LIMITED COLLUSION

AND ENTRY DETERRENCE

by

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Abstract

Recent papers which have been concerned with entry deterrence in markets initially inhabited by an oligopoly have assumed either that the oligopoly acts in a fully collusive fashion, or that the oligopoly behaves in a purely noncooperative fashion. An older branch of the Industrial Organization literature suggests that oligopolies are frequently able to collude on price, while collusion on other variables is unlikely. This paper looks at this type of limited collusion in an oligopoly-entry deterrence setting. One important point which emerges from the analysis is that in such a world the free rider problem is an important determinant of oligopoly behavior. A second important result is that the oligopoly may limit price in an attempt to circumvent the free rider problem.

Much recent theoretical work has been devoted to the role played by entry deterrence in the behavior of pre-established sellers.[1] In looking at markets which are initially inhabited by an oligopoly, these studies have generally taken one of two approaches. First, some studies assume that the oligopoly behaves like a shared monopoly (e.g., Spence 1977). Second, other studies assume that the firms which comprise the oligopoly act in a purely noncooperative manner (e.g., Nti and Shubik 1981). There is an older branch of the Industrial Organization literature which suggests that there is a third approach worth investigating. In this branch it is suggested that oligopolies are frequently able to collude on price, while collusion on other variables is unlikely (see e.g., Chamberlin 1933 and Fellner 1949).[2]

In this paper I attempt to identify some of the issues which arise when an oligopoly, faced with the need to make investments in entry deterrence, is only able to collude in this partial manner. To do this I construct and analyze a simple stochastic model of a pre-established oligopoly faced by a single potential entrant. The model analyzed is similar to models previously analyzed by Spence (1977) and Dixit (1980). The key property of the model is that, as was first captured in Spence's analysis, sunk capacity tends to serve as an investment in entry deterrence. Two important points emerge from the analysis. The first is that when oligopolies can only collude in the partial manner specified above, the free rider problem is an important determinant of oligopoly behavior. That is, because of the free rider problem, an exogenous increase in the number of pre-established sellers frequently results in a decrease in the amount of sunk capacity held by the oligopoly, an increase in the probability of entry, a decrease in expected aggregate profits for the oligopoly, and an increase in expected social welfare. The second point is that this limited ability to collude will sometimes cause the oligopoly to choose a limit price strategy. The key element here is that, because of the relationship between quantity sold and capacity, the oligopoly can ensure a high investment in sunk capacity by choosing a low value for the collusive price. Thus, when the free rider problem is severe, i.e., when the number of pre-established sellers is high, the oligopoly will frequently circumvent the free rider problem by limit pricing.

The outline for the paper is as follows. Section I presents an oligopoly model for which sunk capacity serves as an investment in entry deterrence. Sections II and III analyze the model. In particular, Section III derives some comparative static results concerning the number of pre-established sellers. Section IV presents some concluding remarks.

I. The Model

The model deals with a homogeneous product market inhabited by a pre-established oligopoly which faces a single potential entrant. The oligopoly is composed of n identical risk neutral firms, where n is an exogenous parameter. Furthermore, it is assumed that the oligopoly has the ability to collude on price, but is incapable of colluding on investments in capacity.

The timing of events inside the model is as follows. First, the oligopoly decides on a collusive price, denoted simply as P. Second, each pre-established seller takes P as given, and decides on an initial or sunk investment in capacity. Consistent with the assumption that the oligopoly cannot collude on investments in capacity, it is assumed that for this stage of the game the pre-established sellers behave in a noncooperative or Nash fashion. Third, the potential entrant observes the aggregate sunk capacity of the oligopoly, and enters the market if and only if he anticipates a non-negative level of profits. Fourth, the model culminates with a sales period. It is assumed that if entry has not occurred, then each of the pre-established sellers charges P and each receives $\frac{1}{n}$ of total demand. On the other hand, if entry has occurred, then the collusive agreement breaks down and the n+1 firms in the market become Marshallian price takers.[3]

One problem with the above specification is that the oligopoly's optimal value for P from an ex ante point of view may not be the optimal value from an ex post point of view. That is, the oligopoly may find that, after the pre-established sellers have made their sunk investments in capacity, it would prefer to set a new value for P. If the pre-established sellers have rational expectations concerning such a price change, then allowing the oligopoly to change P in the above manner could actually make the oligopoly worse off. This is because the expectations of the price change would have adverse effects on the oligopoly's aggregate investment in sunk capacity. Throughout most of the paper I will ignore this issue, and assume in an ad hoc manner that the oligopoly can bind itself to its optimal ex ante value for P. At the end of Section III, however, I will come back to this issue. I will there demonstrate that, if the pre-established sellers can write very simple contracts with buyers, there is a non-ad hoc reason for assuming that the oligopoly can bind itself to its optimal ex ante collusive price.

