# COMPETITIVE SIGNALLING RECONSIDERED*

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In markets where product quality is diffuse and verification by buyers at the time of purchase is costly, sellers have an incentive to seek some indirect means of signalling high quality products. As emphasized by Spence, any activity that has a lower marginal cost for a seller of a higher quality product can act as a potential signal. However, with free entry and competition not only over price but also over signals, there is, in general, no Nash equilibrium.

In this paper it is argued that, if the Spencian model is modified in one simple way, then it is possible to derive conditions under which there is a unique Nash equilibrium. Essentially, for a given distribution of types of sellers, a Nash equilibrium obtains as long as the rate at which the marginal cost of signalling declines across types is sufficiently large.

The results are then applied to models of labor and insurance markets.

The large and rapidly growing literature on principal-agent problems is conveniently divided into papers which focus on problems of hidden actions and those which focus on problems of hidden knowledge. The latter are also naturally divided into studies of incentive schemes in which the principal is a monopolist[1] and those in which a large number of principals compete for agents' services. Here we focus on the many principal-many agent problem when knowledge is hidden.

The first formal discussion of the issues is provided by Spence [1973], who examines markets in which sellers (agents) have private information about the quality of their products. There is also some activity which is less costly for sellers with higher quality products. Recognizing that this activity is a potential "signal" of product quality, buyers pay a premium for higher levels of the signal.

Spence modelled all market participants as price takers. Each seller observes the market return to signalling and chooses that signal which is individually optimal. In equilibrium, all those buyers making trades based on signals find that their prior beliefs are confirmed <u>ex post</u>.

While very much in the spirit of traditional Walrasian, "price taking", models, the conclusion that emerges is strikingly different. Instead of there being a unique equilibrium (or possibly a finite set of equilibria), Spence shows that market signalling equilibria form a continuum.

More recent papers by Rothschild and Stiglitz [1976] and Riley [1975, 1979a] make it clear that this result is critically linked to the price taking assumption. In the traditional full information equilibrium, each agent is small relative to the markets in which he trades and there is no incentive to attempt price competition — hence the price taking assumption. However, with the externality that underlies a market signalling equilibrium, it is no longer necessarily the case that price taking behavior is individually rational for all market participants.

Focussing on the application of signalling to the purchase of insurance, Rothschild and Stiglitz consider the simplest case of two types of agents — high and low risk. They show that unless the proportion of high risk types is sufficiently great, all the "Walrasian" signalling equilibria are unstable. That is, there is always some alternative opportunity open to a buyer which, in the absence of reactions by other buyers, generates strictly greater expected profits. Equivalently if the market is modelled as a noncooperative game, in which the buyers (principals) first announce what they will pay for different levels of the signal, and sellers (agents) then respond, there is no Nash equilibrium in pure strategies.[2]

My own papers focus primarily on the opposite polar case — a continuum of agents. Adopting the game theoretic terminology, a central conclusion is that nonexistence is generic in the class of models considered by Spence. In particular raising the price offered to those choosing the lowest observed level of the signal is always profitable. Moreover, price competition may be profitable at higher levels of the signal as well.

There have been several attempts to overcome this failure to explain signalling behavior. Each of these builds on a paper by Wilson [1977] which begins with the premise that agents will anticipate the responses of others when they consider new actions. The least demanding of the alternative equilibrium concepts is the "reactive equilibrium."[3] Loosely, a set of strategies $s_1^*,\dots,s_n^*$ for $n$ competing agents is a reactive equilibrium if two conditions are satisfied. First, for any agent $i$ and any alternative strategy $s_i$ which raises $i$'s payoff there is another agent $j$ who can benefit by reacting at the expense of agent $i$. Second, there is no further reaction by a further agent which can make agent j's reactions unprofitable. The idea then, is that agent i will recognize agent j's clear incentive to react and therefore be deterred from choosing $s_i$ rather than $s_i^*$.

As argued in Riley [1979a], of the sets of signal-price contracts which separate out the different types of agents, there is a unique Pareto dominating set and this is a reactive equilibrium. Moreover, there can be no reactive equilibrium in which high quality-low signalling cost agents are pooled with low quality-high signalling cost agents. Thus the reactive equilibrium is unique.

While the assumption of this greater level of sophistication is plausible for some applications of the theory, there are other applications for which it is possible to take a more skeptical view. Thus in this paper an alternative way out of the nonexistence dilemma is examined. Instead of modifying the equilibrium concept, the route chosen is the adaption of the model itself. It is argued that, despite the negative conclusions of the published literature, there is a family of signalling models which generate an equilibrium satisfying the strong Nash equilibrium condition that all price competition must be unprofitable, in the absence of reactions by other price setters.

These models differ from those appearing in the literature in only one critical way. Rather than assume that all agents would enter a particular market in a world of perfect information it is assumed that, even in such a world, a positive fraction of the agents would choose not to participate. In the labor market, for example, suppose $\theta \in [0,1]$ is the productivity of a given type in the production of a particular commodity. Then, as long as there is some alternative job opportunity offering any worker a wage $w_A$ only those for whom $\theta > w_A$ have an incentive to produce this commodity.

Similarly, in the signalling of project quality by insider stockholding (Leland and Pyle [1980]) and the signalling of loan quality by collateral or loan size (Bestor [1984], Milde and Riley [1984]), those entrepreneurs with sufficiently low quality offerings would not be financed in a world of costless information about quality.

Even in insurance markets, with perfect information about loss probabilities, nonparticipation will often be plausible. Under fair insurance, the risk of loss $L$ with probability $p$ will be fully covered by a premium $pL$. Then those with sufficiently high probabilities of loss may be better off not undertaking the risky activity.

This simple modification of the basic Spencian model is important because it eliminates the profitability of price competition at the lowest observed level of the signal. The primary focus of the paper is then to seek out conditions under which price competition is also unprofitable at higher levels of the signal. As Spence emphasized, an activity is a potential signal if it is less costly for those agents selling products with a higher quality product. The central result of this paper is that if the proportional rate of decrease with respect to quality, of the marginal cost of signalling is sufficiently high, there exists a Nash equilibrium. That is, price competition is never profitable.

The paper is organized as follows. Section I lays out a principal agent model with hidden knowledge. Section II examines, in detail, a simple discrete example in which the quality of the product traded is independent of the level of the signal. This is essentially Spence's earliest model of a labor market, with education providing information but not increasing marginal productivity. Section III considers the same model under the assumption of a continuum of agents and shows that, in this limiting case as well, there are reasonable conditions under which a Nash equilibrium exists. Section IV returns to the discrete model and reexamines the existence question under much less stringent assumptions. It is shown that the insurance market example considered by Rothschild and Stiglitz satisfies the weaker assumptions and the conditions under which a Nash equilibrium exists are then explored. It is argued that, as long as the probability of loss is sufficiently low for all types willing to obtain fair insurance, these conditions are likely to be met.

## I. A Many Principal-Many Agent Model With Hidden Knowledge

Consider a market in which each of the set of potential sellers (agents) can provide one unit of a commodity or service. Sellers can also choose the level, $s$, at which to engage in some sales related activity, that is, to "signal." Differences among sellers are assumed to be parametrizable by a single hidden characteristic $\theta \in \Theta$. Then we shall refer to an agent as being of "type $\theta$."

A contract $\langle s,r \rangle$ between a buyer (principal) and seller is a payment $r$ in return for signal level $s$. If a type $\theta$ agent accepts $\langle s,r \rangle$ the value of his product is $V(\theta,s)$ so that the buyer's profit is

$$\pi(\theta,s,r) = V(\theta,s) - r. \tag{1}$$

Types are parametrized so that higher levels of $\theta$ imply higher product value $V(\theta,s)$. It is also assumed that $V$ is nondecreasing in $s$.

