OIL LEASE AUCTIONS:

RECONCILING ECONOMIC THEORY WITH PRACTICE

by

Marc S. Robinson

Discussion Paper #292
Revised September, 1984

Department of Economics
University of California, Los Angeles
405 Hilgard Avenue
Los Angeles, California 90024

OIL LEASE AUCTIONS:

RECONCILING ECONOMIC THEORY WITH PRACTICE

by

Marc S. Robinson*

University of California, Los Angeles

Revised September 1984

Abstract

Oil lease auctions have been the subject of a number of recent theoretical papers. Many of the conclusions relate to the optimal structure of these auctions from a seller's perspective. In this paper, several contributions are made to auction theory regarding the use of minimum prices. These results, when combined with other considerations relevant to landowners, suggest a structure for oil lease auctions quite different from that currently recommended theoretically, but consistent with that observed in actual auctions — both private and public — in the United States.

Marc Robinson
Department of Economics
405 Hilgard Avenue
University of California
Los Angeles, CA 90024
(213) 206-6732

1. Introduction

Oil lease auctions have been the subject of a number of recent theoretical papers. Many of the conclusions relate to the optimal structure of these auctions from a seller's perspective. They form part of an extensive, and thriving, literature on auction theory. In this paper, several contributions are made to auction theory regarding the use of minimum prices. These results, when combined with other considerations relevant to landowners, suggest a structure for oil lease auctions quite different from that currently recommended theoretically, but consistent with that observed in actual auctions — both private and public -- in the United States.

Though many of the results in this paper have much wider application, the analysis of oil and other mineral rights auctions is itself important for a number of policy issues. Governments are the owners of a large fraction of the mineral rights; outside the U.S., they are typically the sole owners. An analysis of producer behavior under alternative bidding mechanisms can suggest possible leasing strategies. In those jurisdictions where auctions are not used, the analysis provides a basis for comparison with current practice. Secondly, mineral rights can be taxed, in a simple model, without distorting production and, therefore, without deadweight loss. When production complexities, such as uncertainty, are added, devising a tax which accomplishes this for privately-owned minerals may prove difficult. The model in this paper could be used to analyze the effect on exploration of some common taxes. In addition, the analysis of mineral rights auctions helps determine who bears the burden of resource taxation. Finally, to the extent that current practice in bidding for mineral leases leads to inefficient or otherwise socially undesirable outcomes, government intervention may be appropriate.

The following section discusses the literature regarding optimal oil lease auctions, particularly those papers discussing forms of payment — bonus, royalty, profit-share, full-share. The recommended auction from the landowner's perspective has profit-share or full-share bidding.

Section 3 emphasizes three points which suggest that full- and profit-sharing might be dominated by bonus bidding: cost of monitoring, comparability of offers, and risk aversion on the part of the landowner. Section 4 contains the most significant contributions of this paper. The advantage of a minimum price and the effect on the seller's choice of such a reservation price of the possibility of collusion or asymmetries among bidders regarding costs or information is examined. A general theorem on the form the reservation price should take when both seller and buyer can observe something correlated with the value of the object ex-post is proven.

The issues raised in Sections 3 and 4 show that the typical practice in American oil leases — bonus bidding with a fixed minimum royalty payment -- makes the landowner better off than the previous theoretical literature would suggest. The final section summarizes the results and draws additional policy conclusions.

2. Literature on Oil Lease Auctions

A growing body of literature on auction theory has developed from a seminal paper by William Vickery [1961].[1] Auction markets are characterized by a fundamental asymmetry: a single seller of an indivisible object faces a number of potential buyers. Most of the analysis has been focused on the type of auction -- the rules by which bids are presented and the level of payments are calculated from the bids.[2]

Less attention has been paid in the auction literature to the form of payment made between the winner of the auction (and possibly other participants) and the seller. This is primarily because, in many auctions, the seller cannot observe the true value of the auctioned item to the buyer. Maskin and Riley [1982] show that, when buyer and seller are either risk-neutral or risk-averse, it does not pay the seller to merely randomize the payment from the buyer. In mineral rights auctions, however, it is possible for the seller to observe something correlated with at least the ex-post value to the buyer.

Four different forms of payment will be considered in this essay. The first is full sharing -- bids are the fraction of the net value of the prospect that the buyer is willing to give the landowner. The bidder's costs include a market rate of return on any capital inputs, and if costs are greater than revenues the landowner must cover them. The second form of payment is profit (but not loss) sharing, which is identical to full sharing except that the landowner does not cover losses. The third type is royalty bidding, which involve fractions of the gross sales revenue which must be paid to the landowner. The final form of payment that will be considered is bonus bidding, where the payment is an unconditional cash payment, independent of any discovery that might be made. These are the forms of payment which have been discussed in the theoretical literature on oil leases.

An interesting model of rent capture under alternative payment structures has been developed by Reece [1979]. In building on earlier work by Wilson [1977], himself [1978], and others, he considers a prospect with a common, unknown, nonnegative value and known fixed exploration cost. A known number of risk-neutral symmetric bidders get different, unbiased signals about the value. Nash equilibria in the bidding strategies for a sealed-bid first-price auction are examined, with two of the important parameters being the number of bidders and the variance of the signal. The expected results are obtained: as the number of bidders grows or the variance of the signal shrinks, landowner revenue is increased. The model has bidders take into account the information that the winning bidder received the highest value estimate, which is therefore probably optimistic. The expected value of the prospect conditional on being the winning bidder is lower than the value estimate of the winner. Bidders bid even lower than the expected net value of the prospect conditional on winning in the hopes of making money (recall that Reece is considering a sealed high bid auction).

More interesting are the results on how landowner revenue varies with the form of payment. Reece considers three possibilities: profit-share, royalty, and bonus bidding. In all three cases, bidder profits depend on the value of the prospect, though to different degrees. Bidding strategies are different for the various payment structures, but the expected profit to the winning bidder is positive in all cases. Landowners do not receive the full expected value of their mineral rights under any of these payment structures, but Reece demonstrates that they do better under profit-sharing than with royalty bidding, with royalties dominating bonus bids. Intuitively, the ranking is in inverse order of the importance of the value estimate to the bidder.

Leland [1978] considers the three payment structures in a simpler model with risk-averse buyer and seller. Profit sharing results in the landowner sharing dry-hole risks and cost uncertainty with the winning bidder, while under royalty bidding the landowner shares only dry-hole risk with the developer. If the payment is made as a lease bonus, the landowner bears neither risk. If, as Leland expects, bidders are more risk-averse than landowners, he obtains the same ranking as Reece: profit-share, royalty, bonus.