It is now necessary to specify some structure concerning technology and demand. Let x_i denote the output of pre-established seller i, and k_i denote the firm's capacity measured in output units. It is assumed that each pre-established seller has a constant average variable cost of production, denoted w, and a constant cost per unit of capacity, denoted r. That is, total costs of pre-established seller i, denoted $C(x_i,k_i)$, are given by

$$C(x_i,k_i) = \begin{cases} wx_i + rk_i & \text{for } x_i \leq k_i \\ \infty & \text{for } x_i > k_i. \end{cases} \tag{1}$$

It is also assumed that capacity investment is a sunk cost, but after an initial placement a pre-established seller can increase capacity without incurring any penalty. Note, this cost function is the same as the one used in Dixit (1980), except here fixed costs are set equal to zero.

The potential entrant has the same cost function as the pre-established sellers, except the potential entrant faces a minimum investment in capacity, denoted $\hat{k}$. The potential entrant is assumed to know $\hat{k}$ with certainty. On the other hand, the pre-established sellers are assumed to have objective beliefs about $\hat{k}$, which are described by the cumulative distribution function $H(\cdot)$. Furthermore, $H(\cdot)$ is assumed to satisfy the following restrictions: $H(0) = 0$, $H(\bar{k}) = 1$, $H'(\cdot)$ is well defined and is strictly positive in the interval $(0,\bar{k})$, $\lim_{\varepsilon \to 0} H'(\varepsilon) = 0$, and $H''(\cdot)$ is well defined and non-negative in the interval $(0,\bar{k})$. The first two restrictions simply state that $\hat{k}$ falls somewhere between 0 and $\bar{k}$. The third and fourth restrictions eliminate the possibility of a somewhat uninteresting corner solution.[4] The fifth restriction guarantees that the oligopoly faces decreasing marginal returns to the holding of excess sunk capacity — a property necessary for tractability. Notice, there is an asymmetry in the model in that the pre-established sellers do not face a minimum investment in capacity. This is somewhat bothersome, and because of this one might want to interpret the following analysis as only pertaining to a limited range of values for n. That is, the analysis might best be interpreted as only pertaining to values for n which are small enough to guarantee that the pre-established sellers' minimum investment in capacity never becomes an issue.[5]

After pre-established seller i makes his investment in sunk capacity, the value of which is denoted k_i^o, pre-established seller i's marginal cost function is given by Figure 1. Figure 1 is explained as follows. First, marginal cost equals w for levels of production below the firm's sunk investment in capacity. Second, for levels of production above or equal to the firm's sunk investment in capacity, extra units of capacity must be purchased and the value for marginal cost equals $r + w$. Note, with a slight modification Figure 1 can be made to represent the potential entrant's marginal cost function. This is done by substituting $\hat{k}$ for k_i^o.

Finally, $D(.)$ denotes industry demand as a function of price, where $D(r+w) > 0$ and where the marginal revenue curve which $D(.)$ yields is assumed to be continuous and strictly downward sloping. It is also assumed that $\bar{k} > D(r+w) - D(P^M)$, where P^M denotes the collusive price the oligopoly would choose in the absence of an entry threat. This last restriction guarantees that, if the oligopoly were to ignore the entry threat, the probability of entry would still be less than one. The restriction is imposed for tractability reasons.

II. Preliminary Results

Before proceeding to analyze the oligopoly's choice of P, it is necessary to derive some preliminary results. This is done in Propositions 1 through 3. Note, because of the game theoretic nature of the problem, the model will be analyzed in an order which is the reverse of the actual chronological order of events. That is, Propositions 1 and 2 deal with what occurs during the sales period, the Corollary to Proposition 2 deals with the entry decision, and Proposition 3 deals with the oligopoly's aggregate investment in sunk capacity.

Below x_i^N will denote pre-established seller i's output when entry does not take place, k_i^N will denote the firm's final capacity level when entry does not take place, and as indicated previously k_i^o will denote the firm's investment in sunk capacity. In order to make sure this notation is clear I will review for a moment. Each pre-established seller i can invest in capacity at two different points in time. First, he can invest in capacity before the entry decision is made. This is referred to as his investment in sunk capacity and is denoted as k_i^o. Second, he can add to his capacity investment during the sales period. k_i^N simply equals k_i^o plus the addition which is made when entry does not take place. Note, finally, to keep the exposition from becoming overly bogged down in detail, I have relegated all of the proofs to an Appendix.

Proposition 1: $x_i^N = \frac{D(P)}{n}$ and $k_i^N = \max\left\{k_i^o, \frac{D(P)}{n}\right\}$.

