WHY BAD WORKERS RECEIVE RAISES

by

John Haltiwanger

and

Michael Waldman

University of California, Los Angeles

September 1983

Working Paper No. 301

*We would like to thank the Foundation for Research in Economics and Education for financial support.

In a recent article, Medoff and Abraham (1980) presented evidence concerning the relationship between experience, compensation and productivity among managerial employees. Their conclusion was that for workers in the same job category there seems to be first, a strong positive correlation between experience and compensation, and second, no correlation or a negative correlation between experience and productivity. Similar results have been found by, among others, Dalton and Thompson (1971) and Pascal and Rapping (1972). In particular, Dalton and Thompson found that engineers over the age of thirty five were in general below average in terms of productivity, while at the same time being above average in terms of compensation. On the other hand, Pascal and Rapping found that, even after controlling for productivity differences, there seems to be a positive correlation between experience and compensation for major league baseball players.

The above somewhat paradoxical results have brought forth a host of competing explanations. Examples of explanations which have been put forth to explain these results are those of Salop and Salop (1976), Grossman (1977), Lazear (1979, 1981), and Harris and Holmstrom (1982). In Salop and Salop workers vary in terms of an innate quit propensity, and firms in turn employ upward sloping age earnings profiles to screen out potential employees who are 'quitters'. Grossman's argument also relies on quitting behavior, but there the crucial factor is that young workers have on average a higher probability of quitting. This tends to lower wages for young workers, because it increases the probability that in the better states of nature the worker will leave the firm. Lazear's argument is one concerned with shirking. That is, by deferring payments firms can increase the penalty associated with being fired, and in this way deter employees from shirking.[1] Finally, Harris and Holmstrom consider a world where, because workers are unable to irrevocably bind themselves to firms, the long-term implicit contracts which emerge are such that a worker can have his wage bid up when positive information about the worker is revealed, while negative information can never cause the wage to fall. In this world, even for workers of the same perceived ability, there is a positive correlation between experience and compensation. This is due to two factors. First, older workers have had a longer time period in which wages could have been bid up. Second, both because an old worker's ability can be assessed more precisely and because older workers have less time to remain in the market, older workers are required to pay lower insurance premia.[2]

One other explanation has been put forth in an attempt to account for the empirical results reported. This explanation has not drawn as much attention as those listed above, but we find it intuitively appealing. This alternative explanation was put forth by Medoff and Abraham.

> . . . A different rationale for implicit contracts providing that at least some workers be paid less than they produce when young and more than they produce when old might rest on workers being risk-averse and owners of firms being risk-neutral. Not knowing at the beginning of their work lives whether their productivity would grow rapidly or slowly, workers might want a pay scheme that guaranteed annual pay increases of a certain amount independent of whether they turned out to be high-productivity or low-productivity employees . . . (Medoff and Abraham 1980, pp. 733-734)

To put the above argument another way, because of uncertainty a worker may have concerning his own ability, wage increases for workers who are revealed to be of low ability can serve as a form of insurance. Possibly the reason this explanation has not drawn the attention of those listed earlier is that, as opposed to those explanations, this explanation is not backed up by a formal theoretical analysis. In fact, there is a formal analysis which suggests that the intuition behind this argument is <u>invalid</u>. Consider in more detail Harris and Holmstrom's model. That model would seem to satisfy all the requirements necessary to exhibit the insurance effect hypothesized by Medoff and Abraham. For example, the model contains risk averse workers and risk neutral firms, long-term implicit contracts, and workers who are initially uncertain about their own ability. According to the logic of Medoff and Abraham, therefore, in the Harris and Holmstrom model wage increases should be given both to workers for whom positive information is revealed and to workers for whom negative information is revealed. This, however, is not what Harris and Holmstrom found. In their analysis no wage increases were given to workers for whom negative information was revealed.

