OPTIMAL PRICING GIVEN TRANSACTION COSTS:

THE CASE OF RESERVE VERSUS GENERAL ADMISSION SEATING

by

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Abstract

This paper analyzes the maximization problem faced by a theater or sports stadium owner who must decide between reserve and general admission seating, where there are additional transaction costs associated with reserve seating. The results follow. First, an exogenous increase in the valuation consumers place on seat quality tends to increase the number of reserve seats. Second, an exogenous increase in the transaction costs associated with reserve seating tends to decrease the number of reserve seats. Third, with identical consumers, the seller offers the socially optimal number of reserve seats. Fourth, when consumers vary with respect to their time valuations, the seller will frequently offer a higher than socially optimal number of reserve seats.

A frequent objective of analyses of firm behavior has been the derivation of optimal or profit maximizing pricing schemes (see e.g., Mussa and Rosen 1978, Harris and Raviv 1981, and Chiang and Spatt 1982). In deriving these schemes, however, economists have in the main ignored an important aspect of the problem by failing to take explicit account of transaction costs. That is, since transaction costs often vary with the type of pricing scheme employed, the explicit modeling of such costs is necessary for a full understanding of real world pricing schemes. In this paper I attempt to take explicit account of transaction costs inside an analysis of a real world situation wherein such costs have a critical effect on the type of pricing scheme employed. The objective of the analysis is threefold. First, I would like to provide an explanation for the existence of pricing schemes which seem hard to justify in the absence of transaction costs. Second, I would like to see how the choice of pricing scheme depends on the different parameters of the model. Third and most importantly, I would like to investigate the social welfare aspects of the problem. That is, do firms always adopt the pricing scheme which maximizes social welfare, and if not, what is the nature of the biases which exist.[1]

Consider a theater or sports stadium owner who is trying to decide on what type of pricing scheme to employ for an upcoming event. On the one extreme he could treat each seat as a separate commodity, and have each ticket sold correspond to a particular seat. This will be referred to as a pure reserve seating system. At the other extreme he could sell tickets which guarantee access to a seat, but which do not specify a particular seat. This will be referred to as a pure general admission system. Finally, he could choose some intermediate system wherein some seats are sold as reserve seats, while others are sold through a general admission system. Since a pure reserve seating system would allow the owner to take full advantage of quality differentials which exist between seats, in the absence of transaction cost considerations it is hard to justify anything but a pure reserve seating system. Ignoring transaction cost considerations is misleading, however, since reserve seating typically entails more transaction costs than general admission seating. For example, reserve seating requires additional ushers both to help customers find their seats, and to ensure that no individuals attempt to occupy seats other than those they have purchased tickets for. Thus, to fully understand a theater or stadium owner's pricing scheme choice, it is necessary to take account of the transaction costs involved.

This paper constructs and analyzes a model of the multiple seat pricing problem discussed above, which explicitly captures the additional transaction costs associated with reserve seating. The model is analyzed under two different assumptions concerning consumers. It is first assumed that all consumers are identical. For this type of world the analysis suggests the following. First, an exogenous increase in the valuation consumers place on seat quality will tend to increase the number of seats offered as reserve seats. Second, an exogenous increase in the transaction costs associated with reserve seating will tend to decrease the number of seats offered as reserve seats. Third and most importantly, from a social welfare point of view, the private incentive firms face for adopting reserve seating will always be at an optimal level. That is, the number of seats offered as reserve seats will always be the socially optimal number.

The second assumption under which the model is analyzed is that consumers differ, but only with respect to the valuations they place on time. This part of the analysis reaches two basic conclusions. First, the fact that consumers differ with respect to the valuations they place on time has no effect on the type of pricing scheme employed, and therefore the first two results of the previous case continue to hold. Second, the fact that individuals differ in this manner does have an effect on the social welfare aspects of the problem. In particular, it is demonstrated that, given two restrictions on the quality distribution of seats, the number of seats offered as reserve seats tends to be greater than the socially optimal number. However, an example is also presented which demonstrates that, when these two restrictions are not satisfied, it is possible for the number of seats offered as reserve seats to be smaller than the socially optimal number.

The outline for the paper is as follows. Section I presents a model of the multiple seat pricing problem. Section II analyzes this model under the assumption that all consumers are identical. Section III analyzes the model under the assumption that consumers differ, but only with respect to the valuations they place on time. Section IV presents some concluding remarks.

I. The Model

Individuals in this model derive utility from two activities. The first activity is the consumption of a good which is provided via a perfectly competitive market. This good is referred to as B, where b_i denotes the number of units of B consumed by individual i. It is assumed that good B is perfectly divisible, and that it exhibits constant marginal utility. The constant marginal utility assumption simplifies the analysis, while leaving the qualitative nature of the results unchanged. The second activity consists of being a spectator, or in other words occupying a seat, at what for simplicity of exposition is referred to as a sports event. Spectating is obviously an indivisible consumption activity, i.e., each individual can occupy either no seats or one seat. The utility derived from spectating depends both on the quality of the seat occupied, and on the time expended acquiring that seat. The quality of the seat occupied by individual i is denoted Q_i, while the time expended acquiring seat Q_i is denoted t_i.

The following utility function which incorporates the properties just discussed is adopted.

$$(1) \qquad U_i = b_i + vQ_i - c_i t_i,$$

where c_i denotes individual i's valuation for time and where all individuals have the same valuation for seat quality, this valuation being denoted as v. The interpretation of (1) is straightforward. b_i represents the utility derived by individual i from the consumption of good B. vQ_i represents the utility directly derived by individual i from the spectating activity.[2] $c_i t_i$ represents the disutility incurred by individual i, which is due to the time expended acquiring seat Q_i. Note finally, if individual i decides not to attend the sports event, $Q_i = 0$.

