RATIONAL EXPECTATIONS AND THE LIMITS OF RATIONALITY: AN ANALYSIS OF HETEROGENEITY

by

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A recurring controversy in economic thought concerns the conflict between the assumption of rationality and the fact that economic agents have limited capacities to process information.[1] For example, this conflict was a factor in the marginalist debate of the forties, and is a factor in the challenge to standard economic theory of Herb Simon and his followers.[2]. Recent refinements to the concept of rationality have brought this conflict into sharper focus. That is, rationality no longer simply implies that behavior is determined by the maximization of a well ordered function. Rather, it now typically implies the expected utility hypothesis of behavior under uncertainty, and the rational expectations hypothesis for the formation of expectations.

Those individuals who have attempted to provide alternatives to the rationality assumption have in general paid close attention to descriptive realism. In other words, the alternative models of the decision making process which have been provided tend to match, at least to a first approximation, the manner in which decisions are reached in the real world. One aspect of the real world, however, has generally been ignored in these alternative models. Specifically, these models, as well as most models which contain the rationality assumption, ignore the idea that agents tend to be heterogeneous in terms of information processing abilities. That is, some agents in our economy are able to process information in a very sophisticated manner, while others are much more limited in their capabilities. In the present paper we attempt to investigate the ramifications of this type of heterogeneity. We do this by analyzing two simple models wherein agents differ in terms of their ability to form expectations. To keep the analysis tractable, in each of the models it is assumed that only two types of agents exist. Agents of the first type have unlimited abilities to form expectations, and thus have correct or rational expectations. These agents will be referred to as "sophisticated".

Agents of the second type are limited in their ability to form expectations, and thus have incorrect expectations. These agents will be referred to as "naive". Our goal is to characterize equilibria under this type of heterogeneity, and in the process identify situations in which sophisticated agents have a disproportionately large effect on equilibrium, and situations in which naive agents have a disproportionately large effect.[3,4]

One might question our approach of looking at the equilibria of models which contain agents who have incorrect expectations. Some might argue that, since models with learning frequently converge to rational expectations equilibria, in the types of models we are investigating it only makes sense to investigate rational expectations equilibria.[5] We disagree with this argument for three reasons. First, there are many important economic situations which are not repeated for the agents involved, and for these situations the fact that learning models converge to rational expectations equilibria is irrelevant. Examples include the career choice problem faced by the young, and the related decision concerning whether or not to attend college. Second, there are many economic situations which repeat, however, for each repetition there is a proportion of agents who have no previous experience with the situation. An example of this is the problem consumers face when they must choose among different computer hardware systems. That is, for each new generation of computers there is likely to be a proportion of buyers who have not previously bought a computer system. Third and finally, empirical evidence does not support the practice of restricting attention solely to rational expectations equilibria (see Arrow 1982 for a survey of this evidence).

The outline for this paper is as follows. In Section I each agent in the economy is faced with the problem of choosing a single path from a set of two, where the paths exhibit congestion effects. By congestion effects we mean that for any agent i, the higher is the number of other agents who choose the same path as the one chosen by agent i, the less well off is agent i. The model of this section might best be thought of as a stylized model of either the problem of career choice, or the problem of choosing a road with which to reach some final destination. In this setting we find that sophisticated agents have a disproportionately large effect on equilibrium. That is, congestion effects cause the equilibrium to more closely resemble what occurs when all agents have rational expectations than would be suggested by the relative number of sophisticated agents and naive agents in the population.

The intuition behind this result is as follows. Because of incorrect expectations, the allocation of naive agents to paths is biased relative to how agents are allocated when there are no naive agents. Sophisticated agents, on the other hand, anticipate this behavior and, because of congestion effects, compensate by having their behavior being biased in an exactly opposite manner. Or overall, sophisticated agents have a disproportionately large effect on equilibrium because sophisticated agents anticipate the bias of the naive, and compensate in a way which tends to nullify this bias.

In Section II we again look at the model developed in Section I, but this time we change the specification so that the model exhibits the reverse of what we previously referred to as congestion effects. In other words, for any agent i, the higher is the number of other agents who choose the same path as the one chosen by agent i, the <u>better</u> off is agent i. We will say that situations which exhibit this type of property exhibit synergistic effects. Contrary to what we found in Section I, here it is the naive agents who have a disproportionately large effect on equilibrium. That is, synergistic effects cause the equilibrium to more closely resemble what occurs when all agents are naive than would be suggested by the relative number of sophisticated agents and naive agents in the population.

The intuition behind this result is similar to the intuition given above. Because of incorrect expectations, the allocation of naive agents to paths is again biased relative to how agents are allocated when there are no naive agents. Furthermore, sophisticated agents again anticipate this behavior, but because there are synergistic effects, they now compensate by having their behavior being biased in a manner <u>similar</u> to the bias of the naive. Or overall, naive agents have a disproportionately large effect on equilibrium because sophisticated agents anticipate the bias of the naive, and compensate in a way which tends to exaggerate this bias.

In Section III we consider a simple model of the chain-store paradox (see Selten 1978, Kreps and Wilson 1982, and Milgrom and Roberts 1982 for previous papers on this issue). The key property of this model is that it is one where reputation effects are potentially important. The basic result which emerges from the analysis of Section III is that, when reputation effects are potentially important, it is possible for either type of agent to be dominant. That is, under some parameterizations sophisticated agents have a disproportionately large effect on equilibrium, while for other parameterizations the naive agents are the ones who have the disproportionately large effect.

One might ask what conclusions can be drawn from the above results as regards the common practice of assuming that all agents in the economy have rational expectations. Our feeling is that since agents in the real world are obviously heterogeneous in terms of information processing abilities, the practice of assuming rational expectations is relatively more defensible when agents who can process information in a very sophisticated manner have a disproportionately large effect on equilibrium. Thus, our results suggest that for the analysis of situations which exhibit congestion effects, there are relatively strong justifications for assuming rational expectations. However, for the analysis of situations which exhibit either synergistic effects or the possibility for reputation formation, the practice of assuming rational expectations would seem to be less defensible.

I. Choosing Paths with Congestion Effects

In this section we consider the problem of agents choosing between two paths, where the paths exhibit congestion effects. By congestion effects we mean that for any agent i, the higher is the number of other agents who choose the same path as the one chosen by agent i, the worse off is agent i. Examples of real world choice situations which exhibit congestion effects are the career choice problem faced by the young, the related decision concerning whether or not to attend college, and the commuter's problem of choosing between alternate routes.

As indicated above, in this section each agent must choose between two paths. We will denote these two paths simply as path A and path B. Furthermore, it is assumed that this choice of paths is an irreversible choice, and that it is made simultaneously by all the agents in the population. If agent i takes path A, then his utility equals $f_A(N_A) - c_i$, where $f_A' < 0$ and where N_A denotes the total number of agents who choose path A. Similarly, if agent i takes path B, then his utility equals $f_B(N_B) - (C-c_i)$, where $f_B' < 0$ and where N_B denotes the total number of agents who choose path B. It is also assumed that either f_A' or f_B' is always strictly negative. Note, c_i and $(C-c_i)$ can be interpreted as representing either agent i's underlying preferences for the two paths, or the costs incurred by agent i in taking each path. This latter interpretation matches well with our description of the model as a model of commuting. That is, when thought of as a model of commuting, c_i can be interepreted as the distance between agent i's housing location and path A's entry ramp, while $(C-c_i)$ can be interpreted as the distance between the housing location and path B's entry ramp.

