JOB ASSIGNMENTS, SIGNALLING AND EFFICIENCY

by

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Abstract

This paper analyzes a model in which information about a worker's ability is only directly revealed to the firm employing the worker; however, other firms use the worker's job assignment as a signal of ability. Three themes recur throughout the analysis. First, wage rates tend to be more closely associated with jobs than with ability levels. Second, there is frequently an inefficient assignment of workers to jobs, i.e., even when a firm has complete information about a worker's ability, that worker's job assignment doesn't necessarily maximize the worker's output. Third, the severity of this inefficiency tends to be negatively correlated with the level of firm specific human capital in the economy.

I. Introduction

In the labor market, it is frequently the case that during a worker's lifetime information about the worker's ability is gradually revealed to the firms in the economy. In modeling this idea economists have taken one of two approaches. First, some studies allow the information to be revealed in a public manner, i.e., any information held by a single firm is held by all the firms (e.g., Ross, Taubman and Wachter 1981 and Harris and Holmstrom 1982). Second, other studies allow the information to be revealed only to the firm employing the worker (e.g., Jovanovic 1979). We argue in this paper that neither approach completely captures the manner in which such information is typically revealed in our economy. Specifically, although we agree with the second approach's assumption that information about an individual's ability will in general only be directly revealed to the firm employing the individual, we here claim that this does not in itself go far enough. Our position is that firms other than the present employer can gather some information about a worker's ability by considering his job assignment. That is, firms other than the employing firm are frequently at an informational disadvantage when it comes to experienced workers. However, this informational disadvantage is somewhat attenuated by the fact that such firms can use an individual's job assignment as an imprecise signal of the individual's ability.[1]

In this paper we attempt to identify some of the issues which arise when labor market information is revealed in the above manner. We do this by analyzing, under both spot and long-term contracting assumptions, a simple competitive model of the economy in which ability levels are not publicly observable, but job assignments are. Three themes recur throughout the analysis. First, wage rates tend to be more closely associated with jobs than with ability levels. Second, there tends to be an inefficient assignment of workers to jobs, i.e., even when a firm has complete information about a worker's ability, that worker's job assignment doesn't necessarily maximize the worker's output. Third, the severity of this inefficiency tends to be negatively correlated with the level of firm specific human capital in the economy.

The outline for the paper is as follows. Section II presents our competitive model of the economy. Section III analyzes the model under a spot contracting assumption. Section IV analyzes the model under a long-term contracting assumption. Section V presents some concluding remarks.

II. The Model

In this section we list the assumptions which constitute our competitive model of the economy.

Assumptions

1) Within the economy there is only one good produced and the price of this good is normalized to one.
2) Individuals live for two periods, and in each period labor supply is perfectly inelastic and fixed at one unit for each individual.
3) Individuals display no disutility for effort. However, each individual has associated with him or her a value for a variable which will be called ability, and which will be denoted by A. Furthermore, an individual's ability is a random draw from a distribution which is uniform between the values A^H and A^L, where $A^H > A^L$. The assumption of uniformity is not crucial for obtaining the results of our analysis, but rather makes the mathematics easier to follow.

4) Previous to his first period of employment an individual's ability is unknown both to the individual and to all the firms in the economy. However, after a single period of work the individual's ability becomes known to the individual's first period employer.

5) An individual can be assigned to either of two jobs, which we will denote as job 1 and job 2. Output on job 1 is independent of ability, while output on job 2 is a linear function of ability. Furthermore, there is firm specific human capital so that an old individual's output is higher if the individual has not switched firms during his lifetime. To be specific, individual i's output equals,

x	if the individual is assigned to job 1 and if this is the first period the individual has been employed by his current employer.
$x(1+s)$	if the individual is assigned to job 1 and if this is the second period the individual has been employed by his current employer.
A_i	if the individual is assigned to job 2 and if this is the first period the individual has been employed by his current employer.
$A_i(1+s)$	if the individual is assigned to job 2 and if this is the second period the individual has been employed by his current employer.
Note:	A_i refers to the ability level of individual i, and s is restricted to be greater than zero.

6) The mathematics of the model remain relatively simple if we restrict the model so that all young workers are assigned to job 1. We therefore assume $\frac{A^H+A^L}{2} < x$. To see that this is the correct restriction simply note that $\frac{A^H+A^L}{2}$ is the expected output in job 2 of a young individual for whom there is no information about ability. That is, this restriction guarantees that the expected output of a young individual for whom there is no information about ability is higher in job 1 than in job 2, and therefore that all young workers will be assigned to job 1. Finally, to

ensure that the assignment of old workers to jobs is not trivial we also assume $A^H > x$.

7) Ceteris paribus, individuals are indifferent both among job assignments and among firms.

8) The job assignment-wage rate pair offered to an old worker by his first period employer is public information.

9) In agreeing to a contract an individual cannot irrevocably bind himself to a firm.

10) Each firm can hire at most one young worker per period.

11) Firms face a zero rate of interest

12) There is free entry.

III. Analysis Under Spot Contracting

The assumption that firms are restricted to spot contracts means that a firm cannot write contracts in one period which commit the firm to specific actions or policies in a subsequent period. For example, in offering a young worker a contract, a firm cannot bind itself to specific wage policies for when the worker is old.

The first step in our analysis is to analyze the wage setting process for old workers. Our method of analysis is to treat the problem as an extensive form game with imperfect information (see Harsanyi 1967-1968), where the specific game corresponding to the wage setting process for any particular old worker is as follows. "Nature" moves first and selects the worker's ability level according to assumption 3 of Section II. The worker's first period employer then observes the worker's ability level, and subsequently decides on the worker's job assignment-wage rate pair. The market, i.e., the other firms in the economy, then observes this job assignment-wage rate pair, and subsequently makes the worker a competing wage offer. Finally, it is assumed that the worker accepts the highest wage offer he receives, and that given equal wage offers the worker remains with his first period employer.