Preferences over alternative offers $\langle s,r \rangle$ are represented by the utility function $U(\theta,s,r)$, where $U$ is increasing in the return $r$ and decreasing in $s$. For every agent there is also a mutually exclusive alternative to trading in this market which yields a utility level $U_A$.

Finally we assume that the marginal cost of signalling, that is, the increase in return required for an agent to be willing to increase his signalling activity, diminishes with $\theta$. Formally,

$$MC_s = \left.\frac{dr}{ds}\right|_U = - \frac{\frac{\partial U}{\partial s}(\theta,s,r)}{\frac{\partial U}{\partial r}(\theta,s,r)} \qquad \text{decreases with } \theta \tag{2}$$

Condition (2) guarantees that, for any set of offers, the choice of signal level $s(\theta)$ will be nondecreasing in $\theta$.

## II. A Simple Labor Market Example

Rather than discuss this model in abstract terms, we begin with a simple example of labor market signalling. A type $\theta$ worker who chooses signal level $s$ has a value to each of the firms in some industry of

$$V(\theta,s) = \theta. \tag{3}$$

Each worker also has an opportunity to work elsewhere for a wage $r_A$. The cost of signalling at level $s$ is $C(\theta,s)$, thus the net return to type $\theta$ if offered the wage contract $\langle s,r\rangle$ is

$$U(\theta,s,r) = r - C(\theta,s) \tag{4}$$

Condition (2) then reduces simply to the requirement that the marginal cost of signalling $\partial C/\partial s$ be lower for more productive workers.

As Rothschild and Stiglitz showed for the two type case, no contract which attracts more than one type can be part of a Nash (or stable Walrasian) equilibrium. More generally (see Wilson [1977]) we have

<u>Proposition 1.</u> A Nash Equilibrium contains no pools.

Given this result we need only examine sets of contracts which separate out all those types who choose to signal. To simplify the analysis further we assume there are just three types of agents so that $\Theta = \{\theta_0, \theta_1, \theta_2\}$ we further assume that

$$\theta_0 < r_A < \theta_1 < \theta_2 \tag{5}$$

Each worker chooses that contract <s,r> which maximizes his net gain $U(\theta,s,r)$. Moreover, if more than one contract yields the same utility we assume that the contract selected is the one with the lowest level of the signal.[4]

One possible set of contracts which separates out the three types is the set $\{E_0, E_1, E_2\}$ depicted in Figure 1. Type $\theta_0$, with the steepest indifference map, $U^0 = U(\theta_0,r,s)$, chooses the contract $E_0$. Type $\theta_1$, with indifference map, $U^1 = U(\theta_1,r,s)$, chooses $E_1$. Finally, type $\theta_2$, with the least steep indifference map, $U^2 = U(\theta_2,r,s)$, chooses $E_2$.

Note that only those workers with productivity exceeding $r_A$ find signalling desirable. Thus the allocation of workers between the two industries is efficient. Note also that the profit on each contract is zero. Note, finally, that each type $\theta_i$ is indifferent between his choice $E_i$ and the choice $E_{i+1}$ of type $\theta_{i+1}$.

It should therefore be intuitively clear that, of all sets of contracts which separate out the different types, the set $\{E_0, E_1, E_2\}$ is Pareto efficient. Formally, modifying only slightly arguments in Riley [1979a] and Engers and Fernandez [1984] we have the following result.

<u>Proposition 2</u>. Characterization of the Pareto efficient set of separating contracts.

Suppose the hypotheses of Section I are satisfied. Then, of all the sets of contracts which are individually not unprofitable and separate out those

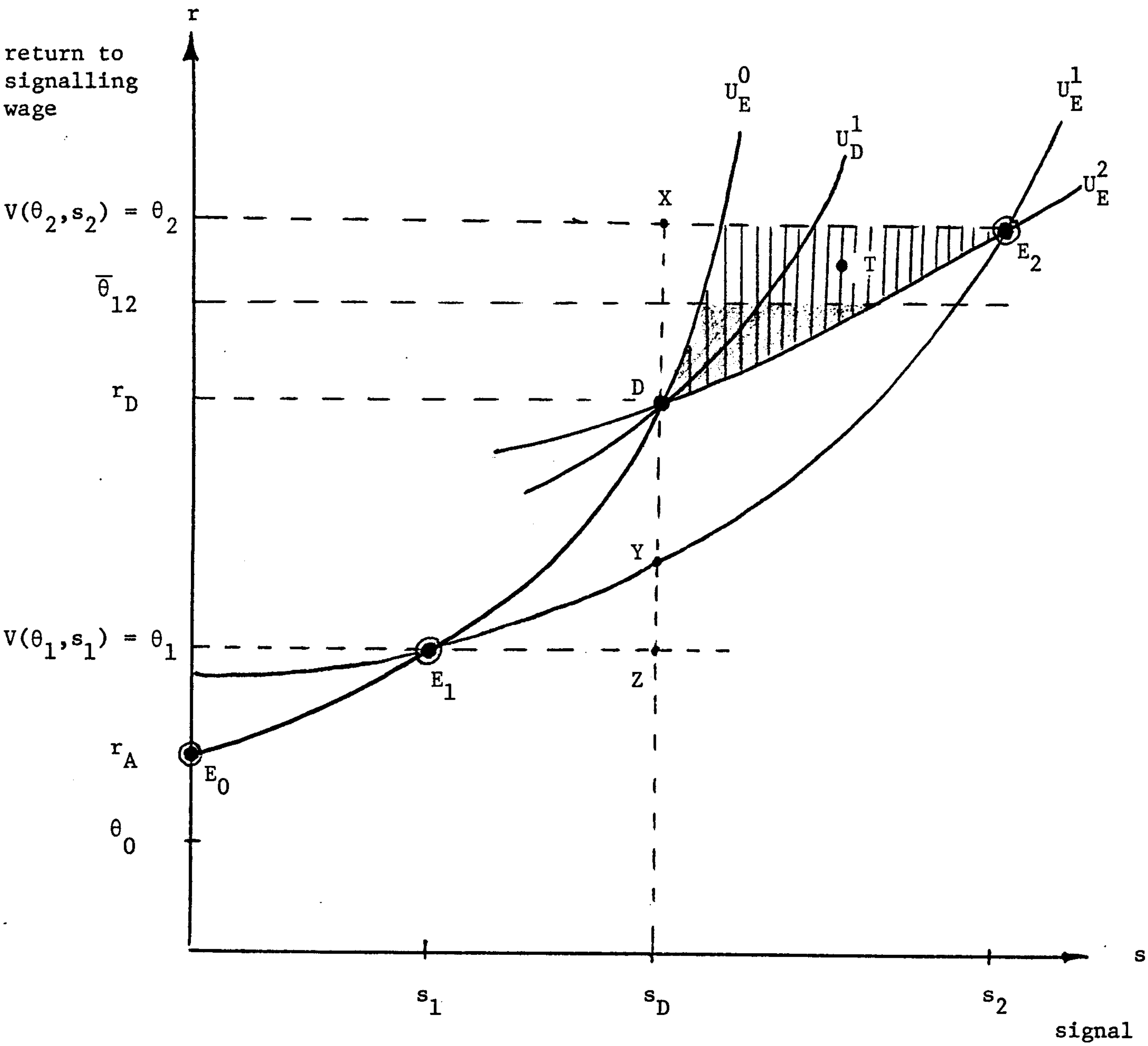


Figure 1: Pareto Efficient Separating Set of Contracts

types who signal, there is a unique set that is Pareto efficient for the agents. This set

(i) allocates types efficiently between those who signal and those who do not,

(ii) generates zero profits on each contract,

and, if the set of types is discrete,

(iii) has the property that if $E_i$ is the choice of type $\theta_i$, $E_i \underset{\theta_i}{\sim} E_{i+1}$.