The population consists of a continuum of agents who vary in terms of their values for c_i. In particular, the distribution of c_i's in the population is described by a density $g(c_i)dc_i$ defined on the interval $[0,C]$, where $g(.)$ is continuously differentiable and nonzero everywhere in the specified interval. As indicated in the Introduction, there are two types of agents in this population: sophisticated and naive. It is assumed that a proportion p of the total population is sophisticated, $0 < p < 1$, while a proportion $(1-p)$ is naive. Formally, this translates into assuming that the distribution of c_i's in the population of sophisticated (naive) agents is described by a density $h(c_i)dc_i$ $(j(c_i)dc_i)$ defined on the interval $[0,C]$, where $h(.) = pg(.)$ $(j(.) = (1-p)\, g(.))$ everywhere in this interval. Note, we also assume $f_A(\int_0^C g(c_i)dc_i) > f_B(0) - C$ and $f_B(\int_0^C g(c_i)dc_i) > f_A(0) - C$. This assumption guarantees that, as long as $0 < p < 1$, the equilibrium for our model will be characterized by some agents taking each path.

The only aspect of the model which remains to be specified concerns expectations. Expectations are relevant in that, prior to deciding which path to take, each agent forms expectations concerning the resulting value for $f_A(N_A) - f_B(N_B)$. Sophisticated agents are assumed to have unlimited abilities to form expectations, and thus have correct or rational expectations concerning the resulting value for $f_A(N_A) - f_B(N_B)$. On the other hand, naive agents are limited in their ability to form expectations, but all have the same incorrect expectations concerning $f_A(N_A) - f_B(N_B)$, where V denotes this expectation.[6]

The above assumption that all naive agents have the same incorrect expectations is basically a simplifying assumption. There is, however, a reason for assuming that all naive agents have similar biases in their expectations. That is, in the studies of cognitive psychologists it is frequently the case that the expectational errors of agents are correlated across individuals (see Kahneman, Slovic and Tversky 1982).

The rest of this section consists of an analysis of the above model.[7] Let $D = (f_A(N_A)-f_B(N_B)+C)/2$ and $D^n = (V+C)/2$. Given that sophisticated agents have rational expectations, sophisticated agent i will choose path A(B) when[8]

$$(1) \qquad c_i < (>) D.$$

On the other hand, given the expectations of the naive, naive agent i will choose path A(B) when

$$(2) \qquad c_i < (>) D^n.$$

(1) and (2) in turn yield[9]

$$(3) \qquad N_A = \int_0^D pg(c_i)dc_i + \int_0^{D^n} (1-p)g(c_i)dc_i$$

and

$$(4) \qquad N_B = \int_D^C pg(c_i)dc_i + \int_{D^n}^C (1-p)g(c_i)dc_i.$$

Let $N_A^s(N_B^s)$ denote the number of agents who choose path A(B) when all are sophisticated, i.e., $p = 1$, $N_A^n(N_B^n)$ denote the number of agents who choose path A(B) when all are naive, i.e., $p = 0$, and D^s denote the value for D when $p = 1$. Note, we refer to the equilibrium when $p = 1$ as the pure rational expectations equilibrium, and the equilibrium when $p = 0$ as the pure limited rationality equilibrium. (3) and (4) yield

(5a) $$N_A^s = \int_0^{D^s} g(c_i)dc_i,$$

(5b) $$N_B^s = \int_{D^s}^{C} g(c_i)dc_i,$$

(5c) $$N_A^n = \int_0^{D^n} g(c_i)dc_i,$$

and

(5d) $$N_B^n = \int_{D^n}^{C} g(c_i)dc_i.$$

(1) through (5) in turn allow us to prove the following.

Proposition I.1: If $0 < p < 1$ and $N_A^s \gtrless N_A^n$, then $N_A \gtrless pN_A^s + (1-p)N_A^n$. Note, this also implies that if $0 < p < 1$ and $N_B^s \gtrless N_B^n$, then $N_B \gtrless pN_B^s + (1-p)N_B^n$.

Proof: Suppose not, i.e., suppose for instance that $N_A^s > N_A^n$ and that there exists a $0 < p < 1$, denoted $\hat{p}$, such that $N_A < \hat{p}N_A^s + (1-\hat{p})N_A^n$. Our suppositions that $N_A^s > N_A^n$ and $N_A < \hat{p}N_A^s + (1-\hat{p})N_A^n$ together with the definitions of D and D^s yield that D at $p = \hat{p}$ must exceed D^s. On the other hand, our two suppositions together with (3), (5a), (5c) and the definition of D^n yield that D^s must exceed or equal D at $p = \hat{p}$. Thus we have a contradiction, which means that if $0 < p < 1$ and $N_A^s > N_A^n$, then $N_A > pN_A^s + (1-p)N_A^n$. The other cases are demonstrated similarly.

The interpretation of Proposition I.1 is straightforward. When agents are heterogeneous in terms of information processing abilities, i.e.,

$0 < p < 1$, sophisticated agents have a disproportionately large effect on the number of agents who take each path. That is, in terms of the number of agents who take each path, the equilibrium more closely resembles the pure rational expectations equilibrium than would be suggested by the relative number of sophisticated agents and naive agents in the population. The next step of the analysis is to consider the social welfare aspects of our model.

Social welfare, denoted simply as W, is here defined as the sum of the utilities of all the agents in the population. Furthermore, to stay consistent with previous notation, W^s will denote social welfare when $p = 1$ and W^n will denote social welfare when $p = 0$. In terms of social welfare, we feel there are two questions which deserve to be addressed. The first question is analogous to the question addressed in Proposition I.1. That is, how does W compare with $\bar{W}$, where $\bar{W} = pW^s + (1-p)W^n$. The second question is how do the reactions of the sophisticated agents, in response to the behavior of the naive, affect social welfare. That is, do the reactions of the sophisticated tend to drive social welfare towards W^s or towards W^n. To answer this second question we compare W with $\hat{W}$, where $\hat{W}$ is the social welfare which results when sophisticated agents behave as if D equals D^s, while naive agents behave as before. In other words, $\hat{W}$ is the social welfare which results when, even though agents are heterogeneous, sophisticated agents behave as if all agents were sophisticated.

Before proceeding to our social welfare proposition, it is necessary to define some terms concerning the optimal allocation of agents to paths. Note, because congestion effects create externalities, the equilibrium when all agents are sophisticated does not in general yield an optimal allocation of agents to paths. Suppose there was a social welfare maximizing government which was all knowing, and which could direct the actions of all the agents in the population. We will denote as $N_A^0(N_B^0)$ the total number of agents such a government would send to path A(B).

We can now make comparisons between W and our two social welfare benchmarks.

Proposition I.2: Suppose $f_A''(\cdot) < 0$, $f_B''(\cdot) < 0$, $0 < p < 1$ and that either $N_A^s, N_A^n < N_A^0$ or $N_A^s, N_A^n > N_A^0$. If $W^s > W^n$, then $W > \bar{W}$ and $W > \hat{W}$. If $W^s < W^n$, then $W < \hat{W}$.

Proof: See Appendix.

Proposition I.2 provides support for the notion that, even in terms of social welfare, sophisticated agents tend to be disproportionately important. That is, given a few restrictions, the following two statements are necessarily true. First, in response to the behavior of the naive, the sophisticated react in a manner which drives social welfare towards W^s and away from W^n. Second, for the case $W^s > W^n$, it is also true that W is closer to W^s than would be suggested by the relative number of sophisticated agents and naive agents in the population. The reason our results are stronger for the case $W^s > W^n$ is the following. As indicated above, given our restrictions we are able to demonstrate that the response of the sophisticated to the behavior of the naive always drives social welfare towards W^s and away from W^n. It is also true that, given our restrictions, $\hat{W}$ is never less $\bar{W}$. Thus, when $W^s > W^n$, $W > \hat{W}$ implies $W > \bar{W}$. However, when $W^s < W^n$ the same logic does not apply.