Given the above, a (pure) strategy for the worker's first period employer is a specification of a job assignment-wage rate pair for every possible realization of the worker's ability level. Additionally, a (pure) strategy for the market is a specification of a competing wage offer for every possible observation of a job assignment-wage rate pair. The typical way of analyzing such a game is to look for pairs of strategies which constitute Nash equilibria. That is, one would look for pairs of strategies such that each strategy maximized the expected profit of the player using it, given that the other player was using his own specified strategy. This approach is inappropriate for our problem, however, because one of the players is not a maximizing agent. That is, the market should not have a strategy which maximizes the market's profits, but should rather have a strategy which is consistent with what would result from competition among a large number of firms. Thus, our approach is to look for pairs of strategies which satisfy a Market-Nash equilibrium, where this is defined by the following. First, given that the market is using its own specified strategy, the strategy for the first period employer must maximize the first period employer's expected profits. This condition is the same as the conditions which define a Nash equilibrium. Second, given that the first period employer is using his specified strategy, the strategy for the market must everywhere be consistent with a zero profit constraint. This second condition is different than the conditions which define a Nash equilibrium, and might best be understood through an example. Suppose the first period employer's strategy is such that a worker is assigned to job 1 at wage rate $\hat{W}$ if and only if the worker's ability is between A^1 and A^2. Given this, the wage offer made to such a worker by the market must equal $\max[x, \frac{A^1+A^2}{2}]$. That is, the market wage offer must equal the expected output of such a worker in a firm other than the first period employer. Note, when we refer to expected output, we mean the expected output given the job assignment which maximizes the expectation. Finally, the third condition is that the market wage offer can never exceed A^H or be less than x. That is, the market wage offer can never be higher than the highest possible value for the expected output of a worker in a firm other than the first period employer, or lower than the lowest possible value for the expected output of a worker in a firm other than the first period employer. This last condition is similar to imposing a perfectness criterion on our equilibrium concept (see Selten 1975 for a discussion of perfectness).[2]

Before proceeding, one final point concerning our equilibrium concept needs to be addressed. As is frequently the case when the Nash equilibrium concept is applied to an extensive form game, our model displays a multiplicity of Market-Nash equilibria. To overcome this problem we place the following two restrictions on the strategies of our players. First, given that the market is employing its specified strategy, a first period employer cannot be indifferent between his own specified strategy and some other strategy. That is, the strategy of a first period employer must be a unique optimal strategy. Second, given the job assignment as fixed, the market wage offer must be a continuous function of the first period employer's wage offer.[3] These two restrictions eliminate what we consider to be implausible equilibria. Note, we will refer to an equilibrium which satisfies these additional restrictions as a Restricted Market-Nash equilibrium.

We can now proceed to analyze the wage setting process for old workers. The analysis consists of the following two Propositions.

Proposition 1: For the extensive form game corresponding to the wage setting process for old workers, there exists a Restricted Market-Nash equilibrium with the following properties.

1) For every realization of a worker's ability level, the worker remains with his first period employer.

2) If the realization of a worker's ability is below A^+, where $A^+ = \frac{A^H+2sx}{2s+1}$, then the worker is assigned to job 1 and is paid x.

3) If the realization of a worker's ability is above A^+, then the worker is assigned to job 2 and is paid $\frac{A^H+A^+}{2}$.[4]

Proof: Let the first period employer's strategy be described by 2) and 3) above. The situation described in Proposition 1 will result if the market's strategy is first, that every individual assigned to job 1 is offered x, and second, that every individual assigned to job 2 is offered $\frac{A^H+A^+}{2}$. Given this, we need simply demonstrate that this pair of strategies satisfy the conditions for a Restricted Market-Nash equilibrium.

It is easily seen that the market's strategy satisfies the required conditions. First, because $\frac{A^H+A^L}{2} < x$, the market wage offered a worker assigned to job 1 at wage x equals the expected output of such a worker in a firm other than the first period employer, i.e., $x = \max[x, \frac{A^L+A^+}{2}]$. Second, because $A^+ > x$, the market wage offered a worker assigned to job 2 at wage $\frac{A^H+A^+}{2}$ also satisfies this condition, i.e., $\frac{A^H+A^+}{2} = \max[x, \frac{A^H+A^+}{2}]$. Note, the fact that the strategy satisfies the required continuity restriction and the restriction concerning allowable wage offers is obvious.

Now we need only verify that, relative to the market's strategy, the first period employer's strategy is a unique optimum. First, it could never be optimal for the firm to assign a worker to a job and then pay the worker less than the wage offered by the market. This is seen by noting that if the firm assigned the worker to job 1 at a wage rate x, then the firm would get $(1+s)x$ in output. Second, since there are no returns to paying wages above the market wage, the first point implies that the firm will always pay the worker the market wage, i.e., if the worker is assigned to job 1 he is paid x, and if he is assigned to job 2 he is paid $\frac{A^H+A^+}{2}$. Third, given this, a worker will be assigned to job 2 if and only if the increase in production exceeds $\frac{A^H+A^+}{2} - x$. This implies there is some critical ability level above which individuals are assigned to job 2, and below which individuals are assigned to job 1. Let us denote this ability level as $\hat{A}$. If we can demonstrate that $\hat{A} = A^+$, then that will complete the proof. We can solve for $\hat{A}$ by finding the ability level at which the increase in wages just equals the increase in production, i.e.,

$$(1) \qquad \frac{A^H+A^+}{2} - x = (1+s)\hat{A} - (1+s)x.$$

Substituting for A^H and simplifying yields $\hat{A} = A^+$. Q.E.D.