As shown in Figure 1, $\{E_0, E_1, E_2\}$ is not a Nash equilibrium. Note that any offer in the vertically shaded region is attractive to types $\theta_1$ and $\theta_2$. Then if the average productivity of these two types, $\overline{\theta}_{12}$, is as depicted, any alternative offer in the interior of the heavily shaded region is strictly profitable.

Given the distribution of types who choose to signal, the question we wish to address here is what are the conditions under which such profitable alternatives do not exist.

First of all, as long as the proportion of types $\theta_0$, who choose not to signal, is sufficiently large, it will never be profitable to make an offer which attracts all three types. Given this assumption, the vertically shaded region is the entire set of potentially profitable alternatives. Since the indifference curve $U(\theta_1,s,r) = U_E^1$, bounding this set is upward sloping, the most profitable of these alternatives is the point D where the indifference curves

$$\begin{aligned} U(\theta_0,s,r) &= U(\theta_0,0,r_A) \equiv U_A^0 \\ U(\theta_2,s,r) &= U(\theta_2,s_2,r_2) \equiv U_E^2 \end{aligned} \tag{6}$$

intersect.

Holding fixed the preferences of type $\theta_0$, we can vary D by altering the shape of the indifference curves (6). We then seek conditions under which $r_D > \overline{\theta}_{12}$. Clearly this will be the case if

$$XD < \theta_2 - \overline{\theta}_{12}$$

that is, if

$$\frac{XD}{XZ} < \frac{\theta_2 - \overline{\theta}_{12}}{\theta_2 - \theta_1} = \frac{f_1}{f_1+f_2}$$

where $f_1$ is the proportion of type $\theta_1$ in the population. Since XZ exceeds XY it follows that a sufficient condition for $r_D$ to exceed $\overline{\theta}_{12}$ is that

$$\frac{XD}{XY} < \frac{f_1}{f_1+f_2} \tag{7}$$

From (4) and (6) we can rewrite (7) as

$$\frac{f_1}{f_1+f_2} > \frac{C(\theta_2,s_2) - C(\theta_2,s_D)}{C(\theta_1,s_2) - C(\theta_1,s_D)} = \frac{\int_{s_D}^{s_2} \frac{\partial C}{\partial s}(\theta_1,s)ds}{\int_{s_D}^{s_2} \frac{\partial C}{\partial s}(\theta_2,s)ds}$$

But

$$\frac{\int_{s_D}^{s_2} \frac{\partial C}{\partial s}(\theta_1,s)ds}{\int_{s_D}^{s_2} \frac{\partial C}{\partial s}(\theta_2,s)ds} < \max_{s\varepsilon[s_D,s_2]} \left\{\frac{\frac{\partial C}{ds}(\theta_2,s)}{\frac{\partial C}{ds}(\theta_1,s)}\right\}$$

Therefore wage competition is unprofitable if

$$\frac{\frac{\partial C}{\partial s}(\theta_2,s)}{\frac{\partial C}{\partial s}(\theta_1,s)} < \frac{f_1}{f_1+f_2}$$

For the three type case we have therefore proved

Proposition 3. Sufficient condition for a Nash Equilibrium

If the proportional rate of decline with $\theta$ of the marginal cost of signalling, $\frac{\partial C}{\partial s}(\theta,s)$ is sufficiently large, the Pareto efficient set of separating contracts is a Nash equilibrium.

With more than three types it should be clear that the same argument will hold for every potential pool of two types. Actually, an almost identical argument can be used for larger pools as well. Thus the proposition is quite general.

What remains unclear, however, is the stringency of the sufficient conditions as the number of types becomes large. In the next section we provide an example with a continuum of agents and show that the sufficient conditions can be readily satisfied in this limiting case as well.

## III. Nash Equilibrium with a Continuous Distribution of Types

Consider again the simple labor market model but this time suppose that the set of workers $\Theta$ is the interval $[a,b]$, with $a < r_A < b$, and the c.d.f. for $\theta$, $F(\theta)$ is twice continuously differentiable and strictly increasing over $[a,b]$. To focus on essentials we make a further simplification and assume that the cost of signalling

$$C(\theta,s) = \frac{s}{\theta^e}$$

Then

$$-\frac{\frac{\partial}{\partial\theta}\left(\frac{\partial C}{\partial s}\right)}{\frac{\partial C}{\partial s}} = \frac{e}{\theta} \tag{9}$$

so that the higher is $e$, the larger is the proportional rate of decline of the marginal cost of signalling.[5]

From Proposition 2 we seek a set of contracts which allocates the workers across industries efficiently, separates out all those types who signal, and generates zero profits. Thus we seek a wage function $r = W(s)$ such that, $s(\theta)$, which solves

$$\underset{s}{\text{Max}} \left\{U(\theta,s,W(s)) = W(s) - \frac{s}{\theta^e}\right\} \tag{10}$$

also satisfies

$$W(s(\theta)) = \begin{cases} r_A , & \theta < r_A \\ \theta , & \theta > r_A \end{cases} \tag{11}$$

Such a wage function is illustrated in Figure 2. Type $\hat{\theta}$, observing $W(s)$, chooses $s(\hat{\theta})$. As required the resulting wage paid, $W(s(\hat{\theta}))$, is equal to this worker productivity, $\hat{\theta}$.

Suppose, as we shall later confirm, $W(s)$ is differentiable. Then, for any type choosing a positive $s$ we require

$$W'(s(\theta)) - \frac{1}{\theta^e} = 0$$

Combining (11) and (12) yields the ordinary differential equation

$$W'(s)\, W(s)^e = 1, \qquad W(0) = r_A$$

Integrating and making use of the bounary condition we obtain

$$\int_{r_A}^{W(s)} w^e dw = s \tag{13}$$

and so

$$\frac{W(s)^{1+e}}{1+e} = \frac{r_A^{1+e}}{1+e} + s \tag{14}$$

Before turning to the existence question it is interesting to see how the equilibrium cost of signalling varies with $e$. Since, in equilibrium