Proposition 1 tells us what happens during the sales period when entry did not occur in the previous stage of the game. The interpretation of Proposition 1 is straightforward. First, each pre-established seller has his output determined by his pro rata share of demand, i.e., $x_i^N = D(P)/n$. Second, a pre-established seller will never have a total investment in capacity which exceeds both the firm's sunk investment in capacity and the firm's output, i.e., $k_i^N = \max\{k_i^o, D(P)/n\}$. Proposition 2 states what happens during the sales period when entry did occur in the previous stage of the game.

Proposition 2: If entry occurs when $\hat{k} + \sum_{i=1}^{n} k_i^o > (<)\ D(r+w)$, then each of the pre-established sellers earns negative (zero) profits and the potential entrant earns negative (zero) profits.[6,7]

Proposition 2 can be explained as follows. When firms act as Marshallian price takers, they operate along their marginal cost schedules. When $\hat{k} + \sum_{i=1}^{n} k_i^o > D(r+w)$, price must equal w for each of the firms to be on its marginal cost schedule. Since w is below the minimum value for long-run average cost, this entails negative profits for the pre-established sellers and the potential entrant. Similarly, when $\hat{k} + \sum_{i=1}^{n} k_i^o < D(r+w)$ price must equal $r + w$, which entails zero profits for the pre-established sellers and the potential entrant.

Given Proposition 2, it is possible to state under what conditions entry occurs, and what the probability of entry is. This is done in the following Corollary.[8]

Corollary to Proposition 2: Entry occurs if and only if $\hat{k} + \sum_{i=1}^{n} k_i^o < D(r+w)$. Also, the probability of entry equals $H(D(r+w) - \sum_{i=1}^{n} k_i^o)$.

The final preliminary result to be derived in this section concerns the oligopoly's aggregate investment in sunk capacity. Proposition 3 deals with this aspect of the model. Note, K will now denote this aggregate investment, i.e., $K = \sum_{i=1}^{n} k_i^o$, and D^* will now denote $D(r+w)$.

Proposition 3: For every $r + w < P \leq P^M$ there is an associated unique value for K, where this value for K satisfies the condition $K \geq D(P)$ and also satisfies equation (2). Furthermore, no other K satisfies (2), and $k_i^o = K/n$ for all i.[9]

$$(2) \qquad H'(D^*-K)(PD(P)-rK-wD(P)) - (1-H(D^*-K))nr \begin{cases} = 0 & \text{if } K > D(P) \\ \leq 0 & \text{if } K = D(P) \end{cases}$$

Proposition 3 is where the assumptions $H''(\cdot) > 0$ in the interval $(0,\bar{k})$ and $\bar{k} > D(r+w) - D(P^M)$ become important. Equation (2) is basically a first order condition (see the proof of Proposition 3 in the Appendix). Furthermore, the obvious way to complete the analysis contained in this paper would be to use this first order condition as a constraint in the oligopoly's maximization problem concerning P. The use of first order conditions in this manner is not, however, a valid procedure unless the first order condition can only be satisfied at a global maximum (see Mirrlees 1975 and Grossman and Hart 1983 for a discussion of this issue). The assumptions $H''(\cdot) > 0$ in the interval $(0,\bar{k})$ and $\bar{k} > D(r+w) - D(P^M)$ ensure that this is indeed the case, and that therefore the procedure of utilizing (2) as a constraint is a valid one. Note, that this procedure is valid is implied by the statement in Proposition 3 that no other K satisfies (2).

III. The Oligopoly's Problem

Given the results of the previous section, it is now possible to analyze the oligopoly's choice of a collusive price, and in turn to derive some results concerning how oligopoly behavior depends on the number of pre-established sellers. The oligopoly's problem is to choose the collusive price which maximizes expected aggregate profits, given the constraint that the resultant aggregate investment in sunk capacity is determined by equation (2). That is, if we let Π denote expected aggregate profits, then in choosing a collusive price the oligopoly faces the following maximization problem.[10,11]

(3) $$\max_{P,K} \Pi \quad \text{s.t. } K \text{ satisfies (2)}$$

Before proceeding to analyze the above maximization problem, I will introduce some new notation. First, S will now denote expected social welfare. Second, B will now denote the probability of entry. Third, from this point on a subscript on a variable will indicate that the variable is associated with the value for n with the same subscript. For example, P_1 denotes the collusive price chosen by the oligopoly when $n = n_1$.

Proposition 4: If n_1 and n_2 are such that $K_1 > D(P_1)$, $K_2 > D(P_2)$ and $n_2 > n_1$, then

i) $P_1 = P_2$

ii) $K_1 > K_2$

iii) $B_1 < B_2$

iv) $\Pi_1 > \Pi_2$

v) $S_1 < S_2$.

