THE PERSISTENCE OF NOMINAL SHOCKS

IN A

PARTICULAR EQUILIBRIUM MODEL

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I. INTRODUCTION

> The [Lucas-Prescott] model is a conclusive counterexample (if one is needed) to the idea that a persistent response to shocks is not possible in a rational-expectations equilibrium.
>
> Lucas, 1981

> There are various attempts to deal with the persistence problem facing the [rational expectations] theory, but these strike me as unconvincing.
>
> Baily, 1982

Can the observed serial correlation in economic aggregates be explained in the context of a rational expectations-equilibrium business cycle model? The quotations above summarize the opinions of the economics profession on this question;. There is no doubt about the correct answer, every economist knows the answer, but no two economists have the same answer.

The cause of this dispersion of opinion is simply our vast and deep ignorance of the behavior of dynamic economic models outside of the steady state. The available (i.e., solvable) models of cyclical behavior are unable to satisfactorily account for the stylized facts of the business cycle. As a result, our theorizing regularly reaches a "jumping-off" point where the model is left behind and our individual, informal notions of plausibility are brought into play.

It would be folly to attempt to resolve this complicated debate in the span of a single paper. Instead, we attempt to answer the following narrow question. Can we choose a model of the investment technology that <u>by itself</u> and in the context of an equilibrium business cycle model generates a hump shaped transfer function between nominal shocks and real output and employment that is similar to the transfer functions estimated by Barro and Rush (1980) and others? In the next section, the nature of this circumscribed question and of the answers one might obtain are explored. In the section following, we formulate a particular equilibrium business cycle model — one, we argue, that is well-suited to answering this question. Finally, the answers supplied by the model are discussed and interpreted.

II. THE PERSISTENCE QUESTION

> One of the few undisputed facts in macroeconomics is that output is humped shaped.
>
> Blanchard, 1981

Lucas's seminal articles (1975, 1977) marked the revival of interest in equilibrium models of the business cycle. The source of the cycle in these modern models is the difference between the general price level and agents' expectations (formed at some previous time) of the price level. These expectation errors induce rational, utility maximizing agents to undertake investments and to supply labor in quantities that differ from the ones they would have chosen had they been blessed with perfect foresight. An implication of the assumption of rational expectations is that expectation errors are uncorrelated over time. This feature has led some critics to conclude that rational expectations models, while they may be able to explain white noise deviations from trend, are unable to account for the persistent deviations from trend of economic aggregates such as output and employment.

To make this criticism more precise, it is useful to complete the quotation that heads this section. Blanchard clarifies his bold contention (are there really any undisputed facts in macroeconomics?) by saying that "more precisely...the distribution of weights of the moving average representation of the deviation of quarterly output from an exponential trend has a hump shape." The weights estimated by Blanchard from postwar U.S. data bear out his statement, and they are listed below.[1]

Quarter	Weight
1	1.00
2	1.32
3	1.47
4	1.50
5	1.30
6	0.99
7	0.80
8	0.79

These weights represent the percentage deviations from trend of real output in response to an exogenous random shock. The size of the shock is chosen to make the first period deviation equal to one percent of the trend value. This gives the weights the interpretation of the elasticity of output with respect to a random shock.

As we said before, the random shock in an equilibrium business cycle model is an expectation error. Hence, if expectation errors could be observed, the regression of the log of (detrended) real output on current and lagged expectation errors should show this same pattern of effects. Robert Barro and Mark Rush (Barro, 1977, 1978; Barro and Rush, 1980) constructed a measure of the error in aggregate expectations of the growth rate of money to use in precisely this type of regression. Listed below is one of their estimates of these regression coefficients rescaled to make them comparable to the weights estimated by Blanchard.[2] The numbers in parentheses are the standard errors of the estimated effects. While there are substantial differences in the size of these effects, the qualitative similarities in the pattern of effects — the hump shape and the timing of the peak effect — are clearly evident.

Quarter	Effect
1	1.00 (0.33)
2	2.22 (0.49)
3	2.55 (0.58)
4	2.98 (0.62)
5	2.98 (0.62)
6	2.15 (0.56)
7	1.45 (0.45)
8	0.60 (0.27)

What now about the contention that uncorrelated expectation errors cannot produce this kind of correlated response in output? Of course, there are no _a priori_ logical grounds for this contention. Ever since the work of Frisch (1933) and Slutsky (1937), economists have realized that uncorrelated "impulses" can generate any arbitrary time series behavior in endogenous variables. At a purely mechanical level, Frisch proposed that we can usefully regard the structural relations of the economy as a filter which converts a stream of uncorrelated exogenous shocks into observable economic variables. By the appropriate choice of filter, any desired behavior in economic variables can be produced. Thus the statement that uncorrelated expectation errors cannot produce correlated movements in economic aggregates is logically incorrect.

