EQUILIBRIUM CONCEPTS IN GAME THEORY:

THE NEED FOR DYNAMICS

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Abstract

The "Nash equilibrium" or "equilibrium point" solution concept (here
called the Nonstrategic Equilibrium NE) is badly flawed. The players must
be myopic, unstrategic actors or else they both count on having the last
move. Either rationale represents an unacceptable implicit dynamic process.
Any eligible process requires rationality on the part of each player, and
consistent rules of play: such a process, together with a given game matrix
of outcomes, leads to a Dynamically Rational Equilibrium (DRE). Among the
elements needed to specify the dynamic process are the initial status quo
position (is it a clean slate, or some particular initial strategy-pair?), the
sequence of moves (do the players move alternately or simultaneously?), and
the termination rules (who may end the haggling process, and under what
conditions?).

Two types of termination rules were analyzed in detail. (1) Exogenously
fixed termination: Here the DRE is always perfectly determinate in the
alternating-move game -- sometimes to the advantage of the last-mover,
sometimes not -- but the simultaneous-move game may not always have a
determinate DRE. (2) "Natural" termination: This occurs when both parties
are satisfied to pass rather than switch. Under natural termination, the
"trap" outcome of the Prisoners' Dilemma is not always the DRE; in
particular, the efficient outcome will be the DRE for any status quo
position in the simultaneous-move game, and for the clean-slate status quo in
the alternating-move game.

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I will be contending that some of the standard solution concepts of "non-cooperative" game theory are defective or misunderstood, for lack of credible dynamics. To ask if a proposed solution is an equilibrium requires us to look at the processes leading toward or away from that outcome. In particular, I will maintain, eligible processes must involve (a) rational decision-making for individuals in a strategic context and (b) mutual consistency among the assumed behaviors of the opposed players.

My objective here has certain parallels with some older and some more recent discussions — including Schelling [1960], Rapoport and Guyer [1966], Stein [1980], Fiorina and Shepsle [1982], and Dixit [1982]. But for lack of space I will go back to first principles rather than attempt a critical comparative analysis. (Some specific points of difference will be indicated below as they arise.)

A. Nonstrategic Equilibrium (NE)

What I will call here the Nonstrategic Equilibrium (NE) — more usually known as the "Nash equilibrium," or sometimes as the "equilibrium point" — is the best-known solution concept. Formally, an outcome is an NE if the payoff to the Row player R is the highest in its column while the payoff to the Column player C is the highest in its row. Thus, at least in the short run, each decision-maker can only lose by revising his chosen strategy (his "try", I will sometimes say). I will usually be assuming, for simplicity,

each should be saying to himself: "If I do this, he might do that, so I ought to...." A solution principle that fails to take account of strategic behavior is inconsistent with the raison d'etre of game theory.

It is difficult even to imagine any dynamic process that might actually lead to the NE solution. Here is one attempt. Let Fate arbitrarily prescribe initial tries for each player, establishing a status quo position. Each player is then told: "You can change your try if you like, but don't worry about your opponent's reaction — he is locked in." Obviously, the statements being made to the two players are inconsistent (the second eligibility principle is violated). Should either party accept the offer and actually switch, therefore, Fate has to renege; the game is called off. Fate repeats this process (the players being memory-less) enough times to cover all the possible strategy-pairs as initial status-quo positions. Those strategy-pairs that survive the ordeal, that is, such that neither player accepts the offer to change, are the NE's. Not very plausible, is it? Here's a variant. Starting as before suppose that, should either player change his try, instead of calling the game off Fate says to the other: "I lied to that player who just switched his strategy — you are not locked in after all, and have the option of another try. But trust me, your opponent is now locked in." Of course, should this other player now accept the offer and make a change, Fate then goes back to the first party with the same story. The game ends when both players in succession, each believing the tale, pass rather than switch. Not a much more credible dynamic than the other!

The NE is plausible only for contexts where the participants are unable to behave strategically. One example might be interactions between organisms

MATRIX 2 [42]				MATRIX 3 [61]			
Row-Dominance				Double Trouble			
*3,3		4,1		*4,4		1,3	
2,2		1,4		3,1		*2,2	

B. Safety Equilibrium (SE) and the Saddle-Point Condition

In elementary presentations of game theory, the tale usually starts with the constant-sum game and two opponents who seem rather over-concerned with playing safe. The Row player R is supposed to look for the row guaranteeing him the greatest minimum payoff (the maximum of the row minima). And similarly the Column player C looks for the maximum of the column minima.[4] Of course, for both players so to behave would be just as unreasonable as the decisions implicit in the NE. The Nonstrategic Equilibrium NE corresponds to two generals each of whom always underestimates his enemy, thinking him incapable of intelligent maneuver. In contrast, the Safety Equilibrium SE corresponds to over-cautious generals, each of whom gives the enemy credit for unlimited ability to optimally counter his own move.