A word is in order concerning our restrictions. First, we assume there are increasing marginal congestion effects, i.e., $f_A''(\cdot) < 0$ and $f_B''(\cdot) < 0$. This assumption basically serves to guarantee that social welfare is a single-peaked function of the number of sophisticated agents who take each path -- a property which is critical for our proof.[10] Second, we assume that either $N_A^s, N_A^n < N_A^0$ or $N_A^s, N_A^n > N_A^0$. The role of this assumption is as follows. Our proof relies on the idea that, when the behavior of the naive drives social welfare down, the sophisticated respond by shifting towards first best optimal behavior -- thus driving social welfare up. On the other hand, when the behavior of the naive drives social welfare up, the sophisticated respond by shifting away from first best optimal behavior -- thus driving social welfare down. The role of the restriction that either $N_A^s, N_A^n < N_A^0$ or $N_A^s, N_A^n > N_A^0$ is primarily to guarantee that the response of the sophisticated, relative to first best optimal behavior, is consistent with the above. For example, suppose the restriction was violated and the naive behaved in a manner which tended to lower social welfare. Even though the sophisticated would respond by having behavior being biased in an exactly opposite manner, given that our restriction is violated, this could actually entail having the behavior of the sophisticated move away from first best optimal behavior.[11]

One interesting implication of the above proposition concerns the following issue. Consider a choice situation where a proportion of the population is naive, and the naive behave in a manner which lowers social welfare. The above proposition tells us that, if the environment exhibits congestion effects, then there may be little reason to be concerned about the incorrect expectations of the naive. That is because, in response to the behavior of the naive, the sophisticated tend to react in a manner which reduces the social welfare loss attributable to these incorrect expectations.

Finally, in concluding this section we would like to point out that in addition to providing insights concerning the interaction of naive and sophisticated agents, this model can also be used to provide insights concerning the interaction of informed and uninformed agents (see Salop and Stiglitz 1977, and Grossman and Stiglitz 1980 for examples of previous analyses concerned with informed and uninformed agents). That is, the model can be reinterpreted as one where all agents have rational expectations, but only some agents are informed about a relevant piece of information. For example, consider the problem of commuters having to choose which of two routes to travel, where only a portion of the agents is aware that one of the routes has a lane blocked off. The model analyzed in this section can be reinterpreted as a model of just this problem. Under this interpretation the sophisticated agents become the agents who are informed of the closed lane, while the naive agents become the uninformed agents. The conclusion which can be drawn concerning the interaction of informed and uninformed agents is basically the same conclusion as drawn above for sophisticated and naive agents. That is, when there are congestion effects, informed agents tend to have a disproportionately large effect on equilibrium.

II. Choosing Paths with Synergistic Effects

In this section we consider the problem of agents choosing between two paths, where the paths exhibit synergistic effects. By synergistic effects we mean that for any agent i, the higher is the number of other agents who choose the same path as the one chosen by agent i, the better off is agent i. An example of a real world choice situation which exhibits synergistic effects is the problem faced by consumers in choosing a computer hardware system. This choice problem exhibits synergistic effects in that the larger the number of individuals who purchase a particular system, the greater will be the subsequent availability of computer peripherals and software for that system. Other examples include the problems faced by consumers in choosing between a cassette and an 8-track tape player, and the choice between the

"Beta" and "VHS" formats for video cassette recorders. Similar to the computer example, these choice problems exhibit synergistic effects because the larger the number of individuals who purchase a particular system, the greater will be the subsequent variety of tapes available for that system.

To investigate the problem of agents choosing between paths when synergistic effects are present, we analyze a variant of the model developed in the previous section. The new assumptions are as follows. First we assume $f_A' \geq 0$, $f_B' \geq 0$ and that one of the two is always strictly positive. Second, we assume $f_A(0) > f_B(\int_0^C g(c_i)dc_i) - C$ and $f_B(0) > f_A(\int_0^C g(c_i)dc_i) - C$. Third, we assume $[f_A'(\int_0^z g(c_i)dc_i) + f_B'(\int_z^C g(c_i)dc_i)]\, g(z)/2 < 1$ for all $0 \leq z \leq C$. The first assumption insures that the model now exhibits synergistic effects. The second assumption is the synergistic analogue to the Section I assumption which guaranteed that, as long as $0 < p < 1$, the equilibrium is characterized by some agents taking each path. The third assumption eliminates the possibility of multiple equilibria (note: in Section I no similar assumption was needed to rule out the possibility of multiple equilibria).

Under this specification equations (1) through (5) of the previous section continue to hold. This in turn allows us to prove the following.

Proposition II.1: If $0 < p < 1$ and $N_A^n \gtrless N_A^s$, then $N_A \gtrless pN_A^s + (1-p)N_A^n$. Note, this also implies that if $0 < p < 1$ and $N_B^n \gtrless N_B^s$, then $N_B \gtrless pN_B^s + (1-p)N_B^n$.

Proof: Suppose not, i.e., suppose for instance that $N_A^n < N_A^s$ and that there exists a $0 < p < 1$, denoted $\hat{p}$, such that $N_A \geq \hat{p}N_A^s + (1-\hat{p})N_A^n$. Given this, (3), (5a) and (5c) yield that D at $p = \hat{p}$ must exceed or equal D^s. On the other hand, our assumption $[f'_A(\int_0^z g(c_i)dc_i) + f'_B(\int_z^C g(c_i)dc_i)]g(z)/2 < 1$ for all $0 < z < C$ combined with (3) yields that N_A is a strictly increasing function of N_A^n. We also know $N_A = N_A^s$ when $N_A^n = N_A^s$, which given the preceding implies N_A at $p = \hat{p}$ must be strictly less than N_A^s. Furthermore, given $f'_A > 0$, $f'_B > 0$ and that one is always strictly positive, the fact that N_A^s exceeds N_A at $p = \hat{p}$ yields a contradiction because it implies D^s exceeds D at $p = \hat{p}$. Thus, when $N_A^n < N_A^s$, it must be the case that $N_A < pN_A^s + (1-p)N_A^n$. The other cases are demonstrated similarly.[12]

The interpretation of Proposition II.1 is straightforward. When agents choose among paths which exhibit synergistic effects, naive agents, rather than sophisticated agents, have a disproportionately large effect on the number of agents who take each path. That is, in terms of the number of agents who take each path, the equilibrium more closely resembles the pure limited rationality equilibrium than would be suggested by the relative number of sophisticated agents and naive agents in the population.

We feel this result is especially interesting in that it provides a rationale for the persistent dominance of IBM in the computer industry. As mentioned earlier, one example of a choice situation which exhibits synergistic effects is the problem of choosing a computer hardware system. This problem exhibits synergistic effects because the availability of computer peripherals and software for a system depends on the number of individuals who purchase the system. Now, suppose each time a new generation of computers comes onto the market, a segment of the population anticipates that the market shares of the different brands will be the same as for the previous generation of computers. Because of the synergistic properties of the market, in such a situation brands that are successful in one period would have a high probability of having their success repeated. There are two reasons for this. First, consumers who extrapolate will tend to purchase brands with previous high market shares. Second, given Proposition II.1, it is also true that more sophisticated consumers will tend to behave in this manner. Thus, the persistent dominance of IBM in the computer industry may very well be due to the synergistic properties of the market.

