DISTINGUISHING CHARACTERISTICS OF  
TEMPORARY AND PERMANENT LAYOFFS

by

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The new "implicit contract theory," whose proponents claim explains temporary layoffs, has received considerable attention. However, in this paper, we argue that essential elements are missing from the contract theoretic explanation of temporary layoffs. The term "temporary layoff" connotes a firm, facing intertemporal fluctuations in demand, finding it optimal in successive periods of time to employ, temporarily lay off, and then recall the same worker. This suggests that a distinguishing characteristic of a temporary layoff is that it is in fact temporary. That the firm chooses to recall the laid off worker when sales recover, rather than hiring a new worker, suggests that there is a value to the firm of a continuing worker-firm attachment. And that the laid off worker is available for recall suggests that continuing the attachment has value to the worker. It is therefore troubling, given this characterization of a temporary layoff as an inherently intertemporal phenomenon involving long term worker-firm attachments, that the prototype implicit contract model is a one period model in which labor is assumed to be homogeneous.

Even the few notable efforts towards putting the contract theoretic explanation of layoffs in to an intertemporal framework (e.g., Baily (1977) and Holmstrom (1981)) have failed to capture the "temporariness" of temporary layoffs. For these latter models only allow separations to occur in the final period of the model. Hence, it is impossible within the context of such models (both the one period and two period) to distinguish between temporary and permanent separations. Accordingly, there has been no formal analysis of either the recall decision by firms and the decision to wait for recall by workers; two seemingly very important factors for analyzing temporary layoffs. In this paper, we develop a multiperiod contract model in which separations (either employer or employee initiated) may occur in any one of

several successive periods. This allows us to distinguish between temporary and permanent layoffs, explicitly analyze the recall decision and the wait for recall decision, and opens up a variety of new issues regarding the type of implicit contracts that emerge under these circumstances.

Consideration of an intertemporal setting in which separations may occur in any one of several successive periods requires re-evaluation of the motivation for long term worker-firm attachments provided by contract theory. Existing contract models have suggested two primary reasons that long term worker firm attachments develop in the labor market. First, contract theorists (e.g., Azariadis (1975) and Baily (1974)) have argued that long term worker firm attachments develop in an uncertain environment because workers are risk averse. They argue that firms act doubly as a worker's employer and as his insurer against fluctuations in the worker's opportunities. Secondly, contract theory (e.g., Baily (1977), Burdett and Mortensen (1980)) borrows from the analysis of Becker (1962) and Oi (1962) by arguing that long term attachments develop because of such factors as hiring costs, training costs, search costs and moving costs. In what follows, we emphasize this latter turnover costs motive for long term worker-firm attachments. Our neglect of risk aversion is consistent with recent developments that question the originally hypothesized critical role of risk aversion in the implicit contract theoretic context. Elsewhere it has been shown that risk aversion is neither necessary (e.g., Baily (1977), Burdett and Mortensen (1980)) nor sufficient (e.g., Akerlof and Miyazaki (1980), and Mortensen (1978)) for explaining the observed occurrence of temporary layoffs. Moreover, risk aversion on the part of workers is clearly not necessary for explaining the existence of long term worker-firm attachments and whether risk aversion, by itself, is sufficient to generate long term worker-firm attachments is

debatable (see Haltiwanger (1982)).<sup>1</sup>

One of the primary issues to be resolved is the form that implicit contracts take in an intertemporal setting. The one and two period contracts of previous analyses do not readily extend to a multiperiod setting. For the contracts in the one and two period setting are based on the presumption of workers and firms being perfectly mobile ex ante (prior to the onset of the contract) and immobile ex post.<sup>2</sup> This ex ante mobility -- ex post immobility is not a reasonable characterization of worker-firm immobility in a multiperiod setting. In particular, while it may be reasonable to argue that prior to any given period new entrants are mobile, certainly workers who have been attached to a specific firm during previous periods are not so mobile. Rather than impose this ex ante, ex post distinction, the analysis in this paper presumes that firms can hire new workers in any given period and workers may quit in any given period as well. However, explicit introduction of hiring and training costs, search costs, and moving costs provides an incentive for long term worker-firm attachments to develop. Given these factors promoting worker-firm immobility, a worker considering "attaching" himself to a particular firm is concerned not only with the coming period but also with the long term. Hence, long term implicit contracts are hypothesized but these long term implicit contracts must take into account the possibility of both temporary and permanent layoffs as well as quits.

Much of the paper is concerned with working out the properties of the model. The model is one in which the firm must make a number of interdependent decisions in each period of time that affect its current and future profits. The firm must decide on whether to permanently lay off or temporarily lay off any of the workers in its existing pool of workers, it must decide on whether or not it should add to this pool by hiring and

training new workers, and although the firm is presumed to be constrained to offer its new workers terms as good as are available elsewhere, the firm has flexibility with regard to how the terms (e.g., wages) are distributed over time. This is important because in this setting, in which job separations may occur in any one of several successive periods, the terms of compensation in a given period play an allocational role not only in that particular period but also in all periods prior to that period. Employee decisions on whether or not to quit, particularly on whether or not to quit when temporarily laid off, play a fundamental role in the analysis as well.

### The Model

Consider a model with the following assumptions:

(i) A worker's productivity increases with firm specific experience.

This is attributed to the natural accumulation of skills with experience and the simple hiring costs associated with processing a new worker with the firm. Worker heterogeneity of this type is incorporated by assuming two classes of workers: senior workers, with at least one period of experience with the firm in period  $i$  (denoted by  $L_i^e$ ) and new entrant workers (denoted by  $L_i^n$ .) As a first approximation, assume that the difference in productivity implies that new entrant workers produce no output in their initial period of experience with the firm and experienced workers produce output according to a strictly concave production function given by:

$$(1) \quad F(L_i^e) \quad \text{where } F' > 0, F'' < 0.$$

Hours per worker are assumed to be fixed and normalized to one.

(ii) Since "experience" is firm-specific, the firm in question has only a limited number of experienced workers available in any given period.

Defining  $m_i^e$  to be the number of available experienced workers in period  $i$

and  $R_i^e$  the number actually contracted for in period  $i$ , it must be true that  $m_i^e > R_i^e$ . The determinants of  $m_i^e$  and  $R_i^e$  are discussed below.

(iii) The firm is assumed to be a price taker for all periods. The type of ex ante-ex post uncertainty posited by the typical contract model is not considered here. That type of uncertainty requires that the firm decide on the number of workers it will contract for in the current period prior to knowing the actual level of demand for the period. Alternatively, we suppose that the decisions made in the current period that take effect in the current period are made with full knowledge of the current period's demand. For instance, the decisions on the number of experienced workers to retain, to temporarily layoff and the number of new workers to hire in the current period are made with full knowledge of the current period's demand. However, the firm may be uncertain in the current period about future demand. Since this is a model in which it takes time for workers to acquire experience, this implies that the firm must, in general, make a decision on the desired size of the available experienced labor pool for a future period prior to knowing the actual level of demand for the period. Formally, the type of uncertainty presumed in this analysis implies that from the perspective of any given period  $i$ , the firm knows with certainty the price in period  $i$ ,  $P_i$ , but  $P_{i+k}$  ( $k > 0$ ) is a random variable.

(iv) The firm faces a competitive constraint for new workers (assumed to be risk neutral) who require that their expected discounted income associated with contracting with the firm is as good as is available elsewhere. This assumption takes the form:

$$(2) \quad W_i^n + V_{i+1}^e \rho > V_i^n, \quad \rho < 1$$

where  $V_i^n$  is the market determined expected discounted income available elsewhere,  $W_i^n$  is the wage promised to the new worker in period  $i$ ,  $V_{i+1}^e$  is

the expected income promised to the workers for the future, and  $\rho$  is the discount factor (i.e., so  $V_{i+1}^e \rho$  is the present discounted value of income for all periods starting with period  $i+1$ ). Problems of enforceability are assumed to be absent.