Critics of the new equilibrium business cycle models are aware of the work of Frisch and Slutsky. Their objections to the modern equilibrium business cycle models derive from their doubts as to the types of "filters" these models are likely to produce. The new models embody the vision of an economy where markets clear instantaneously with respect to expectations of future income and profit streams and where expectations are well-founded, highly informed predictions of future events. Is it reasonable to expect that the structural relations postulated in this type of model will, after the rounds of algebra required to convert the model to its time series representation, produce a filter that is consistent with the observed persistent movements in output and employment?

Proponents of the equilibrium approach argue that there are theoretically plausible features of the new models that can account for persistence. These features can be divided into two categories -- information lags and accelerator effects. (Examples of both these features can be found in Lucas, 1975.) Let us consider information lags first.

The assumption that there are information lags, that agents do not know the values of state variables in previous periods, wins the battle at the cost of losing the war. If there are information lags, then expectation errors will themselves be correlated over time. Thus, assuming information lags does not explain the serial correlation of economic aggregates; it merely leads to ask for an explanation of the serial correlation of expectation errors. More importantly, while information lags may, in fact, play an important role in the economy, we have no theory to guide us in constructing a model of information lags. The use of information lags to drive the dynamic behavior of our macroeconomic models reintroduces, under a different name, exactly the same kind of ad hoc "free parameters" that Lucas (1980) and others have sought to eliminate from business cycle models.

We are left then with accelerator effects. Today's expectation error induces the purchase of an extra machine. Tomorrow, enlightened by the arrival of new information, agents regret this investment. This regret does not, however, reduce the capital stock to its target level, and the higher capital stock implies a higher real wage now and for some time into the future. As a result, the levels of output and employment remain for some time above their steady state growth paths even in the absence of any further expansionary errors in expectations.

This line of argument is plausible, but not conclusive. One can think of many factors that might tend to reduce or overturn this influence for persistence (e.g., the wealth effect of the higher capital stock on labor supply). As a result, variants of and extensions to this simple accelerator mechanism have been suggested. We will consider three of these suggestions.

An extension favored by Sargent (there are examples scattered throughout Sargent, 1979) is the assumption that there are costs associated with changing the stock of capital. These costs are hypothesized to increase with the square of the change in the capital stock. Firms in this model are loath to "undo" misguided investments too rapidly. A problem with this approach is that the existence of adjustment costs leads firms to make smaller initial errors in investment. The existence of inventories creates another channel for accelerator effects. The importance of inventories in rational expectations models has been stressed by Blinder and Fischer (1978). Finally, Kydland and Prescott (1982) have made much of the fact that investment plans take time to come to fruition.

Can any of these mechanisms, in the absence of information lags, generate the type of persistence found by Blanchard and Barro and Rush? This is the question we attempt to answer in this paper. Our answer takes the following form. We construct an equilibrium model that can accommodate all the investment technologies described above. We solve the model for a variety of values of the preference and technology parameters and calculate the impulse response functions of investment, labor supply, and output that correspond to the elasticities estimated by Blanchard et. al. Finally, these response functions are examined to see if they are at all similar to the empirical response functions and, further, to see if the effects of nominal shocks persist for any appreciable time.

A disclaimer is in order at this point. The answers that are produced by this technique are necessarily qualitative and informal. We will propose some quantitative measures that aid in assessing the dynamic behavior of the solved model, but, at bottom, one will be left with the problem of deciding whether or not the effects of lagged shocks in our model "look like" the effects estimated from U.S. data. This task will, fortunately, be quite easy. None of the models produces response functions that are even remotely like the estimated response functions (although the Kydland-Prescott "time to build" model does generate substantial persistence).

Even in the absence of such easily interpreted results, there are persuasive reasons for taking this kind of qualitative evidence quite seriously. First, in order to produce a model that captures, in an essential way, the features of the Lucas business cycle theory and that admits of a solution, one must accept some severe restrictions. No reasonable individual should be tempted to fit such a limited model to the data. (Two eminently reasonable individuals have, however, done precisely that -- see Kydland and Prescott, 1982.) Thus, it is only the readily apparent qualitative features of such a model that merit our attention. The estimated standard errors of gross fictions need not concern us. Second, one should be wary of placing too much emphasis on the estimates of particular coefficients in a time series model of the macroeconomy. At our current state of statistical knowledge, the fitting of time series models to economic data is much more of an art than a science. One hopes that the broad pattern of lag weights is robustly estimated (as it appears to be for this phenomena — hence Blanchard's contention), but one would not want to choose between fundamentally different economic policies (to choose an apt operational example) on the basis of a hypothesis test concerning these coefficients.