The next step of the analysis is to consider again the social welfare aspects of the model. In what ensues W, $\bar{W}$, $\hat{W}$, N_A^0 and N_B^0 will be defined as in Section I. For the analysis of synergistic effects, however, it is also necessary to define some terms concerning the following second best problem. Suppose there was a social welfare maximizing government which was all knowing, but which could only direct the actions of the sophisticated agents in the population. We will denote as $N_A^o(N_B^o)$ the total number of agents who would wind up at path $A(B)$, given a government with this limited ability to direct behavior.

We can now make comparisons between W and our two social welfare benchmarks.

Proposition II.2: Suppose $f_A''(.) < 0$, $f_B''(.) < 0$, $0 < p < 1$ and that either $N_A^s, N_A^n < N_A^0$ and $\hat{N}_A < N_A^o$, or $N_A^s, N_A^n > N_A^0$ and $\hat{N}_A > N_A^o$, where $\hat{N}_A = pN_A^s + (1-p) N_A^n$. If $W^n < W^s$, then $W < \bar{W}$ and $W < \hat{W}$. If $W^n > W^s$, then $W > \hat{W}$.

Proof: See Appendix.

Proposition II.2 provides support for the notion that, even in terms of social welfare, naive agents tend to be disproportionately important. That is, given a few restrictions, the following two statements are necessarily true. First, in response to the behavior of the naive, the sophisticated react in a manner which drives social welfare towards W^n and away from W^s. Second, if $W^n > W^s$, then W is closer to W^n than would be suggested by the relative number of sophisticated agents and naive agents in the population. The reason our results are stronger for the case $W^n < W^s$ is similar to the reason our results in Proposition I.2 were stronger for the case $W^n < W^s$. The only difference is that now $\bar{W}$ is never less than $\hat{W}$.

A word is in order concerning our restrictions. First, we assume there are decreasing marginal synergistic effects, i.e., $f''_A(\cdot) < 0$ and $f''_B(\cdot) < 0$. As was the role of the assumption of increasing marginal congestion effects in Proposition I.2, this assumption guarantees that social welfare is a single-peaked function of the number of sophisticated agents who take each path. Second, we assume that either $N^s_A, N^n_A < N^0_A$ and $\hat{N}_A < N^o_A$ or $N^s_A, N^n_A > N^0_A$ and $\hat{N}_A > N^o_A$. This assumption serves the same role as the assumption in Proposition I.2 that either $N^s_A, N^n_A < N^0_A$ or $N^s_A, N^n_A > N^0_A$. One question which arises is why is the condition more restrictive in Proposition II.2. The answer is that, when there are congestion effects, assuming that $N^s_A, N^n_A <(>) N^0_A$ guarantees that $\hat{N}_A <(>) N^o_A$. A second question is what is the role played by the additional restriction. Propositions I.2 and II.2 work because, under our restrictions, the following statement is satisfied. When the presence of naive agents causes the behavior of the sophisticated to move away from (towards) first best optimal behavior, social welfare tends to be driven down (up) or below (above) our social welfare benchmarks. The role of our additional restriction is simply to guarantee that the above statement is valid. That is, if the restriction is violated, then we have the possibility of a standard second best problem wherein having the sophisticated move away from (towards) first best optimal behavior might actually increase (decrease) social welfare.

As in the previous section, we can consider what our analysis implies for a choice situation where a proportion of the population is naive, and the naive behave in a manner which tends to lower social welfare. The above proposition tells us that, if such a choice environment exhibits synergistic effects, then there may be reason to be quite concerned about the incorrect expectations of the naive. That is because, in response to the behavior of the naive, the sophisticated now react in a manner which tends to increase the social welfare loss attributable to these incorrect expectation.

Finally, in concluding the previous section we noted that in addition to providing insights concerning the interaction of naive and sophisticated agents, the model of Sections I and II can also be used to provide insights concerning the interaction of informed and uninformed agents. That is, the model can be reinterpreted as one where all agents have rational expectations, but only some agents are informed of a relevant piece of information. Our conclusion regarding this point in Section I was that, when there are congestion effects, informed agents tend to have a disproportionately large effect on equilibrium. In a contrary fashion, the conclusion we can draw from the analysis of Section II is that, when there are synergistic effects, the uninformed agents tend to be the ones who are disproportionately important.

III. The Chain-Store Paradox Revisited

In this section we analyze a variant of a game initially studied by Selten (1978). The game is basically described as follows. Let there be a monopolist who operates in T separate markets, and who faces a different potential entrant in each market. In each market, furthermore, let the potential entrant move first by deciding whether or not to enter the market. If the potential entrant decides not to enter, then there are no further moves by players in that market. If entry does occur, however, then the monopolist has to decide whether to cooperate or to act aggressively. Finally, let payoffs satisfy the following three conditions. First, if entry has occurred in market k, then both the monopolist and the entrant receive higher profits in market k if the monopolist acts cooperatively. Second, for each market k, the monopolist receives even higher profits if entry does not occur. Third, for each market k, if the monopolist is sure to act aggressively then the potential entrant is better off not entering.

Selten referred to this game as the chain-store game, and pointed out that the game embodies a paradox. On the one hand, intuition suggests that if such a game were to actually be played, the monopolist would likely act aggressively whenever entry occurs in an early market, and in turn this would limit the number of markets in which entry occurs. On the other hand, game theory predicts that entry will occur in every market, and that the monopolist will cooperate each time entry occurs.

Recently, Kreps and Wilson (1982) and Milgrom and Roberts (1982) have demonstrated one way in which the paradox can be resolved. Their basic idea is that, if there is even a small probability that the monopolist always acts aggressively, then the resulting equilibrium closely resembles the intuitive equilibrium specified above. That is, the monopolist always acts aggressively if entry occurs in an early market, and this in turn deters entry in early markets.[13] In this section we investigate what happens when the decision making process of the potential entrants is varied, rather than the above approach of varying the decision making process of the monopolist. Specifically, we investigate how equilibrium is affected when, for each market, there is some probability that the potential entrant is naive and forms expectations by simply extrapolating from previous behavior. The basic result which emerges from our analysis is that, when reputation effects are potentially important, it is possible for either type of agent to be dominant. That is, under some parameterizations sophisticated agents will have a disproportionately large effect on equilibrium, while for other parameterizations the naive agents will be the ones who are disproportionately important.

As stated previously, we consider a monopolist who operates in T separate markets, and who faces a different potential entrant in each market. It is assumed that the monopolist and the potential entrants are all risk neutral, and that the monopolist faces the T potential entrants in a sequential fashion. By the latter we mean that the monopolist first faces the potential entrant in what we refer to as market 1, and then sequentially faces the potential entrants in markets $2,3...T$. In each market, furthermore, payoffs to the players are as in Figure 1. Note, we consider the case $0 < b < 1$ and $a > 1$. Also, it is assumed that when the potential entrant in each market k faces the monopolist, the potential entrant is aware of all moves by players in lower numbered markets.

To complete the model we must specify the manner in which players form expectations. With probability one the monopolist is sophisticated and thus has rational expectations. On the other hand, for each market k there is a probability p that the potential entrant is sophisticated, and a probability $(1-p)$ that the potential entrant is naive. If a potential entrant is sophisticated, then he also has rational expectations. If a potential entrant is naive, then he simply extrapolates from previous behavior. Specifically, if the last entry brought forth a cooperative (aggressive) response from the monopolist, then a naive potential entrant anticipates that with probability one a further entry will bring forth a cooperative (aggressive) response.[14] If there are no prior entries to extrapolate from, however, then a naive potential entrant anticipates that with probability π entry is met by cooperation and with probability $(1-\pi)$ entry is met by aggression. Finally, to keep the analysis simple we place the following two restrictions on the parameters. First, $\pi b + (1-\pi)(b - 1) < 0$, i.e., a naive potential entrant does not enter if there are no prior entries to extrapolate from.[15] Second, for every integer k it is the case that $\sum_{i=1}^{k} (1-p)^i a \neq 1$.