The formulation of the constraint (2) with the decision variables  $W_i^n$  and  $V_{i+1}^e$  introduces a convenient analytical device. Instead of the firm specifying to new workers the initial wage along with an explicit specification of future wages and future permanent and temporary layoff probabilities, the firm simply promises that the expected discounted income in the future will equal or exceed  $V_{i+1}^e$ . In accordance with this, it is assumed that in each future period the firm will explicitly specify the wage and the probabilities of permanent and temporary layoffs for that particular period along with a promise of what expected discounted income will be in the future. This "rolling" process works in the following manner. Suppose that period 0 is the initial period of the firm's existence implying that the firm has no available experienced workers in period 0 (i.e.,  $m_0^e = 0$ ). The firm hires  $L_0^n$  new workers in period 0 by promising them expected discounted income  $V_0^n$ . Since new workers in period 0 have an expected income as good as is available elsewhere, no new workers quit in period 0 and hence  $m_1^e = L_0^n$ . In period 1, the firm faces the constraint that the expected income starting with period 1 must equal  $V_1^e = (V_0^n - W_0^n)/\rho$ . This constraint takes the form:

$$(3) \quad \frac{R_1^e}{m_1^e} \left\{ \frac{L_1^e}{R_1^e} [(1-\gamma_1^e)(W_1^e + V_2^e \rho) + \gamma_1^e Y_1] \right. \\ \left. + (1 - \frac{L_1^e}{R_1^e}) [(1-\gamma_1^u)(B_1 + V_2^e \rho) + \gamma_2^u Y_1] \right\}$$

$$+ (1 - \frac{R_1^e}{m_1^e}) Y_1 > V_1^e$$

It is helpful to explain (3) in a piecemeal fashion. A worker who is in the available experienced labor pool in period 1 faces a number of possibilities. The worker may be employed, permanently laid off, temporarily laid off or may choose to quit. Permanent and temporary layoffs are distinguished in this analysis by specifying that a worker who is permanently laid off is permanently separated from the firm with the provision that the worker cannot be recalled. Alternatively, a worker who is temporarily laid off is assumed to be in the pool of available experienced workers for the following period (given that the worker does not quit). This distinction essentially represents a difference in commitment on the firm's part between telling a worker when laid off that the firm has no future plans that include the worker and telling a worker when laid off that the firm expects to be able to recall the worker within a reasonable period of time.

Since within the class of experienced workers all workers are assumed to be homogeneous, permanent layoffs are made randomly out of the experienced labor pool.<sup>3</sup> This implies that the probability of being permanently laid off is given by  $1 - (R_1^e/m_1^e)$ .  $Y_1$  represents the expected discounted income available to a permanently laid off worker through the best use of the worker's time given that he will not be recalled. In what follows, the firm takes  $Y_1$  to be exogenous where  $Y_1 < V_1^n$  due to search and mobility costs. For analytical simplicity,  $Y_1$  is assumed to be non-stochastic.

A worker who is not permanently laid off, may be temporarily laid off, employed or choose to quit. Temporary layoffs are made randomly out of the retained experienced labor pool,  $R_1^e$ . The probability of being temporarily



laid off is given by  $1 - (L_1^e/R_1^e)$ . A worker who is temporarily laid off may either "quit" (in which case by definition he is not available for recall) or may wait for recall. The quit function of temporarily laid off workers, but not the actual number of quits, is taken by the firm as given. The number of quits will depend on the firm's wage policy. Formally, denoting  $\gamma_1^u$  as the probability that a worker temporarily laid off in period 1 will quit,  $\gamma_1^u$  is assumed to depend positively on the net gain associated with quitting given by  $G_1^u = Y_1 - (B_1 + V_2^e\rho)$ .  $B_1$  is the expected income available to a temporarily laid off worker in period 1 given that the worker remains available for recall and  $V_2^e$  is the expected income promised by the firm in period 1 to the worker for the future.<sup>4</sup> Since  $Y_1$  is assumed to be non-stochastic,  $\gamma_1^u$  is a simple step function taking a value of 0 when  $G_1^u < 0$  and a value of 1 when  $G_1^u > 0$ .

A worker who is offered employment in period 1 may nevertheless decide to quit as well. Letting  $\gamma_1^e$  be the probability that a worker with an employment offer in period 1 will quit,  $\gamma_1^e$  is assumed to depend positively on  $G_1^e = Y_1 - (W_1^e + V_2^e\rho)$  (where  $W_1^e$  is the wage paid to a worker employed in period 1). Analogously to  $\gamma_1^u$ , given that  $Y_1$  is non-stochastic,  $\gamma_1^e = 0$  if  $G_1^e < 0$  and  $\gamma_1^e = 1$  if  $G_1^e > 0$ .

In period 1, the firm may be hiring new workers as well as rehiring experienced workers. The contract constraint the firm faces for new workers in period 1 is equation (2) moved one period forward. Observe that this implies that the firm's promise made to both new workers and experienced workers in period 1 for future expected income starting in period 2 is the same. This is reasonable in this context because starting in period 2, workers hired in either period 0 or 1 are by assumption identical in terms of productivity. Hence, for any given period  $i$ , it is assumed that the firm

faces the following two constraints for new workers and experienced workers, respectively:<sup>5</sup>

$$(2') \quad W_i^n + V_{i+1}^e > V_i^n, \quad V_i^n > Y_i$$

$$(3') \quad \frac{R_i^e}{m_i^e} \left\{ \frac{L_i^e}{R_i^e} [(1-\gamma_i^e)(W_i^e + V_{i+1}^e) + \gamma_i^e Y_i] \right. \\ \left. + (1 - \frac{L_i^e}{R_i^e}) [(1-\gamma_i^u)(B_i + V_{i+1}^e) + \gamma_i^u Y_i] \right\} \\ + (1 - \frac{R_i^e}{m_i^e}) Y_i > V_i^e, \quad V_i^e > Y_i, \quad Y_i > B_i + Y_{i+1}^\rho$$

(v) The available experienced labor pool in period  $i$ ,  $m_i^e$ , consists of the experienced workers who were actually employed in period  $i-1$ , the experienced workers who were temporarily laid off but did not quit in period  $i-1$ , and the new, entry level workers hired in period  $i-1$ . This implies  $m_i^e = L_{i-1}^{e*} + (1 - \gamma_{i-1}^u) (R_{i-1}^e - L_{i-1}^e) + L_{i-1}^n$  where  $L_i^{e*} = (1 - \gamma_i^e)L_i^e$ .

Given the above assumptions, it is now possible to fully specify the firm's maximization problem. The firm in period  $t$  is concerned with maximizing the sum of expected discounted profits from period  $t$  onwards given by:

$$(4) \quad P_t F(L_t^{e*}) - W_t^e L_t^e - W_t^n L_t^n + E_t \sum_{k=t+1}^T [P_k F(L_k^{e*}) - W_k^e L_k^{e*} - W_k^n L_k^n] \rho^{k-t}$$

where  $E_t$  is the expectational operation conditional on information known at time  $t$  and  $T$  is arbitrarily large but finite ( $t < T$ ). Conceptually, the firm's maximization problem can be described in the following manner. The firm at the beginning of the current period  $t$  inherits a pool of experienced workers  $m_t^e$  and takes as given the labor cost parameters  $V_t^e$ ,  $Y_{t+k}$  and  $B_{t+k}$  (for all  $k > 0$ ). The firm knows  $P_t$  with certainty but is uncertain

about  $P_{t+k}$  for  $k > 0$ . The firm must make decisions on the number of experienced workers to retain, to temporarily lay off and the number of new workers to hire in the current period. It must also decide on the intertemporal distribution of wages between the current period and the indefinite future. These are the binding decisions that the firm must make because these are decisions that take effect in the current period. It makes these binding decisions based on its knowledge of current demand and its expectations of future demand. The firm specific capital accumulation and the resulting long term implicit contracts necessitate the firm considering the impact of the current decisions on expected discounted future profits. Hence the intertemporal maximization problem. Formally, the firm maximizes (4) subject to the constraints (2)', (3)',  $m_i^e > R_i^e$  and  $R_i^e > L_i^e$ .<sup>6</sup> The firm chooses  $L_i^e, L_i^n, W_i^e, W_i^n, V_{i+1}^e$  and  $R_i^e$  ( $V_i > t$ ) contingent on the realization of product demand in period  $i$ .

Derivation and analysis of the optimality conditions (which with these discrete quit decision rules involves dividing the problem into mutually exclusive regimes, determining the optimality conditions under each regime, and then comparing expected profits across regimes -- see Appendix B) allows us to prove the following proposition.