Proposition 2: For the extensive form game corresponding to the wage setting process for old workers, the Restricted Market-Nash equilibrium described in Proposition 1 is unique.

Proof: See Appendix.

To complete our analysis we need to derive the wage rate for young workers. To do this it is first necessary to introduce a bit of new notation. Let W^Y denote the wage rate for young workers, and let

$B = \frac{1}{A^H - A^L}$. Note, this last bit of notation makes $(A^+ - A^L)B$ the probability a young worker will be assigned to job 1 when he becomes old, and $(A^H - A^+)B$ the probability a young worker will be assigned to job 2 when he becomes old.

To now solve for W^Y we simply need recall that all young workers are assigned to job 1 because of our assumption $\frac{A^H + A^L}{2} < x$. Given this and Proposition 1, W^Y can be derived through the use of a zero profit condition (note: assumption 10 of Section II specified that there is a zero rate of interest), i.e.,

$$(2) \qquad W^Y + (A^+ - A^L)Bx + (A^H - A^+)B\left(\frac{A^H + A^+}{2}\right) = x + (1+s)(A^+ - A^L)Bx + (1+s)(A^H - A^+)B\left(\frac{A^H + A^+}{2}\right),$$

or

$$(3) \qquad W^Y = x + (A^+ - A^L)Bsx + (A^H - A^+)Bs\left(\frac{A^H + A^+}{2}\right).$$

(3) completes our characterization of the equilibrium under spot contracting, and we can now discuss our results. Consistent with the discussion in the Introduction, the spot contracting equilibrium displays the following three properties. First, even though an old worker's ability level is known to the firm employing the worker, wage rates for old workers are assigned to jobs rather than to ability levels. This result is consistent with the observation of internal labor market theorists that wage rates are more closely associated with job assignments than with ability levels (see e.g., Doeringer and Piore 1971).[5] The second property is that from an efficiency standpoint there is a misassignment of old workers to jobs. That is, in terms of efficiency any old worker whose ability level is greater than x should be assigned to job 2. However, in the equilibrium we derived every old worker whose ability level was less than A^+ was assigned to job 1, and since $A^H > x$ and $s > 0$, A^+ is strictly greater than x. Third, the

severity of this inefficiency is negatively related to the level of firm specific human capital. That is, the probability that an individual is misassigned, $\frac{A^+-x}{A^H-A^L}$, is a decreasing function of s.

The intuition underlying the first property is clear. Since ability levels are not publicly observable, firms other than the first period employer use an individual's job assignment as an imprecise signal of ability. Furthermore, when attempting to rehire a worker, each firm only matches what other firms are willing to pay the worker. And subsequently, this causes wage rates to be assigned to jobs rather than to ability levels. A question arises as to why there exists no equilibrium where the wage rate itself serves as an additional or independent signal of ability. The reason for this is that, given a worker's job assignment, the optimal wage will in general not vary with the worker's ability level. Thus, any strategy wherein a worker's wage provides an additional signal of ability can never be part of an equilibrium pair of strategies.

The intuition behind the second property is also straightforward. When a worker is assigned to job 2 he is signalled to be of high ability, and it is necessary to pay him a corresponding high wage. For those workers who are just slightly more productive in job 2 than in job 1, the increase in production is not high enough to compensate the firm for the necessary increase in the wage. Thus, there is a misassignment of old workers to jobs because such a worker is not assigned to the job which maximizes his output.

Finally, to understand the third property on an intuitive level one must understand the role played by firm specific human capital in our model. Suppose our model had no firm specific human capital. Under this condition, the misassignment of old workers to jobs would degenerate to the point where no one but a worker of the very highest ability would ever be assigned to job

2. This can be seen by observing that as s approaches zero, A^+ approaches A^H. What happens is that the wage offered by the market to old workers assigned to job 2 is driven so high that only the very highest ability old workers are so assigned. This is a result of the increase in output from a "correct" old worker job assignment being the same for a first period employer as for the other firms in the economy. Now, by having firm specific human capital we are able to move away from this degeneracy, because firm specific human capital makes a first period employer get a higher return from a "correct" assignment than do the other firms in the economy. To see this consider a worker of ability level A^*, where $A^* > x$. If the first period employer assigns the individual to job 2 rather than job 1, the increase in output is $(1+s)(A^*-x)$. If, on the other hand, another firm assigns this worker to job 2 rather than job 1, the increase in output is only (A^*-x). This then explains our third property. The higher is the level of firm specific human capital, the farther does the model move away from a degenerate assignment of old workers to jobs. In turn, the farther does the model move away from this degeneracy, the less severe is the remaining misassignment of old workers to jobs. Or to put these two notions together, the higher is the level of firm specific human capital, the less severe is the misassignment of old workers to jobs.

Before concluding this Section we need to discuss one last point. Empirical studies have almost always found that real world age earnings profiles are upward sloping. Our spot contracting equilibrium, however, is frequently characterized by downward sloping age earnings profiles. For example, any old worker assigned to job 1 has a downward sloping age earnings profile. What is happening is the following. Because of firm specific human capital, firms correctly anticipate that a young worker employed in this period will yield a quasi-rent in the subsequent period. Each firm also correctly anticipates that if it were to hire an old worker with no previous experience at the firm, no such future quasi-rents would be forthcoming. Therefore, since in rehiring workers firms only match the wage offers of other firms, this tends to cause wages to be negatively correlated with age.