The following four propositions characterize perfect Nash equilibria for our model as a function of the exogenous parameters. By restricting the analysis to perfect Nash equilibria we eliminate the possibility that sophisticated players maintain threats which they would not find rational to carry out (see Selten 1978 for a discussion of perfectness in this context).

Proposition III.1: If $p = 1$, then

i) entry occurs in every market

ii) the monopolist cooperates every time entry occurs.

Proof: Consider first market T. If entry occurs the only rational response for the monopolist is to cooperate. Given this, the only rational move for the market T potential entrant is to enter. Now consider market k, where it is known that entry will necessarily occur in every higher numbered market. If entry occurs in market k, given that entry is going to occur in every higher numbered market, the only rational response for the monopolist is to cooperate. This in turn yields that the only rational move for the market k potential entrant is to enter. Finally, the above two ideas yield that the only perfect Nash equilibrium is for entry to occur in every market, and for the monopolist to cooperate every time entry occurs.

Proposition III.1 tells us that in the absence of any limited rationality our model yields Selten's chain store paradox. That is, when with probability one each potential entrant is sophisticated, entry occurs in each market and the monopolist cooperates each time entry occurs. We now analyze the other polar case.

Proposition III.2: If $p = 0$, then entry never occurs.

Proof: This follows immediately from our assumption $\pi b + (1-\pi)(b-1) < 0$.

Proposition III.2 indicates that when with probability one each potential entrant is naive, the model yields results which are exactly opposite from those which held when with probability one each potential entrant was sophisticated. Note, although the proof depends on the assumption $\pi b + (1-\pi)(b-1) < 0$, the equilibrium would only change slightly with the opposite assumption. Specifically, if $\pi b + (1-\pi)(b-1) > 0$, equilibrium is characterized by the following three statements. First, the period 1 potential entrant would enter. Second, the monopolist would respond by acting aggressively. Third, entry would not occur in any other market.

We have now analyzed the two polar cases of pure rational expectations and pure limited rationality. Our next two propositions consider restrictions on the parameter space which are between these two polar cases.

Proposition III.3: If $\sum_{i=1}^{T-1} (1-p)^i a < 1$, then

i) entry first occurs in the lowest numbered market which contains a sophisticated potential entrant

ii) entry occurs in every succeeding or higher numbered market

iii) the monopolist cooperates each time entry occurs.

Proof: Consider market T. If entry occurs, the only rational response for the monopolist is to cooperate. Given this, the market T potential entrant will enter if he is sophisticated. Now consider market T - 1. Given the above and the fact that $(1-p)a < 1$, the only rational response for the monopolist is again to cooperate. This in turn yields that the market T - 1 potential entrant will enter if he is sophisticated. Furthermore, successively repeating this argument yields that entry will occur in each market which contains a sophisticated potential entrant, and the monopolist will cooperate every time entry occurs.

To complete the proof we need only demonstrate that a naive potential entrant will (will not) enter if there is (is not) a lower numbered market which contains a sophisticated potential entrant. Consider first a naive potential entrant for whom there is a lower numbered market which contains a sophisticated potential entrant. For this potential entrant extrapolation yields that the monopolist cooperates, and thus such a potential entrant will enter. Now consider a naive potential entrant for whom there is no lower numbered market which contains a sophisticated potential entrant. For this potential entrant there will be no prior entries to extrapolate from, and thus such a potential entrant will not enter.[16]

Propositions III.1 and III.2 indicate that, for neither type of agent to be disproportionately important, entry should occur on average in pT markets. However, Proposition III.3 indicates that when $\sum_{i=1}^{T-1} (1-p)^i a < 1$, entry occurs on average in $\sum_{i=1}^{T} (1-(1-p)^i)$ markets, and it is easily shown that $\sum_{i=1}^{T} (1-(1-p)^i)$ always exceeds pT and in fact it frequently exceeds pT by a wide margin. For example, if $T = 10$ and $p = 1/2$, then $pT = 5$ and $\sum_{i=1}^{T} (1-(1-p)^i) \cong 9$. Thus, for the parameterizations covered by Proposition III.3, sophisticated agents are disproportionately important.

Proposition III.4: If $\sum_{i=1}^{T-1} (1-p)^i a > 1$, then

i) entry does not occur in markets 1 through $T - z$, where z is the lowest integer for which $\sum_{i=1}^{z} (1-p)^i a > 1$

ii) for markets numbered above $T - z$

a) entry first occurs in the lowest numbered market which contains a sophisticated potential entrant

b) entry occurs in every succeeding or higher numbered market

c) the monopolist cooperates each time entry occurs.

Proof: Suppose i) is true. Given this, ii) follows from the same logic as in the proof of Proposition III.3. Thus, we need only prove i).

Consider a sophisticated potential entrant located in a market k, where $1 < k < T - z$. If this potential entrant were to enter, the monopolist would act aggressively because the expected return from the deterrence of future naive potential entrants would exceed the immediate return to cooperating, i.e., $\sum_{i=1}^{T-k} (1-p)^i a > 1$. Thus, the only rational move for such a sophisticated potential entrant would be not to enter. Furthermore, given this, it follows immediately that any naive potential entrant located in a market k, where

$1 < k < T - z$, would also decide not to enter.[17]

As stated previously, Propositions III.1 and III.2 indicate that, for neither type of agent to be disproportionately important, entry should occur on average in pT markets. However, Proposition III.4 tells us that, for some parameterizations, entry occurs on average in less than pT markets. For example, if $(1-p)a > 1$, then entry occurs on average in p markets, and p is obviously less than pT. Thus, Proposition III.4 indicates that there are some parameterizations for which naive potential entrants have a disproportionately large effect on equilibrium.

Proposition III.4 concludes our analysis. In summary, we have found that in a repeating game situation where reputation is a potential factor, it is possible for either type of agent to dominate. That is, for some parameterizations sophisticated agents will have a disproportionately large effect on equilibrium, while for other parameterizations the naive agents will be the ones with the disproportionately large effect.

Finally, given that either type of agent can dominate, a question arises as to which type of agent will in general dominate in such repeated game situations. Although it is difficult to quantify, we feel that at least for chain-store like games the naive agents will in general be the dominant ones. There are two pieces of evidence which point in this direction. First, one interpretation of the Kreps and Wilson, and Milgrom and Roberts results is that when the probability for limited rationality is introduced on the side of the monopolist, then it is almost always the probability of naiveté which is disproportionately important. Second, in a richer version of the model analyzed in this section, the restriction needed for the naive agents to be dominant becomes much less stringent. Specifically, consider a model where there is some uncertainty concerning what the monopolist receives in a market when entry does not occur. Preliminary analysis suggests that as long as there is some probability that (1-p) times this return is greater than one, the naive agents will be the ones who dominate.

