

A SIMPLE DURABLE GOODS MARKET*

by

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A SIMPLE DURABLE GOODS MODEL

ABSTRACT

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I examine the market for a durable which I call cars. The durability of cars has two implications. First, they can be stored in inventories by producers. Second, since they provide a stream of services to consumers, consumers may wish to defer purchases to take advantage of price fluctuations. I analyze a simple market with serially independent demand fluctuations. This market has a unique rational expectations equilibrium. When demand is unexpectedly high, price is high and some consumers defer purchases. When demand is unexpectedly low, price is low and producers hold inventories. Prices, output, inventories and notional supply are all serially independent. Sales, and the notional demand, however, exhibit a positive serial correlation. Intuitively, this is because large sales mean that demand is unexpectedly high, that as a result some consumers defer purchase, and that these consumers increase demand -- and sales -- next period.

1. INTRODUCTION

Since inventories appear empirically to play an important role in the business cycle considerable effort has been made to study the theoretical aspects of inventory fluctuations. Blinder and Fischer show how inventories can cause output to be serially correlated over time, even though demand shocks are not. Interest has also focused on "price stickiness" or the "insensitivity" of prices to demand shocks. Blinder [1981A, 1981B], Reagan [1982] and Reagan and Weitzman [1982] have all examined this issue.

While addressing the same set of issues this paper attempts to remedy a serious defect of previous inventory models. In all these models demand is taken to independent of past and anticipated future prices (although Blinder [1981A] does allow the demand innovations to have serial correlation). With a durable good this assumption makes little sense: typically not only is a durable storable for the firm (frozen orange juice) but also provides a relatively long stream of services to the consumer (cars). Thus the consumer must decide when to buy the durable, and anticipated future prices matter. The timing of consumer purchases appears to be an important part of the business cycle and is thus deserving of study in its own right. In this paper I study the simultaneous decisions of firms to hold inventories and consumers to defer purchases in a simple rational expectations model.

Except for the Reagan/Weitzman paper (and to some extent Blinder), the inventory literature has followed Zabel [1972] and focused on the inventory decisions of a single monopolistic producer. This is because at the firm level inventories in a competitive market are not well-behaved. However, there is no problem at the industry level, and thus I elect to study what is probably the more interesting case -- that of perfect competition. I also assume constant marginal cost of production, zero storage cost (except for

foregone interest) and serially independent demand shocks. There are frequent assumptions except in Blinder and the earlier Arrow/Karlin/Scarf literature.

In equilibrium I show that sales do have a positive serial correlation even though demand shocks do not. Interestingly this is because of consumer choice of time-of-purchase, not firm inventory holding. Unlike Reagan and Weitzman I find that the sensitivity of prices to demand shocks does not depend on the size of the shock to any great extent. However this is due largely to the fact that all consumers are identical in the model. Finally, I am able to show that the rational expectation equilibrium is pareto efficient by showing that it maximizes expected consumer utility.

2. THE MODEL

At time t cars sell for p_t . There are C_t cars available for sale (notional supply) and D_t consumers wishing to buy a car (notional demand). Excess demand is $X_t \equiv D_t - C_t$. Sales of cars are S_t . Borrowing and lending can take place in a market with discount factor $0 < \beta < 1$. Cars last forever and there is no resale market.

Cars are produced by a single risk neutral competitive producer who views both price and market inventories as outside his control. Output of cars at t is Y_t ; the cost of producing a car is c and is incurred at time $t-1$ when the production decision must be made. Thus, cars are produced at constant marginal cost with a one period lag. Inventories available for sale at t are I_t . Notional supply is just

$$(2-1) \quad C_t = Y_t + I_t$$

while inventories are determined by

$$(2-2) \quad I_t = C_{t-1} - S_{t-1}.$$

The firm begins in period zero with no cars. Demand begins one period later.

Each car buyer demands exactly one car, and after purchase keeps it forever. In period t ε_t new car buyers arrive at the market. These are i.i.d. nonnegative random variables with continuous c.d.f. $F(\varepsilon)$. The stock of potential car buyers is

$$(2-3) \quad D_t = D_{t-1} - S_{t-1} + \varepsilon_t.$$

Consumers are identical and risk neutral and receive a money equivalent utility of $u - p$ in the period they buy a car for price p ; they receive zero in all other periods.

A car produced in period $t-1$ costs c to produce and if consumed when available in period t yields a money equivalent utility of u . Discounting this back to $t-1$ shows that if the value of cars is to exceed their production cost, then

$$(2-4) \quad \beta u > c.$$

I shall always assume this to be the case.

A rational expectations competitive equilibrium of this market is characterized by

- (A) Rational Expectations - probability distribution perceived by agents are the same as those generated by the model.
- (B) Zero Profits - the firm has expected present value of zero.