Proposition 1 The optimal wage and employment decisions in period  $i$  are characterized by the following properties (for  $t < i < T$ )

$$(5) \quad W_i^e + V_{i+1}^e \rho > Y_i \quad (=> \gamma_i^e = 0 \text{ and } L_i^{e*} = L_i^e)$$

$$(6) \quad B_i + V_{i+1}^e \rho > Y_i \quad (=> \gamma_i^u = 0)$$

$$(7) \quad \text{if } R_i^e > L_i^e, \text{ then}$$

$$(a) \quad \Pr(m_{i+1}^e = R_{i+1}^e) > 0$$

$$(b) \quad P_i F'(L_i^e) = B_i$$

$$(c) \quad B_i + E_i \left( \sum_{k=i+1}^T P_k F'(L_k^e) \rho^{k-i} \right) > P_i F'(R_i^e) + E_i \left( \sum_{k=i+1}^T P_k F'(L_k^e) \rho^{k-i} \right)$$

$$(d) \quad B_i + E_i \left( \sum_{k=i+1}^T P_k F'(L_k^e) \rho^{k-i} \right) > Y_i$$

(8) if  $m_i^e > R_i^e$ , then

$$(a) \quad P_i F'(L_i^e) + E_i \left( \sum_{k=i+1}^T P_k F'(L_k^e) \rho^{k-i} \right) = Y_i$$

$$(b) \quad L_i^n = 0$$

(9) if  $L_i^n > 0$ , then

$$(a) \quad E_i \left( \sum_{k=i+1}^T P_k F'(L_k^e) \rho^{k-i} \right) = V_i^n$$

$$(b) \quad \Pr(m_{i+1}^e = R_{i+1}^e) > 0$$

Proof: See Appendix A.

Conditions (5) and (6) of Proposition 1 imply that the firm finds it optimal to induce workers with employment offers not to quit and to induce workers who have been temporarily laid off to wait for recall. This is of interest given that many previous contract analyses artificially impose immobility on workers who contract with the firm. In the present analysis, in

which worker mobility is explicitly allowed, if the firm has made a decision to retain the worker, then the worker is induced to be immobile. Inducing temporarily laid off workers to wait for recall requires, by condition (6), a contractual wage structure that rises with firm specific experience. A wage structure that rises with firm specific experience has been motivated in other contexts to, for instance, prevent worker shirking or to induce employed workers not to quit (e.g., Lazear (1980), Carmichael (1981)). However, the present analysis emphasizes a different motive, namely, to induce laid off workers to wait for recall.

Condition (7a) of Proposition 1 indicates that temporary layoffs are optimal in period  $i$  only if there is a positive probability associated with the firm recalling all laid off workers in period  $i + 1$ . This suggests that temporary layoffs may be understood as a means of temporarily "storing" workers during temporary downturns in demand. That the firm only lays off a worker temporarily if there is a positive probability of recall suggests that what is relevant for temporary layoffs is not so much that current demand is lower than expected but rather that current demand is low relative to both past and expected future demand. An implication of this is that uncertainty of product demand is not a necessary factor for explaining temporary layoffs in this context. Temporary layoffs are seen to be optimal in this context as a means of the firm holding inventories of experienced attached workers during intertemporal downturns in demand whether these downturns are anticipated or unanticipated. This represents a departure from the typical contract theoretic explanation for layoffs which relies heavily on demand being uncertain. This is not meant to deny that temporary layoffs may be explained as a result of the firm holding an inventory of workers against potential unexpected short run variations in demand (which is the basis of the standard

contract theoretic explanation) but that temporary layoffs may also be explained as a means of the firm smoothing the variations in its attached, experienced labor pool by using temporary layoffs during intertemporal fluctuations in demand (whether these fluctuations are anticipated or not.)

Condition (7b) of Proposition 1 reveals that the marginal worker who has been retained should only be temporarily laid off if the value of his marginal product falls below the opportunity cost of his time (given that he remains available for recall). At first glance, this appears to be essentially the same condition for the optimality of temporary layoffs found in previous contract analyses (e.g., Baily (1977), Burdett and Mortensen (1980)). However, there is a subtle but important difference that reveals the manner in which the distinction between temporary and permanent layoffs has been blurred in previous analyses. The difference is that in the present analysis, although worker mobility is allowed, the relevant opportunity cost of a worker's time for determining the optimality of temporary layoffs excludes the opportunity of alternative permanent employment. In contrast, the recent work of Baily (1977), and Holmstrom (1981) predicts that the probability of "layoffs" (imprecisely defined) at a given firm should simultaneously be an increasing function of the valuation of leisure by workers, the level of imperfectly experienced rated unemployment benefits and the expected income from alternative permanent employment opportunities for workers. To illustrate the problems with this prediction, observe that condition (7b) of Proposition 1 implies that  $B_1 > 0$  is a necessary condition for temporary layoffs to be optimal. This implies that if  $B_1 = 0$ , then regardless of a worker's alternative permanent employment opportunities, temporary layoffs will not be optimal. In other words, an improvement in alternative permanent employment opportunities while perhaps increasing the probability of a

permanent layoffs may not change the probability of temporary layoffs. In fact, in a more general model, one might argue that an improvement in alternative permanent employment opportunities for workers may actually lower the probability of temporary layoffs. This is because an improvement in alternative employment opportunities for workers will, other considerations apart, increase the likelihood that a temporarily laid off worker will quit and thereby will increase the costs associated with temporary layoffs.

Conditions (7c) and (7d) of Proposition 1 are best interpreted by considering the best use of a worker's time in terms of maximizing the expected joint income to both the firm and the worker. The expected joint income associated with the marginal worker being temporarily laid off in period  $i$  given that the worker does not quit is given by

$B_i + E_i \left( \sum_{k=i+1}^T P_k F'(L_k^e) \rho^{k-1} \right)$ . The expected joint income associated with the

marginal worker being permanently laid off is given by  $Y_i$ . The expected joint income associated with the marginal worker being employed (given that all retained workers are employed) is given by  $P_i F'(R_i^e) +$

$E_i \left( \sum_{k=i+1}^T P_k F'(L_k^e) \rho^{k-1} \right)$ . Hence, conditions (7c) and (7d) indicate that a

worker should only be temporarily laid off if this represents the best use of the worker's time relative to either being employed or permanently laid off.

Condition (8a) of Proposition 1 indicates that experienced workers should be permanently laid off only if the present discounted value of the marginal product associated with retaining all of the available experienced workers

falls below the expected discounted income a worker can earn elsewhere (net of search and moving costs). Condition (8d) reveals that if any experienced workers are permanently laid off in period  $i$  then no new workers are hired in period  $i$ . This is understood by considering the relevant benefits and costs of retaining an experienced worker relative to hiring a new worker. Since hiring and training costs are present, retraining and employing an experienced worker clearly dominates in terms of a higher present discounted value of marginal product. The question is what are the relevant costs of retaining an experienced worker relative to hiring a new worker. If the firm retains and employs the marginal experienced worker in period  $i$  then the labor cost is essentially  $W_i^e + V_{i+1}^e$ . However, if the firm permanently lays off the marginal experienced worker in period  $i$ , then this raises the probability of any given experienced worker being permanently laid off, and due to the implicit contract the firm will have to provide compensation for this higher probability of layoffs. Adjusting the cost to the firm of retaining a worker for compensation costs the firm incurs if it does not retain the worker yields an effective retention cost of  $Y_i$  for the marginal worker. Since  $V_i^n > Y_i$ , this means that not only does retaining an experienced worker yield higher productivity but the effective cost to the firm of retaining an experienced worker is lower than the cost of hiring a new worker.

The optimality of seniority based hiring demonstrated here suggests that in a more general setting, if many different classes of experience (and hence skill) levels were considered, then permanent layoffs on a seniority basis would be optimal. However, since the necessary conditions (9a) and (9b) for  $L_i^n > 0$  and the necessary conditions (7a)-(7d) for  $R_i^e > L_i^e$  are not inconsistent, it may be optimal in certain situations for the firm to be



hiring new workers at the same time it is temporarily laying off experienced workers. For example, consider a situation in which the firm has an expected long term trend growth rate of demand that is relatively high but is experiencing what is perceived to be a temporary negative deviation from this trend. Then an optimal strategy for the firm may be to temporarily lay off experienced workers (if they can be induced to wait for recall) but hire and employ new workers during the temporary downturn so that these new workers can acquire firm specific skills that would otherwise be relatively scarce when demand recovers. This suggests that in a more general setting with several different experience level classes, such a situation might imply that inverse seniority temporary layoffs are optimal. Hence, distinguishing between temporary and permanent layoffs suggests that while turnover costs imply that seniority based permanent layoffs are optimal, turnover costs may imply that inverse seniority temporary layoffs are optimal.

The firm in this analysis has an incentive to maintain a pool of experienced workers consistent with long run trend demand. The problem the firm faces when suffering a downturn in demand in the current period is deciding on whether the downturn is permanent or transitory. In view of this signal extraction problem, it is helpful to consider the firm's optimal strategy in response to the limiting cases of known permanent and known transitory fluctuations in demand. The following proposition helps to characterize the firm's response to known permanent changes in product demand.