Each individual i also faces a budget constraint, which is written below with B being the numeraire good.

$$(2) \qquad b_i + e_i < Y$$

It is being assumed in (2) that each individual i has the same income, this income being denoted simply as Y. Additionally, e_i denotes the price paid for a ticket to the sports event by individual i. Now, it is easily demonstrated that (2) must hold as an equality, and therefore the budget constraint can be substituted directly into the utility function, i.e.,

(3) $$U_i = Y - e_i + vQ_i - c_i t_i.$$

The only aspect of the model which remains to be described is the market for seats at the sports event. All such seats are controlled by a single economic agent, i.e., the seats are monopolistically provided. Furthermore, as indicated previously these seats are assumed to vary in terms of quality. In particular, the distribution of seat qualities is described by a density $g(Q)dQ$ defined on the interval $[Q^L, Q^H]$. Note, also, the function $g(.)$ is assumed to be nonzero and finite everywhere in the specified interval. Now, there are three different ways that the monopolist can sell the seats. First, he could have each ticket sold correspond to a particular seat. This will be referred to as a pure reserve seating system. Second, he could sell tickets which guarantee access to a seat, but which do not specify a particular seat. This will be referred to as a pure general admission system. Third, he could choose some intermediate system wherein some seats are sold as reserve seats, while the rest are sold through a general admission system. When seats are sold as reserve seats, the monopolist incurs costs which are not incurred when the seats are sold through a general admission system. This is due to the additional transaction costs associated with reserve seating. In particular, equation (4) describes the reserve seating costs, denoted Z, incurred when all seats between the quality levels Q_1 and Q_2 are sold as reserve seats.[3]

(4) $$Z = \int_{Q_1}^{Q_2} zg(Q)dQ$$

It is now necessary to specify how seats are allocated when they are sold under a general admission system. The assumption made is that the seats are allocated according to the order in which individuals arrive. Specifically, first arrivals get first choice of seats, while later arrivals must choose from among the seats which have not already been claimed.

Finally, two restrictions are placed on the type of selling strategy the monopolist is allowed to employ. The first restriction is that, if the monopolist offers general admission seats, then the seats offered must comprise a closed compact interval of the quality distribution of seats. This restriction guarantees that a set of equilibrium strategies exist for the consumers, and that an optimal general admission price exists for the monopolist. The second restriction concerns the following problem. Given that there is no monitoring of general admission seats, i.e., no ushers, an individual could purchase a reserve seat ticket and sit in a general admission seat. To preclude this possibility the monopolist is restricted from employing a strategy wherein a reserve seat purchaser prefers to sit in a general admission seat. In other words, it must be incentive compatible for each reserve seat purchaser to sit in the seat purchased.

II. Analysis with Identical Time Valuations

In this section the model presented in the previous section is analyzed under the assumption that all consumers have the same valuation for time, i.e., $c_i = c$ for all i. Before proceeding with the analysis, however, it is necessary to clearly spell out the timing of market activity. To make the analysis simple to follow, it is assumed that purchasing a ticket and arriving at the stadium are two distinct activities. More specifically, the timing of market activity is assumed to consist of the following two stages. First, at some unspecified time before the sports event begins, the monopolist sells tickets to the event. Second, individuals who purchase tickets during the first stage arrive at the sports event, either arriving just as the event begins or arriving some point earlier. Given this specification for the timing of market activity, it is easy to see that only individuals who purchase general admission tickets will ever show up early. The reason such individuals might show up early is that, as indicated previously, they can improve the quality of their seats by doing so. Individuals with reserve seat tickets would never show up early because each reserve seat ticket specifies a particular seat.

An alternative specification for the timing of market activity would be to have consumers purchase tickets when they arrive at the stadium. Under the additional assumption that the monopolist cannot have the price for general admission tickets depend on arrival times because of high transaction costs, this alternative specification yields exactly the same results as the specification actually employed. As is indicated above, the advantage of the specification employed is that proofs are less complex and are easier to follow.

It is now possible to proceed with the analysis. The problem faced by the monopolist is threefold. He must decide which seats to sell as reserve seats, and which seats to sell as general admission seats. He must decide upon a price schedule for reserve seats. He must decide upon a price for general admission seats.

One thing can be immediately derived about the monopolist's optimal strategy. That is, if a seat of quality Q_j is sold as a reserve seat, then the price of that seat is vQ_j. This is due to the fact that when a seat is sold as a reserve seat, there is nothing to stop the monopolist from capturing all the possible rents associated with that seat. The rest of the analysis consists of a sequence of propositions. Note, in the following P will denote the price for general admission seats.

Proposition II.1: If the monopolist offers all seats between the quality levels Q^+ and Q^* as general admission seats, where $Q^* > Q^+$, then the following will be true.

1) $vQ^+ < P < vQ^*$
2) No seat of quality less than Q^+ will be offered as a reserve seat.
3) Q^+ and Q^* will necessarily be such that the monopolist will want to sell all higher quality seats as reserve seats.
4) A general admission seat will be occupied if and only if P is less than or equal to v times the quality of the seat.
5) Each individual i who purchases a general admission ticket will have his arrival time such that $P + ct_i = vQ_i$.

Proof: See Appendix.