Proposition 4 compares two different values for n for which the oligopoly holds excess sunk capacity, i.e., $K > D(P)$. This Proposition demonstrates how oligopoly behavior can be affected by the free rider problem. That is, given two such values for n, the free rider problem causes the aggregate investment in sunk capacity to be lower when n is higher, i.e., $K_1 > K_2$. Furthermore, this in turn causes the probability of entry to be positively correlated with n $(B_1 < B_2)$, expected aggregate profits for the oligopoly to be negatively correlated with n $(\Pi_1 > \Pi_2)$, and expected social welfare to be positively correlated with n $(S_1 < S_2)$. The next Proposition deals with values for n for which the oligopoly does not hold excess sunk capacity.

Proposition 5: If n_1 and n_2 are such that $K_1 = D(P_1)$ and $K_2 = D(P_2)$, then

i) $P_1 = P_2$

ii) $K_1 = K_2$

iii) $B_1 = B_2$

iv) $\Pi_1 = \Pi_2$

v) $S_1 = S_2$.

This Proposition states that, given values for n for which no excess sunk capacity is held, oligopoly behavior will be independent of n. One might ask why the free rider problem does not cause oligopoly behavior to vary across such values for n. The answer is this. When the oligopoly does not hold excess sunk capacity, the oligopoly is basically at a corner solution. Furthermore, because it is a corner solution, changes in the incentive to hold sunk capacity will not affect behavior unless the change moves the oligopoly away from the corner solution.

One interpretation for what is happening when the oligopoly winds up holding no excess sunk capacity is that the oligopoly is limit pricing in an attempt to circumvent the free rider problem. The logic behind this interpretation is as follows. The oligopoly knows that no matter what price it chooses, the pre-established sellers will make an investment in sunk capacity which is at least equal to the demand which will result if entry does not occur. Thus, by choosing a relatively low price, or what I refer to as limit pricing, the oligopoly can circumvent the free rider problem by directly forcing the pre-established sellers to make a large investment in sunk capacity. The following two Propositions provide evidence in support of this interpretation.[12]

Proposition 6: If n_1 is such that $K_1 > (=) D(P_1)$, then for any value n_2, $n_2 < (>) n_1$, it must be the case that $K_2 > (=) D(P_2)$.

Propositon 7: If n_1 and n_2 are such that $K_1 > D(P_1)$ and $K_2 = D(P_2)$, then $P_1 > P_2$.

Proposition 6 states that excess sunk capacity will tend to be held when n is low, while no excess sunk capacity will tend to be held when n is high. Given the above interpretation of what is happening when the oligopoly winds up not holding any excess sunk capacity, this Proposition is easily understandable. That is, when the free rider problem is severe, i.e., when n is high, the oligopoly tends to circumvent the free rider problem by limit pricing.

Proposition 7 compares the collusive prices which emerge in the two types of solutions. The interpretation of this Proposition is also straightforward. That is, when the oligopoly limit prices it sets the collusive price at a relatively low value. The next Proposition compares social welfare across the two types of solutions.

Proposition 8: Suppose n_1 and n_2 are such that $K_1 > D(P_1)$ and $K_2 = D(P_2)$. If $D(.)$ is a linear function, then $S_1 < S_2$.

Proposition 8 simply states that, given a linear demand curve, social welfare is higher when the oligopoly limit prices. This Proposition completes the analysis of (3). I will end this section by reconsidering an issue first mentioned in Section I.

Till this point I have assumed in an ad hoc manner that the oligopoly can bind itself to its optimal ex ante value for P. Some readers may find this objectional since, after the pre-established sellers have made their sunk investments in capacity, the oligopoly might find that it prefers to set a different value for P (note: this could only be the case if the solution to (3) has $K = D(P)$). I will here attempt to justify the approach taken in this paper.

Let price ceiling contracts be contracts firms offer to buyers which put the buyers under no obligation, but which state that at the option of the buyer the firm will sell the good at a price specified in the contract, denoted the option price. Proposition 9 demonstrates that, if the pre-established sellers have the ability to offer these simple contracts, then there is a non ad hoc reason for assuming that the oligopoly can bind itself to its optimal ex ante price.

Proposition 9: If, prior to the entry decision, pre-established sellers offer price ceiling contracts which specify the oligopoly's optimal ex ante value for P as the option price, then it is rational for all of the pre-established sellers to behave as if they had bound themselves to the oligopoly's optimal ex ante value for P.