- (C) Optimal Production - the firm believes that it can sell all it wants at the prevailing market price. Given this belief, it cannot profit by altering its production plan.
- (D) Voluntary Exchange - if the firm sells a car, it prefers this to selling later; if a buyer buys a car, he prefers this both to waiting and to not buying a car at all.
- (E) Feasibility - $0 < S_t < \min(D_t, C_t)$.

Let H_t be all information available to agents at the close of period t -- all current and past output, sales and especially notional supply and demand (which are presumed to be observable). In accordance with (A) all random variables conditioned on H_t have the distribution generated endogenously by the model.

3. THE EQUILIBRIUM THEOREM

The market described above has a unique rational expectations equilibrium. The equilibrium production plan is

$$(3-1) \quad Y_1 = \gamma \equiv F^{-1}(1-\lambda) \quad \text{where } \lambda \equiv c/\beta u$$

$$Y_t = \epsilon_{t-1} \quad t > 1.$$

Sales are determined by

$$(3-2) \quad S_t = \min(D_t, C_t).$$

and prices by

$$(3-3) \quad \text{If } \epsilon_t > \gamma \quad p_t = \bar{p} \equiv c + u(1-\beta)$$

$$\epsilon_t < \gamma \quad p_t = \underline{p} \equiv c.$$

Note that $\text{pr}(\epsilon_t = \gamma) = 0$ so that (3-3) almost surely determines prices.

As a preliminary to proving this theorem, I first develop some of its qualitative properties. The probability that the high price \bar{p} occurs is $1 - F^{-1}(\gamma) = \lambda$ by the definition of γ . Expected net period price is

$$(3-4) \quad E(p_{t+1} | H_t) = \lambda \bar{p} + (1-\lambda)p = c/\beta$$

which is the cost of producing a car this period in next period dollars. It follows from (3-4) that $E(p_{t+1} | H_{t-k}) = c/\beta \quad k > 1$.

The low price $\underline{p} = c$ is less than the production cost c/β . However, the firm is indifferent between selling at \underline{p} and selling next period for the expected price of c/β . The high price $\bar{p} = c + u(1-\beta)$ is, by virtue of (2-4), above production cost and leaves the consumer indifferent between buying now for a present value of $u - \bar{p}$ or next period for the expected present value of $\beta[u - (c/\beta)]$.

Finally, by a series of algebraic manipulations, using (3-2), (2-1) and (2-3) equilibrium excess demand is

$$(3-5) \quad X_t = \epsilon_t - \gamma.$$

Thus, when $p_t = \bar{p}$ the firm sells all its cars and some consumers wait to buy a car; when $p_t = \underline{p}$ all consumers buy a car and the firm holds inventories.

I now prove that the system of equations above does indeed define an equilibrium. I consider conditions (B) and (C) and (D). Condition (E) follows direction from (3-2). Later in the section I prove that this is the only equilibrium.

Define V_t to be the expected present value of the selling price of a car stocked at time t conditional on H_{t-1} .

LEMMA (3-1): $V_t = c/\beta$.

Proof: Let f_t be the probability that the car is sold now when \underline{p} occurs. Then

$$V_t = \lambda \bar{p} + (1-\lambda)f_t \underline{p} + (1-\lambda)(1-f_t)\beta V_{t+1}.$$

Algebraic manipulation of (3-6) making use of (3-4) shows that if

$Z_t \equiv \beta V_t - \underline{p}$ then

$$Z_{t+1} = [\beta(1-\lambda)(1-f_t)]^{-1} Z_t.$$

However, Z_t is bounded since $0 < V_t < \bar{p}$, and the only solution of (3-7) that remains bounded is $Z_t \equiv 0$. Since $Z_t = \beta V_t - \underline{p}$

$$V_t = \underline{p}/\beta = c/\beta.$$

QED

The lemma implies that the expected selling price of a car equals its cost of production. Thus the zero profit condition (B) holds and any production plan is profit maximizing given the perception that the firm can't control prices and can sell all it wants when \bar{p} occurs. Thus (C) holds. Finally, (D) holds for the firm. If $p_t = \bar{p}$ it sells all its cars and strictly prefers to sell them now. If $p_t = \underline{p}$ it is indifferent between selling now and waiting

and is willing to hold the required inventories. A similar argument shows that exchange is voluntary for the consumer.

This proves that the equations above do indeed give rise to an equilibrium. I now show that this is the only equilibrium.

Observe that

$$(3-6) \quad E(p_{t+1} | H_{t-k}) = c/\beta \quad k > 0$$

(provided there is a positive probability of sale in period $t+1$). For if $E(p_{t+1} | H_t) > c/\beta$ the firm will wish to produce infinitely many cars at t . Thus, $E(p_{t+1} | H_{t-k}) < c/\beta$. But if strict inequality holds with positive probability of a sale, the present value of the firm is negative: it never expects a profit on a sale, but sometimes expects a loss. Thus (3-6) holds.