Using the results contained in Proposition II.1, it is possible to set up the monopolist's maximization problem. Note, Q^* will continue to denote the highest quality seat which the monopolist offers as a general admission seat.

(5a) $$\max_{Q^*,P} \int_{P/v}^{Q^*} Pg(Q)dQ + \int_{Q^*}^{Q^H} vQg(Q)dQ - \int_{Q^*}^{Q^H} zg(Q)dQ$$

(5a) can be explained as follows. $\int_{P/v}^{Q^*} Pg(Q)dQ$ is the revenue from general admission seats, where P/v is the quality of the lowest quality general admission seat occupied and Q^* is the quality of the highest. $\int_{Q^*}^{Q^H} vQg(Q)dQ$

is the revenue from reserve seats, where each reserve seat's price equals v times the quality of the seat. $\int_{Q^*}^{Q^H} zg(Q)dQ$ is the cost of making all the seats between the quality levels Q^* and Q^H reserve seats (see equation (4)).

Denote as Q' the quality of the lowest quality general admission seat occupied, i.e., $Q' = P/v$. Instead of having the monopolist maximize over P, one could equivalently have the monopolist maximize over Q'. This substitution reduces (5a) to (5b).

$$\text{(5b)} \qquad \max_{Q^*,Q'} \int_{Q'}^{Q^*} vQ'g(Q)dQ + \int_{Q^*}^{Q^H} vQg(Q)dQ - \int_{Q^*}^{Q^H} zg(Q)dQ$$

(5b) yields (6) and (7) as first order conditions.

$$\text{(6)} \qquad (vQ'-vQ^*+z)g(Q^*) = 0$$

$$\text{(7)} \qquad \int_{Q'}^{Q^*} vg(Q)dQ - vQ'g(Q') = 0$$

Note, also, (5b) can be written as (8).

$$\text{(8)} \qquad \max_{Q^*} \int_{Q'}^{Q^*} vQ'g(Q)dQ + \int_{Q^*}^{Q^H} vQg(Q)dQ - \int_{Q^*}^{Q^H} zg(Q)dQ$$

$$\text{s.t. } Q' = \arg\max_{\hat{Q}} \int_{\hat{Q}}^{Q^*} v\hat{Q}g(Q)dQ$$

The specification in (8) is convenient for some of the remaining proofs. Now, with these equations in hand, the analysis can proceed to the remaining propositions.

Proposition II.2: Starting from a parameterization for which the monopolist does not choose a pure general admission system, i.e., $Q^* < Q^H$, an increase in v will necessarily result in a decrease in Q^*.[4]

Proof: I will first demonstrate that such an increase in v cannot result in Q^* being unchanged, and I will then demonstrate that such an increase cannot result in Q^* rising. Consider initially the constraint contained in (8). The value for v never affects the value of Q' chosen. Now, suppose an increase in v, from a parameterization where $Q^* < Q^H$, could result in Q^* being unchanged. Given that v has no effect on how Q' is dependent on Q^*, it should then be possible for the partial derivative of the left hand side of (6) with respect to v to be equal to zero. This derivative, however, equals $(Q'-Q^*)g(Q^*)$, which (6) itself implies is negative. Thus, such an increase in v cannot leave Q^* unchanged.

Now, let Q_1', Q_1^* be the values for Q' and Q^* chosen when $v = v_1$, where v_1 is such that $Q_1^* < Q^H$. Also, let Q_2', Q_2^* be the values for Q' and Q^* chosen when $v = v_2$, where $v_2 > v_1$. Given the definition of a maximum, (5b) implies both (9) and (10).

$$(9) \qquad v_1\left[\int_{Q_1'}^{Q_1^*} Q_1' g(Q)dQ + \int_{Q_1^*}^{Q^H} Qg(Q)dQ\right] - \int_{Q_1^*}^{Q^H} zg(Q)dQ$$

$$> v_1\left[\int_{Q_2'}^{Q_2^*} Q_2' g(Q)dQ + \int_{Q_2^*}^{Q^H} Qg(Q)dQ\right] - \int_{Q_2^*}^{Q^H} zg(Q)dQ$$

$$(10) \qquad v_2[\int_{Q_1'}^{Q_1^*} Q_1' g(Q)dQ + \int_{Q_1^*}^{Q^H} Qg(Q)dQ] - \int_{Q_1^*}^{Q^H} zg(Q)dQ$$

$$< v_2[\int_{Q_2'}^{Q_2^*} Q_2' g(Q)dQ + \int_{Q_2^*}^{Q^H} Qg(Q)dQ] - \int_{Q_2^*}^{Q^H} zg(Q)dQ$$

(10) can be rewritten as,

$$(11) \quad v_1[\int_{Q_1'}^{Q_1^*} Q_1' g(Q)dQ+\int_{Q_1^*}^{Q^H} Qg(Q)dQ]-\int_{Q_1^*}^{Q^H} zg(Q)dQ+(v_2-v_1)[\int_{Q_1'}^{Q_1^*} Q_1' g(Q)dQ+\int_{Q_1^*}^{Q^H} Qg(Q)dQ]$$

$$< v_1[\int_{Q_2'}^{Q_2^*} Q_2' g(Q)dQ+\int_{Q_2^*}^{Q^H} Qg(Q)dQ]-\int_{Q_2^*}^{Q^H} zg(Q)dQ+(v_2-v_1)[\int_{Q_2'}^{Q_2^*} Q_2' g(Q)dQ+\int_{Q_2^*}^{Q^H} Qg(Q)dQ].$$

If $Q_2^* > Q_1^*$, then $[\int_{Q_1'}^{Q_1^*} Q_1' g(Q)dQ + \int_{Q_1^*}^{Q^H} Qg(Q)dQ] > [\int_{Q_2'}^{Q_2^*} Q_2' g(Q)dQ + \int_{Q_2^*}^{Q^H} Qg(Q)dQ]$.