IV. Conclusion

Recent papers which have considered the oligopoly-entry deterrence issue have assumed either that the oligopoly acts like a shared monopoly, or that the oligopoly acts in a purely noncooperative manner. An older branch of the Industrial Organization literature suggests that oligopolies are frequently able to collude on price, while collusion on other variables is unlikely.

This paper analyzed this type of limited collusion inside an oligopoly model which has the property that sunk capacity serves as an investment in entry deterrence. Two major results emerged from the analysis. First, in such a setting the free rider problem is an important determinant of oligopoly behavior. That is, because of the free rider problem, an exogenous increase in the number of pre-established sellers frequently resulted in a decrease in the amount of sunk capacity held by the oligopoly, an increase in the probability of entry, a decrease in expected aggregate profits for the oligopoly, and an increase in expected social welfare. Second, the oligopoly's limited ability to collude sometimes resulted in the oligopoly utilizing a limit price strategy. The logic here is that, because of the relationship between quantity sold and capacity, the oligopoly could circumvent the free rider problem by choosing a relatively low price and in this way directly force the pre-established sellers to make a large investment in sunk capacity.

Appendix

Proof of Proposition 1: Pre-established seller i faces the following maximization problem when entry has not occurred.

$$\max_{k_i^N, x_i^N} \; Px_i^N - r(k_i^N - k_i^o) - wx_i^N \tag{4}$$

$$\text{s.t.} \quad k_i^N > k_i^o$$

$$k_i^N > x_i^N$$

$$x_i^N < \frac{D(P)}{n}$$

If the first two constraints are satisfied as inequalities, then differentiating with respect to k_i^N yields $-r$. Therefore, one of these two constraints must be binding, i.e., $k_i^N = \max\{k_i^o, x_i^N\}$.

If the third constraint isn't binding, then differentiating with respect to x_i^N yields

$$P - w - \begin{cases} r & \text{if} \quad x_i^N > k_i^o \\ 0 & \text{if} \quad x_i^N < k_i^o. \end{cases} \tag{5}$$

Since P is a collusive price it must necessarily exceed $r + w$. Therefore, (5) yields that the third constraint is binding, i.e., $x_i^N = D(P)/n$. Furthermore, substituting this result into the previous result yields $k_i^N = \max\{k_i^o, D(P)/n\}$.

Proof of Proposition 2: Consider first the case $\hat{k} + \sum_{i=1}^{n} k_i^o < D(r+w)$. If price exceeds $r + w$, then each firm will want to supply an infinite quantity. This will obviously not clear the market. If price is less than r

$+ w$, each pre-established seller i will want to supply no more than k_i^o and the entrant will want to supply no more than $\hat{k}$. Since $\hat{k} + \sum_{i=1}^{n} k_i^o < D(r+w)$, this will also not clear the market. If price equals $r + w$, each pre-established seller i will be willing to supply any quantity in the interval $[k_i^o,\infty)$ and the entrant will be willing to supply any quantity in the interval $[\hat{k},\infty)$. Since $\hat{k} + \sum_{i=1}^{n} k_i^o < D(r+w)$, this price is obviously consistent with the market clearing. Furthermore, since long-run average cost equals $r + w$, the pre-established sellers and the potential entrant will earn zero profits in this case.

Finally, the proof for the case $\hat{k} + \sum_{i=1}^{n} k_i^o > D(r+w)$ follows along similar lines and is therefore omitted.

Proof of Corollary to Proposition 2: This is a straightforward implication of Proposition 2 and the fact that the potential entrant enters whenever he anticipates non-negative profits.

Proof of Proposition 3: In this situation pre-established seller i faces the following maximization problem.

$$\max_{k_i^o} \; (1-H(D^*-k_i^o-\sum_{j\neq i} k_j^o))(\frac{PD(P)}{n} - r \max \{k_i^o, \frac{D(P)}{n}\} - \frac{wD(P)}{n}) \tag{6}$$

Differentiating yields the following first order condition.

$$H'(D^*-K)(\frac{PD(P)}{n} - r \max \{k_i^o, \frac{D(P)}{n}\} - \frac{wD(P)}{n}) - (1-H(D^*-K))\begin{cases} 0 & \text{if} \quad k_i^o < \frac{D(P)}{n} \\ r & \text{if} \quad k_i^o > \frac{D(P)}{n} \end{cases} < 0 \tag{7}$$

Since $P > r + w$, the left hand side of (7) is non-negative whenever $k_i^o < D(P)/n$. Thus, a firm will never have a value for k_i^o where $k_i^o < D(P)/n$ (note: it is being assumed here that if a pre-established seller is indifferent between investing in capacity before and after the entry decision, then he makes the investment before the entry decision). This allows (7) to be rewritten as,

$$(8) \qquad H'(D^*-K)\left(\frac{PD(P)}{n} - rk_i^o - \frac{wD(P)}{n}\right) - (1-H(D^*-K))r \begin{cases} = 0 & \text{if } k_i^o > \dfrac{D(P)}{n} \\ \leq 0 & \text{if } k_i^o = \dfrac{D(P)}{n}. \end{cases}$$

Notice, (8) cannot hold for all pre-established sellers simultaneously unless $k_i^o = K/n$ for all i. Taking this into account reduces (8) to (2).