III. A PARTICULAR EQUILIBRIUM MODEL

> The view of the prototypical individual decision problem taken by modern capital theory is a useful point of departure for considering behavior over the cycle.
>
> Lucas, 1977

There are two main ways to construct an equilibrium business cycle model. One way, exemplified by Sargent (1976), is to estimate a time series model that incorporates, by way of *a priori* restrictions, the rational expectations-natural rate hypothesis. The other way is to specify the fundamental structural relations of the economy, the preference and technology functions, and then to solve for the time series representation of the model. Both approaches have advantages and disadvantages. The first approach allows the model to fit the observed data more closely. The second approach enables the researcher to correctly evaluate the effects of alternative policies, i.e., the second approach is immune to the Lucas critique (Lucas, 1976). However, the analytical difficulties of solving for the equilibrium of a structural model limit the types of models that can be considered.

Only the second approach is of use in answering our question about persistence. The persistence question is really a question about the effect on the time series representation of a model of differences in the structural form. Thus, we accept, albeit grudgingly, the limitations associated with producing a solvable model in order to gain some insight into this difficult question.

A. Model A

A common way of proceeding,[3] and the one which we adopt, is to model the economy as though it consisted of a single representative individual. This technique avoids the difficult problem of integrating across individuals who may differ both in their preferences and their productive oppportunities. Our representative individual produces a single commodity, X, according to the linearly homogeneous function

$$X = f(K,N,Y)Z_1 \tag{1}$$

where K is the capital stock, N is the fraction of time spent in production, Y is the stock of inventories, and Z_1 is a random variable that represents the stochastic element in production. Both stocks are measured as of the beginning of the period. For the simplest version of the model that we will consider (denoted Model A), $f(\ )$ is specified as

$$f_A(K,N,Y) = K^{\gamma} N^{(1-\gamma)} \tag{2}$$

that is, the production function is Cobb-Douglas and the possibility of holding inventories is excluded in Model A. The maximum amount of labor time available per period is normalized to equal one.

In every version of the model, the economy is closed and there is no government sector. In Model A, which incorporates none of the extensions to the basic accelerator model, this implies that output is divided each period into consumption and gross investment, i.e.,

$$X_t = C_t + I_t. \tag{3}$$

Letting δ be the constant rate of physical depreciation, capital accumulation is given by

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{4}$$

Let β be a constant discount factor. The representative individual seeks to maximize the expected value of the sum

$$\sum_{t=o}^{\infty} \beta^t \, u(C_t, A(L)(1-N_t)) \tag{5}$$

where $u(\ )$ is the one period utility function. $A(\ )$ is a polynomial in the lag operator L. This lag polynomial is restricted so that the α_i sum to one and

$$\alpha_i = (1-\nu)^{i-1} \, \alpha_1 \tag{6}$$

for all i greater than or equal to one. It follows that

$$A(L)\ (1-N_t) = 1 - \alpha_o N_t - (1-\alpha_o)\nu M_t \tag{7}$$

where M_t is defined as

$$M_t = \sum_{i=1}^{\infty} (1-\nu)^{i-1} N_{t-i}. \tag{8}$$

This definition allows one to write the recursive relationship

$$M_{t+1} = (1-\nu)M_t + N_t. \tag{9}$$

Finally, it is assumed that u( ) has the CES form

$$\begin{aligned} u(C_t, A(L)(1-N_t)) \\ &= [\lambda C_t^{-\rho} + (1-\lambda)A(L)(1-N_t)^{-\rho}]^{-1/\rho} \\ &= [\lambda C_t^{-\rho} + (1-\lambda)(1-\alpha_o N_t - (1-\alpha_o)\nu M_t)^{-\rho}]^{-1/\rho}. \end{aligned} \tag{10}$$

This formulation of preferences is adapted from the formulation given in Kydland and Prescott (1982). Several researchers have suggested that additive separability of utilities across time periods may be inconsistent with observed labor market behavior (e.g., see the discussion in Barro and King, 1982). The assumption that a one-sided distributed lag of leisure is an argument of the current period flow-of-utility function, rather than simply the value of leisure in the current period, is an attempt to accommodate this concern. The degree of intertemporal substitutability of leisure is controlled by the parameters α_o and ν, both of which are assumed to be positive and less than or equal to one. The closer α_o is to one, the smaller is the effect of past leisures on current utility; if α_o is equal to one, the utility function reduces to the standard, time-separable form. The value of ν determines the weight given to distant leisures in the sum M_t. As ν approaches zero, the effect of past leisures decays more slowly; if ν is equal to one, M_t is simply equal to N_{t-1}. By adjusting α_o and

ν, the sensitivity of persistence effects to different degress of non-separability across time in utility can be explored.

To close the model, the information available to the representative individual, or, equivalently, the representation of nominal shocks in this economy, must be specified. We assume that Z_1 cannot be observed directly. Instead, the agent in this economy observes an indicator variable, Z_3, which is the product of Z_1 and a "noise" variable, Z_2. We assume that both Z_1 and Z_2 have the log-Normal distribution; as a result, the indicator is also distributed as log-Normal.