Proposition 2: If beginning in period  $t$  the firm expects that the exogenous product demand and labor cost variables that it will face will be both intertemporally constant and non-stochastic (i.e.,  $\bar{P} = P_t = P_{t+j}$ ,

$\bar{B} = B_t = B_{t+j}$ ,  $V_{t+j}^n = \sum_{k=t+j}^T \bar{V}^n \rho^{k-j-t}$ ,  $Y_{t+j} = \sum_{k=t+j}^T \bar{Y} \rho^{k-j-t}$ , for all  $t < t+j < T$ , then:

$$(i) \quad L_t^e + L_t^n = L_{t+j}^e \quad (\text{for all } t < t+j < T)$$

$$(ii) \quad R_{t+j}^e = L_{t+j}^e \quad (\text{for all } t < t+j < T)$$

$$(iii) \quad L_{t+j}^n = 0 \quad (\text{for all } t < t+j < T)$$

$$(iv) \quad \text{if } m_t^e > L_t^{\max}, \text{ then } L_t^e = L_{t+j}^e = L_t^{\max} \text{ and } L_{t+j}^n = 0 \quad (\text{for all } t < t+j < T)$$

$$(v) \quad \text{if } m_t^e < L_t^{\min}, \text{ then } L_t^e + L_t^n = L_{t+j}^e = L_t^{\min}, L_t^n > 0, \text{ and } L_{t+j}^n = 0 \quad (\text{for all } t < t+j < T)$$

$$(vi) \quad \text{if } L_t^{\max} > m_t^e > L_t^{\min}, \text{ then } L_t^e = L_{t+j}^e = m_t^e, \text{ and } L_{t+j}^n = 0 \quad (\text{for all } t < t+j < T)$$

where  $L_t^{\max}$  and  $L_t^{\min}$  are defined by:

$$\bar{P}F'(L_t^{\max}) = \bar{Y}, \quad \bar{P}F'(L_t^{\min}) = \bar{V}^n \left( 1 / \sum_{j=t+1}^T \rho^{j-t} \right) = \bar{V}_t^n, \quad \bar{V}^n > \bar{Y}$$

Proof: See Appendix A.

Proposition 2 establishes that if beginning in period  $t$  the firm expects constant exogenous product demand and labor costs for the indefinite future, then the firm will immediately "jump" to the optimal stationary state experienced labor pool and will fully employ this labor pool in all periods. This implies that if the initial experienced labor pool in period  $t$  is larger than the optimum, the firm will permanently lay off the excess in period  $t$ . Similarly, if the initial labor pool in period  $t$  is smaller than the optimum, the firm will hire the requisite number of new workers in

period  $t$  in order to have the optimum number of experienced workers for periods  $t+1$  onwards.

Focusing on the costs of compensating workers for a lower probability of being employed helps explain the result that in the face of a permanent downturn in demand it is always optimal for the firm to reduce the employed work force through permanent layoffs rather than through temporary layoffs. The reason is that, for a given permanent reduction in the optimal employed work force, the cost of compensating workers for what would be an increase in the probability of temporary layoffs in every period exceeds the cost of compensating workers for a one time increase in the probability of permanent layoffs in period  $t$  (given  $B_1 < Y_1 - Y_{1+1}\rho$ ).

The problem, of course, is that the firm may be uncertain with respect to whether a given current downturn in demand is temporary or permanent. Its temporary and permanent layoff decisions are obviously dependent on its expectations of the future path of demand. The following proposition formally characterizes the firm's behavior in light of this signal extraction problem.

Proposition 3 Let  $B_t = B_{t+j} = \bar{B}$ ,  $Y_t = \sum_{j=t}^T \bar{Y}_t \rho^{j-t}$ ,  $V_t^n = \sum_{j=t}^T \bar{V}_t^n \rho^{j-t}$ ,

$P_t = \bar{P} + \epsilon_t$  where  $\epsilon_t < 0$ , and let  $P_{t+k} = \bar{P} + \alpha^k \epsilon_t$  ( $0 < \alpha < 1$ ). Then as  $\alpha$  increases, the probability of temporary layoffs in period  $t$  decreases while the probability of permanent layoffs increases.

Proof: See Appendix A.

The parameter  $\alpha$  in Proposition 3 provides a measure of the firm's expectation of the duration of a given downturn in current demand. The proposition suggests that as the firm becomes more pessimistic with respect to the duration of a given downturn in demand the firm substitutes permanent

layoffs for temporary layoffs. This proposition provides further support for our earlier contention that what matters for temporary layoffs is not so much that current demand is lower than expected but rather that current demand is low relative to both past and future levels of demand. An additional implication of this proposition is that if one empirically observes the incidence of unemployed workers who perceive themselves to be on temporary layoff declining then this may signal a worsening of the economy rather than a recovery. For it may be that underlying this reduction in temporary layoffs is a shift towards permanent layoffs as firms become more pessimistic with respect to the duration of a given downturn.

#### Concluding Remarks

This paper develops a theory of the firm's demand for labor in an intertemporal context in which the firm faces a variety of adjustment costs for varying its labor force. The adjustment costs are attributable to the natural accumulation of firm specific skills by workers, hiring costs, and the costs imposed upon a firm through having to pay higher wages on average to workers when employment at that firm is unstable. The presence of these adjustment costs imply the development of mutually advantageous long term attachments in the labor market. These long term attachments are modeled in this analysis by hypothesizing that the firm makes fully enforceable long term implicit contracts with the workers. The long term implicit contracts are such that a worker in joining the firm is promised an expected discounted income that is at least as good as is available elsewhere.

This multiperiod implicit contract framework, which allows for separations to occur in each of several successive periods, enables us to identify distinguishing characteristics of temporary and permanent layoffs that were heretofore blurred or distorted in previous contract theoretic

analyses. Temporary layoffs are shown to be optimal in response to downturns in demand of sufficiently temporary duration. In this regard, we find that what matters for temporary layoffs is not so much that demand is lower than expected but rather that demand is low relative to both past and future levels of demand. Moreover, we find that a necessary condition for temporary layoffs to be utilized is that the firm is able to induce temporarily laid off workers to wait for recall. We argue that the firm may accomplish this inducement by specifying a compensation structure that rises with firm specific experience. This suggests that the observed existence of deferred compensation schemes (e.g., non-vested pensions, seniority provisions) may be at least partially justified as a means of inducing temporarily laid off workers to wait for recall even though such waiting might imply a short run loss of income.

Permanent layoffs, on the other hand, are shown to be optimal only in response to downturns in demand of sufficiently permanent duration. In response to the probable signal extraction problem that firms face in determining the "temporariness" of a given downturn, we demonstrate that as the firm becomes increasingly pessimistic with respect to the duration of the downturn, the firm substitutes permanent layoffs for temporary layoffs.

In distinguishing between temporary and permanent layoffs, we find that it is helpful to differentiate between the opportunity cost of a worker's time including and the opportunity cost excluding the possibility of alternative permanent employment. The likelihood of temporary layoffs is demonstrated to be positively related to the opportunity cost of each worker's time excluding the possibility of alternative permanent employment whereas permanent layoffs are shown to be positively related to improvements in workers' alternative permanent employment opportunities. However, the likelihood of temporary

layoffs is argued to be either unrelated or negatively related to improvements in workers' alternative permanent employment possibilities.

The existence of hiring costs, training costs, search costs, and moving costs implies that there is an incentive for a firm to maintain a pool of experienced workers consistent with the long term expected growth rate of demand. In accord with this, we demonstrate that permanent layoffs should be made on a seniority basis. However, we argue that these same turnover costs may imply that temporary layoffs should be made on an inverse seniority basis. The reasoning is that there are circumstances under which it may be optimal for the firm to temporarily lay off senior workers who have already received training while at the same time the firm employs and trains junior workers for the future. Such circumstances obviously require the expected long term growth rate of demand to be relatively high while current demand is relatively low.

An obvious limitation of the analysis in this paper is that we only help explain "efficient" turnover. Recent developments in the implicit labor contract literature have begun to examine the impact of introducing asymmetric information and moral hazard problems which appear to yield interesting new insights towards providing an explanation for "inefficient" turnover.<sup>7</sup> However, it is important to note that these recent developments are (like their full information predecessors) based on one and two period contract models in which separations are only allowed in the final period of the analysis. Hence, the multiperiod contract model presented in this paper in which separations may occur in each of several successive periods may be viewed as a necessary first step before addressing the issues raised by asymmetric information and moral hazard problems in a more complex intertemporal setting. In this regard, the moral hazard and asymmetric

information problems associated with implicit labor contracts may very well be exacerbated in this intertemporal setting because of the intertemporal incentive problems associated with inducing temporarily laid off workers to wait for recall. This would appear to be a fertile area for future research.