The idea that our model displays a property so contrary to empirical evidence, might influence some readers to discount it. However, we would argue that there are two reasons why our model should not be discounted simply because it displays this property. First, our model did not include any general as opposed to firm specific human capital, and the inclusion of general human capital would tend to make the age earnings profiles upward sloping. Second, when in the next section we analyze our model under a long-term contracting assumption, the equilibrium is never characterized by downward sloping age earnings profiles.

IV. Analysis Under Long-Term Contracting

By long-term contracting we mean that a firm can offer a contract in one period, which binds the firm to specific actions or policies in the subsequent period. For example, in offering a young worker a contract, a firm can specify in a binding manner how the worker's wage will depend on his job assignment when the worker is old. Note, however, because an old worker's ability is private information to the worker's first period employer, a contract cannot specify that the firm's actions will depend on the worker's ability in a manner which is not incentive compatible for the firm.

Under spot contracting it was not necessary to specify a number of things concerning how income fluctuations affect a worker's utility. That is, the analysis of the previous Section is correct for any such specification. The spot contracting analysis worked this way because under spot contracting a worker could not affect the wage he would receive when he was old, through his choice of a spot contract when he was young. With long-term contracting a worker can affect the wage he receives when old through his choice of a contract when young, and thus it is necessary that these things now be specified in more detail. Our first assumption is that workers do not have access to either capital markets or insurance markets, and therefore a worker's utility in each period is simply a function of the wage received in that period. Second, we assume that workers have von Neumann-Morgenstern expected utility functions which are consistent with workers being risk averse over wages. That is, if a worker receives W_t in period t, then utility equals $U(W_t)$, where $U' > 0$ and $U'' < 0$. Third, it is assumed that workers have a zero rate of discounting.

The typical way of analyzing a free entry model under a long-term contracting assumption, is to find the contract which maximizes a worker's expected lifetime utility, given the constraint that the contract satisfies a zero expected profit condition. To take this approach for our model, however, poses a difficult problem. That is, it would entail determining the outcome of the extensive form game corresponding to the wage setting process for old workers, for every conceivable contract. This is a problem becaue for some contracts an equilibrium for this game might not exist, while for others there might be multiple equilibria. To make the problem tractable we therefore restrict the problem in the following two ways. First, as in Section III, it is assumed that if for a given contract there exists one or more Restricted Market-Nash equilibria, then the outcome of the game for that contract is one of those Restricted Market-Nash equilibria. Second, we restrict our search to contracts for which there exists at least one Restricted Market-Nash equilibrium. Note, in the following we will refer to these two restrictions as our long-term contracting restrictions.

The outline for this section is as follows. We first confine ourselves to a class of contracts which we will refer to as Class M contracts. We then look at the extensive form game corresponding to the wage setting process for old workers, and solve for the (unique) Restricted Market-Nash equilibrium associated with each contract in this class. Having done this, we are then able to demonstrate that, given our long-term contracting restrictions, the equilibrium long-term contract for our model is necessarily a member of Class M. Finally, again given our long-term contracting restrictions, this in turn enables us to investigate the equilibrium properties of our model.

A Class M contract is a long-term contract offered to young workers which specifies three wage rates, denoted W^Y, W_1^0 and W_2^0, and which binds the firm offering the contract in the following ways. First, the firm is obligated to pay a worker accepting the contract the wage W^Y during the worker's first period of employment. Second, the firm is restricted from firing such a worker after the worker's first period of employment. Third, if the firm assigns the worker to job 1 (job 2) when he is old, then the firm is obligated to offer the worker the wage W_1^0 (W_2^0). Finally, a Class M contract also satisfies the following two restrictions on wages.

1) $W_1^0 > x$

$$2)\quad W_2^0 > \begin{cases} \dfrac{A^H + A'}{2} & \text{if} \quad 2x - A^H < A' < A^H \\ A^H & \text{if} \quad A' > A^H \\ x & \text{if} \quad A' < 2x - A^H \end{cases}, \quad \text{where} \quad A' = \frac{W_2^0 - W_1^0}{1+s} + x.$$

Consider now a worker who accepts a Class M contract. As indicated earlier, the wage this worker receives when old is determined by the outcome of an extensive form game similar to the game analyzed in Section III. Specifically, this new extensive form game is as follows. "Nature" again moves first and selects the worker's ability level according to assumption 3 of Section II. The worker's first period employer then observes the worker's ability level, and subsequently decides on the worker's old age job assignment. Note, the long-term contract constrains the wage offers the first period employer can make at this point. That is, if the worker is assigned to job 1 then the firm is constrained to offer the wage W_1^0, while if the worker is assigned to job 2 then the firm is constrained to offer the wage W_2^0. In the next stage of the game the market, i.e., the other firms in the economy, observes this job assignment and subsequently makes the worker a competing wage offer. Finally, it is again assumed that the worker accepts the highest wage offer he receives, and that given equal wage offers the worker remains with his first period employer.

For the extensive form game described above, a (pure) strategy for the worker's first period employer is a specification of a job assignment for every possible realization of the worker's ability level. Additionally, a (pure) strategy for the market is a specification of a competing wage offer for each of the two possible job assignments. Given this, we can now proceed to analyze the wage setting process for an old worker who accepts a Class M contract. The analysis consists of the following two Propositions.

Proposition 3: Consider a worker who accepts a Class M contract. For the extensive form game which determines the wage received by this worker when he is old, there exists a Restricted Market-Nash equilibrium with the following properties.