The representative individual in this economy wishes to observe Z_1, the productivity shock, in order to choose the optimal level of labor supply and investment. The noise variable, Z_2, that obscures Z_1 represents the expectation error that, in a richer model, would measure the uncertainty injected into the economy by the monetary growth process. This formulation of the agent's signal extraction problem is a stereotype in the rational expectations literature.[4]

B. Alternative Investment Technologies

We denote by Model B the version of this model that incorporates a cost to adjusting the stock of capital. Model B is obtained by replacing (3) in Model A with

$$X_t = C_t + I_t + \theta(I_t - dK_t)^2. \tag{11}$$

The value of the parameter θ determines the severity of the adjustment costs.

Model C extends Model A by allowing for the holding of inventories. Following Kydland and Prescott (1982), the production function is changed to

$$f_C(K,N,Y) = [(1-\tau)K^{-\omega} + \tau Y^{-\omega}]^{-\gamma/\omega} N^{(1-\gamma)}. \tag{12}$$

Equation (3) is also changed to

$$X_t = C_t + I_t + (Y_{t+1} - Y_t). \tag{13}$$

The formulation of Model C requires further comment. In the real economy, inventories are not a homogeneous item. Some inventories are goods-in-process, some are buffer stocks of finished goods, and some are speculative stocks of inputs. Some of these types of inventories may legitimately enter the production function by making larger production runs possible and by reducing uncertainty in input supply. In the model presented here, howver, the more natural interpretation is that inventories are buffer stocks of finished goods held as insurance against unfavorable productivity shocks. In periods where a great deal of uncertainty attends the investment-consumption decision, the ability to hold inventories allows the representative individual to defer a decision until more information is available at the cost of only one period's foregone utility. Thus it would be desirable to model inventories as valuable only in their ultimate use as either consumption goods or capital equipment.

Unfortunately, the mathematical tools at our disposal break down under this more palatable formulation. The reason for this breakdown is easy to understand. When there are no costs to adjusting the stock of inventories, an agent can have infinite consumption every period by simply decumulating inventories at an infinite rate. Outlawing this nonsensical type of behavior amounts to imposing a non-negativity constraint on inventories. The solution method we will use on this model cannot accommodate inequality constraints.

As a compromise, productive potential is attributed to inventories to "persuade" the representative individual to hold a non-negative stock of them.

Unless a period is defined to be a very long unit of time, it is unlikely that a very large percentage of the current period's investment expenditures will be embodied in on-line capital equipment by the start of next period. Model A can be generalized in the following manner to allow for this fact. Imagine in this new model, called Model D, that there is a fixed gestation time of J periods for investment projects. Let $S_{j,t}$ be the real value of projects which, at time t, are j periods from completion. For convenience, suppose that the resource requirement of an unfinished investment project is equal to 1/J of the value of the investment. If this is the case, then gross investment in period t is simply the average, over j, of $S_{j,t}$. Capital accumulation takes place according to

$$K_{t+1} = (1-\delta)K_t + S_{1,t} \tag{14}$$

and

$$S_{j,t+1} = S_{j+1,t}. \tag{15}$$

C. Solution by Linear-Quadratic Approximation

It is difficult, if not impossible, to solve for the agent's decision rules in Models A-D. If utility were quadratic and the laws of motion of the model were linear, we could utilize well-known iterative algorithms to calculate numerical solutions for these rules for any arbitrary choice of the parameter values (see Chow (1975) for a description of these techniques). This consideration leads us to the following solution strategy.

First, we use the income identity to replace consumption in the utility function. This substitution has the effect of embedding the production function, the only other non-linear relationship, in the utility function. Next, we take a second-order Taylor series approximation to this "concentrated" utility function. We center the approximation about the non-stochastic steady state of the model. This approximation to the original model has linear laws of motion and a quadratic welfare functional and can be solved by standard techniques.

Once the model is in linear-quadratic form, it is particularly easy to obtain its time series representation. It is instructive to outline the necessary steps. We write the general linear model as

$$y_t = Ay_{t-1} + Bx_t + z_t. \tag{16}$$

y is a vector of state variables, x is a vector of variables which the representative individual chooses (labor supply, inventory investment, and investment in physical capital in our Models A-D), and z is a vector of random disturbances. The agent's welfare functional is a discounted sum of quadratic forms in y. The approximation strategy described above allows us to recast all of our models in this form.

It can be shown (Chow, 1975) that the agent's decision rule for choosing x takes the linear form[5]

$$x_t = Gy_{t-1} + g. \tag{17}$$

Substituting (17) into (16) yields the first-order, vector autoregressive representation

$$y_t = [A+BG]y_{t-1} + Bg + z. \tag{18}$$

In Models A-D, the disturbance vector, z, contains the expectation error, Z_3. The behavior of output and the other variables in the system in response to an expectation error is thus revealed by equation (18). The transfer weights analogous to those estimated by Blanchard, Barro, and Rush are given by

$$y_t = DBg + Dz \tag{19}$$

where

$$D = \{I - [A+BG]L\}^{-1}. \tag{20}$$

This formulation of the persistence question allows for a precise statement of the objection raised by critics of the equilibrium business cycle model. The matrix D determines the dynamic behavior of the economy. Two components of D -- the matrices A and B — reflect exogenous features of the environment. The other component, G, embodies the solution of the representative individual's optimization problem. Hence, the critics of these models claim that, in the context of these models, a rational agent will always choose G so as to offset those aspects of the environment that tend to perpetuate disturbances.