## FOOTNOTES

<sup>1</sup>The reasoning for this is quite simple. Risk aversion on the part of workers, in the absence of other factors promoting worker-firm immobility, should imply long term attachments between workers and their insurance agents but not between workers and particular firms. The efficient risk shifting arrangement under these circumstances would entail a worker's insurance-employment agent providing a guaranteed income to the worker and then placing the worker at each moment in time in the activity that yields the highest value. For further discussion of this point, in particular the potential role played by enforcement and monitoring costs, see Haltiwanger (1982).

<sup>2</sup>By ex post worker firm immobility we mean that a firm may only employ workers from the pool of workers that it contracted with ex ante and workers may only be employed with the firm with whom they contracted with ex ante.

<sup>3</sup>This is a consequence of the assumption of only two classes of workers: new and experienced. A more general model with many different experience (and hence skill) levels would not have this property. However, this model is sufficiently general to address the issues of seniority based permanent and temporary layoffs because one can pose the question in terms of whether the firm will be laying off (either permanently or temporarily) any of its attached workers while it is hiring new workers.

<sup>4</sup>In general,  $B_1$  can be thought to consist of unemployment benefits and the income equivalent of the value of the additional leisure a temporarily laid off worker acquires. It is assumed that the government financed unemployment benefits are financed by a tax system that is not experienced rated.



<sup>5</sup>Observe that it is assumed that  $Y_i > B_i + Y_{i+1}\rho$ . It is obvious that  $Y_i > B_i + Y_{i+1}\rho$  because  $Y_i$  is the opportunity cost of a worker's time including the alternative permanent employment opportunities and  $B_i$  excludes alternative permanent employment opportunities. It is assumed that this inequality is strict because the market is such that an unattached worker would always prefer to work rather than be idle.

<sup>6</sup>For period  $T$ , the constraints (2') and (3') must be modified slightly since  $V_{T+1}^e$  is not well-defined. The constraints (2') and (3') become, respectively,

$$(2'*) \quad W_T^n = V_T^n$$

and

$$(3'*) \quad \frac{L_T^e}{m_T^e} [(1-\gamma_T^e)W_T^e + \gamma_T^e Y_T] + (1 - \frac{L_T^e}{m_T^e})Y_T > V_T$$

Note that  $\gamma_T^u = 1$  and by definition  $Y_T > B_T$ . Moreover,  $\gamma_T^e = 1$  if  $Y_T > W_T^e$  and  $\gamma_T^e = 0$  if  $W_T^e > Y_T$ . Essentially the problem in the last period reduces to a standard one period contract problem and thus there is no way to distinguish between temporary and permanent layoffs in this last period. This latter point is of interest because it highlights our argument that with previous analyses, which only allow separations to occur in the final period of the analysis, temporary and permanent layoffs cannot be distinguished. For this reason, in the analysis that follows, we focus our attention on period  $i$ , where  $i < T$ , so that a meaningful distinction can be made.

<sup>7</sup>For a survey of this recent work see Hall and Lazear (1982).

## APPENDIX A

The optimality conditions (derived in Appendix B) are (for  $t < i < T$ )

$$(A1) \quad W_i^e + V_{i+1}^e \rho > Y_i$$

$$(A2) \quad B_i + V_{i+1}^e \rho > Y_i$$

$$(A3) \quad P_i F'(L_i^e) = B_i + \mu_i$$

$$(A4) \quad P_i F'(L_i^e) + E_i \left( \sum_{j=i+1}^T P_j F'(L_j^e) \rho^{j-i} \right) = Y_i + \delta_i$$

$$(A5) \quad Z_i = \left( \sum_{j=i+1}^T P_j F'(L_j^e) \rho^{j-1} - V_i^n \right) < 0, \quad Z_i \cdot L_i^n = 0$$

$$(A6) \quad \mu_i (R_i^e - L_i^e) = 0, \quad R_i^e > L_i^e$$

$$(A7) \quad \delta_i (m_i^e - R_i^e) = 0, \quad m_i^e > R_i^e$$

Proof of Proposition 1: (5) -- by (A1); (6) -- by (A7); 7(a) -- when  $R_i^e > L_i^e$ , by (A6),  $\mu_i = 0$ , combining (A3) and (A4) when  $\mu_i = 0$  yields:

$$(A8) \quad Y_i - B_i - Y_{i+1} \rho - E_i(\delta_{i+1}) \rho + \delta_i = 0$$

Since by assumption  $B_i + Y_{i+1} \rho < Y_i$ , this implies  $E(\delta_{i+1}) > 0$ ; 7(b) -- by (A3) and (A6); 7(c) -- by 7(b) and (A4); 7(d) -- by 7(b) and (A4); 8(a) -- by (A4) and (A7); 8(b) -- since  $V_i^n > Y_i$ , by (A4) and (A5); 9(a) -- by (A5); 9(b) -- by (A4), we have:

$$(A9) \quad P_{i+1} F'(L_{i+1}^e) + \sum_{j=i+2}^T P_j F'(L_i^e) \rho^{j-(i+1)} = Y_{i+1} + \delta_{i+1}$$

Multiplying both sides of (A9) by  $\rho$  and taking expectations conditional on information known in period  $i$  yields:

$$(A10) \quad E_i \left( \sum_{j=i+1}^T P_{i+1} F'(L_j^e) \rho^{j-i} \right) = Y_{i+1} \rho + E(\delta_{i+1}) \rho$$

By (A5) and (A10) if  $L_i^n > 0$ , then  $V_i^n = Y_{i+1} \rho + E(\delta_{i+1}) \rho$ . Hence,  $E(\delta_{i+1}) > 0$ .

Proof of Proposition 2: We prove (i) - (vi) by considering the following three exhaustive characterizations of the initial condition for  $m_t$ : (I)  $m_t^e > L^{\max}$ ; (II)  $m_t^e < L_t^{\min}$ ; and (III)  $L_t^{\min} < m_t^e < L^{\max}$ . Note that the problem is now non-stochastic.

(I)  $m_t^e > L^{\max}$ . Suppose  $R_t^e > L^{\max}$ . If  $R_t^e > L^{\max}$ , then  $P_t F'(R_t^e) < \bar{Y}$ . This implies  $\delta_t - \delta_{t+1} \rho < 0$  by (A4) if  $R_t^e = L_t^e$ . If  $R_t^e > L_t^e$ , then  $\bar{P} F'(L_t^e) = \bar{B} < \bar{Y}$  so that  $\delta_t - \delta_{t+1} \rho < 0$  even if  $R_t^e > L_t^e$ . Hence,  $\delta_t - \delta_{t+1} \rho < 0$  if  $R_t^e > L^{\max}$ . Since  $\delta_{t+1} > 0$ ,  $m_{t+1}^e = R_{t+1}^e$ . Now  $m_{t+1}^e = R_t^e + L_t^n$ . Can  $L_t^n > 0$ ? No, since  $L_t^n > 0$  implies that  $R_{t+1}^e > L^{\max}$  and by the above argument  $\delta_{t+2} \rho > \delta_{t+1}$  (if  $t+2 < t$ ) or  $\delta_{t+1} = \delta_T = \bar{P} F'(R_{t+1}^e) - \bar{Y} < 0$  otherwise. The latter is impossible. For the former, when  $L_t^n > 0$ ,  $\delta_{t+1} \rho = V_t^n - Y_{t+1} \rho$ . This implies  $\delta_{t+2} \rho > V_{t+1}^n - Y_{t+2} \rho$  which by (A4) and (A5) yields a contradiction. Hence  $L_t^n = 0$ .

With  $L_t^n = 0$ ,  $m_{t+1}^e = R_{t+1}^e = R_t^e$ . Hence,  $R_{t+1}^e > L^{\max}$  and by the arguments above  $\delta_{t+2} \rho > \delta_{t+1}$ .

Following similar arguments, it can be shown that  $R_t^e > L^{\max}$  implies  $\delta_T > 0$  and  $R_T^e > L^{\max}$ . However, this yields a contradiction because this implies  $0 > \bar{P}F'(R_T^e) - \bar{Y} = \delta_T$ . Hence, if  $m_t^e > L^{\max}$ , then  $R_t^e < L^{\max}$ .