1) For every realization of the worker's ability level, the worker remains with his first period employer.

2) If the realization of the worker's ability is below A', where $A' = ((W_2^0 - W_1^0)/(1+s)) + x$, then the worker is assigned to job 1 and is paid W_1^0.

3) If the realization of the worker's ability is above A', then the worker is assigned to job 2 and is paid W_2^0.

Proof: This proof follows along the same lines as the proof of Proposition 1, and is therefore omitted.

Proposition 4: The Restricted Market-Nash equilibrium described in Proposition 3 is unique.

Proof: This proof follows along the same lines as the proof of Proposition 2, and is therefore also omitted.

The above two Propositions tell us how any particular Class M contract translates into expected lifetime utility for a worker accepting the contract, and how it translates into expected profits for the firm offering the contract. Now, suppose for the moment that the contract offered in equilibrium is a Class M contract. This means that the wage rates specified in the contract must be the wages which solve the following maximization problem.[6]

$$\max_{W^Y, W_1^0, W_2^0} \quad U(W^Y) + (A'-A^L)BU(W_1^0) + (A^H-A')BU(W_2^0)$$

$$\text{s.t.} \quad x + (A'-A^L)B(1+s)x + (A^H-A')B(1+s)\left(\frac{A^H+A'}{2}\right) > W^Y + (A'-A^L)BW_1^0 + (A^H-A')BW_2^0$$

$$A' = \frac{W_2^0-W_1^0}{1+s} + x$$

$$W_1^0 > x$$

$$W_2^0 > \begin{cases} \frac{A^H+A'}{2} & \text{if} \quad 2x - A^H < A' < A^H \\ A^H & \text{if} \quad A' > A^H \\ x & \text{if} \quad A' < 2x - A^H \end{cases}$$

The above objective function simply states that the contract will maximize a worker's expected lifetime utility. The first constraint ensures that the firm's expected profitability is not negative. The second constraint represents the assignment of old workers among jobs which characterized the Restricted Market-Nash equilibrium. Finally, the third and fourth constraints are simply the constraints on wages contained in the definition of a Class M contract.

The statement of the above maximization problem enables us to proceed to Proposition 5.

Proposition 5: Given our long-term contracting restrictions, the equilibrium long-term contract for our model can be represented as a Class M contract.

Proof: See Appendix.

Furthermore, Proposition 5 tells us that, again given our long-term contracting restrictions, it is possible to characterize the equilibrium long-term contract for our model by characterizing the solution to the maximization problem on p. 17. This is done through the presentation of two propositions.

Proposition 6: If $x + (x-A^L)B(1+s)x + (A^H-x)B(1+s)((A^H+x)/2) > A^H + x$, then the equilibrium long-term contract is characterized by, $W^Y = W_1^0 = W_2^0 = [x + (x-A^L)B(1+s)x + (A^H-x)B(1+s)((A^H+x)/2)]/2$.

Proof: See Appendix.

The intuition behind Proposition 6 is straightforward. Consider first that if all wage differentials were eliminated, A' would be equal to x. Given this, the expression $x + (x-A^L)B(1+s)x + (A^H-x)B(1+s)((A^H+x)/2)$ simply represents the expected lifetime production of a young individual when all wage differentials have been eliminated. On the other hand, if $A' = x$ and if the worker is assigned to job 2 when he is old, then his expected output at a firm other than the first period employer is $\frac{A^H+x}{2}$. Thus, the strategy of the market must be to offer $\frac{A^H+x}{2}$, if the worker is assigned to job 2 when he is old. Therefore, when $x + (x-A^L)B(1+s)x + (A^H-x)B(1+s)((A^H+x)/2) > A^H + x$, each firm will realize that it is not constrained from eliminating all wage differentials by the strategy of the market. The result is that firms offer complete insurance and eliminate all wage differentials. Finally, the value for this single wage is determined by the idea that free entry creates a zero profit constraint.

Notice, for the parameterizations covered in Proposition 6, the properties which characterized the spot contracting equilibrium no longer hold. First, wage rates for old workers are now only assigned to jobs in a trivial fashion. Second, the absence of wage differentials yields $A' = x$, and thus there is no misassignment of old workers to jobs.

Proposition 7: If $x + (x-A^L)B(1+s)x + (A^H-x)B(1+s)((A^H+x)/2) < A^H + x$, then the equilibrium long-term contract is characterized by, $W^Y < W_1^0 < W_2^0 = \frac{A^H+A'}{2}$, where $x < A' < A^H$.

Proof: See Appendix.

We can begin to understand Proposition 7 on an intuitive level through the following argument. $x + (x-A^L)B(1+s)x + (A^H-x)B(1+s)((A^H-x)/2)$ again represents the expected lifetime production of a young individual when all wage differentials have been eliminated. On the other hand, if $A' = x$ and if the worker is assigned to job 2 when he is old, then the strategy of the market must be to offer $\frac{A^H+x}{2}$. Therefore, when $x + (x-A^L)B(1+s)x + (A^H-x)B(1+s)((A^H+x)/2) < A^H + x$, each firm will realize that it is constrained from eliminating all wage differentials by the strategy of the market. This makes complete insurance infeasible and the resulting sequence of wages is instead, $W^Y < W_1^0 < W_2^0 = \frac{A^H+A'}{2}$.