Furthermore, this would imply that (9) and (11) are contradictory, and therefore it must be true that $Q_2^* \not> Q_1^*$. Q.E.D.

Proposition II.3: Starting from a parameterization for which $Q^* < Q^H$, an increase in z will necessarily result in an increase in Q^*.

Proof: This proof follows along the same lines as the proof of Proposition II.2, and is therefore omitted.

The intuition for the above results is straightforward. An increase in the valuation consumers place on seat quality increases the private incentive for adopting reserve seating, thus resulting in an increase in the number of reserve seats — Proposition II.2. On the other hand, an increase in the transaction costs associated with reserve seating decreases the private incentive for adopting reserve seating, thus resulting in a decrease in the number of reserve seats — Proposition II.3. I will now consider the social welfare aspects of the problem.

It is obvious that the situation described above is not a first best result. For example, it is easy to construct an alternative set of arrival times for general admission ticket purchasers which pareto dominates the arrival times which emerge endogenously.[5] A more interesting question is whether the private incentive the monopolist faces for adopting reserve seating is always at a socially optimal level. I will investigate this question by seeing whether social welfare could be improved by letting the government set Q^*. Note, social welfare is defined here simply as monopoly profits plus the sum of the utilities of all the consumers in the economy.

Proposition II.4: Social welfare could not be improved by letting the government set Q^*.

Proof: I will prove the proposition by demonstrating that the maximization problem faced by such a government is exactly the same as the maximization problem faced by the monopolist. The maximization problem faced by such a government is,

$$(12)\quad \max_{Q^*} \int_{Q'}^{Q^*} vQ'g(Q)dQ + \int_{Q^*}^{Q^H} vQg(Q)dQ - \int_{Q^*}^{Q^H} zg(Q)dQ + \int_{Q'}^{Q^*} (vQ-P-ct(Q))g(Q)dQ$$

$$\text{s.t. } Q' = \arg\max_{\hat{Q}} \int_{\hat{Q}}^{Q^*} v\hat{Q}g(Q)dQ,$$

where $t(Q)$ is the waiting time of the individual whose seat is of quality

Q. (12) can be explained as follows. The first three terms of the objective function represent monopoly profits, while $\int_{Q'}^{Q^*} (vQ-P-ct(Q))g(Q)dQ$ is the consumer surplus general admission ticket purchasers derive from the spectating activity. Additionally, the constraint simply states that, given the government's choice of Q^*, the monopolist will maximize over Q'. Note, given the price schedule for reserve seats, it is clear that reserve seat purchasers derive no consumer surplus from the spectating activity.

Proposition II.1 states that for each general admission seat, $P + ct(Q) = vQ$. Substituting this into (12) yields,

$$(13) \qquad \max_{Q^*} \int_{Q'}^{Q^*} vQ'g(Q)dQ + \int_{Q^*}^{Q^H} vQg(Q)dQ - \int_{Q^*}^{Q^H} zg(Q)dQ$$

$$\text{s.t. } Q' = \arg\max_{\hat{Q}} \int_{\hat{Q}}^{Q^*} v\hat{Q}g(Q)dQ.$$

Equations (13) and (8) are exactly equivalent. Q.E.D.

The intuition for this result comes straight out of the mathematics of the proof. Because the price of each reserve seat equals v times the quality of the seat, reserve seat ticket purchasers get no consumer surplus from the spectating activity. On the other hand, because arrival times are such that for each general admission seat, $P + ct(Q) = vQ$, general admission ticket purchasers also get no consumer surplus from the spectating activity. Together these two facts imply that as Q^* is varied, changes in social welfare will translate one for one into changes in monopoly profits. Furthermore, this in turn implies that the monopolist will choose the value for Q^* which maximizes social welfare.[6]

III. Analysis with Different Time Valuations

In this section the model presented in Section I is analyzed under the assumption that individuals vary with respect to their valuations for time. In particular, the distribution of time valuations in the population is described by a density $h(c)dc$ defined on the interval $[\underline{c},\bar{c}]$. Note, also, the function $h(.)$ is assumed to be nonzero and finite everywhere in the specified interval.[7]

As in Section II, one thing can be immediately derived about the monopolist's optimal strategy. That is, it is again the case that if a seat of quality Q_j is sold as a reserve seat, then the price of that seat is vQ_j. The next step in the analysis is to present a proposition which is analogous to Proposition II.1.

Proposition III.1: If the monopolist offers all seats between the quality levels Q^+ and Q^* as general admission seats, where $Q^* > Q^+$, then the following will be true.

1) $vQ^+ < P < vQ^*$
2) No seat of quality less than Q^+ will be offered as a reserve seat.
3) Q^+ and Q^* will necessarily be such that the monopolist will want to sell all higher quality seats as reserve seats.
4) A general admission seat will be occupied if and only if P is less than or equal to v times the quality of the seat.
5) Denote again as Q' the quality of the lowest quality general admission seat occupied, i.e., $Q' = P/v$, and let $c(Q)$ be the time valuation of the individual whose seat is of quality Q. It is necessarily true that each individual i who purchases a general admission ticket will have his arrival time such that

$$t_i = \int_{Q'}^{Q_i} \frac{v}{c(Q)}\, dQ, \quad \text{where} \quad c(Q) \quad \text{is always such that}$$

$$\int_{Q}^{Q^*} g(Q_j)dQ_j = \int_{\underline{c}}^{c(Q)} h(c)dc.$$

Proof: See Appendix.