Suppose  $R_t^e < L^{\max}$ . This implies  $R_t^e < m_t^e$  and hence  $\delta_t = 0$ . However,  $R_t^e < L^{\max}$  by (A4) implies  $\bar{P}F'(L_t^e) = \bar{Y} + \delta_t - \delta_{t+1}\rho > \bar{Y}$ . Yet this implies  $\delta_t > 0$ , a contradiction. Hence, if  $m_t^e > L^{\max}$ , then  $R_t^e = L^{\max}$ .

Given that  $R_t^e = L_t^e = L^{\max}$ , suppose  $L_t^e < R_t^e$ . This would imply  $\bar{P}F'(L_t^e) = \bar{B} > \bar{Y}$  which is a contradiction.

Given that  $R_t^e = L_t^e = L^{\max}$ , suppose  $L_t^n > 0$ . This implies by (A4) and (A6) that  $\delta_{t+1} = V_t^n - Y_{t+1}\rho$  and thus  $m_{t+1}^e = R_t^e + L_t^n = R_{t+1}^e$ . If  $R_{t+1}^e > L_{t+1}^e$ , then  $\bar{P}F'(L_{t+1}^e) = \bar{B} = \bar{Y} + \delta_{t+1} - \delta_{t+2}\rho$  if  $t+2 \leq T$  or  $\bar{P}F'(L_T^e) = \bar{B} = \bar{Y} + \delta_T$ . The latter is impossible. For the former, this implies  $\delta_{t+2}\rho > \delta_{t+1} = (V_t^n/\rho) - Y_{t+1}$  which is impossible. Using similar arguments, it can be shown that  $R_{t+1}^e = L_{t+1}^e$  and  $L_t^n > 0$  yields a contradiction as well. Hence,  $L_t^n = 0$ .

In total, then, if  $m_t^e > L^{\max}$ , it must be the case that  $R_t^e = L_t^e = L^{\max}$  and  $L_t^n = 0$ .

(II)  $m_t^e < L^{\min}$ . Since  $m_t^e > R_t^e > L_t^e$ , by (A4),  $\bar{Y} + \delta_t - \delta_{t+1}\rho > V_t^n$  which implies  $\delta_t > 0$ . Moreover, since  $\bar{V}_t^n > \bar{B}$ , this implies by (A3) that  $R_t^e = L_t^e$  and  $\mu_t > 0$ .

Suppose  $L_t^n = 0$ . For  $t+1 = T$ , this implies  $\delta_{T\rho} > \bar{V}^n(1+\rho) - \bar{Y}\rho$  but by (A4) and (A5)  $\delta_{T\rho} < \bar{V}^n(1+\rho) - \bar{Y}\rho$ . Hence we have a contradiction. For  $t+2 \leq T$ , this implies by (A4) and (A5) that  $\bar{P}F'(L_{t+1}^e) = \bar{Y} + \delta_{t+2}\rho > \bar{V}_t^n$ . Let  $L_{t+1}^n > 0$ , then  $\delta_{t+2}\rho = V_{t+1}^n - Y_{t+2}\rho$ . This in turn implies  $\delta_{t+1} > V_t^n - Y_{t+1}\rho$  which is impossible. Otherwise, if  $L_{t+1}^n = 0$  and if  $t+2 = T$ , then by the arguments above  $\delta_T > \bar{V}^n(1+\rho) - \bar{Y}\rho$  which is a

contradiction. If  $L_{t+1}^n = 0$  and  $t + 2 < T$ , then by the above arguments a contradiction is forthcoming if  $L_{t+2}^n > 0$ . Following this line of reasoning, this implies  $L_{t+k}^n = 0$  for all  $t + k < T$ , which in turn implies  $\delta_T > \bar{V}^n(1+\rho) - \bar{Y}\rho$ , a contradiction. Hence,  $L_t^n > 0$ . Following similar arguments, it can be shown that  $L_t^n + m_t^e > L_t^{\min}$ .

Suppose  $L_t^n + m_t^e > L_t^{\min}$ . If  $t + 1 = T$ , then this implies  $\bar{P}F'(L_T^e) = \bar{V}(\frac{1}{\rho} + 1)$ . Yet, since  $R_T^e = L_t^n + m_t^e > L_t^{\min}$ , this implies  $\bar{P}F'(R_T^e) < \bar{V}(\frac{1}{\rho} + 1)$  for  $R_T^e = L_T^e$  or  $\bar{P}F'(L_T^e) = \bar{B} < \bar{V}(\frac{1}{\rho} + 1)$  for  $R_T^e > L_T^e$ . Hence we have a contradiction for  $t + 1 = T$ . Now suppose  $t + 2 = T$ . This implies  $\bar{P}F'(R_{T-1}^e) = \bar{V}(1 + \rho + \frac{1}{\rho}) - \bar{Y}\rho - \delta_{T-1}\rho < \bar{V}(1 + \frac{1}{\rho + \rho})$ . Suppose  $\delta_T = 0$ . This implies that  $\bar{Y}\rho + \bar{V}(1 + \frac{1}{\rho + \rho}) > \bar{V}(1 + \rho + \frac{1}{\rho})$  which is clearly impossible. Suppose  $\delta_T > 0$ . This implies  $\bar{P}F'(L_T^e) < \bar{V}(1 + \frac{1}{\rho + \rho})$  and hence  $\bar{P}(F'(L_{T-1}^e) + F'(L_T^e)\rho) = \bar{V}(1 + \rho + \frac{1}{\rho}) < \bar{V}(1 + \frac{1}{\rho + \rho})(1+\rho) = \bar{V}(1 + \rho + \frac{1}{\rho})$  which again is impossible. Hence, for  $t + 2 = T$ ,  $L_t^e + L_t^n = L_t^{\min}$ . Following similar reasoning, it can be shown that for all  $t + k < T$ ,  $k > 0$ ,  $L_t^e + L_t^n = L_t^{\min}$ .

In total, then, when  $m_t^e < L_t^{\min}$ ,  $m_t^e = R_t^e = L_t^e$  and  $m_t^e + L_t^n = L_t^{\min}$ .

(III)  $L_t^{\min} < m_t^e < L_t^{\max}$ . Suppose  $R_t^e < m_t^e$ . This implies  $\delta_t = 0$  and by (A4)  $\bar{P}F'(L_t^e) = \bar{Y} - \delta_{t+1}\rho$ . However, since  $L_t^e < R_t^e < m_t^e < L_t^{\max}$ ,  $\bar{P}F'(L_t^e) > \bar{Y}$  which is a contradiction. Hence,  $R_t^e = m_t^e$ .

Suppose  $R_t^e > L_t^e$ . This implies  $\bar{P}F'(L_t^e) = \bar{B}$ . However, since  $L_t^{\max} > m_t^e = R_t^e > L_t^e$ ,  $\bar{P}F'(L_t^e) > \bar{Y} > B_t$  which is a contradiction.

Given that  $m_t^e = R_t^e = L_t^e$ , suppose  $L_t^n > 0$ . This implies  $\delta_{t+1} = V_t^n(1/\rho) - Y_{t+1}$  which implies by (A4) that  $\bar{P}F'(L_{t+1}^e) = (V_t^n/\rho) - Y_{t+2}\rho - \delta_{t+2}\rho$ . Moreover, since  $m_t^e > L_t^{\min}$ , and  $R_{t+1}^e = m_{t+1}^e = m_t^e + L_t^n$ ,  $\bar{P}F'(L_{t+1}^e) < V_t^n / (\sum_{j=t+1}^T \rho^{j-t})$ . By the arguments used in part (ii) of this

proposition this yields a contradiction.