If the landowner is risk-neutral, no bonus payment is desirable.

Given the rank ordering suggested by these two models, it seems that the current practice of bonus payments subject to bidding combined with a fixed royalty rate (which is usually either 12.5 percent or 16.67 percent of the gross revenue) is decidedly suboptimal from the landowner's point of view. This apparent inferiority of payment structure becomes stronger when one considers full sharing of profits and losses, which under either of the above models leads to complete capture of the resource rent by the landowner.[3] Costs, again, include a market rate of return on all capital inputs so any bidder would be willing to undertake the project if the landowner had a 100 percent share. This would be the universal bid if there were competitive bidding, and so full sharing would amount to a hiring of the developers by the landowner. The remainder of this essay is devoted to theoretical results which provide possible explanations for the observed choice of payment structure.

3. The Effect on Payment Structure of Monitoring Costs, Offer Comparability, and Landowner Risk Aversion

The simplest, but not least important, explanation for the absence of full sharing and profit sharing payment structures is the cost of monitoring these arrangements.[4] Both plans require that the landowner keep track of output, arms-length prices, capital costs (including an agreed rate of return), operating costs and overhead. Under profit sharing, the developer has an incentive to shift as much cost as possible from dry holes to successful wells, since he does not get reimbursed for the losses suffered on dry holes. If all bidders knew that they could hide the same portion of their profits, they would raise their bid correspondingly, but, even in this case, there would be a wasteful duplication of accounting services. In addition, the developer has less incentive to control costs than if there were no sharing, since he retains only a fraction of the benefits of such efforts. This incentive problem is much worse under full-sharing, since the landowner pays the share of losses, as well as collecting the share of profits. In the extreme case of complete sharing, the producer has no incentive to minimize costs and the landowner must monitor effort as well. Costly bargaining or arbitration may become necessary if disputes arise. These objections are somewhat less critical if the government is the owner of the mineral rights, since it must already examine the firm's books for income tax purposes and may have criminal sanctions for cheaters.[5] The difficulties with assessing project-specific shares and the incentive problems remain even in this case. For royalty payments, the landowner need only watch physical output and some agreed-upon price; bonus payments require no landowner effort.

The need for the landowner to be able to compare alternative offers should also influence the payment structure. If the landowner or bidder is unable to rank offers accurately, expected rent will be lost. This suggest that payment structures that vary in two or more dimensions would be avoided, since the landowner lacks the information to make the appropriate evaluation. It is another argument against profit or full sharing if producer's costs can differ. An offer of 80 percent of the profits from a high cost bidder may yield less revenue tha a ten percent offer from an efficient one. A related issue creates problems for full-share bidding. The landowner will not learn from an unrestricted auction whether he will make money by undertaking the project; in fact, if the project is a loser, bidders would be willing to offer more than a 100 percent share for the rights. The landowner would need to restrict bids to below 100 percent. If operating costs can differ, royalty bidding will be less desirable in terms of comparability than bonus bidding, since production paths might be affected and royalty collections vary.

Finally, the need for comparability of offers suggests an explanation for the existence of diligence clauses in oil leases. If the seller is due any contingent payment, he will need to assess its present value, both to determine his own wealth and to choose among firms. A diligence clause will assure that potential bidders have similar development plans. Such a clause would specify that various activites must be completed by a certain date or the lease will be forfeit. Under profit sharing, the landowner, even if he could compare offers, might want a diligence clause if he had a higher discount rate than the firm. With royalty bidding, another possible reason is that the development schedule maximizing profits may not be the same as the one maximizing the present value of royalties. If the only payment made by the bidder is an unconditional bonus, the only incentive for a diligence clause is one common to all: the land has a valuable alternative use.

A third factor in the choice of payment structure might be risk aversion on the part of landowners or bidders. Two kinds of uncertainty are present in mineral rights auctions: the value of the prospect is uncertain and a bidder might lose the auction when the winning bid is lower than what he is willing to pay.

Leland [1978] assumes that firms have identical information and are competitive, thus avoiding the second type of uncertainty. He is concerned with the optimal degree of risk-sharing between the government as landowner and the developer. The principal comment to be made on his results in the case of private oil lease auctions is that the landowner, rather than the bidders, is likely to exhibit the greater degree of risk aversion. Bidders are able, through joint ventures if sheer size does not suffice, to effectively diversify their oil prospects to insure against dry-hole risk.

Even if the companies themselves cannot accomplish this diversification internally, shareholders may do it for them. As for the risk arising from uncertain oil prices, the stock market may prefer that the company not attempt a partial internal diversification, especially since oil price movements may be negatively correlated with market movements. When the variation in oil price is due to demand shifts, one would expect a positive correlation with the general market, but when, as in the two price shocks of the last decade, there are supply shifts, there should be a negative relationship between oil industry profits and the rest of the market.

For landowners, including some governments, the prospect represents a large fraction of wealth. It cannot be diversified out of by selling a share in the land, since that amounts to accepting a bonus bid for the mineral rights. Therefore, this gives the landowners a reason to pass the dry hole and price and cost uncertainty risk onto the bidder by taking payment in the form of a bonus, with royalty bidding, profit sharing, and full sharing being ranked below it in that order. A full development of the optimal auction under both types of uncertainty, with correlation of value estimates, under risk aversion has been attempted neither here nor in the literature, so the results above remain suggestive, not conclusive.

Nevertheless, the three considerations discussed in this section lead to the same rank ordering of payment structures: bonus, royalty, profit-share, and full-share. Since this is the reverse of that suggested in Reece [1979] and Leland [1978], the conclusions of those papers are weakened to the extent that these considerations are important. They also help explain some of the observed features of oil lease auctions, especially when combined with the next section.

4. Minimum Prices

The use of minimum (or reservation) prices greater than the value of item to a seller has a social cost. When the seller retains the item which is valued more highly by a bidder in the independent values model,[6] a social loss occurs. The equivalent loss in the mineral rights model[7] occurs when a prospect with positive expected net present value is undeveloped.

Yet Riley and Samuelson [1981] and Myerson [1981] have shown that it is optimal, under certain conditions, for the seller to risk this loss. The compensating private (but not social) gain arises from extracting additional revenue from the lonely high bidder -- that is, when exactly one bidder attaches a value to the item higher than the reservation price.