As opposed to the parameterizations covered by Proposition 6, the parameterizations covered by Proposition 7 are consistent with the first two properties which characterized the spot contracting equilibrium. That is, for old workers wage rates are assigned to jobs rather than to ability levels. Also, since $x < A' < A^H$, there is again a misassignment of old workers to jobs. Finally, it should be noticed that taken as a whole, the long-term contracting analysis also yields a result which is similar to the third property which characterized the spot contracting equilibrium. This third property was that the severity of the misassignment of old workers to jobs was negatively related to the level of firm specific human capital. The similar result in the long-term contracting analysis is based on the derivative of $x + (x-A^L)B(1+s)x + (A^H-x)B(1+s)((A^H+x)/2)$ with respect to s being positive. That is, the sign of this derivative implies that an equilibrium with inefficiency will tend to hold when there is little firm specific human capital, while an equilibrium without inefficiency will tend to hold when there is much firm specific human capital.

V. Conclusion

When one firm attempts to hire a worker away from another firm, the hiring firm is typically at an informational disadvantage as regards the worker's ability level. To overcome this problem we argue that such firms will attempt to use the individual's job assignment as an imprecise signal of the individual's ability. That is, the informational asymmetry between present employers and potential employers is somewhat attenuated by the idea that job assignments are frequently publicly observable, and that therefore potential employers can use job assignments as signals of ability levels.

This paper set up a simple model consistent with the above intuition, and then analyzed the model under both spot contracting and long-term contracting assumptions. The spot contracting analysis yielded three interesting results. First, even though an old worker's ability level was known to the firm employing the worker, wage rates for old workers were assigned to jobs rather than to ability levels. Second, from an efficiency standpoint there was a misassignment of old workers to jobs, i.e., not every old worker was assigned to the job which maximized the worker's output. Third, the severity of this inefficiency was negatively related to the level of firm specific human capital. As for long-term contracting, our model was capable of supporting two different types of equilibria. When in offering a long-term contract a firm was constrained in a binding manner by what other firms would offer old workers in the following period, the resulting equilibrium was consistent with the first two properties listed for the spot contracting equilibrium. On the other hand, when other firms' future wage offers did not serve as a binding constraint, wage differentials were eliminated and the resulting equilibrium did not at all resemble our spot contracting equilibrium. Finally, taken as a whole, the long-term contracting analysis also yielded a result which was consistent with the third property listed for the spot contracting equilibrium. That is, when there was little firm specific human capital an equilibrium with inefficiency tended to hold, while much firm specific human capital typically caused an equilibrium without inefficiency to hold.

As evidenced above, modeling the labor market under the assumption that job assignments as opposed to ability levels are public information yields a number of interesting results. However, because of the simple structure of our model, we consider the present paper to be only a preliminary step in the investigation of this idea. That is, we believe it would be worthwhile investigating refinements of our model. Two examples come to mind. First, we could relax the assumption that firms can hire at most one young worker per period. In the long-term contracting analysis this would allow firms to contract directly over the proportion of workers who are to be promoted in any one period, and we believe the outcome would be a decrease in the misassignment of old workers to jobs. Second, we could allow individuals either to know their own ability levels initially, or to learn their ability levels after a single period of employment. In moving in this direction, however, a choice has to be made as to the type of long-term contracting which will be allowed. One alternative is to follow the approach taken by implicit contract theorists (see e.g., Azariadis 1975 and Baily 1974), who to this point have made no distinction between what is feasible in an explicit contracting world and what is feasible in an implicit contracting world. On the other hand, given that the labor market is typically not characterized by explicit long-term contracts, an attractive second alternative is to restrict contracts to those which are conceivably supportable through a reputation mechanism (note: this alternative is similar to a path of inquiry suggested in Holmstrom 1980). The reason this choice is important is that when we move to a world where workers have some knowledge concerning their own ability levels, the results of the analysis depend significantly on which alternative is chosen. To see this, consider the restriction on contracting which seems consistent with a reputation world, that firms can only have contingencies in contracts depend on things that are public knowledge. Under the assumption that individuals learn their own ability levels after one period of work, the results of our model change not at all under the restriction we just mentioned, while the results change significantly when there are no restrictions on long-term contracting.

Appendix

Proof of Proposition 2: Let us first restrict ourselves to equilibria where there is no turnover. Given this restriction, the first period employer cannot have a strategy where there is a positive probability of assigning the worker to job i at wage W^1 and a positive probability of assigning the worker to job i at wage W^2 where $W^2 \neq W^1$. That is, for the ability levels where one of these two assignments are made, the firm would necessarily always prefer to offer the worker the lower wage. Thus, if a worker is assigned to job 1 then he must be paid some single wage which we will denote as $\underline{W}_1^0$, and if he is assigned to job 2 then he must be paid some single wage which we will denote as $\underline{W}_2^0$. This in turn tells us that there is some critical ability level, denoted $\hat{A}$, above which the worker is assigned to job 2, and below which the worker is assigned to job 1.

Now, given that the strategy of the first period employer is as described above, the strategy of the market must be consistent with offering a worker assigned to job 1 at wage $\underline{W}_1^0$, $\max[x, \frac{A^L+\hat{A}}{2}]$, and offering a worker assigned to job 2 at wage $\underline{W}_2^0$, $\max[x, \frac{A^H+\hat{A}}{2}]$. Furthermore, this combined with our continuity restriction on the market's strategy yields, $\hat{A} = A^+$, $\underline{W}_1^0 = x$ and $\underline{W}_2^0 = \frac{A^H+A^+}{2}$. That is, the only equilibrium without turnover is the equilibrium presented in Proposition 1.

An equilibrium with turnover must have the market wage offer be greater than the first period employer's wage offer whenever the first period employer both assigns the worker to job 1 and offers the worker a wage below $(1+s)x$. This means that when assigning a worker who winds up not staying with the firm, the first period employer must be indifferent among a large number of job assignment-wage rate pairs. This violates our uniqueness restriction on the strategy of the first period employer. Q.E.D.