In this paper we investigate the insurance effect hypothesized by Medoff and Abraham. To do this we set up and analyze a simplified version of the Harris and Holmstrom model. Harris and Holmstrom analyzed their model under a very restrictive capital market assumption, i.e., workers were completely restricted from borrowing. In order to facilitate comparisons with the Harris and Holmstrom analysis, therefore, we first analyze our model under the assumption that workers cannot borrow. Not surprisingly, under this assumption the model does not exhibit the insurance effect hypothesized by Medoff and Abraham. We then analyze our model under capital market assumptions wherein at least some type of borrowing is allowed. Here we find that our model does exhibit the insurance effect hypothesized by Medoff and Abraham. That is, when borrowing is allowed, pay raises are even given to workers for whom negative information is revealed. Or in other words, it seems that the Harris and Holmstrom analysis did not yield results consistent with the insurance effect hypothesized by Medoff and Abraham because of the restrictive capital market assumption employed by Harris and Holmstrom.

The outline for the paper is as follows. Section I sets forth a simplified version of the Harris and Holmstrom model. Section II analyzes this model under the Harris and Holmstrom capital market assumption, and under two assumptions wherein at least some type of borrowing is allowed. Section III presents some concluding remarks, including a discussion of empirical tests that might be used to discriminate among the various explanations which have been proposed.

I. The Model

In this section we list the assumptions which constitute our model.

Assumptions

1) Within the economy there is only one good produced and the price of this good is normalized to one.
2) Workers live for two periods, and in each period labor supply is perfectly inelastic and fixed at one unit for each worker.
3) Workers display no disutility for effort. However, each worker has associated with him or her a value for a variable which will be called ability, and which will be denoted by A.
4) A worker's output at a firm simply equals the value of his ability.
5) Previous to his first period of employment a worker's ability is unknown both to the worker and to all the firms in the economy. However, a worker's output in every period is public information, which in turn yields that after a single period of employment a worker's ability becomes public knowledge.
6) Each worker's value for A is a draw from a random variable which equals A^H with probability p, and equals A^L with probability $(1-p)$, where $A^H > A^L$.

7) A worker's preferences over the consumption stream (c_1,c_2) are given by

$$U(c_1,c_2) = \mu(c_1) + \beta\mu(c_2),$$

where $\mu' > 0$, $\mu'' < 0$, and $\beta < 1$. This simply states that workers are risk averse with a discount factor equal to β.

8) Firms are risk neutral, where a firm's valuation over the profit stream (c_1,c_2) is given by

$$\Pi(c_1,c_2) = c_1 + \beta c_2.$$

9) In agreeing to a contract a worker cannot irrevocably bind himself to a firm.

10) A worker can change firms after his first period of employment without incurring any costs. However, for expositional simplicity it is assumed that, given equal wage offers prior to his second period of employment, a worker will choose to remain with his first period employer.

11) There is free entry.

Before ending this section, a word is in order concerning the substantive differences between our model and the Harris and Holmstrom model. There are three such differences. First, our workers live for two periods, while their workers live for T periods. Second, in our model a worker's output equals his ability, while in their model a worker's output equals his ability plus a random disturbance term. Third, in our model the distribution of A is discrete, while in their model A is normally distributed. The purpose of each of these differences is the same. That is, each allows us to more easily concentrate on why the Harris and Holmstrom analysis did not yield results consistent with the insurance effect hypothesized by Medoff and Abraham.

II. Analysis

Before proceeding to analyze the model, it is necessary to stipulate a contracting environment. Consistent with the Harris and Holmstrom analysis, it is assumed that firms offer young workers long-term or implicit contracts which specify three wage rates, denoted W_1, W_2^L and W_2^H. These contracts bind the firm in the following ways. First, the firm is obligated to pay a worker accepting the contract the wage W_1 during the worker's first period of employment. Second, the firm is restricted from firing such a worker after the worker's first period of employment. Third, if the worker is revealed to be of low (high) ability, then the firm is obligated to offer the worker the wage $W_2^L(W_2^H)$. Finally, the contract must also satisfy the restriction on wages, $W_2^j > A^j$ for $j = L,H$. This restriction guarantees that second period wages are high enough to stop the worker from being bid away by another firm.[3]

Following the Harris and Holmstrom analysis, we first analyze our model under the assumption that workers can lend any amount they choose at an interest rate equal to $(1-\beta)/\beta$, but that they are completely restricted from borrowing. Under this capital market assumption, equilibrium is characterized by the wages and consumption levels which solve the following maximization problem. Note, below $c_2^L(c_2^H)$ denotes the second period consumption of a worker who is revealed to be of low (high) ability.