One of the interesting characteristics of the Riley and Samuelson result is that the optimal reservation value did not depend on the number of potential bidders or on the type of auction (within a wide class of auction rules). When the valuations of the bidders are not independent, these results must be modified.

In the common value model, the expected value of the item to a bidder depends on the number of bidders as well as the bidder's value estimate, since the winner's curse must be considered. If the value estimates are independent draws from some underlying distribution and if the only additional information conveyed by winning the auction is that the other potential bidders had lower value estimates, the expected value of the item conditional on winning with bid b is:

$$\frac{\int V F_n(b|V) r(V|S) dV}{\int F_n(b,V) r(V|S) dV} \qquad (1)$$

where $F_n(b,V)$ is the probability that the firm's opponents will bid less than b for a lease with true value V, V is the true value, s is the value signal received, and $r(V|s)$ is the conditional probability density function of the true value V given signal s.[8] The denominator is the probability of winning the auction when the value estimate received is s. The numerator is the expected value won when bid b is submitted.

The expected value of the prospect for the winning bidder in the common value model depends on the type of auction as well as the number of bidders. As noted by Milgrom and Weber [1982], some auctions convey more information regarding value estimates than others.

Fortunately, some general statements regarding reservation values can be made for the common value model, at least when the auction rule is in the class analyzed by Riley and Samuelson. The optimal reservation price depends on the distribution of the expected value of the object to the winning bidder given the information available to him. The optimal reservation value therefore depends on both the number of bidders and the auction type through that distribution.

The model has n symmetric bidders, each of whom gets an independent, unbiased signal s, drawn from an identical distribution, of the unknown true value V of the single, indivisible object. The bidders and the seller are assumed to be risk-neutral. The bidders are assumed to behave noncooperatively, following, in equilibrium, some common bidding strategy $b(E(V))$ where $E(V)$ is the expected value of the object to the bidder conditional on the information available to the winner. The distribution of these conditional expected values $G(E(V))$ is assumed to be strictly increasing and differentiable over the interval $[E(\underline{v}), E(v')]$ with $G(E(\underline{v})) = 0$ and $G(E(v')) = 1$. The seller's valuation of the object is v_0, which for analytical convenience is assumed to be independent of the bidder's value estimates.[9]

The seller is assumed to choose an auction rule which satisfies four properties. First, a buyer can make any bid above some minimum reservation price set by the seller. Second, the buyer making the highest bid is awarded the object. Third, the auction rule is anonymous, treating all bidders alike. Fourth, a common equilibrium bidding strategy $b(E_i(V))$ exists with bids being a strictly increasing function of the expected value of the object conditional on the information available to the winner. Following Riley and Samuelson [1981], a rule with these properties shall be called a member of the family A of auction rules.

The optimal reservation value for the seller for this model is shown in the following theorem. The theorem and proof follow that of Riley and Samuelson modified for the common value model. The proof is in the Appendix.

<u>THEOREM 1</u>: If buyers are risk-neutral in the common value model, the members of the family A of auction rules which maximize the expected gain of the seller are those for which the reservation expected value of a bidder conditional on the information available to a winner $E(v_*)$, below which it is not worthwhile bidding, satisfies:

$$E(v_*) = v_0 + [1 - G(E(v_*))]/G'(E(v_*)). \tag{2}$$

Several observations can be made regarding Theorem 1. First, as in the independent values model, the seller should set a reservation conditional expected value strictly higher than his own personal valuation. The intuition is the same as in the independent values model. At reservation conditional expected values just above the seller's own valuation, the situations where the seller loses (the highest conditional expected value being between v_0 and $E(v_*)$) arise less frequently than the situations where the seller gains (the highest conditional expected value being above v_* with the second highest being below v_0), while the magnitudes of the gains and losses are approximately the same.

It is also worth noting that the proof of Theorem 1 is virtually identical to the proof of the corresponding proposition in Riley and Samuelson. There is a critical difference between the two models, however. The distribution $G(E(V))$ depends on the information available to the winning bidder. It therefore is different for different n, since the information conveyed by winning varies with the number of bidders. This is developed in Theorem 2.

The information received by the winning bidder also depends on the type of auction. In the sealed-bid first-price auction, for example, the only information conveyed is that the winning bidder had the highest value estimate, while in the idealized English auction the winner knows all lower value estimates. The proof of Theorem 1 therefore did not contradict the Milgrom and Weber [1982] result that, under the assumptions of the theorem, English auctions have higher expected seller revenue than sealed high bid auctions. The two auctions have different distributions $G(E(V))$, and therefore may have different expected seller revenue even if the reservation conditional expected value is the same. Moreover, the optimal reservation value in the common value model may vary with auction type as well.

The following theorem, as noted earlier, develops the effect of the number of bidders on the minimum price:

<u>THEOREM 2</u>: In the model of Theorem 1, the reservation conditional expected value which maximizes expected seller revenue will depend on n, the number of potential bidders. The optimal reservation conditional expected value will increase or decrease with the number of bidders according to whether:

$$G''(E(v_*)) + [G'(E(v_*))]^2/[1 - G(E(v_*))] \tag{3}$$

is positive or negative.

<u>PROOF</u>: The proof just involves partially differentiating the expression for the optimal reservation conditional expected value from Theorem 1 with respect to n. The derivative of v_0 with respect to n is zero by the assumption that the seller's valuation is independent of the bidder. The partial derivative of the second term of Equation (2) is:

$$[-\frac{G'(E(V_*))}{G'(E(V_*))} - \frac{(1-G(E(V_*)))G''(E(V_*))}{[G'(E(V_*)]^2}] \frac{\partial E(V_*)}{\partial n} \tag{4}$$

Now the expected value of the object to a bidder, conditional on his value estimate and winning the auction, is a decreasing function of the number of bidders. Having the highest value estimate is worse news as the number of potential bidders increases. The sign of the derivative of the optimal reservation conditional expected value is therefore opposite to the sign of the expression in the brackets. The first term is just -1, so the sign of the bracketed expression depends on whether the second term is greater or less than 1. It will be less than 1, and therefore the optimal reservation conditional expected value will increase with the number of bidders, whenever the expression in the statement of the theorem is positive; it will be greater than 1 when it is negative. Q.E.D.