IV. Conclusion

Papers which provide alternatives to the rationality assumption, as well as papers which contain the rationality assumption, have in general ignored the idea that agents tend to vary in terms of information processing abilities. In this paper we have attempted to investigate the ramifications of this type of heterogeneity. We did this by analyzing two simple models in which the population is composed of two groups. Agents in the first group were characterized by rational expectations and were referred to as sophisticated, while agents in the second group were characterized by incorrect expectations and were referred to as naive. The analysis yielded three major results. First, in a world characterized by congestion effects, sophisticated agents tend to have a disproportionately large effect on equilibrium. Second, in a world characterized by synergistic effects, naive agents tend to be disproportionately important. Third, in a repeating game situation where reputation is a potential factor, it is possible for either type of agent to be dominant. Finally, the analysis also yielded insights concerning the interaction of informed and uninformed agents, where all agents have rational expectations. Specifically, the analysis suggests that, for situations which exhibit congestion effects, informed agents tend to have a disproportionately large effect on equilibrium. On the other hand, for situations which exhibit synergistic effects, the analysis suggests that uninformed agents are the ones who are disproportionately important.

The research presented in this paper could be extended in a number of directions. One direction of particular interest concerns the applicability of the results contained herein to the rational expectations challenge to Keynesian macroeconomics. This challenge has left macroeconomics in a state of flux. The reason being that, although the challenge has brought to the fore the theoretical inconsistencies inherent in Keynesian macroeconomics, simple rational expectations macroeconomic models do not yield predictions consistent with empirical observation. The profession has responded to this situation by embedding a variety of imperfections and rigidities into rational expectations models (e.g., staggered long-term contracts are introduced in Taylor 1980, lags in the production process are introduced in Kydland and Prescott 1982, and uncertainty with Bayesian learning is introduced in Frydman and Phelps 1983). The analysis in this paper suggests a different response. That is, rather than assuming rational expectations and then introducing imperfections, why not start with the more realistic assumption that agents are heterogeneous in terms of their information processing abilities. We feel such an approach shows particular promise because of the following. First, given the similarity between our description of synergistic interaction among agents and old style Keynesian multiplier analysis, it would not be surprising if macroeconomic models with this type of heterogeneity exhibited synergistic effects. Second, the results in this paper suggest that if this is indeed the case, then it may very well be that the naive agents are the ones who would be dominant in such a model.[18]

Appendix A

Proof of Proposition I.2: Note, first, all the restrictions in the statement of the proposition are being assumed throughout the proof. Now, we know that

$$\text{(A1)} \qquad W^s = \int_0^{D^s}(f_A(N_A^s) - c_i)g(c_i)dc_i + \int_{D^s}^{C}(f_B(N_B^s) - (C-c_i))g(c_i)dc_i,$$

$$\text{(A2)} \qquad W^n = \int_0^{D^n}(f_A(N_A^n) - c_i)g(c_i)dc_i + \int_{D^n}^{C}(f_B(N_B^n) - (C-c_i))g(c_i)dc_i,$$

$$\text{(A3)} \qquad \hat{W} = p[\int_0^{D^s}(f_A(\hat{N}_A) - c_i)g(c_i)dc_i + \int_{D^s}^{C}(f_B(\hat{N}_B) - (C-c_i))g(c_i)dc_i]$$

$$+ (1-p)[\int_0^{D^n}(f_A(\hat{N}_A) - c_i)g(c_i)dc_i + \int_{D^n}^{C}(f_B(\hat{N}_B) - (C-c_i))g(c_i)dc_i],$$

and

$$\text{(A4)} \qquad W = p[\int_0^{D}(f_A(N_A) - c_i)g(c_i)dc_i + \int_{D}^{C}(f_B(N_B) - (C-c_i))g(c_i)dc_i]$$

$$+ (1-p)[\int_0^{D^n}(f_A(N_A) - c_i)g(c_i)dc_i + \int_{D^n}^{C}(f_B(N_B) - (C-c_i))g(c_i)dc_i],$$

where $\hat{N}_A = \int_0^{D^s} pg(c_i)dc_i + \int_0^{D^n}(1-p)g(c_i)dc_i$ and $\hat{N}_B = \int_{D^s}^{C}pg(c_i)dc_i + \int_{D^n}^{C}(1-p)g(c_i)dc_i$.

Consider a social welfare maximizing government which could direct the actions of all the agents in the population. Such a government would choose D^* to maximize:

$$\text{(A5)} \qquad W^* = \int_0^{D^*}(f_A(N_A^*) - c_i)\, g(c_i)dc_i + \int_{D^*}^{C}(f_B(N_B^*) - (C-c_i)\, g(c_i)dc_i,$$

where $N_A^* = \int_0^{D^*} g(c_i)dc_i$ and $N_B^* = \int_{D^*}^{C} g(c_i)dc_i$ Differentiation yields that

W^* is a single-peaked function of D^*, i.e., $(\partial W^*/\partial D^*) > 0$ for $D^* < D^0$ and $(\partial W^*/\partial D^*) < 0$ for $D^* > D^0$, where $D^0 = \arg\max_{D^*} W^*$.

Now consider a social welfare maximizing government which could only direct the actions of the sophisticated agents in the population. Such a government would choose D^+ to maximize:

$$\text{(A6)} \qquad W^+ = (1-p)\ [\int_0^{D^n} (f_A(N_A^+)-c_i)\ g(c_i)dc_i + \int_{D^n}^{C} (f_B(N_B^+) - (C-c_i))\ g(c_i)dc_i]$$

$$+\ p[\int_0^{D^+} (f_A(N^+) - c_i\ g(c_i)dc_i + \int_{D^+}^{C} (f_B(N_B^+) - (C-c_i))\ g(c_i)dc_i],$$

where $N_A^+ = \int_0^{D^n} (1-p)\ g(c_i)dc_i + \int_0^{D^+} pg(c_i)dc_i$ and $N_B^+ = \int_{D^n}^{C} (1-p)\ g(c_i)dc_i + \int_{D^+}^{C} pg(c_i)dc_i$. Differentiation yields that W^+ is a single-peaked function of D^+.

Suppose $W^n < W^s$ and $N_A^s > N_A^n$. Given the single-peaked property of W^*, this implies $N_A^n < N_A^s \leq N_A^0$. Given Proposition I.1, this in turn implies $N_A > \hat{N}_A$. Utilizing the expression for N_A and solving for N_A^o from the maximization of W^+, it can be demonstrated that $N_A^n, N_A^s < N_A^0$ implies $N_A < N_A^o$.[19] Hence, we have $N_A^o > N_A > \hat{N}_A$. Given the single-peaked property of W^+, this in turn implies that if $W^n < W^s$ and $N_A^s > N_A^n$, then $W > \hat{W}$. Moreover, given the symmetry of the problem, this result yields that $W^n < W^s$ and $N_A^s < N_A^n$ imply $W > \hat{W}$.

Given $f_A''(.) \leq 0$ and $f_B''(.) \leq 0$, (A1-3) and the definition of $\bar{W}$ yield:

$$\text{(A7)} \qquad \hat{W} - \bar{W} \geq p(1-p)(N_A^s - N_A^n)(f_A(N_A^n) - f_A(N_A^s))$$

$$+\ p(1-p)(N_B^s - N_B^n)(f_B(N_B^n) - f_B(N_B^s)) \geq 0.$$

Given the preceding result, (A7) yields that $W^s > W^n$ implies $W > \bar{W}$.

Finally, the proof that $W^n > W^s$ implies $W < \hat{W}$ follows along similar lines.

Proof of Proposition II.2: Note, first, all the restrictions in the statement of the proof are assumed throughout the proof. Given these restrictions we know (A1-6) continue to hold. Furthermore, W^* remains a single-peaked function of D^* and W^+ a single-peaked function of D^+.