Proof of Proposition 5: The proof of Proposition 5 is quite long and tedious, so we will just outline it here. Consider first contracts which satisfy a zero profit constraint, and which result in no turnover. Class M contracts were constructed so as to necessarily include the best such contract for which there is a single wage associated with each job. For the best such contract to not be a Class M contract, therefore, the first period employer's strategy associated with the best such contract must have some job where there is a positive probability of assigning the worker to the job at some wage W^1 and a positive probability of assigning the worker to the job at some wage W^2, $W^2 \neq W^1$. However, given the same logic as in the proof of Proposition 2, this cannot be the case. Thus, the best such contract is a Class M contract.

Now consider contracts which satisfy a zero profit constraint, and which result in turnover. The logic contained in the proof of Proposition 2 combined with our restriction that the first period employer's strategy must be a unique optimal strategy yields the following. First, if the worker is assigned to job 1 by the first period employer, then he is bid away by the market at some wage which does not vary with the worker's ability. Second, if the worker is assigned to job 2, then he stays with the first period employer at some wage which does not vary with the worker's ability. Given this, it is easy to demonstrate that for any such contract there necessarily exists a Class M contract which yields higher expected utility and which satisfies a zero profit constraint. Specifically, consider such a contract, where the wage paid the worker if he is assigned to job 2 equals W^2. There necessarily exists a Class M contract which dominates the specified turnover contract, where $W_1^0 = (1+s)x$, $W_2^0 = W^2$ and W^Y is set so as to make the zero profit constraint hold as an equality. Q.E.D.

Before proving Propositions 6 and 7 we will derive three things about the maximization problem on p. 17. First, the first constraint must hold as an equality. This is obvious because if the constraint did not hold as an equality, then W^Y could be raised without violating any of the constraints. Second, this in turn yields that if there is an absence of wage differentials, then $W^Y = W_1^0 = W_2^0 = [x + (x-A^L)B(1+s)x + (A^H-x)B(1+s)((A^H+x)/2)]/2$. Note, from this point on we will denote this wage rate as W^*. Third, the left hand side of the first constraint is maximized when $W_1^0 = W_2^0$. This can be seen by noting that the derivative of the left hand side of the first constraint with respect to W_1^0 equals $(W_2^0-W_1^0)B/(1+s)$.

Proof of Proposition 6: From the above we know that if Proposition 6 is incorrect, then the contract must have wage differentials. Additionally, the restriction contained in the statement of Proposition 6 yields that the set of wages, $W^Y = W_1^0 = W_2^0 = W^*$, satisfy all the constraints. Now, denote as $\hat{W}^Y$, $\hat{W}_1^0$, $\hat{W}_2^0$, a random set of wages which satisfy the first constraint and for which there are wage differentials. The above two things imply that we can prove Proposition 6 by demonstrating that for $W^Y = W_1^0 = W_2^0 = W^*$, the value of the objective function is necessarily higher than the value at the wages $\hat{W}^Y$, $\hat{W}_1^0$, $\hat{W}_2^0$. At $W^Y = W_1^0 = W_2^0 = W^*$ the left hand side of the first constraint is at a maximum. Therefore,

$$(4) \qquad 2W^* \geq \hat{W}^Y + R\hat{W}_1^0 + (1-R)\hat{W}_2^0,$$

where R equals the probability a worker accepting a contract with wages $\hat{W}^Y$, $\hat{W}_1^0$, $\hat{W}_2^0$ is assigned to job 1 when he is old. Furthermore, since (4) can only hold as an equality if $\hat{W}_1^0 = \hat{W}_2^0$, the concavity of U( ) implies,

$$(5) \qquad 2U(W^*) > U(\hat{W}^Y) + RU(\hat{W}_1^0) + (1-R)U(\hat{W}_2^0). \qquad \text{Q.E.D.}$$

Proof of Proposition 7: We will prove Proposition 7 by demonstrating that every other possible sequence of wages yields a contradiction.

Case 1: $W^Y = W_1^0 = W_2^0$

We know the first constraint must hold as an equality. Therefore, $W^Y = W_1^0 = W_2^0 = W^*$. For the range of parameterizations specified in Proposition 7, however, this set of wages does not satisfy the fourth constraint, i.e., a contradiction.

Case 2: $W^Y > W_1^0, W_2^0$ (strictly greater for at least one of the two and where $A^L < A' < A^H$)

The impossibility of all of the rest of the cases stems from the idea that there is a way to vary the wage rates, such that the objective function moves in a positive direction and all the constraints continue to be satisfied. This variation can either take the form of an alternative set of wages, or a direction of variation. Because the arguments are basically all the same, we will only present the argument in detail for Case 2. Note, since the argument is somewhat more complex when it relies on a direction of variation, we work through a case which utilizes this method.