The next step is to show that there is always a unique K which satisfies (2), and that this K represents a global maximum for each of the pre-established sellers. Existence and uniqueness are implications of the following two facts. First, in the limit as K approaches D^*, the left hand side of (2) must be negative because $H(D^*-K)$ and $H'(D^*-K)$ both approach zero. Second, because $H''(\cdot) \geq 0$ in the interval $(0,\bar{k})$ and $\bar{k} > D(r+w) - D(P^M)$, in the relevant region the derivative of the left hand side of (2) with respect to K always exists and is always negative. Finally, that this K represents a global maximum for each of the pre-established sellers is an implication of the following fact. Because $H''(\cdot)$ exists and is non-negative in the interval $(0,\bar{k})$, in the relevant region the partial derivative of the left hand side of (8) with respect to k_i^o always exists and is always negative.

Proof of Proposition 4: When n is such that the resulting solution has $K > D(P)$, the constraint in (3) must hold as an equality. Thus, whenever this is the case, the solution to (3) can be characterized by setting up a Lagrangian multiplier and deriving first order conditions.

(9) $$L = \Pi + \lambda(H'(D^*-K)(PD(P)-rK-wD(P)) - (1-H(D^*-K))nr)$$

(10) $$L_K = H'(D^*-K)(PD(P)-rK-wD(P)) - (1-H(D^*-K))r$$
$$+ \lambda(-H''(D^*-K)(PD(P)-rK-wD(P)) - H'(D^*-K)r(n+1)) = 0$$

(11) $$L_P = (1-H(D^*-K))(D(P)+PD'(P)-wD'(P))$$
$$+ \lambda(H'(D^*-K)(D(P)+PD'(P)-wD'(P))) = 0$$

(12) $$L_\lambda = H'(D^*-K)(PD(P)-rK-wD(P)) - (1-H(D^*-K))nr = 0$$

Consider first (10). The constraint together with our assumption concerning $H''(.)$ yields $\lambda > 0$. In turn, substituting this into (11) yields $D(P) + PD'(P) - wD'(P) = 0$. Notice, this condition does not depend on n, i.e., $P_1 = P_2$. Now consider (12). Our assumption concerning $H''(.)$ yields that the partial derivative of the left hand side of (12) with respect to K is negative, while direct calculation yields that the partial derivative with respect to n is also negative. Thus, the fact that $P_1 = P_2$ yields $K_1 > K_2$ and $B_1 < B_2$. The next step is to note that as long as the partial derivative of Π with respect to K is positive, increases in K increase profits. Furthermore, (12) yields that this derivative is positive (zero when $n = 1$), which means that $K_1 > K_2$ implies $\Pi_1 > \Pi_2$. Finally, it is clear that when entry occurs there is no social welfare loss. Thus, the expected social welfare loss equals $(1 - B)$ times the social welfare loss which occurs when entry does not take place. Given $B_1 < B_2$ and $K_1 > K_2$, the above implies $S_1 < S_2$.

Proof of Proposition 5: Consider the following maximization problem.

$$(13) \qquad \max_{P,K} \Pi \quad \text{s.t.} \quad K = D(P)$$

It is obvious that the solution to this problem, denoted P^L, K^L, does not depend on n. Thus, I can prove Proposition 5 by demonstrating that whenever n is such that the resulting solution has $K = D(P)$, this solution solves (13).

Suppose there exists an n for which this is not the case. This says that if at this value for n the oligopoly were to set $P = P^L$, then the resulting K would not equal K^L. Otherwise there is a contradiction because, by the definition of P^L, the oligopoly could have done better than its optimal strategy by setting $P = P^L$. Now, note that when $P = P^L$ the partial derivate of Π with respect to K is positive for all values of K less than the K chosen when $P = P^L$, but greater than $D(P^L)$. This implies that if at this value for n the oligopoly were to set $P = P^L$, then the resulting value for Π would be higher than when $P = P^L$ and $K = K^L$. This in turn yields a contradiction because it implies that the oligopoly could have done better than its optimal strategy by setting $P = P^L$.