Given the results contained in Proposition III.1, it is a trivial matter to set up the monopolist's maximization problem. That is, Propositions II.1 and III.1 are exactly the same except for the specification of arrival times, and arrival times do not enter into the monopolist's maximization problem. Thus, the monopolist in this section faces the same maximization problem as he faced in the previous section, i.e., equation (5b) continues to be the relevant equation. Furthermore, given that this is true, it must also be the case that Propositions II.2 and II.3 continue to be valid. In other words, an exogenous increase in the valuation consumers place on seat quality still tends to increase the number of seats offered as reserve seats, while an exogenous increase in the transaction costs associated with reserve seating still tends to decrease the number of seats offered as reserve seats.

As opposed to the conclusions reached above, the social welfare implications of this section are not the same as those of the previous section. In the previous section the monopolist always offered the socially optimal number of reserve seats. I will now demonstrate that, given two restrictions on the density function $g(.)$, the monopolist here tends to offer more reserve seats than is socially optimal.[8] Note, in the following Q_M^* will denote the Q^* selected when the choice of Q^* is in the hands of the monopolist, and Q_G^* will denote the Q^* selected when the choice is in the hands of a social welfare maximizing government.

Proposition III.2: If the following two restrictions on the density function $g(.)$ are satisfied and $Q_M^* < Q^H$, then $Q_G^* > Q_M^*$.

1) $g(Q_2) \geq g(Q_1)$ for any Q_1, Q_2 pair, where $Q^L \leq Q_1 < Q_2 \leq Q^H$

2) $g(Q^H) \leq 2g(Q^L)$

Proof: See Appendix.

The intuition for this result is as follows. As opposed to what was true for general admission ticket purchasers in Section II, general admission ticket purchasers here do derive consumer surplus from the spectating activity. That is, for each consumer i who purchases a general admission ticket, it is the case that $P + c_i t_i < vQ_i$.[9] Now, as the monopolist increases the number of reserve seats, given restrictions 1) and 2), he will also necessarily decrease the number of general admission ticket purchasers. This suggests that as Q^* is varied in a negative direction, there will be a negative change in social welfare which the monopolist will not internalize. Furthermore, this in turn implies that the monopolist will tend to set Q^* at a value which is below what would be socially optimal.[10]

Proposition III.2 states that, when two restrictions on the density function $g(.)$ are satisfied, the monopolist tends to have more than the socially optimal number of reserve seats. Now consider the following example. Let $v = 1$, $z = 10$, $Q^L = 0$, $Q^H = 25$, $g(Q) = 1$ for all $0 \leq Q \leq 21$, and $g(Q) = 3$ for all $21 < Q \leq 25$. Also, let time valuations be distributed such that there is a mass point at $c = 1$ with weight 11, and the rest of the population has a time valuation equal to 10. In the Appendix it is demonstrated that for this example, $Q_G^* < Q_M^*$. This tells us that, when restrictions 1) and 2) are not satisfied, it is possible for the number of seats offered as reserve seats to be less than the socially optimal number. The intuition for what is going on in this example is roughly the following. For each consumer i who purchases a general admission ticket, the value of $(Q_i - Q')$ will tend to be positively related with the amount of consumer surplus this individual derives from the spectating activity. When restrictions 1) and 2) are satisfied, each consumer i who purchases a general admission ticket will have $(Q_i - Q')$ be a nondecreasing function of Q^*. Thus, this factor serves as an additional rationale for why, when 1) and 2) are satisfied, the monopolist has too high an incentive for adopting reserve seating. In the above example, however, restriction 2) is violated. The result is that over a range of Q^*, $(Q_i - Q')$ tends to be a decreasing function of Q^*. Furthermore, for the above example this effect dominates, the result being that $Q^*_G < Q^*_M$.[11]

IV. Conclusion

Most studies concerned with deriving profit maximizing pricing schemes abstract away from the existence of transaction costs. Given that transaction costs often vary with the type of pricing scheme employed, however, such abstractions can easily lead to incorrect conclusions concerning the form of real world pricing schemes. In this paper I attempted to take explicit account of transaction costs inside an analysis of a real world situation wherein such costs have a critical effect on the type of pricing scheme employed. In particular, I analyzed the maximization problem faced by a theater or sports stadium owner who must decide between reserve and general admission seating, where there are additonal transaction costs associated with reserve seating. The major results of the analysis are as follows. First, an exogenous increase in the valuation consumers placed on seat quality tended to increase the number of seats offered as reserve seats. Second, an exogenous increase in the transaction costs associated with reserve seating tended to decrease the number of seats offered as reserve seats. Third, under the assumption that consumers are perfectly identical, the seller always offered the socially optimal number of reserve seats. Fourth, when consumers were allowed to vary with respect to the valuations they placed on time, but the quality distribution of seats satisfied two restrictions, the seller tended to offer a higher than socially optimal number of reserve seats. Fifth, when these two restrictions were not satisfied, it was possible for the number of seats offered as reserve seats to be smaller than the socially optimal number.