Actually, however, the usual verbal tale is quite misleading. It generally serves only as a steppingstone toward the "saddle-point" solution, which in general is not the SE. Compare Matrices 4 and 5 below:

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[4]The usual presentation begins with the zero-sum case where only the payoffs for R need be shown, C's payoffs being the negative of these. C would then be described as looking for the minimum of the column maxima. To avoid confusion, and as the zero-sum game is rarely important here, I will always indicate the payoffs for the two parties explicitly.

the converse does not hold, the dubious tale about the over-cautious generals was entirely beside the point. If the saddle-point condition is to be the criterion in the constant-sum game, we might as well have started right off looking for the Nonstrategic Equilibrium NE.

For the non-constant-sum game, the picture is entirely different. First of all, the NE solution need not be unique. And if the saddle-point concept is generalized to the non-constant-sum case, so as to refer as before to an SE solution where the two safety levels are actually achieved, then an NE (whether or not unique) need not be a saddle-point. Matrix 6 below is an example. It has two NE's (marked *), a single SE (marked #) distinct from either of these, and furthermore the SE is a saddle-point (marked %) while the NE's are not!

MATRIX 6 [68]

*3,4	2,2#%
1,1	*4,3

I conclude that there is nothing intrinsically compelling about either the Nonstrategic Equilibrium NE or the Safety Equilibrium SE (whether or not incorporating the saddle-point feature). Nor do these have any general tendency to coincide (except for the constant-sum case where NE and saddle-point are equivalent). To arrive at a satisfying solution concept we will have to pay serious attention to the dynamic processes; solutions cannot

cell (r''',c''') in the same row as (c',c') and the same column as (r'',c''). Using the constant-sum property, $r^o > r'$ implies $c' > c^o$. And $c^o > c''$ implies $r'' > r^o$. And so if (r''',c''') is also an NE, then $r''' > r'' > r^o$ and $c''' > c' > c^o$. But since $r''' + c''' = r^o + c^o$, this is a contradiction. Hence there cannot be a second NE. (This proof, while it applies only to the constant-sum case, is not limited to 2x2 games.)

every possible strategy-pair in turn) is the status quo.[7] The two cases are quite different. For one thing, an equilibrium might be _retentive_ though not _attractive_: once there you stay there, but starting anywhere else you don't get there.[8]

2. The sequentiality procedure

There are two main cases: the players may move (choose tries) in alternation or simultaneously. But even if you move later in time than your opponent, if your choice of try must be made in ignorance of his you are really moving simultaneously from the decisional point of view. This opens up the possibility of intermediate cases, where you have some but only partial information as to the other's preceding try. (For example, if he has options 1,2,3 you may be able to observe whenever he chooses 3 but be unable to distinguish between his 1 and 2.) As another intermediate case, there might be switches between simultaneous and alternating moves for successive rounds of play. I will deal here only with the two polar cases: strict alternation versus strict simultaneity.

3. The rules for termination

Here there are an indefinite number of distinguishable and interesting possibilities. The following suggests some of the major categories.

(1) _Fixed termination_: How the game ends may be established by some exogenously pre-ordained formula. An example for an alternation game: "Row has a try, then Column, then Row once more — after which play ends." (But I

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[7]Compare my attempt above to find a plausible dynamic for the NE. Rapoport and Guyer [1966], p. 206, were also led to such an assumption in attempting to arrive at a concept of stability: "...the players will be assumed to play the game after the outcome of the initial game is known."

[8]See Fiorina and Shepsle [1982].

D. Principle of Solution

Having built this foundation, I can finally begin to discuss solutions. Of course, there are too many distinguishable possibilities for completeness to be a practical goal.[9] After a brief discussion of the principles of solution here, I will go on to slice away at the Gordian knot of possibilities in a number of directions. Specifically, I will examine some special important game matrices that warrant detailed treatment (e.g., Prisoners' Dilemma), and some classes of termination rules that are amenable to generalized solutions.

The main principles of solution were suggested in the opening discussion. Any acceptable solution must involve: (a) <u>individually rational play</u>: each player maximizes his payoff, and is fully able to compute even very long and multiple chains of possibilities (on the order of "If I do this and then he does that....") (b) <u>Consistency of the dynamic process</u>: Not only the players but the process must make sense. For example, <u>both</u> players cannot be given the last move. This elementary condition, as already seen, invalidates the usual dynamic offered for the Nonstrategic Equilibrium NE. (c) <u>Adequate knowledge</u>: I will assume throughout that the rules of play are fully known to both parties, as is the underlying game matrix. The history of tries on previous rounds will also be assumed known.