IV. RESULTS

A. Final Specification

In order to solve for the decision rule (17), we must choose values for all the parameters in the model. This task poses some difficulties. There is little in the way of either theory or empirical evidence to guide us in the choice of some of the parameters of the model. For these parameters, a range of values is used. In order to keep the number of different versions of the model within reasonable bounds, only a few different values of these variables are tried. Values are chosen to include both extreme values and at least one intermediate value. The table below lists the values assigned to all the parameters in the model.

TECHNOLOGY PARAMETERS

Parameter	Values	Description
δ	0.025	depreciation
γ	0.36	production function
θ	(0.01, 0.2)	adjustment cost
J	4	gestation period
τ	0.0000028	production function
ω	4.0	production function

PREFERENCE PARAMETERS

α_o	(0.1, 0.5, 1.0)	non-separability
β	0.99	discount factor
ρ	(-0.9, 1.0, 249.0)	utility function
λ	0.33	utility function
ν	(0.1, 0.5, 0.9)	non-separability

The values of the discount factor, β, and the depreciation factor, δ, depend on the length of the period defined for the model. In order to get a reasonably fine-grained view of the dynamic behavior of the variables, a period is specified to be one quarter of a year. Kydland and Prescott (1982) quote estimates of approximately four-percent per year for the average return on all capital. Using this figure for the real interest rate implies that β is roughly 0.99. Depreciation varies widely with the type of capital considered. Since there is only one type of physical capital in this model, a compromise value of ten percent annual depreciation is used.

Labor's share of output has consistently been estimated to be in the neighborhood of sixty-four percent. Labor's share in the model of this paper is $1-\gamma$, hence γ is set to 0.36. Kydland and Prescott (1982) estimated the other production function parameters from quarterly U.S. data and obtained point estimates of 0.0000028 for τ and 4.0 for ω. Those estimates are used here.

As with the rate of depreciation, the gestation period for capital investment, J, varies with the type of capital considered. Mayer (1960) and Hall (1977) find gestation periods of seven or eight quarters for investment in plant and equipment. This period must certainly be shorter for some consumer durables. Again the single sector nature of the model suggests a compromise value of four quarters.

The parameter λ in the utility function can be considered to be a "loading factor" which determines the weights of consumption and leisure in the utility function. The crucial parameter in the utility function is ρ which determines the elasticity of substitution between consumption and leisure. To concentrate attention on this latter parameter and to reduce the number of models to be solved, λ is set to 0.33.

The remaining parameters, α_0, θ, ρ, and ν, constitute the group of parameters about which little is known. As mentioned above, ρ controls the elasticity of substitution between consumption and leisure. The elasticity of substitution, call it σ, can be shown to be

$$\sigma = 1/(1+\rho). \tag{21}$$

To observe differences in the behavior of the model as the elasticity of substitution varies, low ($\sigma = 0.004$, $\rho = 249.0$), moderate ($\sigma = 0.5$, $\rho = 1.0$), and high ($\sigma = 10.0$, $\rho = -0.9$) elasticities of substitution are used.

The parameters α_o and ν control the non-separability of leisure over time. Recall that

$$0 < \alpha_o, \quad \nu < 1. \tag{22}$$

Values of 0.1, 0.5, and 1.0 are used for α_o. Values of 0.1, 0.5, and 0.99 are used for ν. The parameter θ, which appears only in Model B, determines the penalty for adjusting the capital stock. Values of 0.01 and 0.2 are used for θ.

Each of the models A-D must be solved for each combination of parameter values. The parameter θ does not appear in Models A, C, and D, so only combinations of α_o, ρ and ν need be considered for these models. Each of these parameters takes three different values which yields twenty-seven combinations. However, when $\alpha_o = 1.0$, the welfare functional is separable over time and the value of ν is irrelevant. Thus there are really only twnety-one combinations of these parameters. Since, for Model B, the two settings of θ must be taken into account, there are 105 different versions of the model to solve and analyze.

B. Measures of Persistence

How can we measure the persistence of the effects of nominal shocks? In the empirical literature, many of the measures of persistence are measures more of the precision of the estimates of transfer function rather than measures of some feature of the transfer function itself. Thus, a common measure of the persistence of nominal shocks is the last lag for which the estimated coefficient is significantly different from zero. Analogously, a statistic often reported in studies of cyclical behavior is the Q-statistic which is used to test the hypothesis that further autocorrelations of a variable are statistically insignificant.