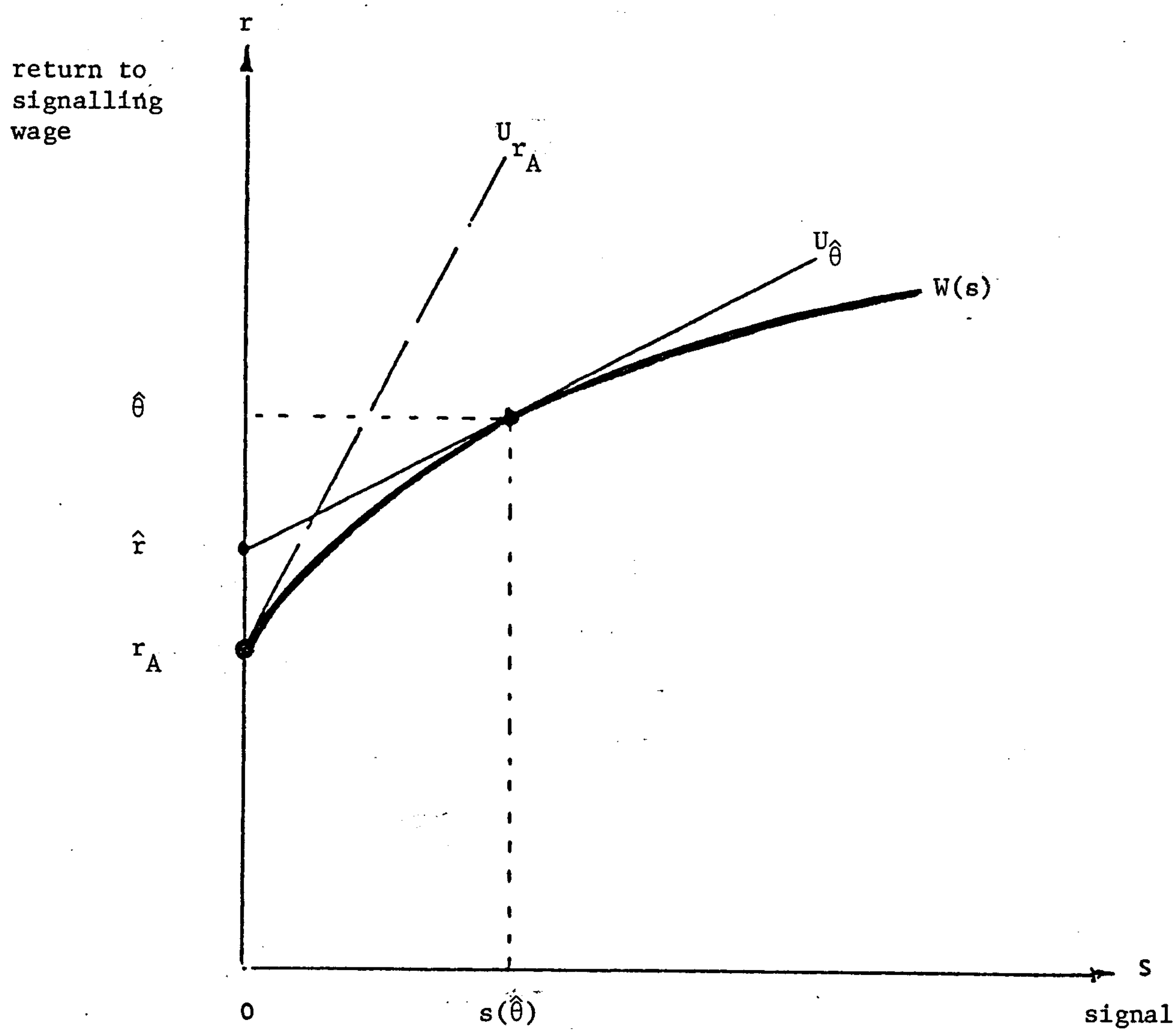


Figure 2: Pareto Efficient Separating Wage Function

$s(\theta) > 0 \rightarrow W(s(\theta)) = \theta$, it follows from (13) that

$$s(\theta) = \int_0^\theta w^e dw$$

Then the equilibrium cost of signalling for type $\theta$

$$C(\theta, s(\theta)) = \frac{s(\theta)}{\theta^e} = \int_0^\theta \left(\frac{w}{\theta}\right)^e dw .$$

We therefore have

Proposition 4. Ranking Signalling Technologies

With signalling cost function $C(\theta,s) = s/\theta^e$ and productivity independent of $s$, the higher is $e$ (and hence the larger is the proportional rate of decline in the marginal cost of signalling), the greater is the equilibrium return to all those signalling.

We now seek condition under which $W(s)$, given by (14), is a Nash equilibrium. Initially we consider only small perturbations, that is, new wage offers in the neighborhood of the wage function $W(s)$. First we consider a new wage offer $\hat{r}$ with no signal required. This is also depicted in Figure 2. Since an agent of type $r_A$ is just indifferent between signalling and not signalling, his indifference curve through $\langle 0, r_A \rangle$ must, as depicted, be tangential to $W(s)$ at $s = 0$. Of course with $\hat{r} > r_A$ type $r_A$ strictly prefers the new offer. Indeed there is an interval of types $[r_A, \hat{\theta})$ who are strictly better off under the new offer than if they signal. In Riley [1979a] it is established that, for $\hat{r}$ sufficiently small, this new offer will attract an interval of types with an average productivity in excess of the offered wage. However, with the the alternative opportunity yielding a wage $r_A < \hat{r}$ all those types on the interval $[a, r_A]$ also find the new offer attractive. Then the average productivity of those accepting the new offer is

$$\int_a^{\hat{\theta}} \theta dF/F(\hat{\theta}) \ .$$

As $\hat{r} \to r_A$, $\hat{\theta} \to r_A$ and hence the average productivity approaches $\int_a^{r_A} \theta dF/F(r_A)$ which is strictly less than $r_A$. Then for $\hat{r} > r_A$ and sufficiently close the new offer is unprofitable.

The other alternative is a new offer $\langle\hat{s},\hat{r}\rangle$ designed to attract all those types on some interval $(\beta,\gamma)$. This is illustrated in Figure 3. An agent of type $\beta$, with indifference curve $U_\beta$ through his best signalling point $\langle s(\beta),\beta\rangle$ is just indifferent between the latter and the new alternative. Similarly as agent of type $\gamma$ is just indifferent, while all those for whom $\theta \varepsilon (\beta,\gamma)$ strictly prefer $\langle\hat{s},\hat{r}\rangle$.

We next obtain an expression for $\hat{r}$ in terms of $\beta$ and $\gamma$ and then compare this new offer with the average productivity of those accepting it.

From (10) the steepness of an indifference curve for type $\theta$ is $\theta^{-e}$. Then $\langle\hat{s},\hat{r}\rangle$ must satisfy

$$\frac{\hat{r} - \beta}{\hat{s}-s(\beta)} = \beta^{-e}, \ \frac{\hat{r} - \gamma}{\hat{s}-s(\gamma)} = \gamma^{-e}. \tag{15}$$

Eliminating $\hat{s}$ we then obtain

$$\hat{r}[\gamma^e - \beta^e] = \gamma^{1+e} - \beta^{1+e} - (s(\gamma)) - s(\beta)) \ . \tag{16}$$

But, from (11) and (13)

$$\begin{aligned} s(\gamma) - s(\beta) &= \int_\beta^\gamma w^e dw \\ &= \gamma^{1+e} - \beta^{1+e} - \int_\beta^\gamma e \ w^e dw \ . \end{aligned} \tag{17}$$