Define three wage rates $\hat{W}^Y$, $\hat{W}_1^0$, $\hat{W}_2^0$, such that $\hat{W}^Y = W^Y - z$, $\hat{W}_1^0 = W_1^0 + z$ and $\hat{W}_2^0 = W_2^0 + z$, and substitute these wages into the objective function and the constraints. Now, taking the derivative of the left hand side of the first constraint with respect to z (evaluated at $z = 0$) yields zero, and taking the derivative of the right hand side of the first constraint with respect to z (evaluated at $z = 0$) yields zero. That is, as we vary z in a positive direction from $z = 0$, the first constraint will continue to be satisfied. Now we take the derivative of the objective function (evaluated at $z = 0$), i.e.,

$$(6)\left.\frac{dO.F.}{dz}\right|_{z=0} = \left.\frac{-dU}{dW}\right|_{W=W^Y} + (A'-A^L)\, B\left.\frac{dU}{dW}\right|_{W=W_1^0} + (A^H-A')\, B\left.\frac{dU}{dW}\right|_{W=W_2^0}.$$

Because we are dealing with the sequence of wages $W^Y > W_1^0, W_2^0$ (strictly greater for at least one of the two), and because $U(\ )$ is concave, (6) implies $\left.\frac{dO.F.}{dz}\right|_{z=0} > 0$. Therefore, z can be varied in a positive direction from $z = 0$, and the value of the objective function will rise while all the constraints continue to be satisfied, i.e., a contradiction. Note, the third and fourth constraints are still satisfied because the wages are being varied in a positive direction.

Case 3: W_1^0 and W_2^0 are such that $A' < A^L$

Define $\hat{W}^Y, \hat{W}_1^0, \hat{W}_2^0$, such that $\hat{W}^Y = W^Y$, $\hat{W}_1^0 = W_2^0 + \varepsilon$ and $\hat{W}_2^0 = \infty$. The argument follows as for Case 2.

Case 4: W_1^0 and W_2^0 are such that $A' > A^H$

Define $\hat{W}^Y, \hat{W}_1^0, \hat{W}_2^0$, such that $\hat{W}^Y = W^Y$, $\hat{W}_1^0 = W_1^0$ and $\hat{W}_2^0 = \max[W_1^0, A^H]$. The argument follows as for Case 2. Note, the rest of the cases have an implicit restriction on wages, such that $A^L < A' < A^H$.

Case 5: $W^Y < W_1^0 < W_2^0 > \frac{A^H+A'}{2}$

Define $\hat{W}^Y, \hat{W}_1^0, \hat{W}_2^0$, such that $\hat{W}^Y = W^Y + zB(A^H-A')$, $\hat{W}_1^0 = W_1^0$ and $\hat{W}_2^0 = W_2^0 - z$. The argument follows as for Case 2.

Case 6: $W_1^0 \geq W^Y \geq W_2^0$ (strictly greater for at least one of the two)

Define $\hat{W}^Y$, $\hat{W}_1^0$, $\hat{W}_2^0$, such that $\hat{W}^Y = W^Y - zB(A^H-A')$, $\hat{W}_1^0 = W_1^0$ and $\hat{W}_2^0 = W_2^0 + z$. The argument follows as for Case 2.

Case 7: $W_2^0 > W^Y \geq W_1^0$

Define $\hat{W}^Y$, $\hat{W}_1^0$, $\hat{W}_2^0$, such that $\hat{W}^Y = W^Y - zB(A'-A^L)$, $\hat{W}_1^0 = W_1^0 + z$ and $\hat{W}_2^0 = W_2^0$. The argument follows as for Case 2.

Case 8: $W_1^0 = W_2^0 > W^Y$

Define $\hat{W}^Y$, $\hat{W}_1^0$, $\hat{W}_2^0$, such that $\hat{W}^Y = W^Y + zB(A'-A^L+(A'-A^H)\frac{1}{1+2s})$, $\hat{W}_1^0 = W_1^0 - z$, and $\hat{W}_2^0 = W_2^0 + \frac{z}{1+2s}$. The argument follows as for Case 2.

Case 9: $W_1^0 > W_2^0 > W^Y$, where $A' \geq 2x - A^H$

Define $\hat{W}^Y$, $\hat{W}_1^0$, $\hat{W}_2^0$, such that $\hat{W}_1^0 = (W_1^0+W_2^0)/2$, $\hat{W}_2^0 = (W_1^0+W_2^0)/2$, and $\hat{W}^Y = W^Y + (2x-(A^H+A^L))B(W_1^0-W_2^0)/2$. The argument follows as for Case 2.

Case 10: $W_1^0 > W_2^0 > W^Y$, where $A' < 2x - A^H$

Define $\hat{W}^Y$, $\hat{W}_1^0$, $\hat{W}_2^0$, such that $\hat{W}^Y = W^Y + zB(A'-A^L)$, $\hat{W}_1^0 = W_1^0 - z$, and $\hat{W}_2^0 = W_2^0$. The argument follows as for Case 2. Q.E.D.

Footnotes

[1]We are familiar with one previous reference to the idea that job assignments can be used as signals of ability. Specifically, in a comment somewhat critical of the internal labor market literature, Stiglitz stated, "...if information about individual qualities can be ascertained from the jobs to which they are assigned, and long-term contracts are not enforceable, in later stages of individuals' lives, they will receive a pay commensurate with their abilities." (Stiglitz 1975, p. 556.)

[2]The results found in this paper could be derived using the Nash equilibrium concept by modeling each firm in the market as a separate player. The reason we did not use this alternative specification is that the proofs with this alternative specification are very tedious.

[3]This restriction is similar to a restriction suggested in Milgrom and Roberts (1982) -- see their Footnote 10.

[4]Proposition 1 does not specify to which job an old worker of ability level A^+ is assigned to. We can ignore this situation because it occurs with probability zero.

[5]Williamson, Wachter and Harris (1975) and Miyazaki (1977) also explained the assignment of wage rates to jobs through contracting arguments. However, neither of these previous papers utilized the idea that job assignments were acting as signals.

[6]The expression representing the probability a young worker will be assigned to job 1 when he is old, the corresponding expression for job 2, and the expression representing the expected output of a worker assigned to job 2 when he is old, are all written for a contract where $A^L < A' < A^H$.

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