There are a number of different ways that the analysis contained in this paper could be extended. Three particular examples come to mind. First, consumers could be allowed to vary in terms of their valuations on seat quality. With this type of variation the seller would need to be concerned with self-selection constraints, an issue central to of a number of recent papers (e.g., Mussa and Rosen 1978, Spence 1978, Chiang and Spatt 1982, and Maskin and Riley 1982). The solution technique used in these previous papers, however, is not necessarily applicable to the model contained in this paper. The reason for this is that all of the previous papers on the subject have had endogenously specified product characteristic distributions, while this paper has an exogenously specified distribution. Such an extension is therefore of interest both in terms of seeing how the results of this paper relate to a world where individuals vary with respect to their quality valuations, and in terms of finding a solution technique for the case where the product characteristic distribution is specified exogenously. Second, in this paper the seller did not have the ability to vary performance quality. For some situations this is probably a realistic way of modeling the seller's problem;

however, in other situations choosing an optimal performance quality would seem to be one of the key decisions the seller must make. Thus, a second worthwhile extension might be to make performance quality a choice variable for the seller. Third, this paper investigated how and when a government could improve social welfare by regulating the number of reserve seats. In reality, however, directly regulating the number of reserve seats would seem to be beyond the scope of feasible government activity. On the other hand, there does exist one government activity which could be used to affect the mix of reserve and general admission seats. That is, the government could affect this mix by differentially taxing reserve and general admission seats. A third worthwhile extension, therefore, might be to incorporate sales taxes into the model, and then derive optimal tax rates for the two types of seating.

Appendix

Proof of Proposition II.1: Suppose the number of general admission tickets purchased is equal to the number of seats between the quality levels $\hat{Q}$ and Q^*, where $\hat{Q} > Q^+$. Since general admission ticket purchasers claim the best general admission seat available upon arriving at the stadium, this implies all seats between the quality levels $\hat{Q}$ and Q^* will be occupied. Furthermore, it is also the case that this situation has associated with it a unique Nash equilibrium in arrival times, where this Nash equilibrium is defined by the following two properties. First, if individuals i and j are general admission ticket purchasers, then $c(t_i - t_j) = v(Q_i - Q_j)$. If this were not the case, then one of the individuals could make himself better off by showing up the instant before the other individual shows up. Second, the last general admission ticket purchaser who arrives must have his waiting time equal zero. An earlier arrival could not be part of an equilibrium because then the individual could make himself better off by showing up later and claiming the same seat.

It is now a fairly straightforward exercise to prove Proposition II.1. First, if $vQ^+ < P < vQ^*$, then the above implies that the Nash equilibrium in purchase strategies will be such that P equals v times the quality of the lowest quality general admission seat occupied. Second, given this, any price outside this interval is necessarily dominated by a price inside the interval. These two points in combination with the points made in the first paragraph prove 1), 4) and 5). Now, if the monopolist offered a seat of quality lower than Q^+ as a reserve seat, then the specification of arrival times implies that a purchaser of such a seat would prefer to claim a general admission seat. This proves 2) since the monopolist was a priori restricted from employing such a strategy. Finally, consider a case where seats immediately above Q^* are not offered as reserve seats. Given what has already been proven, the monopolist could make himself better off by raising Q^*. Therefore, the monopolist must find it optimal to offer the seats immediately above Q^* as reserve seats. Furthermore, given that the monopolist finds it optimal to offer these seats as reserve seats, he must find it optimal to offer even higher quality seats as reserve seats. This proves 3). Q.E.D.

Proof of Proposition III.1: Suppose the number of general admission tickets purchased is equal to the number of seats between the quality levels $\hat{Q}$ and Q^*, where $\hat{Q} > Q^+$. Since general admission ticket purchasers claim the best general admission seat available upon arriving at the stadium, this implies all seats between the quality levels $\hat{Q}$ and Q^* will be occupied. Additionally, since only general admission ticket purchasers have early arrival times, the set of general admission ticket purchasers must be comprised of the individuals in the economy with the lowest time valuations. Finally, it is also the case that this situation has associated with it a unique Nash equilibrium in arrival times, where the equilibrium is defined by the following three properties. First, if individuals i and j are general admission ticket purchasers where $c_i > c_j$, then $Q_i < Q_j$. If this were not the case, then one of the individuals could make himself better off by showing up the instant before the other individual shows up. Notice, this means that the time valuation of the individual who occupies a seat of quality Q, denoted $c(Q)$, is defined by the following: $\int_{Q}^{Q^*} g(Q_j)dQ_j = \int_{\underline{c}}^{c(Q)} h(c)dc.$

Second, let $t(Q)$ specify waiting time as a function of the quality of seat acquired. For the individual who acquires seat Q to find his arrival time optimal, it must be that $\frac{dt(Q)}{dQ} = \frac{v}{c(Q)}$. If the derivative had a lower value, then the individual acquiring seat Q would prefer to arrive earlier and get a higher quality seat. If the derivative had a higher value, then the individual acquiring seat Q would prefer to arrive later and get a lower quality seat. Third, the last individual who arrives must have his waiting time equal zero.

The proof now follows along the same lines as the proof of Proposition II.1. Note, if $\frac{dt(Q)}{dQ} = \frac{v}{c(Q)}$ and $t(Q') = 0$, then $t(Q) = \int_{Q'}^{Q} \frac{v}{c(Q_j)} dQ_j$.