$$(1) \qquad \max_{\substack{W_1, W_2^L, W_2^H \\ c_1, c_2^L, c_2^H}} \mu(c_1) + \beta[p\mu(c_2^H) + (1-p)\mu(c_2^L)]$$

$$\text{s.t. } p[A^H - W_1 + \beta(A^H - W_2^H)] + (1-p)[A^L - W_1 + \beta(A^L - W_2^L)] \geq 0$$

$$W_2^j \geq A^j \quad \text{for} \quad j = L,H$$

$$(1/\beta)\ (W_1 - c_1) + W_2^j - c_2^j \geq 0 \quad \text{for} \quad j = L,H$$

$$c_1 \leq W_1$$

Equation (1) is explained as follows. The objective function simply states that the equilibrium contract will maximize a worker's discounted expected lifetime utility. The first constraint ensures that the discounted expected profits for the firm offering the contract are non-negative. The second constraint is simply our earlier mentioned restriction on wages which guarantees that, after his first period of employment, a worker accepting the contract is not bid away by another firm. The third constraint states that the consumption stream can never exceed what is affordable given the wage stream. The fourth constraint rules out borrowing. The following proposition characterizes the solution to (1). Note, to keep the exposition from becoming bogged down in detail, we have relegated all proofs to an Appendix.

Proposition 1: When workers are completely restricted from borrowing, then

i) $A^L < W_1 = W_2^L < W_2^H = A^H$

ii) $c_1 = W_1$, $c_2^L = W_2^L$, $c_2^H = W_2^H$.

The results in Proposition 1 are quite similar to the results contained in the Harris and Holmstrom analysis. First, a worker who is revealed to be of high ability receives a raise which is just sufficient to stop the worker from being bid away by another firm. Second, a worker who is revealed to be of low ability receives the same wage he received in the previous period. Third, workers find it optimal not to lend. Notice, as mentioned previously, the results under this capital market assumption are not consistent with the insurance effect hypothesized by Medoff and Abraham. That is, according to Medoff and Abraham, a worker who is revealed to be of low ability should also receive a raise.

We now analyze the model under what we refer to as a perfect capital market assumption. Specifically, workers are allowed to lend <u>and</u> borrow any amount they choose at an interest rate equal to $(1-\beta)/\beta$. Under this capital market assumption equilibrium is characterized by the same maximization problem as previously, except now the last constraint no longer applies. The following proposition characterizes the solution to this new maximization problem.[4,5]

Proposition 2: When workers have access to a perfect capital market, then

i) $W_1 < W_2^L = W_2^H > A^H$

ii) $c_1 > W_1$, $c_2^L < W_2^L$, $c_2^H < W_2^H$

iii) $c_1 = c_2^L = c_2^H$.

Proposition 2 tells us that, when workers face a perfect capital market, then the outcome is a first best result. That is, workers face no risk because the second period wage received is independent of the ability revealed, while borrowing allows workers to smooth out their consumption stream. As opposed to the results under the no borrowing assumption, the results here are consistent with the insurance effect hypothesized by Medoff and Abraham. That is, even workers who are revealed to be of low ability receive raises.

One consequence of assuming a perfect capital market is that workers revealed to be of low ability receive the same raises as workers revealed to be of high ability. Since this result does not seem consistent with either previous empirical findings or casual observation, this result casts doubt on the validity of the perfect capital market assumption. For this reason, we now consider an intermediate assumption between the perfect capital market assumption and the no borrowing assumption. This intermediate assumption is intended to reflect the casual observation that workers face a higher interest rate when they borrow than when they lend. Formally, we assume that workers can lend as much as they choose at the interest rate $(1-\beta)/\beta$. However, if workers choose to borrow then they face the interest rate $((1-\beta)/\beta) + I(c_1-W_1)$, where $I(0) = 0$, $I'(0) = 0$ and $I'(x) > 0$ for all $x > 0$. Under this capital market assumption, equilibrium is characterized by the following maximization problem.

$$(2) \qquad \max_{\substack{W_1, W_2^L, W_2^H \\ c_1, c_2^L, c_2^H}} \mu(c_1) + \beta[p\mu(c_2^H) + (1-p)\mu(c_2^L)]$$

$$\text{s.t. } p[A^H - W_1 + \beta(A^H - W_2^H)] + (1-p)[A^L - W_1 + \beta(A^L - W_2^L)] > 0$$

$$W_2^j > A^j \quad \text{for } j = L,H$$

$$[(1/\beta) + I(c_1 - W_1)](W_1 - c_1) + W_2^j - c_2^j = 0 \quad \text{for } j = L,H$$

The following proposition characterizes the solution to (2).