Substituting (17) into (15), we obtain

$$\hat{r} = \frac{\int_\beta^\gamma e w^e dw}{\gamma^e - \beta^e}. \tag{18}$$

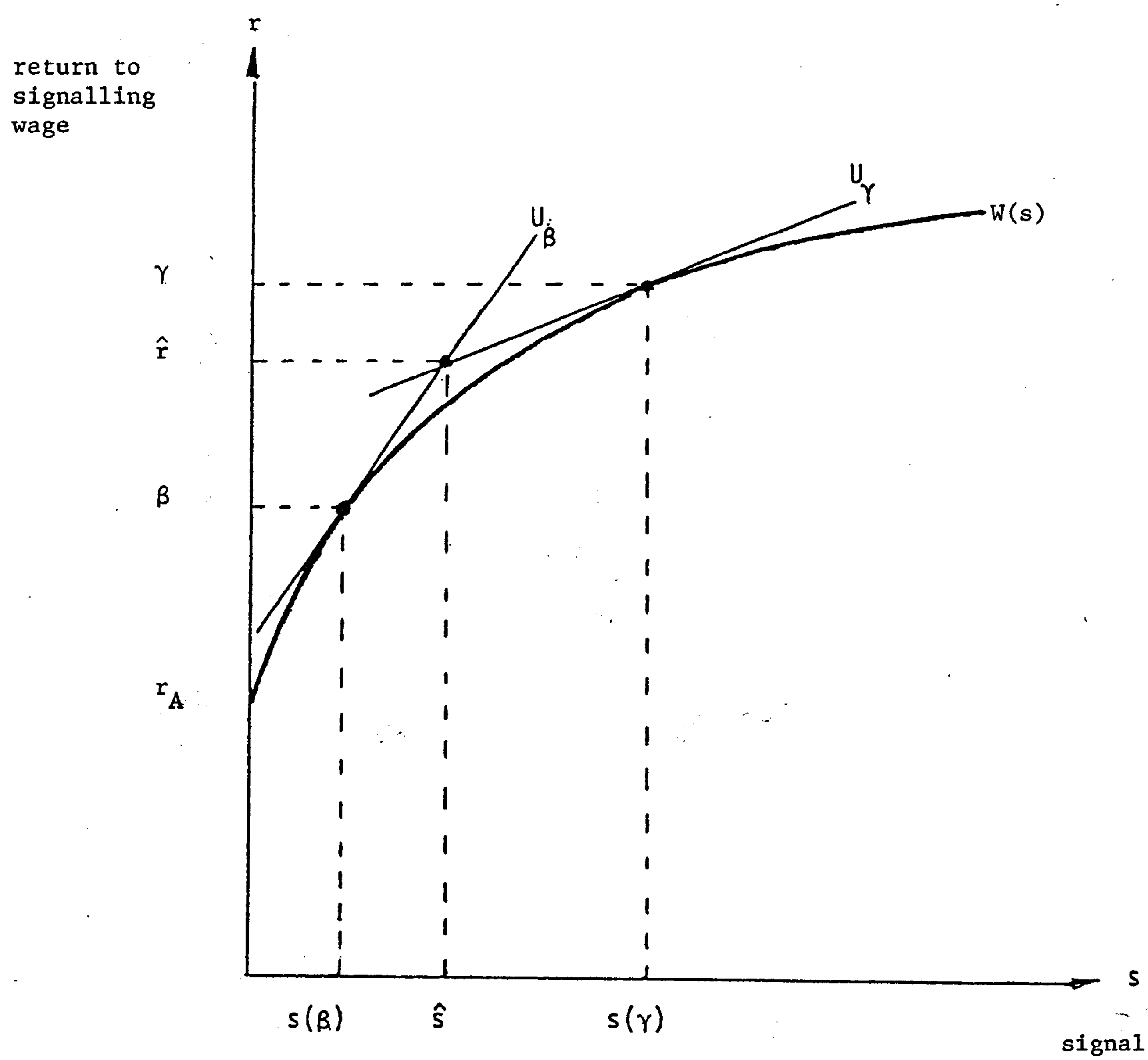


Figure 3: Interior Wage Competition

Thus, to attract workers with productivity in the interval $(\beta,\gamma)$ an employer announces the new offer $\langle\hat{s},\hat{r}\rangle$ where $\hat{r}$ satisfies (18) and $\hat{s}$ satisfies (15).

To determine the profitability of such an offer we must compare it with the average productivity of those accepting, that is

$$\bar{\theta} = \frac{\int_{\beta}^{\gamma}\theta F'(\theta)d\theta}{F(\gamma) - F(\beta)} . \tag{19}$$

To do this we fix $\beta$ at some arbitrary level and examine the change in $\hat{r}$ and $\bar{\theta}$ with $\gamma$ as $\gamma \downarrow \beta$. Appealing to l'Hôpital's Rule we have the following useful result which is proved in the Appendix.

<u>Lemma 1</u>

Define $y(\gamma) = \frac{\int_{\beta}^{\gamma} \theta H'(\theta)d\theta}{H(\gamma) - H(\beta)}$, where $H(.)$ is twice continuously differentiable. Then

$$\text{(i)}\quad y'(\beta) = \frac{1}{2} \quad\text{and}\quad \text{(ii)}\quad y''(\beta) = \frac{1}{6}\frac{H''(\beta)}{H'(\beta)} .$$

From (19), $\bar{\theta}(\gamma)$ is in the family of functions defined in the statement of Lemma 1. Also, setting $H(\theta) = \theta^e$ it follows from (18) that $\hat{r}(\gamma)$ is also in this family. Therefore $\hat{r}(\gamma)$ and $\bar{\theta}(\gamma)$ both increase with $\gamma$ at the same rate of 1/2 when $\gamma = \beta$. Then local wage competition is unprofitable if and only if $\hat{r}''(\beta) > \bar{\theta}''(\beta)$. From Lemma 1 we than obtain

<u>Proposition 5</u>. Necessary and Sufficient Condition for Locally Unprofitable Wage Competition.

With signalling cost function $C(\theta,s) = s/\theta^e$ and productivity independent of $s$, local wage competition is unprofitable if and only if

$$e > 1 + \frac{\theta F''(\theta)}{F'(\theta)}$$

The final step is to establish conditions under which any nonlocal wage-price pair is unprofitable. Since the arguments are somewhat intricate, the proof of the following result is relegated to the Appendix.

Proposition 6. Sufficient Conditions for Globally Unprofitable Wage Competition.

Under the hypotheses of Proposition 5 the Pareto efficient separating wage schedule is a Nash equilibrium if, for all $\theta$

$$e > \max \left\{\frac{\theta F'(\theta)}{F(\theta)}, 1 + \frac{\theta F''(\theta)}{F'(\theta)}\right\} \tag{20}$$

To illustrate the stringency of the constraint (20), note that if $F(\theta)$ is concave $\theta F'(\theta) < F(\theta)$ and $F''(\theta) < 0$. Thus, as long as $e$ exceeds unity, the Pareto efficient separating wage schedule is a Nash equilibrium.

Finally, comparing Propositions 4 and 6 we note that those values of $e$ which generate sufficiently large potential gains from signalling, also lead to existence of a Nash equilibrium. Thus, at least in this labor market example, the equilibrium problems tend to arise when the potential gains from signalling are small.

## IV. Nash Equilibrium for the General Model — the Case of Insurance

The example considered in the previous sections is special in two respects. First the value of the contract, $V(\theta,s)$, is independent of the level of the signal. Second, the utility function is additively separable in income so that the marginal cost of signalling is independent of the return to signalling, $r$.

In most hidden knowledge problems neither of these conditions are met. Consider, for example, a human capital variant of Spence's labor market model. A worker of type $\theta$ can attain an educational level $s$ in time $t(\theta,s)$. His lifetime product, discounted to the date of entry into the work-force is then $V(\theta,s)$. If such a worker is offered a lifetime income, also discounted to the date of entry into the workforce, of $r$, and $\rho$ is the interest rate, the worker's present value of lifetime income

$$U(\theta,s,r) = re^{-\rho t(\theta,s)}.$$

It is a straightforward matter to confirm that, as long as the extra time needed to increase educational achievement, $\partial t(\theta,s)/\partial s$, is decreasing in $\theta$, the assumptions laid out in Section I are satisfied.