There is no estimation problem in this paper. All the parameter values in the model are selected a priori. Once selected, these values imply sets of lag coefficients which can, for all intents and purposes, be calculated exactly. The significance, or lack thereof, of coefficients beyond ac ertain lag cannot be inferred by the usual expedient of comparing the magnitude of the coefficient to its estimated standard error simply because there is no error (other than the rounding error involved in calculation).

We could of course stochastically simulate the behavior of our models and use this simulated data in regressions patterned after those found in the literature. However, there are problems with this approach. In real empirical studies, the data available to the researcher never correspond precisely to the variables for which a theoretical relationship is posited. Furthermore, many of the variables which are observed are measured with error. Neither of these conditions afflict the model of this paper. Every variable in the model is available and all variables are known exactly including the expectation errors. Previous research along precisely these lines (Becketti, 1980) indicates that these considerations are important; estimates from simulated data were consistently unrealistically precise.

What measures of persistence can we use to evaluate these models? First, the eigenvalues of the matrix [A+BG] give some clues to the dynamic behavior of the models. There is no way to link particular eigenvalues to components of the model, but the number of large, in modulus, eigenvalues, and the presence or absence of complex eigenvalues are informative.

The autocorrelation matrix function of the model can be calculated from [A+BG]. Thus, we can observe the cross correlations, at various lags, between the variables in the system and expectation errors. The cross correlations of output with the expectation errors cannot be obtained by this means,[6] but the cross correlations of labor supply with the errors are available, and these cross correlations have much the same interpretation.

The evidence that is most closely comparable to the estimates examined in Section II of this paper is provided by the following conceptual experiment. Let the economy be at rest in the steady state. Now consider a nominal shock which leads the representative individual to expect a productivity shock that, if input levels remained at their steady state values, would increase output by one percent. Let us call this nominal shock a "one percent nominal shock" remembering that the one percent refers to a hypothetical change in output.

We use equation (18) to trace out the effects of a one percent nominal shock over a period of twenty quarters, i.e., we trace out the impulse response functions. As in Blanchard's and Barro and Rush's work, we measure these transfer functions in the units of percentage deviation from trend.[7] In addition to examining the qualitative appearance of these functions, we use two summary measures, the impact and unitary half-lives, to assess the persistence of the effects of the shock.

The impact half-life of a nominal shock is the period after which the effect of the shock is always less (in absolute value) than one-half the absolute impact effect. The unitary half-life is the period after which the effect of the shock is always less (in absolute value) than one-half percent of the steady state value. Since the initial period is labeled period 1, the impact half-life lies between 1 and 20 inclusive, while the unitary half-life lies between 0 and 20 inclusive.

Some comments on the strengths and weaknesses of these two measures may be helpful. Measures of persistence are obtained by comparing lagged effects of shocks to some scaling variable. In an estimation framework, the scaling variable is the estimated standard error of the lag coefficient. The impact and unitary half-lives simply use the absolute impact effect and a one percent absolute deviation, respectively, as the scaling variable. The virtue of the two half-life measures is that the scaling variable represents some notion of the economic importance of the lagged effect. If one can obtain very precise estimates of lag coefficients, then the last significant lag may be many quarters after the initial shock, even if all the lagged effects are extremely small. The unitary half-life appears to be robust with respect to this problem. The impact of half-life, though, is subject to problems of this sort. If, for example, the impact effect of a shock is violently large, the impact half-life can understate the persistence of economically important effects.

A particularly attractive feature of the half-life measures is that they can easily be calculated for the estimated lags found in the literature, i.e., the half-life measures can be used in a statistical setting even though the statistical measures of significance cannot be applied to the impulse response functions calculated in this study. As an example, let us apply the half-life measures to the estimates of Blanchard and Barro and Rush that are reported in Section II. Because of the normalization used in these estimates, the impact and unitary half-lives are identical in this example. In both sets of estimates, the half-life is at least as long as the last reported lag; for these estimates, then, we would report the half-life of a nominal shock as (at least) eight quarters.

C. Results[8]

The results of this study are quite clear. With the exception of Model D, the equilibrium business cycle models analyzed in this study show no persistent effects of nominal shocks whatsoever. This is true for every combination of parameters considered. Model D does generate substantial persistence, but the pattern of serial correlation produced in Model D does not match the hump shape found in the estimates reported in Section II.

The three figures at the end of this paper help to illustrate these results. These figures display the impulse reponse functions of output, labor supply, and investment for Model A and Model D when $\alpha_o = 0.1$, $\rho = 249.0$, and $\nu = 0.5$. While the figures compare only two versions of the model, almost identical pictures would be obtained by comparing versions corresponding to other sets of parameter values or by comparing Model D to either Model B or Model C.

In all three figures, the effects of the shock are largely dissipated in the first period in Model A. In Model D, the nominal shock initiates a slowly damped pattern of substantial cyclical variations. The large negative troughs in the output and labor supply response functions do not occur in every version of Model D, but the positive peaks do appear in all versions of the model. The lagged investment effects in this version of Model A are roughly the same magnitude as the lagged effects on output and labor supply. However, the scale of the diagram had to be changed so much to accommodate the very large lagged investment effects in Model D that the Model A function is almost coincident with the baseline.