Proof of Proposition III.2: The government's maximization problem can be written as,

$$\max_{Q^*} \int_{Q'}^{Q^*} vQ'g(Q)dQ + \int_{Q^*}^{Q^H} vQg(Q)dQ - \int_{Q^*}^{Q^H} zg(Q)dQ + J \tag{14}$$

$$\text{s.t.} \quad Q' = \arg\max_{\hat{Q}} \int_{\hat{Q}}^{Q^*} v\hat{Q}g(Q)dQ,$$

where J is the consumer surplus general admission ticket purchasers derive from the spectating activity. The first order condition for the maximization problem in the constraint is,

$$\int_{Q'}^{Q^*} vg(Q)dQ - vQ'g(Q') = 0. \tag{15}$$

Given restrictions 1) and 2), (15) yields $\frac{dQ'}{dQ^*} < 1$. This combined with 1) yields that as Q^* is increased, the number of general admission ticket purchasers also increases. This in turn implies that, if as Q^* is increased each consumer i who was initially a general admission ticket purchaser has his consumer surplus from spectating increase, then $\frac{dJ}{dQ^*} > 0$.

Now, given the specification of arrival times in Proposition III.1, the time valuation of a consumer who sits in a general admission seat of quality Q can be written as a function $\tilde{c}(\int_Q^{Q^*} g(\hat{Q})d\hat{Q})$, where $\tilde{c}' > 0$. Thus, again given the specification of arrival times in Proposition III.1, the consumer surplus for such an individual i, denoted CS_i, can be written as,

$$(16) \qquad CS_i = \int_{Q'}^{Q_i} v[1 - \frac{c_i}{\tilde{c}(\int_{Q_j}^{Q^*} g(\hat{Q})d\hat{Q})}] \, dQ_j .$$

This can be rewritten as,

$$(17) \qquad CS_i = \int_0^a v[1 - \frac{c_i}{\tilde{c}(\int_{Q'+b}^{Q^*} g(\hat{Q})d\hat{Q})}] \, db,$$

where $a = Q_i - Q'$. Taking the derivative with respect to Q^* yields,

$$(18) \qquad \frac{dCS_i}{dQ^*} = \int_0^a \frac{vc_i}{(\tilde{c}(\int_{Q'+b}^{Q^*} g(\hat{Q})d\hat{Q}))^2} \frac{d\tilde{c}(\int_{Q'+b}^{Q^*} g(\hat{Q})d\hat{Q})}{dQ^*} \, db.$$

Restriction 1) and the fact that $\frac{dQ'}{dQ^*} < 1$ yields $\frac{dCS_i}{dQ^*} > 0$. Thus, $\frac{dJ}{dQ^*} > 0$.

Now it is possible to proceed to the first order condition for (14). This equals,

$$(19) \qquad (vQ' - vQ^* + z) \, g(Q^*) + \frac{dJ}{dQ^*} = 0.$$

Since $\frac{dJ}{dQ^*} > 0$, it must be the case that at $Q^* = Q_G^*$, $(vQ' - vQ^* + z) < 0$. (6) yields that at $Q^* = Q_M^*$, $(vQ' - vQ^* + z) = 0$. Since $\frac{dQ'}{dQ^*} < 1$, it must be the case that $\frac{d(vQ' - vQ^* + z)}{dQ^*} < 0$. This in turn implies $Q_G^* > Q_M^*$.

Q.E.D.

Analysis of example on p. 16: Let $v = 1$, $z = 10$, $Q^L = 0$, $Q^H = 25$, $g(Q) = 1$ for all $0 \leq Q \leq 21$, and $g(Q) = 3$ for all $21 < Q \leq 25$. Also, let time valuations be distributed such that there is a mass point at $c = 1$ with weight 11, and the rest of the population has a time valuation equal to 10. The same argument as in the proofs of Propositions II.1 and III.1 yields that 1) through 4) of Proposition II.1 are still valid, while arrival times are now as follows. Working down from Q^*, general admission seats will be filled by consumers with time valuations equal to 1 until either no such consumers remain or Q' is reached. Furthermore, if consumers i and j are two such consumers, then $t_i - t_j = v(Q_i - Q_j)$. Now, if there are more general admission seats to be occupied than there are consumers with time valuations equal to 1, then remaining seats are filled by consumers with time valuations equal to 10. Furthermore, if consumers i and j are consumers of this type, then $10(t_i - t_j) = v(Q_i - Q_j)$. Note, finally, consumers who sit in seats of quality Q' will have zero waiting times.

Given the above, (6) and (7) yield that this example has a local maximum at $Q^* = 20$, $Q' = 10$ and a local maximum at $Q^* = 22$, $Q' = 12$. There is also a local maximum at $Q^* = 25$, $Q' = 16.5$. Comparing monopoly profits for the three local maxima yields $Q_M^* = 25$.

Now, given that $Q_M^* = 25$, it cannot be the case that $Q_G^* > Q_M^*$. I will now demonstrate that $Q_G^* < Q_M^*$ by showing that at $Q^* = 25$ the left hand side of (19) is negative. At $Q^* = 25$, $(vQ' - vQ^* + z)\, g(Q^*) = 4.5$. The specification of arrival times yields that only general admission ticket purchasers with time valuations equal to 1 will get consumer surplus from the spectating activity. The consumer surplus such an individual derives from spectating, denoted $CS_{c=1}$, can be written as,

$$CS_{c=1} = \int_0^w [1 - \frac{1}{10}]db, \tag{20}$$

where w equals the width of the quality range of general admission seats which are occupied by consumers with time valuations equal to 10. At $Q^* = 25$, $\frac{dCS_{c=1}}{dQ^*} = \frac{-9}{20}$. Integrating this over all general admission ticket purchasers with time valuations equal to 1 yields that at $Q^* = 25$, $\frac{dJ}{dQ^*} = \frac{-99}{20}$. Thus, at $Q^* = 25$, the left hand side of (19) is negative, and therefore it must be the case that $Q_G^* < Q_M^*$.