<u>Remark</u>: Since $1 > (1-G(E(v_*))) > 0$ and since $(G')^2$ is positive, the second term is positive and it is likely that the optimal reservation price increases with the number of bidders in the common value model when symmetric risk-neutral bidders behave non-cooperatively.

Both the Riley and Samuelson results and the preceding theorems on minimum prices were derived under the assumptions that bidders were symmetric and behaved non-cooperatively. When these assumptions are relaxed, expected seller revenue falls due to reduced effective competition. The seller should probably respond to this by increasing his minimum price. The intuition is straightforward: the benefits of increasing the minimum price have grown, since the likelihood of a lonely high bidder is greater, while the expected cost of raising the price has remained unchanged. The following theorems state this point more rigorously for the case of collusive behavior.

In what follows, I shall restrict the family of auction rules somewhat further. The family B of auction rules satisfies all the requirements of the family A and, in addition, has the property that only the winner of the auction makes any payment. This restriction ensures that the reservation value (in the independent values model) or the reservation conditional expected value (in the common value model) is equal to the minimum price set by the seller. All of the four common auction types (English or oral, Dutch, sealed first-price, and sealed second-price) are members of the family B.

For the independent values model, the following assumptions shall be made. The values of the item to the n bidders are independently and identically distributed, drawn from a common distribution $F(v)$ with $F(\underline{v}) = 1$, and $F(v)$ strictly increasing and differentiable over the interval $[\underline{v}, v']$. The seller's personal valuation is v_0 and the reservation value or minimum price is v_*. Then,

THEOREM 3: In the independent values model, if the seller believes that there is a positive probability of the bidders forming a stable cartel, the reservation value which maximizes expected seller revenue, in the family B of auction rules, is higher than if he believed that the probability was zero. Moreover, the optimal reservation value is an increasing function of the perceived probability of a stable cartel.

PROOF: The formal proof is given in the Appendix. Intuitively, whenever the cartel forms, the seller gets his reservation price if the highest valuation is above that price. In this case a higher reservation value increases seller revenue by the amount of the increase whenever the highest valuation is above the reservation value. By contrast, in the non-cooperative equilibrium, the seller gains the increase only when exactly one bidder has a higher reservation value. The loss from a higher reservation price occur with the same frequency and are of the same magnitude in the two cases, since the probability that the highest valuation is below a given reservation price is the same. Since the gain from a higher reservation value at any reservation value is strictly greater when there is a positive probability of a stable cartel and the loss is the same, a seller believing that the probability is positive will increase his expected revenue by raising the reservation value above the optimal reservation price for the non-cooperative case. Since the gain increases as the probability of a cartel increases, so does the optimal reservation value.

THEOREM 4: In the common value model, as described in Theorem 1, if the seller believes that there is a positive probability that there is a stable cartel among the bidders, the reservation conditional expected value or minimum price which maximizes expected seller revenue, for an auction in the family B, is higher than if the seller believed that the probability was zero. The optimal minimum price is an increasing function of the perceived probability.

PROOF: The proof is similar to the previous one; the formal proof is, once again, in the Appendix. If there is a stable cartel, the seller gains from the higher reservation conditional expected value whenever the expected value of the object conditional on the information available to the cartel is greater than that value. This has higher frequency than the corresponding gain when bidders behave non-cooperatively -- when exactly one bidder has a higher conditional expected value. The expected loss is the same in the two cases since the highest conditional expected value among n non-cooperative bidders following the equilibrium strategy is an unbiased estimator of the expected value of the object conditional on the information possessed by the cartel. In either case, therefore, the frequency of the expected loss is equal to the expected probability that the highest conditional expected value is below the reservation value and the expected magnitude of the loss will be identical. Since the gain from a higher reservation conditional expected value is greater for all reservation values while the loss is the same, the seller will increase his expected revenue when there is a positive probability of a stable bidder cartel by raising his reservation conditional expected value above the optimal level for the non-cooperative auction. Since the gain is monotonically increasing in the probability of a stable cartel, so is the optimal reservation conditional expected value. Q.E.D.

The logic of Theorems 3 and 4 could be applied to other circumstances when there is reduced effective competition. Consider an item with a common value about which there is only public information. If there is more than one bidder, the optimal minimum price is zero. If one of the bidders has private information, the bidders with less information must bid more conservatively. Hansen [1984] shows that the only evolutionarily stable equilibrium in the

English and second-price auction is for the bidders with only public information to bid the minimum possible value given the private information. Milgrom and Weber [1981] show that the bidders with public information should follow a conservative mixed strategy which leaves the bidder with private information positive profits. In either case, a positive minimum price is now optimal for the seller.

These results suggest that reservation prices are quite likely to be observed in oil lease auctions, and that these may be substantially above the landowner's personal valuation. An important question remains regarding the form of payment a seller should choose for the reservation price.

The next theorems address this point. Consider a reservation value of the form $v_* + f(V-v_*)$ where $1 > f' > 0$ and $f(0) = 0$. Examples of such reservation prices would be minimum royalty payments or profit (but not loss) shares. It will be shown that sellers have higher expected revenues and the optimal break-even value v_* is lower when reservation values can take this form than when they must be independent of the ex-post value for the four common auction types. Since the optimal break-even value is lower, society will be better off as well when reservation values can take this form.

THEOREM 5: In the independent values model, a reservation value of the form:

$$v_* + f(V-v_*),\ 1 > f' > 0,\ f(0) = 0, \tag{5}$$

where V is the valuation of the winning bidder, will have higher seller revenue than when f is identically zero for the sealed high bid, second-price, Dutch and English auctions. In addition, the optimal v_* when the reservation value has this form is lower than when f is identically zero in those auctions.

PROOF: By the construction of $f(x)$, the incentive to participate in the four auctions is unaffected by the existence of f. Since only the winning bidder pays the seller under these auctions, and since the reservation value including f is less than V, the value of the item to the winner, for $V > v_*$, a bidder will submit a bid if and only if his valuation is greater than v_*, exactly as if f were identically zero.

For the second-price and English auctions, the common equilibrium bidding strategy is the same regardless of f, bidders bidding (or having as their dropout price) their valuations. The seller will therefore never lose when f is of the prescribed form. The seller will gain when all but the winning bidder have valuations lower than $v_* + f(V-v_*)$. The probability of this is greater than zero and the gain over f identically zero is strictly positive by the definition of f.