The table at the end of the paper lists some of the measures discussed above. The first line of the table gives the number of models of each type that were successfully solved.[9] (Model B1 refers to those versions of Model B where $\theta = 0.01$. Model B2 refers to those versions of Model B where $\theta = 0.2$.) Much of the rest of the table reports the maximum and median impact and unitary half-lives of a one percent nominal shock on output, labor supply, and investment. According to these measures, there are no perceptible persistent effects of nominal shocks on either output or labor supply in Models A, B, and C. With the exception of Model B2, there is some evidence of persistence in the transfer function for investment. In Model D, on the other hand, these measures show substantial persistence in all three response functions.

The final two rows of the table list the number of models in which a pair of complex eigenvalues occurred and the largest modulus of a complex eigenvalue for each model. While complex eigenvalues appear in each model type, those appearing in Models A, B2, and C are so small in modulus that they are most likely the result of rounding error rather than evidence of even modest cyclical tendencies. Every version of Model D exhibits a pair of complex eigenvalues with modulus in the neighborhood of 0.90. Furthermore, there is an even larger real root in every version of Model D.

The cross correlation functions for each model echo the results listed in the table and figures. In Models A, B, and C, the flow variables exhibit very weak correlations with lagged shocks. These correlations are generally negative in contrast to he positive correlations estimated from U.S. data. The capital stock shows moderate to strong positive correlations with lagged shocks. In Model D, the lagged cross correlations of the flow variables are substantial, but they exhibit the unusual "seasonal" pattern that appears in the three figures.

There are some differences in the behavior of Models A, B, and C that are noteworthy. The impact effects of nominal shocks are much smaller in Model B than in any of the other models. The impact effects are even smaller in Model

B2 than in Model B1. In Model B, in order to avoid the adjustment costs incurred by mistaken investments, the representative individual simply doesn't respond to indications of productivity changes. Beyond the decrease in the impact multipliers though, the dynamic behavior of Model B is almost identical to that of Model A.

The behavior of Model C is troubling. The impact effect of an expansionary nominal shock in Model C is a substantial run-up in the stock of inventories at the expense of both consumption and investment in physical capital. (The impact effect on utility is unambiguously negative — both consumption and leisure fall initially.) This increase in inventories is used in future periods to finance increased leisure and a rebuilding of the capital stock. This pattern is clearly evident in the cross correlation functions. The negative cross correlation of inventory investment decays slowly. The cross correlations of the capital and inventory stocks are approximate mirror images of each other. This pattern of lagged effects remains constant over all combinations of the parameters. As we noted above, the specification of Model C falls short, for pragmatic reasons, of depicting what we believe is the role of inventories of final goods. Thus, the odd behavior of Model C may simply reflect the inadequacy of this specification.

V. CONCLUDING OBSERVATIONS AND SPECULATIONS

The results of this study are most damaging to the belief that adjustment costs can help to explain the observed serial correlation in economic aggregates. In this study, adjustment costs virtually eliminate not only the lagged effects of nominal shocks, but their contemporaneous effects as well. The behavior of Model B2 suggests that this perverse behavior is more pronounced the higher are the costs of adjustment.

The addition of inventories to the optimal growth model does not generate serially correlated movements in output either. However, the inexplicable behavior of some components of the inventory model call into question the specification used here. Further work is needed to determine the source of this puzzling behavior.

The initial results of assuming that capital takes "time to build" are encouraging. The pattern of lagged effects, the strong "seasonal" spikes in between flat stretches, does not mimic the hump shape found in the U.S. data, nor does the peak effect occur with a lag, but the basic fact of persistence can be reproduced within the framework of this model. In a multi-sector model, where the gestation period varies across sectors, it should be possible to produce a hump-shaped pattern of lagged effects, although this extension alone would not suffice to delay the peak effect. Nonetheless, these early results suggest that the "time to build" model merits further study.

To this point, our comments and conclusions have ranged only within the narrow confines of the models presented here. We may be forgiven then for closing with the following bit of speculation. The results of this study and of others that employ very different approaches (e.g., Mankiw et al., 1982) cast doubt on the ability of the rational expectations-equilibrium business cycle model to explain the observed dynamic behavior of economic aggregates when the structural model consists of a one-sector, individual decision problem. Other researchers (Lilien, 1982 and Long and Plosser, 1980) have suggested that the basic features of macroeconomic fluctuations are explicable only in the context of a multi-sector model of the economy.

Let us assume, for the sake of argument, that these suggestions are correct. If this is the case, then much of the appeal of the equilibrium business cycle model is dissipated. This is not because the model is "wrong" in any way, but simply because it becomes too unwieldy to use for a wide variety of important analyses, particularly policy analyses. Much further research is needed to answer this question, but clearly the intellectual stakes are high.