A second example, and one which we shall focus on here, is provided by the insurance model of Rothschild and Stiglitz. To obtain full coverage against a loss $L$, an individual with von Neumann Morgenstern utility function $u(.)$, and wealth $\omega$, must pay a premium $p$. Alternatively, by coinsuring, that is, accepting a deductible of $s$, the individual receives a premium reduction of $r$. In the no loss state, which occurs with probability $\theta$, wealth is therefore

$$\omega - (p-r) \equiv n + r, \quad \text{where} \quad n \equiv \omega - p$$

In the loss state wealth is

$$\omega - L + (L-s) - (p-r) = n + r - s$$

Expected utility can then be expressed as

$$U(\theta,s,r) = \theta u(n+r) + (1-\theta)\, u(n+r-s) \tag{21}$$

If a type $\theta$ individual accepts the insurance contract $\langle s,r \rangle$ the expected profit on this contract

$$\pi(\theta,s,r) = p-r - (1-\theta)(L-s) = V(\theta,s) - r \tag{22}$$

From (21) the marginal cost of signalling is

$$\begin{aligned} \left.\frac{dr}{ds}\right|_{U=U(\theta,s,r)} &= -\frac{\partial U}{\partial s}\Big/\frac{\partial U}{\partial r} \\ &= \frac{(1-\theta)\, u'(n+r-s)}{\theta u'(n+r) + (1-\theta)\, u'(n+r-s)} \\ &= 1/[1 + \frac{\theta}{1-\theta}\,\frac{u'(n+r)}{u'(n+r-s)}] \end{aligned} \tag{23}$$

It follows immediately that, as required, the marginal cost of signalling declines with the quality of the insurance risk (the probability of no loss, $\theta$).

We can now state the generalization of Proposition 3.

Proposition 7. Sufficient condition for a Nash Equilibrium.

Suppose that, for the general model of Section I, the marginal cost of signalling is, for each of $n$ types of agent, nonincreasing in the return to signalling, $r$. Then the Pareto efficient set of separating contracts is a Nash equilibrium whenever the proportional rate of decline in the marginal cost of signalling with respect to $\theta$

$$-\frac{\dfrac{\partial}{\partial\theta}\left.\dfrac{dr}{ds}\right|_{U(\theta,s,r) = \overline{U}}}{\left.\dfrac{dr}{ds}\right|_{U(\theta,s,r) = \overline{U}}}$$

is sufficiently large.

Proof. As in Section II we analyze the special case with just three types. The generalization to $n$ types is straightforward. Consider Figure 1 again.

Since $V(\theta,s)$ is nondecreasing in $s$, the average value of types $\theta_1$ and $\theta_2$, if they both choose the contract D, is no greater than

$$\overline{V}_{12}(s_2) \equiv (f_1 V(\theta_1,s_2) + f_2 V(\theta_2,s_2))/(f_1+f_2)$$

Then, just as in our earlier argument, there are no profitable alternatives if XD/XY is sufficiently small. But

$$XD = \int_{s_D}^{s_2} \left.\frac{dr}{ds}\right|_{U^2} ds \ , \quad \text{where the integral is along the arc } U^2 = U_E^2$$

and

$$XY = \int_{s_D}^{s_2} \left.\frac{dr}{ds}\right|_{U^1} ds \ , \quad \text{along the arc } U^1 = U_E^1$$

By hypothesis

$$\left.\frac{dr}{ds}\right|_{\overline{U}} \quad \text{is decreasing in } r.$$

Then

$$XD < \int_{s_D}^{s_2} \left.\frac{dr}{ds}\right|_{U^2} ds, \quad \text{along the arc } U^1 = U_E^1,$$

so that

$$\frac{XD}{XY} < \frac{\left.\frac{dr}{ds}\right|_{U^2} ds}{\int_{s_D}^{s_2} \left.\frac{dr}{ds}\right|_{U^1} ds} \ , \quad \text{where both integrals are along the arc } U^1 = U_E^1$$

$$< \max_{s\varepsilon[s_D,s_2]} \frac{\left.\frac{dr}{ds}\right|_{U^2}}{\left.\frac{dr}{ds}\right|_{U^1}} \ , \quad \text{along the arc } U^1 = U_E^1$$

By making the proportional rate of decline in the marginal cost of signalling with respect to $\theta$ sufficiently large, we can make the right hand side of this inequality arbitrarily small. Then we can choose the offer D arbitrarily close to the horizontal line $r = V(\theta_2, s_2)$ and hence above $\overline{V}_{12}(s_2)$. Q.E.D.

We conclude asking under what conditions the hypotheses of this proposition are most likely to be satisfied in an insurance market. From (23), the marginal cost of signalling is nonincreasing in $r$ if $u'(n+r)/\, u'(n+r-s)$ is nondecreasing in $r$, that is if

$$0 < \frac{\partial}{\partial r} \frac{u'(n+r)}{u'(n+r-s)} = [A(n+r-s) - A(n+r)] \frac{u'(n+r)}{u'(n+r-s)} , \qquad (24)$$

where

$$A(\omega) = -\frac{u''(\omega)}{u'(\omega)}$$

is, the degree of absolute aversion to risk at wealth level $\omega$. Assuming, as is usually argued, that absolute aversion to risk does not increase with wealth it follows that inequality (24) is indeed satisftied.

Finally we consider the rate of decline of the marginal cost of signalling. Differentiating the logarithm of (23) by $\theta$ we obtain

$$-\frac{\frac{\partial}{\partial \theta} \left.\frac{dr}{ds}\right|_{\overline{U}}}{\left.\frac{dr}{ds}\right|_{\overline{U}}} = \frac{\frac{1}{(1-\theta)^2} \frac{u'(n+r)}{u'(n+r-s)}}{1 + \frac{\theta}{1-\theta} \frac{u'(n+r)}{u'(n+r-s)}} .$$

$$= \frac{1}{(1-\theta)^2 \frac{u'(n+r-s)}{u'(n+r)} + \theta(1-\theta)}$$

From Proposition 7, a Nash equilibrium exists if this expression is sufficiently large. Note that as the probability of no loss, $\theta$, approaches unity, the denominator approaches zero. Therefore, as long as the loss probabilities are all sufficiently low, the Pareto efficiency separating contract set is a Nash equilibrium.

## V. Concluding Remarks

Taken together, the above results indicate that there are quite reasonable assumptions, for both the insurance market and labor market applications, under which the many principal-many agent problem has a (unique) Nash equilibrium. Furthermore, in the labor market application, the sufficient conditions for a Nash equilibrium are satisfied whenever the gains to those signalling are sufficiently large.

The first point to be emphasized is that the conditions derived are sufficient conditions. It may well be that weaker and more precise necessary and sufficient conditions conditions remain to be discovered. It also remains to examine whether empirically reasonable sufficient conditions can be derived in other applications, such as in the use of insider shareholdings to signal the value of a new stock offering.

Turning to more fundamental theoretical issues, it should be noted that all the published literature makes the key assumption that there is only one hidden characteristic. Therefore a further important step will be to develop models with multiple characteristics. Some preliminary work by Engers [1984] suggest that parallel results are possible with equal numbers of characteristics and signals. However, this area remains largely unexplored.

Another theoretical simplification made in this paper is the assumption that the opportunity cost of choosing to signal at all, is the same for each type. Especially in the labor market case it seems much more plausible that those workers with a high productivity will have a higher opportunity cost. While introduction of a reservation utility $U_A(\theta)$, which varies across types, complicates the technical details, it is clear, from the recent work by Engers and Fernandez, that the conclusions are essentially unchanged.

A further simplification is that, in defining the Nash equilibrium, it is assumed that a principal can only introduce a single new alternative $\langle \hat{s},\hat{r} \rangle$. While a complete analysis would be rather technical, it seems clear, from my preliminary investigation, that the sufficient conditions developed here will also sustain a Nash equilibrium when each principle can introduce multiple new alternatives.