Proof of Proposition 3: Since  $P_{t+j} > P_{t+j-1}$  for  $T > t + j - 1 > t$ , using arguments similar to those found in Proposition 2, it can be shown that  $m_{t+j}^e = R_{t+j}^e = L_{t+j}^e$  for all  $T > t + j > t$ . Now if  $R_t^e = m_t^e$ , then

$\frac{\partial R_t^e}{\partial \alpha} = 0$  since  $m_t^e$  is fixed. Hence, consider a situation in which  $R_t^e < m_t^e$ . Since  $R_t^e < m_t^e$ , by Proposition 1,  $L_t^n = 0$ . Since  $m_{t+j}^e = R_{t+j}^e = L_{t+j}^e$  for all  $T > t + j > t$ ,  $m_t^e > R_t^e$  and  $L_t^n = 0$ , (A4) can be rewritten as:

$$(A12) \quad (\bar{P} + \epsilon_t)F'(L_t^e) + (\bar{P} + \alpha \epsilon_t)F'(R_t^e)\rho + \sum_{j=t+2}^T (\bar{P} + \alpha^j \epsilon_t)F'(R_t^e + \sum_{k=t+1}^{j-1} L_k^n)\rho^{j-t} = Y_t$$

First, suppose  $L_k^n = 0$  for all  $T > k > t + 1$ . Then (A12) becomes:

$$(A13) \quad (\bar{P} + \epsilon_t)F'(L_t^e) + \sum_{j=t+1}^T (\bar{P} + \alpha^j \epsilon_t)F'(R_t^e)\rho^{j-t} = Y_t$$

By (A3) and (A6) either  $R_t^e = L_t^e$  or  $(\bar{P} + \epsilon_t)F'(L_t^e) = \bar{B}$ . If  $R_t^e = L_t^e$ , then

(A13) becomes

$$(A14) \quad (\bar{P} + \epsilon_t)F'(R_t^e) + \sum_{j=t+1}^T (\bar{P} + \alpha^j \epsilon_t)F'(R_t^e)\rho^{j-t} = Y_t$$

$$\text{In this case, } \frac{\partial R_t^e}{\partial \alpha} = - \frac{\sum_{j=t+1}^T j \alpha^{j-1} \epsilon_t F'(R_t^e)\rho^{j-t}}{(\bar{P} + \alpha \epsilon_t)F''(R_t^e) + \sum_{t+1}^T (\bar{P} + \alpha^j \epsilon_t)F''(R_t^e)\rho^{j-t}} < 0.$$

Now, suppose  $R_t^e > L_t^e$ . Then

$$\frac{\partial R_t^e}{\partial \alpha} = - \frac{\sum_{j=t+1}^T j \alpha^{j-1} \epsilon_t F'(R_t^e) \rho^{j-t}}{\sum_{j=t+1}^T (\bar{P} + \alpha^j \epsilon_t) F''(R_t^e) \rho^{j-t}} < 0.$$

Hence, if  $L_k^n = 0$  for all  $T > k > t + k$ , then  $\frac{\partial R_t^e}{\partial \alpha} < 0$ . Now suppose  $L_k^n > 0$  for some  $k$ . Consider the first  $k$  for which this is true. By (A5), this implies

$$(A15) \quad \sum_{j=k+1}^T (\bar{P} + \alpha^j \epsilon_t) F'(R_t^e) + \sum_{h=k}^{j-1} L_h^n \rho^{j-k} = V_k^n$$

In this case, then (A12) becomes:

$$(A16) \quad (\bar{P} + \epsilon_t) F'(L_t^e) + (\bar{P} + \alpha \epsilon_t) F'(R_t^e) \rho + \sum_{j=t+2}^k (\bar{P} + \alpha^j \epsilon_t) F'(R_t^e) \rho^{j-t} = Y_t - V_k^n \rho^{k-t}$$

Now, if  $R_t^e = L_t^e$  this implies:

$$\frac{\partial R_t^e}{\partial \alpha} = \frac{\sum_{j=t+1}^k j \alpha^{j-1} \epsilon_t F'(R_t^e) \rho^{j-t}}{(\bar{P} + \epsilon_t) F''(R_t^e) + \sum_{j=t+1}^k (\bar{P} + \alpha^j \epsilon_t) F''(R_t^e) \rho^{j-t}} < 0$$

or if  $R_t^e > L_t^e$  then we have:

$$\frac{\partial R_t^e}{\partial \alpha} = \frac{\sum_{j=t+1}^k j \alpha^{j-1} \epsilon_t F'(R_t^e) \rho^{j-t}}{\sum_{j=t+1}^k (\bar{P} + \alpha^j \epsilon_t) F''(R_t^e) \rho^{j-t}} < 0$$

Thus, with  $L_k^n > 0$  for some  $k > t + 1$ ,  $\frac{\partial R_t^e}{\partial \alpha} < 0$ . Therefore,  $\frac{\partial R_t^e}{\partial \alpha} < 0$ .

Observe as well that if  $R_t^e > L_t^e$ , then since  $(\bar{P} + \epsilon_t)F'(L_t^e) = \bar{B}$ , we have

$\frac{\partial L_t^e}{\partial \alpha} = 0$ . Otherwise, if  $R_t^e = L_t^e$ , then  $\frac{\partial L_t^e}{\partial \alpha} = \frac{\partial R_t^e}{\partial \alpha}$ . Hence, we have

$$\frac{\partial (m_t^e - R_t^e)}{\partial \alpha} > 0 \quad \text{and} \quad \frac{\partial (R_t^e - L_t^e)}{\partial \alpha} < 0.$$



## APPENDIX B

To derive the optimality conditions, given the discrete decision rules, we consider alternative regimes and then compare expected profits across regimes. It is helpful in this process to consider the time periods in reverse order. First, consider the firm's optimization problem at the beginning of period  $T$ . In period  $T$ , recall that the laid off worker's quit decision is not relevant. The firm maximizes

$$(B1) \quad P_T F(L_T^e(1-\gamma_T^e)) - W_T^e L_T^e(1 - \gamma_T^e) - L_T^n W_T^n$$

subject to:

$$(B2) \quad \frac{L_T^e}{m_T^e} [(1-\gamma_T^e)W_T^e + \gamma_T^e Y_T] + (1 - \frac{L_T^e}{m_T^e}) Y_T > V_T^e$$

and  $m_T^e > R_T^e > L_T^e$  where  $m_T^e$ ,  $V_T^e$ ,  $V_T^n$ ,  $P_T$  and  $Y_T$  are taken as given (as of the beginning of period  $T$ ). Given (B1) and (B2), it is easy to see that  $\gamma_T^e = 0$  must be optimal. Either  $\gamma_T^e = 0$  because otherwise (A2) would be violated (if  $V_T^e > Y_T$ ) or  $\gamma_T^e = 0$  because  $F'(x) \rightarrow \infty$  as  $x \rightarrow 0$  and this implies that positive profits are possible for at least some  $(1-\gamma_T^e)L_T^e > 0$ . Note as well that  $L_T^n = 0$  must be optimal since the value of new workers in period  $T$  (who require a period of training) is zero. Now consider the firm's optimization problem as of the beginning of period  $T - 1$ . By Bellman's principle of dynamic optimality,  $\gamma_T^e = 0$  must still be optimal. Given that  $\gamma_T^e = 0$ , in period  $T-1$ , the optimization problem can be subdivided into four mutually exclusive regimes (with the accompanying restrictions on the terms of the contract): regime I ( $\gamma_{T-1}^e = \gamma_{T-1}^u = 1$ ); regime II ( $\gamma_{T-1}^e = 1, \gamma_{T-1}^u = 0$ ); regime III ( $\gamma_{T-1}^e = 0, \gamma_{T-1}^u = 1$ ), and

regime IV ( $\gamma_{T-1}^e = \gamma_{T-1}^u = 0$ ). First, consider regime I. Regime I is only feasible if  $Y_{T-1} > V_{T-1}^e$ . Suppose this is the case. Under regime I, the optimal contract entails  $L_{T-1}^n(I) > 0$  where  $L_{T-1}^n(I)$  is chosen so that  $E_{T-1}(P_T F'(L_T^e(I)))\rho = V_{T-1}^n$  and  $0 < L_T^e(I) < L_{T-1}^n(I)$  where  $L_T^e(I)$  is chosen so that  $P_T F'(L_T^e(I)) = Y_T + \delta_T(I)$  ( $\delta_T(I)$  is the multiplier associated with the constraint  $L_{T-1}^n(I) = m_T^e(I) > L_T^e(I)$ ). Expected profits under this regime are given by:

$$(B3) \quad E[\pi | I] = -L_{T-1}^n(I)V_{T-1}^n + (L_{T-1}^n(I) - E(L_T^e(I)))Y_{T-1}\rho \\ + E_{T-1}(P_T F(L_T^e(I)))\rho$$

Now consider the follow feasible alternative to the optimal contract under regime I. Suppose that instead of hiring  $L_{T-1}^n(I)$  workers, the firm let  $W_{T-1}^e + V_T^e > Y_{T-1}$  so that  $\gamma_{T-1}^e = 0$ . Under this alternative regime, suppose further that it retained and employed  $R_{T-1}^e(A) = L_{T-1}^e(A)$  workers where  $R_{T-1}^e(A) = L_{T-1}^n(I)$ . Moreover, since  $R_{T-1}^e(A) = L_{T-1}^e(A) = L_{T-1}^n(I)$ ,  $m_T^e(A) = m_T^e(I)$  and hence  $L_T^e(A) = L_T^e(I)$  is feasible. Expected profits under this alternative regime are thus:

$$(B4) \quad E[\pi | A] = -m_{T-1}^e(V_{T-1}^e - Y_{T-1}^e) L_{T-1}^e(A)Y_{T-1} \\ + (L_{T-1}^e(A) - E(L_T^e(A)))Y_{T-1}\rho + P_{T-1}F(L_{T-1}^e(A)) \\ + E_{T-1}(P_T(L_T^e(A)))\rho$$

Since  $R_{T-1}^e(A) = L_{T-1}^e(A) = L_{T-1}^n(I)$ ,  $L_T^e(A) = L_T^e(I)$  and  $V_{T-1}^e < Y_{T-1}$ , comparing (A3) and (B4) reveals  $E[\pi | I] < E[\pi | A]$ . Hence, regime I cannot be optimal.