To prove the dominance over f identically zero when the sealed high bid auction is used, I appeal to another result of Riley and Samuelson. They prove for the independent values model that the common equilibrium bidding strategy in the sealed high bid auction with reservation value v_* is:

$$b(V) = V - \int_{V_*}^{V} F^{n-1}(x)dx/F^{n-1}(V) \tag{6}$$

Introducing f will not cause any bidders to wish to lower their bids, so there is no loss to the seller compared to f identically zero. The introduction of f does create a positive probability that the winning bid will be higher. This can be seen by the above bidding strategy $b(v)$ with respect to v and evaluating at v_*:

$$\left.\frac{\partial b}{\partial V}\right|_{V_*} = 1 - \frac{F^{n-1}(V_*)}{F^{n-1}(V_*)} + \left[\frac{\int_{V_*}^{V_*} F^{n-1}(x)dx(n-1)F'(V_*)}{F^{n-2}(V_*)}\right] . \tag{7}$$

The last term is zero, so the slope of the bid function at v_* is also zero. Since f has a positive slope, at least those bidders with valuations near v_* will wish to increase their bids when f is introduced, since otherwise they have no hope of making positive profits. There is, then, a positive probability that introducing f will strictly increase seller revenue and no possibility that seller revenue will fall, so expected seller revenue is higher than when f is identically zero. The Dutch auction is strategically equivalent in this model, so the proof applies to this auction as well.

The second part of the theorem stated that the seller should choose a lower v_* with f than with f identically zero. Recall that the tradeoff faced by the seller in considering a higher reservation price was the gain from the lonely high bidder and the loss from discouraging all bids. At any given v_*, the loss from increasing v_* is the same whether f identically zero or not; it is equal to $(v_*-v_0)F^n(v_*)$. However, the gain from increasing v_* is smaller when f is positive than when it is identically zero. The seller was already receiving a fraction of the difference between V and v_* when there was a lonely high bidder. Since the expected gain from a higher reservation value is strictly lower when f is positive for all values of v_*, while the expected loss is the same, the optimal v_* is strictly lower when f is possible than when f is identically zero. Q.E.D.

<u>THEOREM 6</u>: In the common value model, a reservation conditional expected value of the form:

$$E(v_*) + f(E(v) - E(v_*)),\ 1 > f' > 0,\ f(0) = 0, \tag{8}$$

where E(v) is the conditional expected value of the winning bidder, will have higher seller revenue in the sealed high bid, second-price, Dutch and English acution that if f were identically zero. The optimal $E(v_*)$ when the reservation conditional expected value has this form is lower than when f is identically zero.

PROOF: The proof follows that of Theorem 5 with only minor alterations in terminology and notation, and so it will not be presented.

The first four theorems of this section provide additional explanations for the use of reservation prices by the seller. Theorems 5 and 6 show that these reservation prices should be conditioned on something correlated with the value of the item to the winning bidder, if that can be observed and other things being equal.

The intuition behind these results can be illustrated with full sharing between landowner (or seller) and bidders. In this case the landowner could insist on anything less than a 100 percent share without causing any prospect with positive expected present value to be abandoned. This is an extreme example of the reservation price varying with the value of the item.

This theorem provides an explanation for the reservation price in oil leases being a fixed royalty rather than a fixed bonus payment. Such an explanation is important, given the discussion in Section 3 which suggested that a bonus payment was best. The argument in favor of the landowner using only bonus payments would have been strengthened by noting that the tax law in the United States favors bonus payments because of the expensing of dry holes and that royalty payments cause a distortion in the starting time for the exploitation of a field and can cause early abandonment of a field if costs rise faster than prices.

The reservation price that has been discussed has been assumed to be binding and public. If it were not binding — that is, if the seller could lower it, if no bids were forthcoming -- then bidders could treat the reservation price as a call in a Dutch auction and decide whether or not to take it on that basis. There is clearly no purpose in not making the existence of a reservation price known, since the advantage of a reservation price lies in altering bidder's behavior. Riley and Samuelson [1981] show that the seller does not gain by concealing the reservation value in the four common auctions in the independent values model with risk-neutral non-cooperative bidders. Milgrom and Weber [1982] show that it does not pay the seller to keep secret any information, including a reserve price, which is correlated with the value estimates for the common value model in the four auctions.

5. Summary of Results and Policy Conclusions

Six major complications to the existing theoretical models of oil lease auctions have been examined:

(1) the cost of monitoring different contracts,

(2) the need for comparability among bids,

(3) risk aversion on the part of the landowner,

(4) allowing sellers to set a minimum price,

(5) the possibility of collusion among bidders, and

(6) the possibility of asymmetry among bidders on costs or information.

These seem to be realistic difficulties faced by owners of mineral rights, especially private individuals holding oil prospects in auctioning off drilling leases. Together, these six considerations -- some of which had previously been considered by theorists but not applied to oil leases -- suggest that the current practice in American oil lease auctions of bonus bidding and a fixed royalty rate is superior, from the landowner's perspective, to the structure suggested in the received theory on oil auctions:

either full sharing or profit sharing.

Cost of monitoring, offer comparability, and landowner risk aversion all tend to imply that bonus bidding is the best for the landowner. All three arguments are more powerful against profit sharing and particularly full sharing than against royalty payments. The seller should set a minimum price above his personal valuation of the item. When collusion or asymmetry are possibilities, the landowner should raise this price still further.

Since the landowner is assumed to have less information than the bidders regarding the value of the prospect, the result that the seller should condition his minimum payment on an observable variable correlated with the value is significant. It helps explain that the minimum payment takes the form of a royalty rather than a bonus. The first three considerations provide explanations for the minimum being a royalty rather than profit or full sharing, as well as justify the use of bonuses for the bid above the reservation price.

It is interesting to note that the possibility of collusion, asymmetry among bidders, and landowner risk aversion should also affect the choice of auctio type. All three considerations suggest that sealed-bid first-price auctions are likely to yield higher seller revenue. Robinson [1984] shows that cartels are stable under second-price or English auctions, but not in sealed first-price auctions. Hansen [1984] shows that sealed first price generates greater revenue when one bidder has superior information; Maskin and Riley [1982] demonstrate the superiority of sealed first-price when either seller or buyer is risk averse.[10] When these considerations are important, the result of Milgrom and Weber [1982] that English auctions generated the highest revenue in a common value model may no longer hold. These same considerations may help explain the observed choice of sealed first price auctions, as well as the payment structure, in U.S. offshore (OCS) oil lease auctions.