Proof of Proposition 6: Consider first the case $K_1 > D(P_1)$. Suppose there exists an n_2, $n_2 < n_1$, such that $K_2 = D(P_2)$. If when $n = n_2$ the oligopoly were to set $P = P_1$, equation (2) yields that the resulting value for K would be greater than K_1 and that expected aggregate profits would be higher than when $n = n_1$. From the proof of Proposition 5, however, we know that if at $n = n_1$ the oligopoly were to set $P = P_2$, the expected aggregate profits for the oligopoly would be greater than or equal to expected aggregate profits when $n = n_2$. Together these two statements yield a contradiction because they imply that either when $n = n_1$ or when $n = n_2$ the oligopoly could do better than its optimal strategy. Finally, the statement in the Proposition concerning the case $K_1 = D(P_1)$ is a direct implication of what has just been proved concerning the case $K_1 > D(P_1)$.

Proof of Proposition 7: First, note that because the partial derivative of the left hand side of (2) with respect to K is negative, we know

$$(14) \qquad H'(D^*-D(P_1))(P_1D(P_1) - rD(P_1) - wD(P_1)) - (1-H(D^* - D(P_1)) r > 0.$$

Now, taking the derivative of (13) with respect to K and rearranging yields the following first order condition.

$$(15) \qquad H'(D^*-D(P)) (PD(P)-rD(P) - wD(P)) - (1-H(D^*-D(P))r + (1-H(D^*-D(P)) (P-w + \frac{D(P)}{D'(P)}) = 0.$$

Since P_1 satisfies the condition $D(P) + PD'(P) - wD'(P) = 0$, it must be the case that $P_1 - w + (D(P_1)/D'(P_1)) = 0$. Thus, (14) implies that substituting P_1 into the left hand side of (15) yields a positive expression. Furthermore, for every $r + w < P < P^M$, the partial derivative of the left hand side of (15) with respect to K is negative (note: footnote 9 discusses why P can be restricted to this range). Or taken together, it must be the case that $P_1 > P_2$.

Proof of Proposition 8: To follow the proof it is necessary to consider Figure 2. Since entry is associated with an absence of social welfare losses and since $K_2 = D(P_2)$, I can prove the Proposition by demonstrating that

$(1-H(D^*-K_1))\ Z_1 > (1-H(D^*-K_2))\ Z_2$ (note: it is being assumed here that consumers have a constant marginal utility for income). From the proof of Proposition 5 we know $\Pi_1 > \Pi_2$, or

(16)
$$(1-H(D^*-K_1))\ ((P_1-(r+w))\ D(P_1) - r(K_1-D(P_1))) > (1-H(D^*-K_2))\ (P_2-(r+w))\ D(P_2).$$

Dropping the excess capacity term and utilizing the facts that $D(P_1) < D(P_2)$ and $\frac{1}{2}(D^*-D(P_1)) > \frac{1}{2}(D^*-D(P_2))$ yields

(17)
$$(1-H(D^*-K_1))\ (P_1-(r+w))\ (D^*-D(P_1)) > (1-H(D^*-K_2))\ (P_2-(r+w))\ (D^*-D(P_2)).$$

Furthermore, substituting into (17) that $Z_1 = (P_1-(r+w))\ \frac{1}{2}(D^*-D(P_1))$ yields $(1-H(D^*-K_1))\ Z_1 > (1-H(D^*-K_2))\ Z_2$.

Proof of Proposition 9: I can prove Proposition 9 by demonstrating that if $n - 1$ of the pre-established sellers behave as if the option price will be the price which holds if entry does not occur, then the remaining seller will not have an incentive to behave in a manner which would result in a different price.

Note, first, the n^{th} pre-established seller could never behave in a manner which results in a higher non-entry price because the option price acts as a price ceiling. Now suppoe that the solution to (3) has $K > D(P)$. In the proof of Proposition 4 it was demonstrated that the value for P in this case equals the lowest possible value for the optimal ex post price. Furthermore, given that the marginal revenue curve is strictly downward sloping, this implies that the oligopoly will necessarily want to go right up to the price ceiling. Thus, the statement is satisfied in this case. Suppose next that the solution to (3) has $K = D(P)$, but $n = 1$ would result in $K > D(P)$. From Proposition 7 we know that in this case the price ceiling is set below the lowest possible value for the optimal ex post price. Again, this means that the oligopoly would necessarily want to go right up to the price ceiling, which in turn implies that the statement is also satisfied in this case. Suppose finally that the solution to (3) has $K = D(P)$, and that $n = 1$ would also result in $K = D(P)$. From Proposition 5 we know that the oligopoly in this case is doing as well as if it were a monopoly. Now, suppose the n^{th} seller has an incentive to behave in a manner that results in a lower non-entry price, which in fact it can only do by holding more sunk capacity than the other $n - 1$ sellers. Given that it is holding more sunk capacity, such behavior could not result in the n^{th} seller being more profitable than the other $n - 1$ sellers. This, however, yields a contradiction since it implies that the oligopoly could do better than the monopoly or unconstrained solution. Thus, the statement is also satisfied in this case.