Proposition 3: The solution to (2) is characterized by,

i) $W_1 < W_2^L < W_2^H = A^H$

ii) $c_1 > W_1$, $c_2^L < W_2^L$, $c_2^H < W_2^H$

iii) $c_1 = c_2^L < c_2^H$.

Proposition 3 tells us the following two things about what happens under our intermediate capital market assumption. First, the model continues to display the insurance effect hypothesized by Medoff and Abraham. Second, low and high ability workers no longer have the same age-earnings profile. In other words, under our intermediate capital market assumption, workers revealed to be of low ability receive raises, but the raises are smaller in magnitude than those received by workers revealed to be of high ability.

III. Conclusion

A number of empirical studies have found evidence consistent with the notion that some or all of the observed correlation between experience and compensation cannot be explained by a correlation between experience and productivity (see e.g., Medoff and Abraham 1980, Dalton and Thompson 1971, and Pascal and Rapping 1972). One of the explanations which has been put forth to account for these results is not backed up by a formal theoretical analysis. This explanation is due to Medoff and Abraham. Stated simply it says that, because of uncertainty a worker may have concerning his own ability, wage increases for workers who are revealed to be of low ability can serve as a form of insurance. In this paper we have attempted to investigate this insurance effect by analyzing a simplified version of a model previously looked at by Harris and Holmstrom. Consistent with Harris and Holmstrom's analysis, we first found that when workers are completely restricted from borrowing, the model does not exhibit the Medoff and Abraham insurance effect. We then analyzed the model under capital market assumptions wherein at least some type of borrowing is allowed. Here it was found that the model does exhibit the Medoff and Abraham insurance effect. That is, when borrowing was allowed, even workers who were revealed to be of low ability received pay raises.

There are two areas in which we feel future research would be productive. The first is concerned with the type of production technology employed in this paper and in most other theoretical papers in this subject area. In general, for the production technologies which have been employed, workers could only be assigned to a single job. A worthwhile exercise might be to investigate a production technology which is more consistent with the existence of a job ladder.[6] We conjecture that the analysis of such a production technology would yield results which more closey match the empirical findings of Medoff and Abraham, and Dalton and Thompson. That is, each of those papers found some evidence that, for a given job assignment, there is actually a negative correlation between experience and productivity. Our conjecture is that an analysis of the type of production technology suggested above would exhibit a result consistent with this empirical finding, because workers who are revealed to be of high ability would find themselves moving up a job ladder.

The second area in which we feel future research would be productive is in the field of empirical testing. Including the explanation investigated in this paper, there are now a number of explanations which can account for why the correlation between experience and compensation does not seem to be explained solely by a correlation between experience and productivity. However, authors who have done empirical work on this subject have not placed much effort into the conducting of tests which might allow us to discriminate among these various explanations. We feel that such effort would now be worthwhile. Below we list two tests which could be used for this purpose. First, testing could be done on groups of workers who all have a very low quit propensity. If results similar to the Medoff and Abraham empirical results were found, we would have an indication that the phenomenon is not due solely to an argument which depends on differences in quit propensities, e.g., the Grossman argument and the Salop and Salop argument. Note, because of the high rents associated with the job, baseball players rarely quit while they are still productive. They are therefore an example of a group which satisfies the specified condition, which in turn implies that the Pascal and Rapping empirical results already give us such an indication. Second, one could test the strength with which compensation increases in the absence of productivity growth. That is, is it the case, as suggested by Harris and Holmstrom, that workers for whom positive information is revealed receive raises, while other workers simply do not receive pay cuts? Or, is it the case, as is suggested by the analysis in this paper, that even workers for whom negative information is revealed receive raises? If the data is more consistent with the former statement, then we would have an indication that the insurance effect investigated in this paper is probably not at work. If, on the other hand, the data is more consistent with the latter statement, then we would have evidence in support of this insurance effect.

Footnotes

[1]In an earlier paper, Becker and Stigler (1974) pointed out that this type of compensation scheme might be an efficient means for compensating law enforcement officials.