In addition, I will be ruling out threat or promise strategies, in which one party commits himself in advance to choose one option or another depending upon his opponent's next move (see Schelling [1960], esp. Ch. 5). Of course I

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[9]The essentially unlimited number of distinguishable dynamic processes has just been indicated. As for the normal-form game matrices, Rapoport and Guyer [1966] show that there are 78 logically distinct 2x2 games under strict preference ordering. This number grows explosively as more strategies are allowed each player, and ditto as more players are admitted.

there is a completely determinate outcome on the final move. But then, given

the information-adequacy assumption, the player with the next-to-last try can

also directly optimize. (This player _is_ behaving strategically, in that he

allows for reaction on the part of the other, but now that reaction is

perfectly predictable.) This logic continues to apply all the way back to the

very first move of the game. Accordingly, in fixed-termination, alternating-

move games the DRE is wholly determinate.

While this procedure is of the simplest, some of the results may be a bit

surprising. For one thing, it is easy to demonstrate that it may not pay a

player to follow a "dominant" strategy of the basic underlying game. In

Matrix 7, Row's R1 is his dominant strategy. Suppose the termination rule

gives Column the last try. If Row chooses R1, then if Column plays

rationally the outcome will be (R1,C1) where Row receives 2. This is also

the NE, as indicated by the symbol *. But should Row choose R2 instead,

the outcome when Column plays rationally (indicated by the symbol +) is at

(R2,C2), where Row receives 3! So (R2,C2) is the DRE solution under the

dynamic process just described. Thus, at least for some dynamic processes one

can do better following a "dominated" strategy.[10] And we have also seen, in

passing, that the DRE solution may be quite distinct from any NE's that

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[10]Rapoport and Guyer [1966, p. 207] make a similar point: they say that,
for certain game matrices (among them Matrix 7 here), the NE is "force-
vulnerable" and so not fully stable. By diverging from his dominant-
strategy NE, a player might be able to coerce the opponent into a consequent
change benefiting the initial diverger. But since R&G never actually
specify the dynamics they have in mind, it remains unclear whether the initial
divergence would actually lead to the desired outcome. A similar objection
can be raised to the related development by Stein [1980, pp. 65ff]. My
analysis of the Matrix 7 case shows that, under the specified dynamic (Row
moves first, then Column has the last try) it is indeed rational for Row to
diverge from his "dominant" strategy. But for other possible dynamic
processes, for example if Row could move again after Column's move, such a
divergence would be a mistake.

last move.

The situation is quite different in Matrix 9 (which is an inessential transposition of the earlier Matrix 4). If Column has the last move, Row's best prior choice is R2. This leads to the outcome (R2,C2) where Row receives only 2 and Column 3. If Row instead has the last move, Column's best prior choice is C2. But then Row can select the outcome (R1,C2) where Row receives 3 and Column only 2. So here the last-trier has the advantage.

Interpreting these results, notice that in Matrix 8 the players' interests are more allied than in conflict. Hence the non-terminating player can rely on the fact that the terminator must automatically help his opponent in trying to help himself. In consequence the former will often find it possible, as in Battle of the Sexes, to capture more of the mutual gain.[11] By contrast, in Matrix 9 the players' interests are much more opposed. (In fact, Matrix 9 is constant-sum.) Here the terminating player, in helping himself on the final move, will tend to injure his opponent, so the non-terminator may be forced to settle for a "safe" but relatively unsatisfactory intermediate payoff.

One other game worthy of special note here is Chicken (Matrix 10). It will be easy to verify that Chicken is like Battle of the Sexes in that the last-mover is at a disadvantage.

MATRIX 10 [66]

Chicken

3,3	*2,4
*4,2	1,1

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[11]On this see also Schelling [1960], p. 143.

as the correct DRE solution at (R2,C2) of the original game.

Nevertheless the solution principle here — "Allow the final-mover to have contingent (follow vs. diverge) strategies, and find the NE of the matrix thus enlarged" — is not really satisfactory. First of all, it compounds the essential illogic of the NE. Where the NE in the basic game has each player acting as if he had the last move, the NE in the expanded game has each player acting as if he had the last choice of strategy. (Once contingency is allowed for, a strategy becomes more than a move.) In particular, the C3 (follower) strategy for Column that enters into the NE of the expanded game is not distinguishable observationally by Row from whichever of the simple strategies C1 or C2 is applicable. If Row chooses R1, Column's C3 in the expanded game requires playing C1 of the basic game in response — and Row has no way of knowing whether it is C3 or C1 of the expanded game that represents Column's true strategy. In short, Column in choosing C3 is attributing to Row the ability to respond optimally not just to a move Column has not yet made but to a strategy that Row could not uniquely distinguish even if he knew Column's move.