Suppose $W^n < W^s$ and $N_A^s > N_A^n$. Given the single-peaked property of W^*, this implies $N_A^n < N_A^s < N_A^0$. In turn, given Proposition II.1 and the single-peaked property of W^+, this yields that when $W^n < W^s$ and $N_A^s > N_A^n$, then $W < \hat{W}$. Moreover, because of the symmetry of the problem, this result yields that $W^n < W^s$ and $N_A^n > N_A^s$ imply $W < \hat{W}$.

(A1-3), and the definition of $\bar{W}$ yield:

$$\text{(A8)} \qquad \hat{W} - \bar{W} = \hat{N}_A f_A(\hat{N}_A) - [pf_A(N_A^s)N_A^s + (1-p)f_A(N_A^n)N_A^n]$$

$$+ \hat{N}_B f_B(\hat{N}_B) - [pf_B(N_B^s)N_B^s + (1-p)f_B(N_B^n)N_B^n].$$

From the definitions, it is necessarily true that $\hat{W} - \bar{W} = 0$ when $N_A^s = N_A^n$. Moreover, differentiation of the RHS of (A8) with respect to N_A^n yields that $\partial(\hat{W}-\bar{W})/\partial(N_A^n) \gtrless 0$, when $N_A^n \lessgtr N_A^s$. Together these two statements imply $\hat{W} < \bar{W}$ when $N_A^s \neq N_A^n$. This, combined with the preceding result, in turn, yields that $W < \bar{W}$ implies $W^s > W^n$.

Finally, the proof that $W^n > W^s$ implies $W > \hat{W}$ follows along similar lines.

Footnotes

[1]This limited ability to process information is sometimes referred to in the literature as bounded rationality. However, because of its frequent assocciation with the related concept termed satisficing, we will refrain from using the term bounded rationality in this paper.

[2]References to the marginalist debate of the forties include Hall and Hitch (1939) and Machlup (1946). References to the work of Herb Simon and his followers include Simon (1959, 1979), Cyert and March (1963), Williamson (1975), and Nelson and Winter (1982). See also Radner (1975a,b) for more explicit modeling of Simon's ideas.

[3]The term equilibrium in this paper simply refers to the outcome of the model, given an exogenous specification for the expectations of the naive.

[4]Three recent papers which do consider the type of heterogeneity we consider are Conlisk (1980), Russell and Thaler (1982), and Akerlof and Yellen (1983). The main difference between our paper and the papers referred to above is that we focus on the conditions which lead to sophisticated agents having a disproportionately large effect on equilibrium, and on the conditions which lead to the naive having a disproportionately large effect — an issue not addressed in the aforementioned papers.

[5]Examples of papers wherein learning causes convergence to rational expectations equilibria include Cyert and DeGroot (1974), and DeCanio (1979).

[6]Under the additional assumption that agents are risk neutral, V can be interpreted as the mean of a distribution which describes the naive agents' expectations. Note, also, because there is no stochastic element in this model, assuming that sophisticated agents have rational expectations is the same as assuming they have perfect foresight.

[7]Our analysis derives properties which an equilibrium for this model must

display. In a mathematical supplement available from the authors upon request we demonstrate that, for the specifications of both Section I and Section II, an equilibrium exists and is unique.

[8]In equations (1) and (2) we do not specify what happens when $c_i = D$. We can ignore this situation because it occurs with probability zero.

[9](3) and (4) are written for the case $0 < D^n < C$.

[10]There is evidence, however, which suggests that at least for our transportation interpretation it is reasonable to assume increasing marginal congestion effects (see Keeler and Small 1977).

[11]An additional reason for imposing the restriction that either $N_A^s, N_A^n < N_A^0$ or $N_A^s, N_A^n > N_A^0$ is that without the restriction some of the comparisons we make in Proposition I.2 can be quite misleading. Consider, for example, the following discussion concerning $\hat{W}$. When the restriction that either $N_A^s, N_A^n < N_A^0$ or $N_A^s, N_A^n > N_A^0$ is satisfied, $\hat{W}$ will necessarily lie strictly between W^s and W^n. Thus, if the restriction is satisfied, our comparison will suggest that the sophisticated are disproportionately important whenever the response of the sophisticated moves social welfare towards W^s. When the restriction is not satisfied, however, $\hat{W}$ can actually lie outside the interval defined by W^n and W^s. Thus, if the restriction is not satisfied, our comparison could actually suggest that the naive are disproportionately important even when it is the case that $W = W^s$.

[12]One point which should be noted is that this proof relies heavily on the assumption employed to eliminate the possibility of multiple equilibria. We conjecture that without this assumption there would always be equilibria consistent with the proposition. On the other hand, without the assumption it would probably also be possible to have equilibria which were inconsistent with the proposition. What this suggests to us is that when there are

synergistic effects it is possible for sophisticated agents to dominate. However, this would only occur when, in a sense, the presence of naive agents caused the economy to jump between different families of equilibria.

[13]The description of Milgrom and Roberts (1982) above is somewhat imprecise. Milgrom and Roberts actually show that the paradox can be resolved by having the potential entrants only perceive that there is a probability that the monopolist always acts aggressively.

[14]This type of oversensitivity by agents to the most recent information available is consistent with empirical evidence (see Arrow 1982), and with the work of cognitive psychologists (see Kahneman, Slovic and Tversky 1982).

[15]This assumption is not a critical one, but rather as mentioned above only serves to simplify the analysis. As we go along we will indicate the results which follow from the alternative that $\pi b + (1-\pi)(b-1) > 0$.

[16]Suppose we had the assumption $\pi b + (1-\pi)(b-1) > 0$. With this alternative assumption Proposition III.3 would state that entry occurs in every market, and that the monopolist cooperates every time entry occurs.

[17]Suppose again we had the assumption $\pi b + (1-\pi)(b-1) > 0$. With this alternative assumption Proposition III.4 would state that: i) entry occurs (does not occur) in market 1 if the market 1 potential entrant is naive (sophisticated); ii) the monopolist acts aggressively if entry occurs in market 1; iii) entry does not occur in markets 2 through $T-z$; iv) for markets above $T-z$, Proposition III.4 of the text is valid.

[18]See Akerlof and Yellen (1983) for macroeconomic examples wherein the presence of naive agents has a critical effect on the form of equilibrium. Note, however, there results are not driven by the presence of either congestion or synergistic effects.

[19]See the mathematical supplement mentioned in footnote 7.

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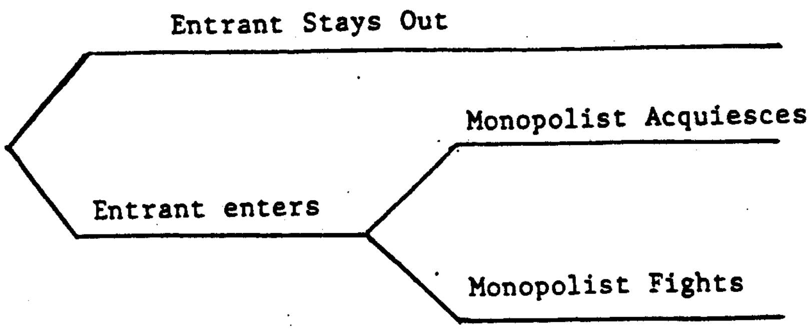
Figure 1



Payoffs

	Entrant's	Monopolist's
Entrant Stays Out	0	a
Monopolist Acquiesces	b	0
Monopolist Fights	b-1	-1

Mathematical Supplement to Rational Expectations and the Limits of Rationality

We will first prove that for the model of Section I a unique equilibrium necessarily exists. Note, we will restrict ourselves to the case $p > 0$ (the case $p = 0$ is trivial).