Finally, it would be incomplete to finish without some comment on how behavior can be modelled when the underlying assumptions imply that no Nash equilibrium exists. As indicated in the introduction, there have been various attempts to model an euilibrium in which principals take into account anticipated reactions when considering alternative actions. To illustrate, consider Figure 1 once more. Since the contract set $\{E_0,E_1,E_2\}$ is the Pareto efficient separating set, any new offer, such as D, which generates strictly positive expected profits, must involve pooling. Then there is always a reaction such as T which skims the cream from the pool. As a result the initial "defection" D generates losses while the reaction T makes profits on every agent who accepts it. It thus seems plausible that the principal considering the defection, D, will recognize that the reaction T poses a serious threat. As a result he will be deterred from choosing to offer D. The Pareto efficient separating set is then a Reactive equilibrium.

The crucial step in this argument is the assumption that at least one other principal will be able to exploit the opportunity arising from the announcement of a new offer, such as D, before the new offer has generated significant profits. (Alternatively, once offered, D cannot quickly be withdrawn, so that the initial profits are offset by later losses as other principals respond.) Therefore, in using a non Nash equilibrium concept to model behavior in some specific market it is important to begin by considering the reasonableness of the quick reaction hypothesis.

## Footnotes

[1]One example is the choice of an income tax scheme by the tax authority when ability is unobservable (Mirrlees [1971]). Another is the choice of an optimal selling scheme by the owner of a unique object (Riley and Samuelson [1981]).

[2]For a discussion of existence in a game theoretic model of a market with differential information, when the informed players move first, see Stiglitz and Weiss [1983].

[3]For a comparison of three alternative non-Nash equilibrium concepts see Riley [1979b].

[4]This essentially technical problem, of nonunique optimal choices, disappears when types are distributed continuously.

[5]The assumption that $C(\theta,s) = s/\theta^e$ is not as restrictive as it might seem. Suppose instead $z$ is the level of the signalling activity with signalling cost $\hat{C}(\theta,z) = A(z)/\theta^e$, where $A(z)$ is strictly increasing. Then we can always define the inverse function $z = A^{-1}(s)$ and define the equivalent signalling cost function $C(\theta,s) \equiv \hat{C}(\theta,A^{-1}(s)) = s/\theta^e$.

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## Appendix

Lemma 1

Define $y(\gamma) = \dfrac{\int_\beta^\gamma \theta H'(\theta)d\theta}{H(\gamma)-H(\beta)}$, where $H(.)$ is twice continuously differentiable. Then

$$\text{(i)}\quad y'(\beta) = \frac{1}{2} \quad\text{and}\quad \text{(ii)}\quad y''(\beta) = \frac{1}{\theta}\,\frac{H''(\beta)}{H'(\beta)}$$

Proof: Define $N(\gamma) = H'(\gamma) \int_\beta^\gamma (H(\theta) - H(\beta))d\theta$

and

$$D(\gamma) = (H(\gamma) - H(\beta))^2 .$$

Taking Taylor's expansions about $\alpha$, we obtain

(a-1)
$$\begin{cases} N(\gamma) = \frac{1}{2} H'(\beta)^2\ (\gamma-\beta)^2 + \frac{2}{3} H'(\beta)\ H''(\beta)\ (\gamma-\beta)^3 + O(\gamma-\beta)^4 \\ D(\gamma) = H'(\beta)^2\ (\gamma-\beta)^2 + H'(\beta)\ H''(\beta)\ (\gamma-\beta)^3 + O(\gamma-\beta)^3 \end{cases}$$

Integrating the numerator of the expression for $y(\gamma)$ by parts we obtain

(a-2)
$$y(\gamma) = \gamma - \int_\beta^\gamma \frac{(H(\theta)-H(\beta))d\theta}{H(\gamma)-H(\beta)}$$

Then differentiating $y(\gamma)$ we obtain

(a-3)
$$y'(\gamma) = H'(\gamma) \int_\beta^\gamma \frac{(H(\theta)-H(\beta))d\theta}{(H(\gamma)-H(\beta))^2}$$

Substituting from (a-1) this can be rewritten as

(a-4)
$$y'(\gamma) = N(\gamma)/D(\gamma)$$

Then

$$y'(\beta) = \lim_{\gamma\downarrow\beta} \frac{N(\gamma)}{D(\gamma)} = \frac{1}{2} .$$

Differentiating (a-4) by $\gamma$ we also have

$$y''(\gamma) = \frac{N'(\gamma)D(\gamma) - D'(\gamma)N(\gamma)}{D(\gamma)^2} .$$

Substituting from (a-1) we then obtain

$$y''(\beta) = \lim_{\gamma \to \beta} \frac{N'(\gamma)D(\gamma) - D'(\gamma)N(\gamma)}{D(\gamma)^2} = \frac{1}{6} \frac{H''(\beta)}{H'(\beta)} .$$

Q.E.D.

Lemma 2

Define $\phi_i(\gamma) = \dfrac{\int_{a_i}^{\gamma} \theta dF_i(\theta) + b_i}{F_i(\gamma) + c_i}$, $i = 1,2.$

Then

$$\phi_1(\gamma) = \phi_2(\gamma) \Rightarrow$$

$$\frac{d\phi_1(\gamma)}{d\gamma} - \frac{d\phi_2(\gamma)}{d\gamma} = (\gamma - \phi_i(\gamma)) \left[\frac{F_1'(\gamma)}{F_1(\gamma) + c_1} - \frac{F_2'(\gamma)}{F_2(\gamma) + c_2}\right] .$$

Proof:

Taking the logarithm of

(a-5) $$\phi(\gamma) = \frac{\int_a^{\gamma} \theta dF(\theta) + b}{F(\gamma) + c}$$

and then differentiating by $\gamma$ we obtain

$$\frac{\phi'(\gamma)}{\phi(\gamma)} = \frac{\gamma F'(\gamma)}{\int_a^{\gamma} \theta dF(\theta) + b} - \frac{F'(\gamma)}{F(\gamma) + c}$$

Then multiplying both sides by $\phi(\gamma)$ and substituting from (a-5) we obtain

$$\phi'(\gamma) = (\gamma - \phi(\gamma)) \frac{F'(\gamma)}{F(\gamma) + c}$$

Q.E.D.

<u>Proposition 6.</u> Sufficient Conditions for Globally Unprofitable Wage Competition.

Under the hypotheses of Proposition 5, the Pareto efficient separating wage schedule is a Nash equilibrium if, for all $\theta$

(20) $$e > \max \left\{ \frac{\theta F'(\theta)}{F(\theta)} , 1 + \frac{\theta F''(\theta)}{F'(\theta)} \right\}$$

<u>Proof.</u> The proposition is derived in three parts. We prove that an alternative $\langle \hat{s},\hat{r} \rangle$ is unprofitable

(i) for $\hat{s} = 0$,

(ii) for $\hat{s} > 0$ and $\langle \hat{s},\hat{r} \rangle$ attracting those who initially so no signalling as well as a subset of those signalling,

(iii) for $\hat{s} > 0$ and $\langle \hat{s},\hat{r} \rangle$ attracting only those who initially choose $s(\theta) > 0$.