FOOTNOTES

*This paper summarizes some of the results of my dissertation research (Becketti, 1983). That research was greatly aided by the guidance and criticism of my committee — Robert Hall, Ben Bernanke, and Paul Evans. The aid of Finn Kydland and Edward Prescott was also essential in completing this project, and I am pleased to have an opportunity to voice my thanks for their substantial help. Many others — too many to list -- kindly read and commented on early versions of this work. However, I must acknowledge particularly fruitful conversation with Charles Plosser and James Powell. Of course, all these individuals are exonerated from any responsibility for the many mistaken notions to which I have clung.

[1]These estimates appear as Column 1, Table 1 in (Blanchard, 1981). The sample covers the period from the third quarter of 1947 to the fourth quarter of 1978.

[2]These estimates are taken from Column 5, Table 2.1 in (Barro and Rush, 1980). The coefficients and standard errors are rescaled so that the first period effect is equal to 1.0.

[3]This model is adapted from the one developed by Kydland and Prescott (1982), and any novel features of the model, unless otherwise noted, are due to Kydland and Prescott. The use made of this model, however, is distinctly different from the use made of it in Kydland and Prescott's paper.

Other models which use the fiction of a representative individual in order to derive aggregate behavioral relationships are Hall (1978) and Mankiw et. al. (1982).

[4]Michael Darby has pointed out, in connection with another paper by this author, that rational expectations models cannot account for an <u>aggregate</u>

productivity shock. This point is correct. In an "islands" model, such as Lucas (1973), the productivity shock would disappear in the integration across islands. Only the average value of the noise variable would appear in the aggregate equations. Making that correction in this model would change none of the results, while retaining an explicit reference to productivity shocks provides a familiar point of reference in what is already a fairly "stripped-down" model.

[5]Actually, in its most general form, the coefficients of this rule vary over time. This is perhaps a convenient point to state, without proof, that the specification of Models A-D guarantees the existence of a unique, time-invariant decision rule of the form of equation (17). In addition, our specification insures that the vector autoregressive process given in equation (18) is stable. Proofs of these statements can be found in Becketti (1983).

[6]Output does not appear in the state vector, y, because output is a non-linear function of the other variables in the system. As a result, while the autocorrelation function for the variables contained in y can be calculated exactly from equation (18), there is no direct way to calculate either the autocorrelations of output or its cross correlations with other variables. These auto— and cross correlations could be estimated by stochastically simulating the model, but, for the reasons stated in the text, that method is not used here.

Developments in the theory of labor contracts suggest that it may be more fruitful to analyze the exact cross correlations of labor supply with nominal shocks than to examine the approximate cross correlations of output with those shocks. In some versions of this theory, labor supply is varied each period to maintain Pareto efficiency. If, in addition, real shocks are uncorrelated, then the time series behavior of labor supply and output should, adjusted for amplitude, be quite similar. This statement holds for labor input measured in efficiency units, hence, it may not fit real world data very well where only hours of labor, and not intensity of labor, are observed. However the labor supply variable in Models A-D is measured in efficiency units, thus the cross correlations of labor supply with nominal shocks should be a good proxy for the cross correltions of output with nominal shocks. In point of fact, the behavior of labor supply in Models A-D turns out to be extremely similar to the behavior of output.

[7]The transfer functions calculated by this method are normalized slightly differently than the estimated functions reported in Section II. Nothing of substance is affected by this change in normalization, and this method allows us to sidestep other ambiguities. See Beckettl (1983) for details.

[8]A one hundred page appendix in Beckettl (1983) contains a much more detailed report on the results of this study. Interested individuals can obtain a copy of this appendix by contacting the author. A small fee will be charged to cover the costs of reproduction and postage.

9Ten of the models yielded apparently unstable solutions. The problem is numerical. The matrix of the quadratic form in the welfare functional is not of full rank, and small rounding errors tend to cause difficulties in these "redundant" dimensions. There is a detailed discussion of this problem in Beckettl (1983). Masanao Aoki has proposed some improvements in the solution technique that overcome these difficulties. These improvements are described in a forthcoming monograph by Aoki.

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MODEL

	A	B1	B2	C	D
Number of models	18	21	21	19	17
Impact half-life					
Output					
maximum	1	1	1	1	5
median	1	1	1	1	1
Labor Supply					
maximum	1	1	1	1	17
median	1	1	1	1	5
Investment					
maximum	2	2	2	1	14
median	1	1	1	1	9
Unitary half-life					
Output					
maximum	1	1	1	2	5
median	1	1	1	1	1
Labor Supply					
maximum	1	1	1	2	13
median	1	1	1	1	5
Investment					
maximum	11	4	1	11	20
median	5	2	1	2	20
Complex eigenvalues					
Number of models	2	2	3	12	17
Maximum Modulus	—	.89	—	—	.93