Secondly, the strategy directive for the last-mover in the NE of the expanded game is conceptually misleading. The C3 column of Matrix 7A only accidentally, so to speak, represents a "follower" strategy. The better description is that, being the last-mover in a terminating game Column is in a position to engage in simple optimization, and it happens to be the case here that last-move-optimizing corresponds to "follower" behavior (C3). But with other game matrices it might correspond to any of the columns C1 through C4. The universal principle "last mover simply optimizes" is a more economical and more accurate description, even in the 2x2 case, than "find the behavior associated with the NE of the game matrix expanded by allowing

MATRIX 3		MATRIX 3A			
Double Trouble		Matrix 3, with			
(Dynamic process: Column has last move)		Added Contingent Strategies			
		C_1	C_2	C_3	C_4
+*4,4	1,3	*4,4	1,3	*4,4	1,3
3,1	*2,2	3,1	*2,2	2,2	3,1

There are two NE's (marked * as usual) in the underlying game matrix. With Column the last-mover, it is evident that (R1,C1) — the mutually preferred of the two NE's — becomes the DRE (marked +). What about the expanded Matrix 3A? As can be seen, not only do both the correct and the incorrect NE's of the original Matrix reappear but a third NE pops up as well! So the expanded-matrix technique implies indeterminacy[14] in what is an entirely straightforward case: Row should pick R1 in the basic matrix, Column's simple-optimizing final-move response is C1, and they both end up with their most-desired payoff.

2. Simultaneous moves

A sketchier treatment will suffice for the fixed-termination, simultaneous-move case. Here on the last (joint) try <u>each</u> player will simply optimize; there is no point doing anything else, since the other player will not be in a position to react. (And, since previous moves do not represent any kind of commitment, only the last try need be considered in finding the

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[14]Of course, the subgame-perfection criterion might then be applied. This would reduce but not eliminate the indeterminacy. In Matrix 3A both (R1,C1) and (R1,C3) are subgame-perfect equilibria in the expanded-strategy matrix, a needless complication since they both correspond to (R1,C1) in terms of <u>moves.</u>

outcomes, which leaves only 15 of the 78 matrices unaccounted for. These 15 cases remaining, including such famous games as Battle of the Sexes (Matrix 8) and Chicken (Matrix 10), do not have a determinate pure-strategy DRE in the fixed-termination simultaneous-move game.

F. "Natural" Termination

A dynamic process "naturally" terminates when neither player wishes to change his move. Once again the equilibria of the alternating-move and simultaneous-move games differ, as will be explained below. To keep length within reasonable bounds, in this section I will discuss only Prisoners' Dilemma -- with emphasis upon the possibility of escaping the famous "trap" solution in which the players end up at a mutually inferior outcome.

1. Alternating moves

In the alternating-move game, once the dynamic process -- the haggling -- is well under way the game naturally terminates when any player passes. The start-up condition also has to be specified, however. If the game starts with a clean-slate status quo, a pass in the first round is not meaningful -- to initialize play, each party has first to choose a row or column, as the case may be. If on the other hand the status quo is some particular cell of the matrix (an exogenously specified initial strategy-pair), the first-mover could pass. But consistent with the previous discussions, the game will not be considered to have ended until the second-trier also has a chance to make a choice of move. Summing up: "natural" termination of the alternating-move game occurs when either party passes after both players have had at least one try.

Fig. 1 pictures the considerations bearing upon equilibrium under "natural" termination of the alternating-move Prisoners' Dilemma. The four

is retentive for Column, but once again neither of these is attractive. So it appears, everything points to a termination at 2,2 —— the "trap" of the Prisoners' Dilemma.

So far, this dynamic analysis supports the traditional solution for the Prisoners' Dilemma: a solution that is not only doubly dominant in strategies but also constitutes a Nonstrategic Equilibrium NE, a Safety Equilibrium SE, and a saddle-point all at once.[15] Nevertheless, the "natural" termination process turns out not always to confirm this solution.