These results have several policy implications. The most obvious relate to the appropriate choice of structure for oil lease auctions on publicly owned land or when the government owns the mineral rights. To the extent that the five considerations are relevant in this situation, the auction structure used in U.S. OCS lease auctions is recommended.

A number of factors are likely to differ when the government is the seller. Perhaps most important, in most countries, contracts between producers and the government are unenforceable if the government breaches, especially when the signatory government loses power. This problem is analyzed in Nellor and Robinson [1984] using a model quite different from that in this paper. Under plausible assumptions, it is found to be in the current government's interest to require a periodic payment of the life of the well, and for the payment to fluctuate with the net revenues from the resource, in order to reduce expropriation risk and coax higher payments from producers. This result is, fortunately, not at all inconsistent with bonus bidding and a fixed royalty rate.

The possibility of collusion and asymmetry among bidders do not become unimportant when the government owns the mineral rights. Therefore, the implications that the seller should utilize sealed-bid first-price auctions and set a reservation price that increases with the value carry through. In practice, the U.S. government reserves the right to refuse to award a prospect to the high bidder, thus adding a second, secret reservation price on top of the fixed royalty rate.

The issues raised in Section 3 are slightly less compelling when the government is the landowner. The government may have a lower cost of monitoring profit share agreements than private individuals particularly since it also collects corporate profits taxes. If the government own a large number of independent prospects, dry hole risks will be small, and if the government is large relative to a single prospect, there is good reason to expect landowner risk aversion to be relatively minor. Past experience with bidders may diminish the obstacles to comparing offers.

For these reasons, the case for the reservation price taking the form of a royalty, with its attendant distortions, rather than a fraction of profits or full ownership is weak when the government owns the rights. Certainly, all three options merit further analysis in a practical situation.

It is worth noting that, under any of the recommended policies, the government does not capture all the expected scarcity rent. The producers will earn positive profits when they have private information or when collusion occurs. The reservation price will discourage some development of profitable fields, so some rent will be lost to both government and producer. If bidders do not collude, some rent will be lost to both even when there is no reservation price if various bidders have private information. Since the information is not shared, some projects which are unprofitable, given all available information, are undertaken while some good prospects go undeveloped.

Taxes on a mineral project or company may be viewed as a reservation price set by the government. An ad valorem severance tax is equivalent to a royalty payment at the same rate. The American "windfall profits" tax is a piecewise-linear royalty with different rates applying to different pools. The "resource rent" tax proposed by Garnaut and Clunies Ross [1975] and Garnaut [1978] is equivalent to profit sharing, since losses are not reimbursed. The Brown [1948] or cash flow tax is a form of full sharing.

The six considerations apply to these, and other, taxes when the government owns the mineral rights. If the government is equally likely to change the rates at some future time, a bidder will not care whether the government call the reservation price a severance tax or a royalty. The arguments of Section 3, therefore, weigh against the Brown and resource rent taxes when the government owns the mineral rights.

When the rights are privately owned, the analysis of the effects of taxes on landowners and producer becomes quite complex. Landowner revenue will fall as government tax revenue increases, since the landowner was previously maximizing, but it is still possible for the landowner to be better off if there is sufficient reduction in the cost of monitoring. A more specific model is needed to do a useful analysis of the incidence of various taxes when there is private ownership, perhaps a modified version of that of Reece [1979].

Much of the analysis of this paper has more general application than oil lease, or even mineral rights, auctions. In common value situations (e.g., art auctions when resale is important or authenticity in question), a minimum price above the seller's personal valuation should be set. The possibility of collusion implies the minimum should be higher; when the seller is less informed than the buyer, the reserve price should be an increasing function of the ex-post value. These results, together with others in the literature, suggest that examination of markets with different characteristics would be fruitful.

FOOTNOTES

*Discussions with John Haltiwanger, Robert Hansen, Benjamin Klein, Edward Leamer, Paul Milgrom, Michael Porter, Steve Salant, John Shoven and David Starrett are gratefully acknowledged. Michael Boskin, Timothy Bresnahan and John Riley made many useful suggestions. The usual caveat applies. Financial support was received from the Center for Economic Policy Research (Stanford), the Foundation for Research in Economics and Education (UCLA), and the Centre of Policy Studies (Monash).

[1] See Milgrom [1979] and Engelbrecht-Wiggans [1980].

[2] Four types of auctions are commonly considered (though some of the later results have wider application): English (or oral), Dutch, sealed-bid first-price (or high bid), and sealed-bid second price. In English auctions, the price rises incrementally until only one bidder remains, who wins the object for that price. For ease of analysis, a pure form of the auction is often considered: everyone intersted in purchasing the object would raise his hand, dropping it only when the auctioneer had increased the price beyond that which they were willing to pay. The auction would end when only one hand was up. In Dutch auctions, the price starts at a very high level, then falls until someone bids. That person wins the prize. It is usually assumed that bidders can observe each other in English and Dutch auctions. In sealed-bid first-price auctions, potential buyers submit closed offers, with the award going to the high bidder. The same is true in second-price auctions, except that the high bidder pays only what the second-highest bidder offered.

[3] See Brown [1948] for a proposal along these lines.

[4] McDonald [1979] and Mikesell [1984] also note that monitoring costs may explain the absence of profit-sharing.

[5]Mikesell [1984] emphasizes this.

[6]In the independent values model, a bidder's value for the auctioned item is independent of the preferences or valuations of the seller or other bidders. For a more formal description, see Riley and Samuelson [1981] and below.

[7]In the mineral rights or common value model, the value of the item is the same to all bidders who, however, have different estimates of the value. See Milgrom and Weber [1982] and below for a more formal description.

[8]Derived by Reece [1978] and previously by Wilson [1977]. Different notation is used below.

[9]The seller's valuation would be independent of the bidder's value estimates if, for example, the seller would use the land for another purpose, such as farming.

[10]For other discussions of the effects on auction type of risk aversion, see Milgrom and Weber [1982]. See Milgrom and Weber [1981] for the effects of asymmetric information and Riley [1983] for the effects of asymmetric beliefs regarding costs.

APPENDIX

PROOF OF THEOREM 1: The expected return for a bidder is equal the expected value of the prospect conditional on the information available to a winner times the probability of winning minus the expected payment. Following Riley and Samuelson's proof for the independent value model, I shall derive expressions for the expected return to a bidder and the probability of winning. The expected payment of a bidder can then be derived using the above relationship. Given the symmetry of bidders and of the auction rule, expected seller revenue will be n times this payment. The optimal $E(v_*)$ can then be determined from the first-order condition.