[2]In an earlier paper, Freeman (1977) analyzed a similar but somewhat more restrictive model and derived similar results.

[3]If for some realization of the worker's ability the second period wage did not satisfy the restriction, then the worker would be bid away and in terms of worker utility and firm profits it would be as if the restriction was satisfied as an equality. Thus, following Harris and Holmstrom, we simply assume that the contract always satisfies the restriction.

[4]Harris and Holmstrom discuss how the posting of bonds may become important when workers have access to capital markets. In our model, as well as theirs, however, by appropriately varying the contractual wages it is possible to mimic without using a bond anything that can be done using a bond. Along this line, if in the optimal contract $W_1 < 0$, then the first period wage might best be thought of as including a bond which is returned with interest if the worker remains with his first period employer.

[5]There are multiple wage profiles which solve this new maximization problem. In Proposition 2 we simply present properties which all such wage profiles exhibit.

[6]One of us has already conducted an analysis somewhat along these lines (see Waldman 1983). It should be noted, however, that the analysis in that paper differs considerably from the present analysis in that the no borrowing assumption is employed throughout, and information about worker abilities is not revealed in a public manner.

Appendix

Proof of Proposition 1: The optimality conditions from (1) reduce to:

$$\mu'(c_1) = p\mu'(c_2^H) + (1-p)\,\mu'(c_2^L) + \lambda_1 \tag{A1}$$

$$\lambda_2 = \mu'(c_1) - \mu'(c_2^H) \tag{A2}$$

$$\lambda_3 = \mu'(c_1) - \mu'(c_2^L) \tag{A3}$$

$$\lambda_2[\beta p(W_2^H - A^H)] = 0, \quad \lambda_2 \geq 0 \tag{A4}$$

$$\lambda_3[\beta(1-p)\,(W_2^L - A^L)] = 0, \quad \lambda_3 \geq 0 \tag{A5}$$

$$\lambda_1\,[W_1 - c_1] = 0, \quad \lambda_1 \geq 0 \tag{A6}$$

and the constraints in (1), (where λ_1, λ_2 and λ_3 are the Kuhn-Tucker multipliers associated with the constraints $W_1 \geq c_1$, $W_2^H \geq A^H$, and $W_2^L \geq A^L$ respectively).

To prove the proposition, first note that both the worker's budget constraint and the firm's non-negative profit constraints must hold as equalities. The worker's budget constraint must be binding because otherwise the worker's utility could be raised by increasing the consumption in either period. Given this, the expected profits must be zero because otherwise the worker's utility could be raised by increasing the wage in either period.

The proposition is proven by considering the feasibility of the following four mutually exclusive characteristics of λ_2 and λ_3 that completely exhaust the possible combinations: (a) $\lambda_2 > 0$, $\lambda_3 > 0$; (b) $\lambda_2 = 0$, $\lambda_3 > 0$; (c) $\lambda_2 = 0$, $\lambda_3 = 0$; (d) $\lambda_2 > 0$; $\lambda_3 = 0$.

First, consider (a). If $\lambda_2 > 0$ and $\lambda_3 > 0$, then by (A4) and (A5), $W_2^H = A^H$ and $W_2^L = A^L$. This implies by the zero expected profit constraint that $W_1 = pA^H + (1-p)A^L$. Hence, $W_2^H > W_1 > W_2^L$. However, with $\lambda_2 > 0$ and $\lambda_3 > 0$, by (A2) and (A3), $c_1 < c_2^H$ and $c_1 < c_2^L$. This implies by (A1) that $\lambda_1 > 0$ which in turn implies $c_1 = W_1$. With $c_1 = W_1$, then by the budget constraints $c_2^H = W_2^H$ and $c_2^L = W_2^L$. Yet, this implies $W_2^L > W_1$ which involves a contradiction.

Now, consider (b). If $\lambda_3 > 0$, then $W_2^L = A^L$. If $\lambda_2 = 0$, then by (A2) $c_1 = c_2^H$. Moreover, with $\lambda_3 > 0$, $c_2^L > c_1$ by (A3). Taken together this implies with (A1) that $\lambda_1 > 0$ and thus $W_1 = c_1$. With $W_1 = c_1$, then by the budget constraints, $c_2^H = W_2^H$ and $c_2^L = W_2^L$. Since $c_1 = c_2^H$, this implies that $W_1 = W_2^H > A^H$. Since $W_2^L = c_2^L > c_2^H = c_1 = W_1$, this implies $W_2^L > A^H$. Taken together, $W_2^L > W_1 = W_2^H > A^H$ violates the non-negative expected profit constraint.