Footnotes

[1]Recent papers concerned with the entry deterrence issue include Dixit (1979, 1980), Eaton and Lipsey (1980, 1981), Grossman (1981), Kreps and Wilson (1982), Milgrom and Roberts (1982a,b), Nti and Shubik (1981), Perrakis and Warskett (1983), Salop (1979), Spence (1977, 1979), and Spulber (1981). Earlier important work in the field includes Bain (1956), Modigliani (1958), Sylos Labini (1969), Gaskins (1971), and Kamien and Schwartz (1971).

[2]Chamberlin (1933) actually only suggests that oligopolies will frequently be able to achieve a collusive result as regards price, i.e., he does not discuss the possibility of collusion on other variables. His argument, however, depends on the speed with which price can be changed. That is, he implies that if a variable can only be changed with a lag, as would definitely be the case with the non-price variable in this paper (investments in sunk capacity), then a collusive result as regards that variable is unlikely.

[3]A different game theoretic assumption which implies similar behavior for the firms is the assumption that a Bertrand equilibrium is established after entry. Unfortunately, for this model a Bertrand equilibrium sometimes fails to exist (see footnote 7 for a further discussion of this point).

[4]The corner solution eliminated is one where the oligopolists hold enough sunk capacity to drive the probability of entry to zero.

[5]Waldman (1982) specifies the relevant restriction on n, and discusses this issue in somewhat more detail.

[6]If entry occurs when $\hat{k} + \sum_{i=1}^{n} k_i^o = D(r+w)$, there are multiple prices which clear the market. Because of this and because $\hat{k} + \sum_{i=1}^{n} k_i^o = D(r+w)$ occurs with probability zero, I will ignore this case.

[7]For the case $\hat{k} + \sum_{i=1}^{n} k_i^o < D(r+w)$, assuming that a Bertrand equilibrium is established after entry is equivalent to assuming that the firms become Marshallian price takers. For the case $\hat{k} + \sum_{i=1}^{n} k_i^o > D(r+w)$, assuming that a Bertrand equilibrium is established after entry may lead to a non-existence problem. The reason for the non-existence problem is that if one firm were to deviate from a price of w, the other firms might not be willing to serve enough of the market to make the deviation unprofitable (see Grossman 1983 for a further discussion of this issue).

[8]It might be argued that a drawback of the model is that the potential entrant receives zero profits, as opposed to positive profits, when entry occurs. There is, however, a slight modification of the model which would not change any of the results, except that entry would be accompanied by positive profits. Specifically, simply assume that the potential entrant's cost per unit of his first $\hat{k}$ units of capacity is $r-\varepsilon$ rather than r.

[9]The analysis can be restricted to collusive prices between $r+w$ and P^M. The logic for the restriction $P > r+w$ is simply that, given Proposition 2, any other value for P would yield non-positive profits for the oligopoly. The logic behind $P < P^M$ is that it is possible to demonstrate that $P = P^M$ dominates any value for P above P^M.

[10]There is an implicit restriction in (3) that $r + w < P < P^M$. See footnote 9 for the logic behind this restriction.

[11]If we set $n=1$, then (3) tells us what behavior is like if the market is initially inhabited by a monopoly. This is clear since if $n=1$, then (2) simply becomes one of the first order conditions of the unconstrained problem.

[12]This interpretation only makes sense if, when $n = 1$, excess sunk capacity winds up being held.

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Figure 1

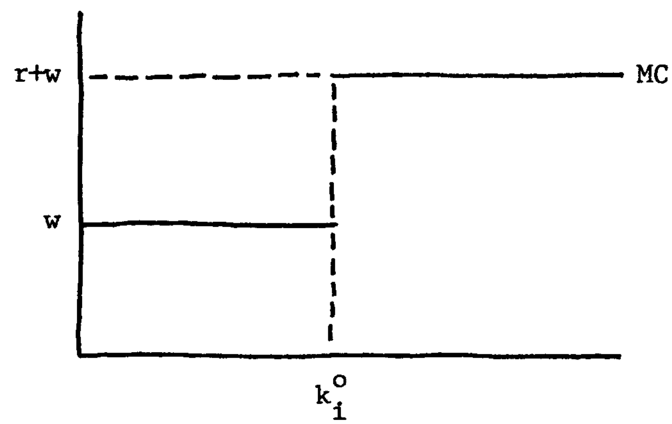

Figure 2