The problem of existence is basically a fixed point problem. That is, to prove existence we must show that there exists a value for D, denoted $\underline{D}$, such that the values for N_A and N_B which result when $\underline{D}$ is substituted into (3) and (4), denoted $\underline{N}_A$ and $\underline{N}_B$, satisfy $\underline{D} = (f_A(\underline{N}_A)-f_B(\underline{N}_B)+C)/2$. Suppose first $\underline{D} = 0$. Substituting into (3) and (4) yields $0 < \underline{N}_A, \underline{N}_B < \int_0^C g(c_i)dc_i$. Given $f_A(\int_0^C g(c_i)dc_i) > f_B(0) - C$, this in turn yields that $\underline{D} < (f_A(\underline{N}_A)-f_B(\underline{N}_B)+C)/2$. Now suppose $\underline{D} = C$. Substituting into (3) and (4) again yields $0 < \underline{N}_A, \underline{N}_B < \int_0^C g(c_i)dc_i$. Given $f_B(\int_0^C g(c_i)dc_i) > f_A(0) - C$, this in turn yields that $\underline{D} > (f_A(\underline{N}_A)-f_B(\underline{N}_B)+C)/2$. Furthermore, the structure of (3) and (4) yields that $(f_A(\underline{N}_A)-f_B(\underline{N}_B)+C)/2$ varies in a continuous manner with changes in D, and thus there must exist a value for D, denoted $\underline{D}$, such that $\underline{D} = (f_A(\underline{N}_A)-f_B(\underline{N}_B)+C)/2$.

Let $\underline{D}^1$ be such that $\underline{D}^1 = (f_A(\underline{N}_A^1)-f_B(\underline{N}_B^1)+C)/2$, where $\underline{N}_A^1$ and $\underline{N}_B^1$ are the values for N_A and N_B which result when $\underline{D}^1$ is substituted into (3) and (4). To prove uniqueness all we need show is that there cannot be another value for D which satisfies the specified condition. Consider a random value for D, denoted $\underline{D}^2$, which satisfies $\underline{D}^2 > \underline{D}^1$. (3) and (4) immediately yield $\underline{D}^2 > (f_A(\underline{N}_A^2)-f_B(\underline{N}_B^2)+C)/2$. Thus, there cannot be a higher value for D which is consistent with equilibrium. Finally, a similar argument yields that there also cannot be a lower value for D which is consistent with equilibrium.

We will now prove that for the specification of Section II a unique equilibrium necessarily exists. Note, we again restrict ourselves to the case $p > 0$.

To prove existence we must again show that there exists a value for D, denoted $\underline{D}$, such that $\underline{D} = (f_A(\underline{N}_A)-f_B(\underline{N}_B)+C)/2$. Suppose first $\underline{D} = 0$. Substituting into (3) and (4) yields $0 < \underline{N}_A,\underline{N}_B < \int_0^C g(c_i)dc_i$. Given $f_A(0) > f_B(\int_0^C g(c_i)dc_i) - C$, this in turn yields $\underline{D} < (f_A(\underline{N}_A)-f_B(\underline{N}_B)+C)/2$. Now suppose $\underline{D} = C$. Substituting into (3) and (4) again yields $0 < \underline{N}_A,\underline{N}_B < \int_0^C g(c_i)dc_i$. Given $f_B(0) > f_A(\int_0^C g(c_i)dc_i) - C$, this in turn yields $\underline{D} > (f_A(\underline{N}_A)-f_B(\underline{N}_B)+C)/2$. Furthermore, the structure of (3) and (4) again yields that $(f_A(\underline{N}_A) - f_B(\underline{N}_B)+C)/2$ varies in a continuous fashion with changes in D, and thus there must exist some value for D, denoted $\underline{D}$, such that $\underline{D} = (f_A(\underline{N}_A)-f_B(\underline{N}_B)+C)/2$.

Let $\underline{D}^1$ be such that $\underline{D}^1 = (f_A(\underline{N}_A^1)-f_B(\underline{N}_B^1)+C)/2$. To prove uniqueness all we need show is that there cannot be another value for D which satisfies the specified condition. Consider a random value for D, denoted $\underline{D}^2$, which satisfies $\underline{D}^2 > \underline{D}^1$. (3) and (4), together with the assumption $[f_A'(\int_0^z g(c_i)dc_i) + f_B'(\int_z^C g(c_i)dc_i)]g(z)/2 < 1$ for all $0 < z < C$, yield $\underline{D}^2 > (f_A(\underline{N}_A^2)-f_B(\underline{N}_B^2)+C)/2$. Thus, there cannot be a higher value for D which is consistent with equilibrium. Finally, a similar argument yields that there also cannot be a lower value for D which is consistent with equilibrium.

<u>Lemma</u> If $N_A^n,N_A^s <(>) N_A^0$, then $N_A <(>) N_A^o$.

Proof: Consider the case of $N_A^n,N_A^s < N_A^0$. Note that if $N_A^n = N_A^0$, then $N_A^n = N_A^o$. Also, note that:

$$\frac{\partial(N_A^o)}{\partial N_A^n} = \frac{(1-p)}{1 - pg(D^o)\frac{\partial D^o}{\partial N_A^n}}$$

where

$$\frac{\partial D^o}{N_A^n} = \frac{1}{2}(1-p)\ [2(f_A'(N_A^o) + f'(N_B^o)) + (1-p)(f_A''(N_A^o)N_A^n + f_B''(N_B^o)N_B^n$$

$$+ p(f_A''(N_A^o) \cdot N_A^o + f_B''(N_B^o)N_B^o] < 0$$

Since f_A', f_B', f_A'' and f_B'' are all non-positive, $\frac{\partial D^o}{\partial N_A^n} < 0$ and hence $1 > \frac{\partial N_A^o}{\partial N_A^n} > 0$. This implies that if $N_A^n < N_A^0$, then $N_A^n < N_A^o$.

Now, consider that:

$$\frac{\partial N_A}{\partial N_A^n} = \frac{(1-p)}{1-pg(D)\ \frac{\partial D}{\partial N_A^n}}$$

where
$$\frac{\partial D}{\partial N_A^n} = \frac{1}{2}(1-p)\ [f_A'(N_A) + f_B'(N_B)]$$

Since f_A', $f_B' \leq 0$, $\frac{\partial D}{\partial N_A^n} < 0$ and hence $1 > \frac{\partial N_A}{\partial(N_A^n)} > 0$. Given that $N_A = N_A^s$, this implies that $N_A < N_A^s$ when $N_A^n < N_A^s$.

Together, the above two arguments imply that if $N_A^0 > N_A^n > N_A^s$, then $N_A^o > N_A$. Now consider the case when $N_A^n < N_A^s < N_A^0$. Suppose that in this latter case, there existed an N_A^n such that $N_A > N_A^o$. Since we know that $N_A < N_A^o$ when $N_A^0 > N_A^n > N_A^s$, for this to be true there must exist an N_A^n such that $N_A = N_A^o$ and $\frac{\partial N_A}{\partial N_A^n} < \frac{\partial N_A}{\partial N_A^o}$ evaluated at this point. However, the expressions for $\frac{\partial D^o}{\partial N_A^n}$ and $\frac{\partial D^o}{\partial N_A^o}$ reveal that $\frac{\partial N_A}{\partial N_A^n} > \frac{\partial N_A^o}{\partial N_A^n}$ when both are evaluated at a point where $N_A = N_A^o$. Hence, we have a contradiction and $N_A < N_A^o$.

The case where $N_A^n, N_A^s > N_A^O$ is proven in a similar manner.