Next consider (c). If $\lambda_2 = 0$ and $\lambda_3 = 0$, then by (A2) and (A3), $c_1 = c_2^H = c_2^L$. By the budget constraints, this implies $W_2^H = W_2^L$. Since $W_2^H > A^H$, this implies by the non-negative expected profit constraint that $W_1 < W_2^H = W_2^L$. Since $c_1 = c_2^H = c_2^L$, this implies by the budget constraints that $c_1 > W_1$. Yet this violates the constraint $c_1 > W_1$. Hence, (c) cannot hold.

Thus, the only feasible combination of λ_2 and λ_3 is (d) (it is easily demonstrated that (d) involves no contradiction). Given that $\lambda_2 > 0$ and $\lambda_3 = 0$, this implies by (A2) that $c_1 < c_2^H$, by (A3) that $c_1 = c_2^H$, and by (A4) that $W_2^H = A^H$. Since $c_2^H > c_1 = c_2^L$, by (A1), $\lambda_1 > 0$. Hence, $c_1 = W_1$. Since $c_1 = W_1$, by the budget constraint, $c_2^L = W_2^L$. Taken together, we have $A^H = W_2^H > W_2^L = W_1$, $c_1 = W_1$, $c_2^H = W_2^H$ and $c_2^L = W_2^L$. Given this, since the non-negative expected profit constraint holds as an equality, $W_2^L = W_1$

$> A^L$. This completes the proof of the proposition.

Proof of Proposition 2: The optimality conditions in this case reduce to (A1)-(A5) (where $\lambda_1 = 0$) and the constraints in (1) (excluding $W_1 > c_1$). As previously, the worker's budget constraint and the firm's non-negative profit constraint must hold as equalities. Now, to prove the proposition, we will first prove that $c_1 = c_2^H = c_2^L$. By (A1), observe that if $c_2^H \neq c_2^L$, then either $c_2^H > c_1 > c_2^L$ or $c_2^L > c_1 > c_2^H$. Yet, if $c_2^H > c_1 > c_2^L$, then by (A3), $\lambda_3 < 0$ which is contradictory. Similarly, if $c_2^L > c_1 > c_2^H$, then by (A2), $\lambda_2 < 0$ which is contradictory. Hence, $c_2^H = c_2^L$. By (A1), this implies $c_1 = c_2^H = c_2^L$. Moreover, $c_1 = c_2^H = c_2^L$ implies by the budget constraints that $W_2^H = W_2^L$. By (A4) and (A5) this implies $W_2^L = W_2^H > A^H$. Given this, the zero expected profit constraint implies $W_1 < A^H < W_2^H = W_2^L$. Since $c_1 = c_2^H = c_2^L$ and $W_1 < W_2^H = W_1^L$, this implies by the budget constraints that $c_1 > W_1$, $c_2^H < W_2^H$ and $c_2^L < W_2^L$. This completes the proof of the proposition.

Proof of Proposition 3: The optimality conditions from (2) reduce to (A2)-(A5),

$$\text{(A6)} \qquad \mu'(c_1) = [p\mu'(c_2^H) + (1-p)\ \mu'(c_2^L)]\ [1 + \beta[I + I'.(c_1-W_1]],$$

and the constraints in (2). As previously, the worker's budget constraint and the firm's non-negative expected profit constraint must hold as equalities. To prove the proposition, first observe that (A2) and (A3) together imply $c_1 < c_2^H$ and $c_1 < c_2^L$. Given this, we now prove that $W_2^H > W_2^L > W_1$. First, suppose that $W_2^L > W_2^H$. In this event, since by (A4), $W_2^H > A^H$, this implies

$\lambda_3 = 0$. By (A3), this implies $c_1 = c_2^L$. Note, as well, that $W_2^L > W_2^H$ implies $c_2^L > c_2^H$ by the budget constraints. Since $c_1 = c_2^L > c_2^H$ and $c_2^H > c_1$ this implies $c_1 = c_2^H = c_2^L$. The zero expected profit constraint implies $W_1 < A^H$ when $W_2^L > W_2^H$. Hence, with $c_1 = c_2^H = c_2^L$ and $W_2^L > W_2^H > W_1$, by the budget constraints, $c_1 > W_1$. Yet, if $c_1 = c_2^H = c_2^L$ and $c_1 > W_1$, then the optimality condition (A6) is violated. Thus, $W_2^L > W_2^H$ yields a contradiction.