Now consider regime II. Under regime II, the optimal contract entails  $R_{T-1}^e(\text{II}) - L_{T-1}^e(\text{II}) > 0$ ,  $L_{T-1}^n(\text{II}) > 0$  only if  $m_{T-1}^e = R_{T-1}^e - L_{T-1}^e$ ,  $L_T^e(\text{II}) > 0$  and chosen so that  $P_T F'(L_T^e(\text{II})) = Y_T + \delta_T(\text{II})$ . Expected profits under regime II are given by:

$$\begin{aligned}
 \text{(B5)} \quad E[\pi | \text{II}] &= E_{T-1} [P_T(L_T^e(\text{II}))\rho - L_{T-1}^n(\text{II})V_{T-1}^n \\
 &+ [R_{T-1}^e(\text{II}) - L_{T-1}^e(\text{II}) + L_{T-1}^n(\text{II}) - E(L_T^e(\text{II}))]Y_{T-1}\rho \\
 &- \max[(R_{T-1}^e(\text{II}) - L_{T-1}^e(\text{II}))(Y_{T-1} - B_{T-1}), m_{T-1}^e V_{T-1}^e + \\
 &(R_{T-1}^e(\text{II}) - m_{T-1}^e)Y_{T-1} + (L_{T-1}^e(\text{II}) - R_{T-1}^e(\text{II}))B_{T-1}]
 \end{aligned}$$

It is easy to show that there is an alternative feasible contract that dominates the contract under regime II. For instance, consider the alternative in which  $R_{T-1}^e(\text{A}) = R_{T-1}^e(\text{II}) - L_{T-1}^e(\text{II})$ ,  $L_{T-1}^n(\text{A}) = L_{T-1}^n(\text{II})$ ,  $L_T^e(\text{A}) = L_T^e(\text{II})$ ,  $W_{T-1}^e(\text{A}) + V_T^e(\text{A})\rho > Y_{T-1}$  so that  $\gamma_{T-1}^e = 0$ , and  $L_{T-1}^e(\text{A}) > 0$  but such that  $P_{T-1} F'(L_{T-1}^e(\text{A})) > B_{T-1}$ . Expected profits under this alternative contract would be:

$$\begin{aligned}
 \text{(B6)} \quad E[\pi | \text{A}] &= P_{T-1} F(L_{T-1}^e(\text{A})) + E_{T-1} (P_T F(L_T^e(\text{A})))\rho \\
 &- L_{T-1}^n(\text{A})V_{T-1}^n + (R_{T-1}^e(\text{A}) - L_{T-1}^e(\text{A}) - E(L_T^e(\text{A})))Y_{T-1}\rho \\
 &- \max (R_{T-1}^e(\text{A}) (Y_{T-1} - B_{T-1}) + B_{T-1}L_{T-1}^e(\text{A})), \\
 &m_{T-1}^e V_{T-1}^e + (R_{T-1}^e(\text{A}) - m_{T-1}^e)Y_{T-1} + [L_{T-1}^e(\text{A}) - R_{T-1}^e(\text{A})]B_{T-1}
 \end{aligned}$$

Comparing (B5) and (B6) indicates that as long as  $P_{T-1} F'(L_{T-1}^e(\text{A})) > B_{T-1}$ ,

then  $E[\pi | A] > E[\pi | III]$ .

Consideration of regime III reveals a similar pattern. The optimal contract under regime III calls for  $R_{T-1}^e(III) > L_{T-1}^e(III) > 0$ ,  $L_{T-1}^n(III) > 0$  only if  $L_{T-1}^e(III) = m_{T-1}^e$ ,  $L_T^e(III) > 0$  where these decision variables are chosen so that:

$$(B7) \quad P_{T-1} F'(L_{T-1}^e(III)) = Y_{T-1} - Y_T \rho + \delta_{T-1}(III) - \delta_T(III) \rho$$

$$(B8) \quad P_T F'(L_T^e(III)) = Y_T + \delta_T(III)$$

$$(B9) \quad Z_{T-1} = P_T F'(L_T^e(III)) \rho - V_{T-1}^n < 0, \quad Z_{T-1} L_{T-1}^n(III) = 0$$

$$(B10) \quad \mu_i(III) = \delta_i(III) \quad \text{for } i = T-1, T.$$

$$(B11) \quad \delta_i(III)(m_i^e(III) - R_i^e(III)) = 0 \quad \text{for } i = T-1, T$$

$$(B12) \quad \mu_i(III)(R_i^e(III) - L_i^e(III)) = 0 \quad \text{for } i = t-1, T$$

Expected profits are given by:

$$E[\pi | III] = P_{T-1} F(L_{T-1}^e(III)) + E_{T-1} [P_T(L_{T-1}^e(III)) \rho - [L_{T-1}^n(III) V_{T-1}^n + Y_T (E(L_T^e(III)) - L_{T-1}^e(III) - L_{T-1}^n(III))] - \max [m_{T-1}^e (V_{T-1}^e - Y_{T1}^e) + L_{T-1}^e(III) Y_{T-1}, L_{T-1}^e(III) Y_{T-1}].$$

Now consider the following alternative to regime III. Let  $B_{T-1} + V^e(A) > 0$  so that  $\gamma_{T-1}^u = 0$ . Under this alternative regime, if  $P_{T-1} F'(L_{T-1}^e(III)) > B_{T-1}$ , then leaving the employment decisions unchanged from regime III will yield the same expected profits as regime III. However, if

$$P_{T-1} F'(L_{T-1}^e(III)) < B_{T-1}, \quad \text{then let } R_{T-1}^e(A) = L_{T-1}^e(III), \quad P_T F'(L_{T-1}^e(A)) =$$

$B_{T-1}$  and leave other employment decisions unchanged. This change implies a

revenue reduction of  $P_{T-1}(F(L_{T-1}^e(\text{III})) - F(L_{T-1}^e(\text{A})))$  and a labor cost reduction of  $B_{T-1}(L_{T-1}^e(\text{III}) - L_{T-1}^e(\text{A}))$  relative to regime III. Given the strict concavity of  $F$ , since  $P_{T-1}F'(L_{T-1}^e(\text{III})) < B_{T-1} = P_{T-1}F'(L_{T-1}^e(\text{A}))$  this implies that the revenue reduction is more than offset by the cost reduction. Hence, regime III is dominated by this alternative regime and thus cannot be optimal.

Hence, we have shown that regimes I, II, and III are all dominated by alternative regimes. Observe as well that the dominating alternative regime in all cases is essentially regime IV in which  $W_{T-1}^e + V_{T-1}^e \rho > Y_{T-1}$  and  $B_{T-1} + V_{T-1}^e \rho > Y_{T-1}$ . By similar arguments, it can be shown that  $W_i^e + V_{i+1}^e \rho > Y_i$  and  $B_i + V_{i+1}^e \rho > Y_i$  are optimal for  $t < i < T$ . The resulting optimality conditions are then (for  $t < i < T$ ):

$$(B13) \quad W_i^e + V_{i+1}^e \rho > Y_i$$

$$(B14) \quad B_i + V_{i+1}^e \rho > Y_i$$

$$(B15) \quad P_i F'(L_i^e) = B_i + \mu_i$$

$$(B16) \quad P_i F'(L_i^e) + E_i \left( \sum_{j=i+1}^T P_j F'(L_j^e) \rho^{j-i} \right) = Y_i + \delta_i$$

$$(B17) \quad Z_i = \left( \sum_{j=i+1}^T P_j F'(L_j^e) \rho^{j-i} - V_i^n \right) < 0, \quad Z_i \cdot L_i^n = 0$$

$$(B18) \quad \mu_i (R_i^e - L_i^e) = 0, \quad R_i^e > L_i^e$$

$$(B19) \quad \delta_i (m_i^e - R_i^e) = 0, \quad m_i^e > R_i^e$$

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