Let U_t be the utility of (any) consumer in the market at time t . Let $\bar{U} \equiv \sup_{H_t} E(U_{t+1} | H_t)$. We can characterize \bar{U} by:

LEMMA (3-2): $\beta \bar{U} < u - \underline{p}$.

Proof: We examine $E(U_{t+1} | H_t)$. If H_t is such that C_{t+1} (current inventories plus production) is zero then $U_{t+1} = \beta E[U_{t+2} | H_{t+1}] < \beta \bar{U}$ implying $U_{t+1} < \bar{U}$. Thus \bar{U} is the supremum over H_t with $C_{t+1} > 0$. I claim that this implies

$$U_{t+1} < (u - p_{t+1}) + \max(0, \beta \bar{U} - u + \underline{p}).$$

There are two cases. If $p_{t+1} > \underline{p}$ the firm sells $C_{t+1} > 0$ cars and since consumers buy them voluntarily $U_{t+1} = u - p_{t+1}$. If $p_{t+1} < \underline{p}$ we can write

$$\begin{aligned}
U_{t+1} &< \max(u-p_{t+1}, \beta \bar{U}) \\
&= (u-p_{t+1}) + \max(0, \beta \bar{U} - u + p_{t+1}). \\
&< (u-p_{t+1}) + \max(0, \beta \bar{U} - u + \underline{p}).
\end{aligned}$$

Thus since $E[p_{t+1} | H_t] = \beta/c \bar{U} < (u-\beta/c) + \max(0, \beta \bar{U} - u + \underline{p})$. If $\beta \bar{U} > u - \underline{p}$ (the converse of what we wish to show) then since $\underline{p} = c$

$$\bar{U} < u - c/\beta + \beta \bar{U} - u + c = c - \beta/c + \beta \bar{U}$$

and $(1-\beta)\bar{U} < c - c/\beta < 0$ which contradicts $\beta \bar{U} > u - c > u - c/\beta > 0$.

Thus $\beta \bar{U} < u - \underline{p}$.

QED

Lemma (3-2) implies that all consumers buy cars if $p < \underline{p}$ and we already know that all cars held by the firm are sold if $p > \underline{p}$. We conclude that

$$(3-7) \quad S_t = \min(D_t, C_t).$$

If $X_t > 0$ and there is excess demand, the consumer must be indifferent between buying now and waiting. By virtue of (3-6) this implies $p_t = \bar{p}$. If $X_t < 0$ the firm must be indifferent between selling now and waiting — by (3-6) this implies $p_t = \underline{p}$. Also if λ is the probability of \bar{p} at time t conditional on $t-1$ information by (3-6)

$$(3-8) \quad \lambda \bar{p} + (1-\lambda)\underline{p} = c/\beta$$

which implies $\lambda = c/\beta u$.

It remains only to show that the optimal production plan is unique. Production at $t-1$ must be chosen so that $\text{pr}(X_t > 0) = \lambda$. A bit of algebraic manipulation then yields the production plan in (3-5).

This completes proof of the theorem.

One aspect of the equilibrium deserves note. Firms do not charge inventory carrying costs: the expected price equals production cost. However, the average price paid by a consumer exceeds expected price. This is because when price is high there are (typically) more consumers than when price is low. Since the firm has zero present value, the excess of average price paid by a consumer over expected price (equals production cost) must exactly compensate the firm for its costs of carrying inventories.

4. UNIVARIATE REPRESENTATION

How do the major variables vary over time? By algebraic manipulations each variable can be expressed as a function solely of the random innovations ε_t . Define $\varepsilon_0 \equiv \gamma$, then

$$\begin{aligned}
 (4-1) \quad X_t &= \varepsilon_t - \gamma \\
 &\quad \bar{p} \quad \varepsilon_t > \gamma \\
 p_t &= \quad \underline{p} \quad \varepsilon_t < \gamma \\
 \\
 Y_t &= \varepsilon_{t-1} \\
 I_t &= \gamma - \min(\varepsilon_t, \gamma) \\
 C_t &= \max(\varepsilon_{t-1}, \gamma)
 \end{aligned}$$

$$(4-2) \quad D_t - D_{t-1} = (\varepsilon_t - \gamma) - \min(\varepsilon_{t-1}, \gamma) - \max(\varepsilon_{t-2}, \gamma)$$

$$S_t = \min(\varepsilon_t, \gamma) + \max(\varepsilon_{t-1}, \gamma) - \gamma$$

The variables in (4-1) -- X_t , p_t , Y_t , I_t and C_t -- are all serially independent. The variables in (4-2) -- D_t and S_t -- are not. Since S_t and S_{t-1} are both increasing functions of ε_{t-1} it is straightforward to show that $\text{cov}(S_t, S_{t-1}) > 0$. Thus sales follow an MA(1) process with positive serial correlation, although demand shocks (and prices, production and inventories) are serially independent.

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