With bidders assumed to behave non-cooperatively, a common strategy $b_i = b(E_i(V))$ is a Nash equilibrium if it is the optimal response of a bidder, say bidder 1, when the other bidders adopt that strategy. Since bidder 1 will wish to bid in the range of $b(E(V))$ if the others follow that strategy, it follows that $b(E(V))$ is an equilibrium strategy if bidder 1 can do no better than bid $b(E_1(V))$ among all possible arguments of $b(x)$.

The bid $b(x)$ is the winning bid if and only if all other bidders have made lower bids, by the properties of the auction rule. By those same properties, the equilibrium bid function is strictly increasing in $E(V)$, therefore bidder 1 wins if all other valuations conditional on the information available to the winner are less than x. Since the probability that a bidder has a conditional expected value less than x is $G(x)$, bidder 1 wins with probability $G^{n-1}(x)$. The expected return to bidder 1 bidding $b(x)$ can be expressed as:

$$E_1(V)G^{n-1}(x) - P(x) \tag{A1}$$

where $P(x)$ is the expected payment of bidder 1, given a bid $b(x)$. For

$b(E(V))$ to be an equilibrium strategy, bidder 1's optimal choice must be to set $x = E_1(V)$ and therefore the following first-order condition must hold:

$$E_1(V) \frac{d}{dx} G^{n-1}(x) - P'(x) = 0 \quad \text{at} \quad X = E_1(V) \tag{A2}$$

whenever $E_1(V)$ exceeds $E(v_*)$, the reservation conditional expected value. In order that a bidder be indifferent between not entering the auction and submitting the bid $b(E(v_*))$, $E(v_*)$ satisfies:

$$E(V_*)G^{n-1}(E(V_*)) - P(V_*) = 0. \tag{A3}$$

Integrating the first order condition over $E_1(V)$ and using the boundary condition, the expected payment for the equilibrium bidding strategy can be written as:

$$P(E_1(V)) = E_1(V)G^{n-1}(E_1(V)) - \int_{E(V_*)}^{E_1(V)} G^{n-1}(x)dx. \tag{A4}$$

The next step is to consider the auction from the point of view of the seller. The seller views the bidder's conditional expected value as a random variable, which he knows has a distribution $G(E(V))$. Therefore his expected revenue from bidder 1 is:

$$\bar{P}^1 = \int_{E(V_*)}^{E(V')} P(x)G'(x)dx. \tag{A5}$$

Substituting for $P(E_1(V))$ and integrating by parts, the expected seller revenue from bidder 1 can be written as:

$$\bar{P}^1 = \int_{E(V_*)}^{E(V')} [xG'(x) + G(x) - 1]G^{n-1}(x)dx. \tag{A6}$$

Given the symmetry of bidders, bidding strategy, and auction rule, total seller revenue is just n times the above expression.

For any reservation conditional expected value $E(v_*)$, there is a probability of $G^n(v_*)$ that all n buyers will decide not to submit a bid, in which case the gain to the seller will be his own personal valuation v_0. The total expected return to the seller is then:

$$V_0 G^n(E(V_*)) + n \int_{E(V_*)}^{E(V')} [xG'(x) + G(x) - 1]G^{n-1}(x)dx. \qquad (A7)$$

Differentiating with respect to $E(v_*)$, the first-order condition for maximizing expected seller revenue is obtained:

$$n[V_0 G'(E(V_*)) - E(V_*)G'(E(V_*)) - G(E(V_*)) + 1]G^{n-1}(E(v_*)) = 0. \qquad (A8)$$

Therefore the optimal $E(v_*)$ is as stated in the theorem. Q.E.D.

<u>PROOF OF THEOREM 3</u>: If complete collusion occurs and the auction rule is a member of the class B, the expected return to the seller will be

$$v_o F^n(v_*) + v_* (1 - F^n(v_*)) \qquad (A9)$$

since the cartel will pay the reservation value if and only if at least one of its members has a valuation above that reservation value. If the seller believes that the collusion will occur with probability π and that, if it does not occur, the bidders will choose the common equilibrium non-cooperative bidding strategy (as in Riley and Samuelson [1981]), then the total expected revenue to the seller will be

$$\pi[v_o F^n(v_*) + v_* [1 - F^n(v_*)]] \qquad (A10)$$

$$+ (1-\pi) [v_o F^n(v_*) + n \int_{v_*}^{v'} (vF'(v) + F(v) - 1) F^{n-1}(v)dv].$$

Differentiating with respect to v_*, the expected revenue of the seller is maximized for some v_* satisfying the condition

$$\pi[nv_o\ F^{n-1}(v_*)F'(v_*) + (1 - F^n(v_*)) - nv_*\ F^{n-1}(v_*)\ F'(v_*)]$$

$$+ (1-\pi)\ [n[v_oF'(v_*) - v_*F'(v_*) - F(v_*) + 1]\ F^{n-1}(v_*)] = 0. \qquad (A11)$$

Collecting all the terms multiplied by π, equation (A11) can be written

$$n(v_oF'(v_*) - v_*F'(v_*) - F(v_*) + 1)\ F^{n-1}(v_*) \qquad (A12)$$

$$+ \pi[n[v_o-v_*]\ F^{n-1}(v_*)\ F'(v_*) - n[v_o-v_*]\ F^{n-1}(v_*)\ F'(v_*)$$

$$+ (1 - F^n(v_*)) - (1 - F(v_*))\ nF^{n-1}(v_*)] = 0.$$

The first term in equation (A12) is the same as the first-order condition for the reservation value in the Riley-Samuelson case. The expression $n(v_o-v_*)$ $F^{n-1}(v_*)\ F'(v_*)$ cancels out of the second term, reflecting that the loss from increasing the reservation price is the same whether or not the cartel forms. Using the formula for a geometric series,

$$1 - F^n = (1-F)\ [1 + F + F^2 + \ldots + F^{n-1}]. \qquad (A13)$$

So the second term in equation (A12) can be written as

$$\pi(1 - F(v_*))\ [(1 + F(v_*) + \ldots + F^{n-1}(v_*) - nF^{n-1}(v_*)]. \qquad (A14)$$

Since $F(v_*) < 1$, $(1 + F + \ldots + F^{n-1})$ is greater than nF^{n-1} term by term, so the expression (A14) is always positive and is increasing in π. Therefore, the derivative of expected return with respect to v_* is still positive at the optimal reservation price when the seller believes that there is no possibility of collusion. The optimal reservation price for the seller is therefore an increasing function of the probability of collusion.