Since it will simplify the exposition somewhat we define

(a-6) $$G(\theta) = \theta^e$$

Then the utility of type $\theta$ can be written as

$$U(\theta,s,r) = r - s/G(\theta)$$

For $\hat{r}$ sufficiently close to $r_A$ there is some type $\gamma > r_A$ indifferent between signalling and accepting the new offer, that is

$$\hat{r} = W(s(\gamma)) - \frac{s(\gamma)}{G(\gamma)} .$$

Since $W(s(\gamma)) = \gamma$ and $s(\gamma)$ satisfies (13) we can rewrite this as

$$\hat{r} = \gamma - \frac{\int_{r_A}^{\gamma} G(w)dw}{G(\gamma)} ,$$

$$= \frac{\int_{r_A}^{\gamma} wG'(w)dw + r_A G(r_A)}{G(\gamma)} .$$

Also the average productivity of those accepting is

$$\bar{\theta} = \int_a^{\gamma} \frac{\theta F'(\theta)d\theta}{F(\gamma)} .$$

We have already argued that, for $\gamma$ sufficiently close to $r_A$, $\hat{r}$ exceeds $\bar{\theta}$. Then for an offer of this type to be profitable there must be some $\gamma$ such that

$$\hat{r} = \bar{\theta} \quad \text{and} \quad \frac{d\hat{r}}{d\gamma} < \frac{d\bar{\theta}}{d\gamma} .$$

Since $\hat{r}$ and $\bar{\theta}$ have the form of the function defined in Lemma 2 this pair of conditions can only hold if

$$\frac{G'(\gamma)}{G(\gamma)} < \frac{F'(\gamma)}{F(\gamma)} .$$

Then a sufficient condition for all such offers to be unprofitable is that for all $\gamma > r_A$ the inequality is reversed, that is

$$e = \frac{\theta G'(\theta)}{G(\theta)} > \frac{\theta F'(\theta)}{F(\theta)} , \quad \text{for all} \quad \theta > r_A .$$

Q.E.D. (i)

We next consider an alternative $\langle\hat{s},\hat{r}\rangle$ which is strictly preferred by those types $\theta \in (\beta,\gamma)$ where

$$0 < \beta < r_A < \gamma < 1.$$

Then types $\beta$ and $\gamma$ must be just indifferent between the new alternative and their respective optima in the Walrasian signalling equilibrium. Since $\beta < r_A$ the equilibrium choice for type $\beta$ is $\langle 0,r_A\rangle$, while for type $\gamma$ it is $\langle s(\gamma),W(s(\gamma))\rangle$ where this is given by (13) and (14). We therefore have

$$\frac{\hat{r}-r_A}{\hat{s}} = \frac{1}{G(\beta)}, \quad \frac{\hat{r}-\gamma}{s(\gamma)} = \frac{1}{G(\gamma)}.$$

Eliminating $\hat{s}$ and substituting for $s(\gamma)$ from (13) the new wage offer may be written as

$$\hat{r} = \frac{\gamma G(\gamma) - r_A G(\beta) - \int_{r_A}^{\gamma} G(w)dw}{G(\gamma) - G(\beta)},$$

$$= \frac{r_A(g(r_A) - G(\beta)) + \int_{r_A}^{\gamma} wG'(w)dw}{G(\gamma) - G(\beta)}.$$

We now consider $\hat{r}$ as a function of $\gamma$ and compare it with the average productivity of those accepting the new offer,

$$\bar{\theta}(\gamma) = \frac{\int_{\beta}^{\gamma} \theta F'(\theta)d\theta}{F(\gamma) - F(\beta)}.$$

As $\gamma \downarrow r_A$, $\hat{r}(\gamma) \downarrow r_A$. Also $\bar{\theta}(r_A) < r_A$. So, by continuity, $\hat{r}(\gamma) > \bar{\theta}(\gamma)$ for $\gamma$ in some interval $[r_A, r_A + \delta]$. Then, if the alternative offer $\langle \hat{s}, \hat{r} \rangle$ attracting types with $\theta \in (\beta, \hat{\gamma})$ is profitable, there must be some $\gamma$ such that

(a-7) $$\hat{r}(\gamma) = \bar{\theta}(\gamma) \quad \text{and} \quad \hat{r}'(\gamma) < \bar{\theta}'(\gamma).$$

Appealing to Lemma 2, we therefore require that

(a-8) $$\hat{r}'(\gamma) - \bar{\theta}'(\gamma) = (\gamma - \hat{r}(\gamma)) \left[\frac{G'(\gamma)}{G(\gamma)-G(\beta)} - \frac{F'(\gamma)}{F(\gamma)-F(\beta)}\right] < 0.$$

From the definition of $\hat{r}$ we know that the term in parentheses is positive. Moreover the bracketed expression, which we shall denote by $B$, can be rewritten as

(a-8) $$B(\gamma) = \frac{G'(\gamma)(F(\gamma) - F(\beta)) - F'(\gamma)(G(\gamma) - G(\beta))}{(G(\gamma) - G(\beta))(F(\gamma) - F(\beta))}.$$

Taking a Taylor's expansion of the numerator of (a-8), $M(\gamma)$, we obtain

(a-9) $$M(\gamma) = F'(\beta)G'(\beta)\ [\frac{G''}{G'} - \frac{F''}{F'}]\ (\gamma-\beta)^2 + O(\gamma-\beta)^3 .$$

From (a-6) $\theta G''(\theta)/G'(\theta) = e-1$. Thus, by hypothesis $G''(\beta)/G'(\beta) > F''(\beta)/F'(\beta)$, and so $M(\gamma)$ and hence $B(\gamma)$ is positive in the neighborhood of $\beta$. But, for (a-7) to hold, $B(\gamma)$ must be negative for $\gamma = \hat{\gamma}$. Hence there is necessarily some $\gamma \varepsilon (\beta,\hat{\gamma})$ for which
$B(\gamma)$ is zero and decreasing. That is,

(a-10) $$\frac{G'(\gamma)}{G(\gamma)-G(\beta)} = \frac{F'(\gamma)}{F(\gamma)-F(\beta)} ,$$

and

(a-11) $$\frac{d}{d\gamma}\ (\frac{G'(\gamma)}{G(\gamma)-G(\beta)}) < \frac{d}{d\gamma}\ (\frac{F'(\gamma)}{F(\gamma)-F(\beta)}) ,$$

both hold. But (a-11) can be written as

(a-12) $$\frac{G''(\gamma)}{G'(\gamma)}\ (\frac{G'(\gamma)}{G(\gamma)-G(\beta)}) - (\frac{G'(\gamma)}{G(\gamma)-G(\beta)})^2 < \frac{F''(\gamma)}{F'(\gamma)}\ (\frac{F'(\gamma)}{F(\gamma)-F(\beta)}) - (\frac{F'(\gamma)}{F(\gamma)-F(\beta)})^2 .$$

Since (a-10) must hold, it follows from (a-12) that

$$\frac{G''(\gamma)}{G'(\gamma)} < \frac{F''(\gamma)}{F'(\gamma)} ,$$

But $\theta G''(\theta)/G'(\theta) = e - 1$ so this contradicts hypothesis (20). Then (a-10) and (a-11) cannot hold simultaneously and so, in turn, (a-7) cannot hold.

Q.E.D. (ii)

The final step in the proof is to consider the case depicted in Figure 3, that is an alternative offer which is attractive to those types $\theta\ \varepsilon\ (\beta,\gamma)$ where $r_A < \beta < \gamma$. From the previous section we know that condition (20) is sufficient for local stability. Then for any $\beta$ there exists $\delta > 0$ such that $\hat{r}(\gamma) > \bar{\theta}(\gamma)$ for all $\gamma \varepsilon (\beta, \beta + \delta)$. The argument then proceeds exactly as above with an appeal to Lemma 2 to obtain a contradiction.

Q.E.D. (iii)