Footnotes

[1]There are a number of papers which rely on the existence of transaction costs to justify the existence of particular real world pricing schemes (see e.g., Alchian 1970, DeVany 1976, Cheung 1977, Barzel 1982, and Kenney and Klein 1983). In general, however, these papers do not explicitly model the transaction costs involved. One exception to this is Rotemberg (1982). He explicitly models transaction costs in a world where monopolistically competitive firms incur a cost when they change price.

[2]A seemingly more general specification would be to have the utility derived from the spectating activity be a general function $v(Q)$. In actuality, however, this yields little additional generality since it would always be possible to rescale the quality variable to yield the linear form used in this paper.

[3]Cheung (1977) argues that the transaction costs associated with reserve seating can be lowered by underpricing high quality seats. His argument relies on the existence of demand uncertainty. Thus, his argument is not applicable to the model in this paper.

[4]It is not necessary to be concerned with the corner solution $Q^* = Q^L$. This is because such a solution could never be optimal for the monopolist. This can be seen by noting that the left hand side of (6) is positive if $Q^* = Q^L$.

[5]Unfortunately, the only incentive compatible set of arrival times is the set given in Proposition II.1.

[6]One could ask whether similar results obtain under the assumption that the government could set both Q^* and Q'. Note, setting the latter value would be equivalent to regulating the price for general admission seats. In

answer, the proof of Proposition II.4 demonstrates that the government could also not improve social welfare if it had the ability to set both values. The intuition for this result is exactly the same as the intuition for Proposition II.4.

[7]It is also assumed in this section, as it was implicitly in the previous section, that there are more consumers in the population than there are seats in the stadium. The technical condition which ensures this is

$$\int_{\underline{c}}^{\bar{c}} h(c)dc > \int_{Q^L}^{Q^H} g(Q)dQ.$$

[8]One density function which satisfies these two restrictions is the uniform density function.

[9]This is true for all general admission ticket purchasers except those who have zero waiting times.

[10]As in footnote 6, one could ask whether similar results obtain under the assumption that the government could set both Q^* and Q'. In answer to this, consider the following Corollary which can easily be demonstrated. Corollary 1 to Proposition III.2: If restriction 1) of Proposition III.2 is satisfied and $Q_M^* < Q^H$, then the following is true under the assumption that the government could set both Q^* and Q'. Taking as fixed the value of Q' set by the government, the value of Q^* set by the government will be higher than the value of Q^* which the monopolist would prefer.

[11]One might wonder whether the results for this example derive from the fact that the time valuation distribution has mass points and is not continuous or the fact that for this example $Q_M^* = Q^H$ (see Appendix), rather than the fact that $g(.)$ does not satisfy 2). Given that the following can easily be demonstrated, however, the results for this example must derive from $g(.)$ not satisfying 2). Corollary 2 of Proposition III.2: Allowing

time valuation distributions which have mass points and are not continuous, if restrictions 1) and 2) are satisfied, then $Q_G^* > Q_M^*$.

References

Alchian, A. (1970), "Information Costs, Pricing and Resource Unemployment," in Microeconomic Foundations of Employment and Inflation Theory, edited by E. Phelps et al, New York: Wiley.

Barzel, Y. (1974), "A Theory of Rationing by Waiting," Journal of Law and Economics, 17, 73-95.

Barzel, Y. (1982), "Measurement Cost and the Organization of Markets," Journal of Law and Economics, 25, 27-48.

Becker, G. (1965), "A Theory of the Allocation of Time," Economic Journal, 75, 493-517.

Cheung, S. (1974), "A Theory of Price Control," Journal of Law and Economics, 17, 53-71.

Cheung, S. (1977), "Why are Better Seats 'Underpriced'," Economic Inquiry, 15, 513-522.

Chiang, R., and C. Spatt (1982), "Imperfect Price Discrimination and Welfare," Review of Economic Studies, 49, 155-181.

DeVany, A. (1976) "Uncertainty, Waiting Time and Capacity Utilization: A Stochastic Theory of Product Quality," Journal of Political Economy, 84, 523-541.

Harris, M., and A. Raviv (1981), "A Theory of Monopoly Pricing with Demand Uncertainty," American Economic Review, 71, 347-365.

Holt, C., and R. Sherman (1982), "Waiting-Line Auctions," Journal of Political Economy, 90, 280-294.

Kenney, R., and B. Klein (1983), "The Economics of Oversearching: Block Booking by DeBeers, Paramount and Loews," unpublished.

Klein, B., Crawford, R., and A. Alchian (1978), "Vertical Integration, Appropriable Rents, and the Competitive Contracting Process," Journal of Law and Economics, 21, 297-326.

Maskin, E., and J. Riley (1982), "Monopoly Selling Strategies with Incomplete Information," unpublished.

Mussa, M., and S. Rosen (1978), "Monopoly and Product Quality," Journal of Economic Theory, 18, 301-317.

Rotemberg, J. (1982), "Monopolistic Price Adjustment and Aggregate Output," Review of Economic Studies, 49, 517-531.

Spence, M. (1977), "Nonlinear Prices and Welfare," Journal of Public Economics, 8, 1-18.

Williamson, O. (1975), Markets and Hierarchies: Analysis and Antitrust Implications, New York: Free Press.

Williamson, O. (1979), "Transaction-Cost Economics: The Governance of Contractual Relations," Journal of Law and Economics, 22, 233-261.