Q.E.D.

PROOF OF THEOREM 4: Let $G_n(E(v))$ be the cumulative distribution of the expected value of the object conditional on the information available to the cartel. Recall that $G(E(v))$ was the cumulative distribution of the expected value conditional on the information available to the winner, and so $G^n(E(v))$ is the cumulative distribution of the expected value to the winning bidder conditional on the information available to him. The expected value to the winner is an unbiased estimate of the true value. It is also an unbiased estimate of the expected value conditional on the information available to the cartel. Therefore

$$E(G_n(E(v))) = G^n(E(v)) \tag{A15}$$

for all $E(v)$.

If the cartel is complete and stable and the auction is a member of class B, the expected return to the seller will be

$$v_o G_n(E(v_*)) + E(v_*)\ (1 - G_n(E(v_*))) \tag{A16}$$

since the cartel will pay the minimum price $E(v_*)$ if and only if it is less than the expected value conditional on the information available to it. If the seller believes that the cartel will occur with probability π and that, otherwise, bidders will choose the common equilibrium non-cooperative bidding strategy, then the total expected revenue to the seller will be

$$\pi[v_o G_n(E(v_*)) + E(v_*)\ [1 - G_n(E(v_*))]] \tag{A17}$$

$$+ (1-\pi)\ [v_o G^n(E(v_*)) + n \int_{E(v_*)}^{E(v')} (xG'(x) + G(x) - 1)\ G^{n-1}(x)\ dx].$$

Differentiating with respect to $E(v_*)$ and collecting terms, the expected revenue of the seller is maximized for some v_* satisfying the condition

$$n[v_o G'(E(v_*)) - [(v_*)\ G'(E(v_*)) - G(E(v_*)) + 1]\ G^{n-1}(E(v_*)) \tag{A18}$$

$$+ \pi[n(v_o - E(v_*))\ [G_n(E(v_*))]' - n(v_o - E(v_*))\ [G^n(E(v_<))]'$$

$$+ (1 - G_n(E(v_*)) - (1 - G(E(v_*))\ nG^{n-1}(E(v_*))] = 0.$$

The first term in equation (A18) is the same as the first-order conditionin the non-cooperative case (A8). By (A15),

$$E(G_n(E(v_*)))' = [G^n(E(v_*))]' \quad \text{and} \quad E(G_n(E(v_*)) = G^n(E(v_*)).$$

The first two parts of the second term in (A18) cancel (since expected seller revenue is being maximized). The remaining portion of the second term of (A18) can, using the geometric series formula (A13), be rewritten as:

$$\pi(1 - G(E(v_*))\ [1 + G(E(v_*)) + \dots + G^{n-1}(E(v_*)) - nG^{n-1}(E(v_*))]. \tag{A19}$$

Since $G(E(v_*)) < 1$, $(1 + G + \dots + G^{n-1})$ is greater than nG^{n-1} term by term and the expression (A19) is positive and increasing in π. Therefore, the derivative of expected seller revenue with respect to the minimum price (A18) is still positive at the value of $E(v_*)$ which satisfies (A8). The optimal minimum price is therefore an increasing function of the probability of collusion. Q.E.D.

References

Brown, E.C. [1948], "Business-Income Taxation and Investment Incentives," in Income, Employment and Public Policy, Essays in Honour of Alvin, H. Hansen, Norton, New York.

Engelbrecht-Wiggans, R. [1980], "Auctions and Bidding Models: A Survey," Management Science, 26, 119-142.

Garnaut, R. [1978], "The Application of a Resource Rent Tax to the Australian Mining Sector," in Ben Smith (ed.), Taxation of the Mining Industry, Centre for Resource and Environmental Studies, Australian National University

__________, and Clunies Ross, A. [1975], "Uncertainty, Risk Aversion and the Taxing of Natural Resource Projects," Economic Journal, 85, 272-287.

Hansen, R. [1984], unpublished doctoral dissertation, University of California, Los Angeles.

Leland, H.E. [1978], "Optimal Risk Sharing and the Leasing of Natural Resources, With Application to Oil and Gas Leasing on the OCS," Quarterly Journal of Economics, 92, 413-438.

Maskin, E.S. and J.G. Riley [1982], "Optimal Auctions With Risk Averse Buyers," Discussion Paper, Cambridge University.

McDonald, S.L. [1979], The Leasing of Federal Lands for Fossil Fuels Production, Johns Hopkins, Baltimore.

Mikesell, R.F. [1984], Petroleum Company Operations and Agreements in the Developing Countries, Resources for the Future, Washington.

Milgrom, P.R. [1979], The Structure of Information in Competitive Bidding, Garland Publishing Co., New York.

__________, and R.J. Weber [1981], "The Value of Information in a Sealed-Bid Auction," Discussion Paper No. 462, Graduate School of Management, Northwestern University.

__________, and __________ [1982], "A Theory of Auctions and Competitive Bidding," Econometrica, 50, 1089-1122.

Myerson, R. [1981], "Optimal Auction Design," Mathematics of Operations Research, 6, 58-73.

Nellor, D., and M.S. Robinson [1984], "Binding Future Governments: Tax Contracts and Resource Development," UCLA Working Paper #297.

Reece, D.K. [1978], "Competitive Bidding for Offshore Petroleum Leases," Bell Journal of Economics, 9, 369-384.

__________ [1979], "Alternative Bidding Mechanisms for Offshore Petroleum Leases," Bell Journal of Economics, 10, 659-669.

Riley, J.G. [1983], "Auctions With Asymmetric Beliefs," Working Paper No. 254, University of California, Los Angeles.

__________ and W.F. Samuelson [1981], "Optimal Auctions," American Economic Review, 71, 381-392.

Robinson, M.S. [1984], "Collusion and the Choice of Auction," UCLA Working Paper #340.

Stigler, G.J. [1964], "A Theory of Oligopoly," Journal of Political Economy, 72, 44-61.

Vickrey, W. [1961], "Counterspeculation, Auctions, and Competitive Sealed Tenders," Journal of Finance, 16, 8-37.

Wilson, R. [1977], "A Bidding Model of Perfect Competition," Review of Economic Studies, 44, 511-518.