For the alternating-move game, we have not yet taken the start-up cycle into account. The efficient 3,3 outcome for the Prisoners' Dilemma, though not attractive, is retentive. Consequently, if it were the initial status quo neither player would have any incentive to switch away from it. Suppose Column were to make the switch from 3,3 to 1,4. Then Row could advantageously "punish" him by switching in turn to the trap outcome 2,2.[16]

Next consider the asymmetrical 1,4 outcome as a status quo position. Row would of course prefer to stick at 1,4. But since the start-up procedure requires that both players pass, this initial position leads inevitably toward the 2,2 trap. The same holds for 4,1 as the initial status quo. And, of course, the trap itself is fully retentive were it the status quo. Summing up: all roads from initial status quo positions do lead toward the trap, except for start-up at the efficient outcome 3,3.

What about the clean-slate status quo? Here the result is quite different. Each player will realize that, should either diverge in the start-

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[15]Also, as seen above, the traditional solution holds for both the alternating-move and simultaneous-move games under fixed termination.

[16]The efficient (3,3) outcome of the Prisoners' Dilemma is therefore a "reactive equilibrium" in the sense of Riley [1979b].

switch on his next try, and similarly Column has no hope of ever achieving 1,4.)

Under this dynamic, should the players ever find themselves away from the 3,3 position — whether as an initial status quo, or as the consequence of mistaken play earlier on — they can easily locate a path back to the efficient outcome. From the 2,2 "trap" position, for example, as just explained each should attempt to switch toward the efficient outcome — and, if they each make the attempt, they will succeed. And of course either of the asymmetrical outcomes will <u>always</u> cause the aggrieved player to switch. So in general for the natural-termination, simultaneous-move game the Prisoners' Dilemma "trap" is ineffective; the efficient outcome will be achieved as the DRE.[17]

As an overall conclusion, then, consideration of alternative dynamic processes reveals that the Prisoners' Dilemma trap is by no means such an

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[17]Considerable attention has been paid lately to a particular contingent strategy called TIT FOR TAT as being optimal for playing the (simultaneous) Prisoners' Dilemma. In TIT FOR TAT each player engages in "good behavior" on his first try, but thereafter will do unto his opponent as his opponent did toward him on the preceding try (see Axelrod & Hamilton [1981], who credit the strategy to Anatol Rapoport). This strategy is usually described in the context of a <u>repeated game</u>. One example would be where two players, who receive on each round of play the actual payoff outcomes dictated by their strategy choices, know that there is also a certain fixed probability each time of the game being renewed for another round. TIT FOR TAT turned out to do quite well against other proposed strategies in a "contest" with the following environment: a large number of players, meeting pairwise at random, but where each has a certain fixed probability of re-meeting (and thereby being in a position to reward or punish) a partner in a preceding engagement. Neither of these contexts corresponds to that discussed here. The assumption here is that there will be only <u>one round of actual play</u>, so that the one-time payoffs will be those indicated by some one of the four cells of the basic underlying Prisoners' Dilemma matrix. If there is any repetitive aspect here, it has been in the <u>dynamic haggling process</u> — not in the payoffs. (TIT FOR TAT might in some "non-computable" cases turn out to be a good haggling technique, but all the dynamics considered here have been simple enough for the players to find the DRE by a direct reasoning process.)

— for example, a _retentive_ equilibrium is not always _attractive_ and vice versa.

3. Among the elements needed to specify the dynamic process are: (i) The status quo position — is it a clean slate, or some particular initial strategy-pair? (ii) The sequentiality procedure — do the players move alternately or simultaneously? (iii) The termination rules — who may terminate the haggling process, and under what conditions?

4. Suppose termination is _fixed_ by some preordained formula (e.g., the game ends after two rounds of tries for each player). It turn out that, for _all alternating-move games,_ the DRE is perfectly determinate! (The question of when the advantage lies with or against the last-mover is of considerable interest.) When moves are simultaneous, however, the DRE remains indeterminate for a number of game matrices — including such famous ones as Battle of the Sexes and Chicken.

5. The concept of "natural" termination seems to be an important idea overlooked in standard game theory. A game is said to naturally terminate when (after each player has had at least one try) _either_ player chooses to pass rather than switch in the alternating-move game, or _both_ players do so in the simultaneous-move game. Under natural termination, the "trap" solution of the Prisoners' Dilemma turn out _not_ to be the DRE in a considerable range of cases. And in particular, for the simultaneous-move game, the efficient rather than the trap outcome will always be the DRE.

FIG. 1

PRISONERS' DILEMMA, ALTERNATING MOVES

(Natural termination, stability analysis)

PRISONERS' DILEMMA
MATRIX

| 3,3 | 1,4 |
| 4,1 | 2,2 |

----- Row's moves

——— Column's moves