Now suppose that $W_2^L < W_1$. Since $W_2^L > A^L$ by (A4), this implies $W_1 > A^L$. Given that $W_2^H > A^H$ and $W_1 > W_2^L > A^L$, the zero expected profit constraint implies $W_1 < W_2^H$. Hence, $W_2^H > W_1 > W_2^L$. By the budget constraints, this implies $c_2^H > c_2^L$. Since $c_2^H > c_1$ and $c_2^L > c_1$, this implies $c_2^H > c_2^L > c_1$. This in turn implies by (A6) that $c_1 > W_1$. If $c_1 > W_1$, then by the budget constraints $W_2^L > c_2^L$. This implies $W_2^L > c_2^L > c_1 > W_1$. This contradicts $W_1 > W_2^L$. Hence, $W_2^L > W_1$. Thus, we have established that $W_2^H > W_2^L > W_1$.

Next, we prove that $c_1 = c_2^L < c_2^H$. First, since $W_2^H > W_2^L$, we have by the budget constraints, (A2), and (A3) that $c_2^H > c_2^L > c_1$. Suppose $c_2^L > c_1$. Then by (A3), $\lambda_3 > 0$ and by (A5), $W_2^L = A^L$. Since $c_2^H > c_1$, we have by (A2), $\lambda_2 > 0$ and thus $W_2^H = A^H$. By the zero expected profit constraint this implies $W_1 = pA^H + (1-p)A^L > A^L = W_2^L$. This contradicts $W_2^L > W_1$. Hence, $c_2^L = c_1$. Thus, we have established $W_2^H > W_2^L > W_1$ and $c_2^H > c_2^L = c_1$. Note, as well, that $c_2^H > c_1$ implies by (A2) that $\lambda_3 > 0$ and hence $W_2^H = A^H$. Given this, by the arguments above, $W_2^L > A^L$, since otherwise this yields a contradiction. Taken together, we have $W_1 < W_2^L < W_2^H = A^H$ and $c_1 = c_2^L < c_2^H$. By the budget constraint, this implies $c_1 > W_1$, $c_2^L < W_2^L$ and $c_2^H < W_2^H$. This completes the proof of the proposition.

References

Becker, G., and G. Stigler, "Law Enforcement, Malfeasance, and Compensation of Enforcers," Journal of Legal Studies (1974), 1-18.

Dalton, G., and P. Thompson, "Accelerating Obsolescence of Older Engineers," Harvard Business Review, XLIX (1971), 57-67.

Freeman, S., "Wage Trends as Performance Displays Productive Potential: A Model and Application to Academic Early Retirement," Bell Journal of Economics, VIII (1977), 419-443.

Grossman, H., "Risk Shifting and Reliability in Labor Markets," Scandanavian Journal of Economics, LXXIX (1977), 187-209.

Harris, M., and B. Holmstrom, "A Theory of Wage Dynamics," Review of Economic Studies, XLIX (1982), 315-333.

Lazear, E., "Why is There Mandatory Retirement," Journal of Political Economy, LXXXVII (1979), 1261-1284.

Lazear, E., "Agency, Earnings Profiles, Productivity, and Hours Restrictions," American Economic Review, LXXI (1981), 606-620.

Medoff, J., and K. Abraham, "Experience, Performance and Earnings," Quarterly Journal of Economics, XCV (1980), 703-736.

Pascal, A., and L. Rapping, "The Economics of Racial Discrimination in Organized Baseball," in Pascal, A. (ed.) Racial Discrimination in Economic Life, Lexington, MA: Lexington Books (1972), 119-156.

Salop, J., and S. Salop, "Self-Selection and Turnover in the Labor Market," Quarterly Journal of Economics, XC (1976), 619-627.

Waldman, M., "Job Assignments, Signalling, and Efficiency," Working Paper No. 286, University of California, Los Angeles, February 1983.