TAX REFORM AND STRONG SUBSTITUTES*

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Abstract

In this paper, we establish a bridge between the traditional wisdom of advocating broad-based commodity taxes and recent developments in the theory of optimal commodity taxation which have emphasized the non-uniformity of the optimal commodity tax structure. We demonstrate that piecemeal, revenue maintaining movements towards uniform taxation amongst groups of commodities that are strong substitutes improves welfare. Our formal analysis provides precise and empirically testable measures of the degree of substitutability which warrants such piecemeal tax rate "squeezing". An interesting implication of the analysis is that it casts doubt on the commonly held proposition that one should impose the highest tax rates on the commodities with the lowest price elasticities of demand.

INTRODUCTION

The principal aim of this paper is to establish that in a many-good economy where the initial commodity and income tax structure is arbitrarily given and no lump-sum tax is available, welfare can be improved by making tax rates of two strongly substitutable goods closer, while keeping tax revenue constant.

If taxes with different ad valorem rates are imposed on two strong substitutes, a consumer can avoid a substantial amount of tax payment by substituting the good with the lower rate for the one with the higher rate.[1] This will distort resource allocation. Thus, reducing the differential between the rates of strong substitutes is likely to improve welfare. This argument is often used as a justification for broadbased taxes.

This intuitive argument, however, is not universally applicable, since "squeezing" of tax rates of some pairs of substitutes, maintaining the original revenue level, certainly reduces welfare. Indeed, the theory of optimal taxation reveals that the optimal commodity tax structure is generally non-uniform even when all goods in the economy are substitutes.[2] Thus, despite the seemingly convincing intuitive argument for the broad-based tax, we do not know exactly in what situations squeezing of tax rates of substitutable goods can be justified.

In the present paper, we will establish a sufficient condition for the degree of substitutability that warrants such tax-rate "squeezing". This sufficient condition will be in an empirically testable form, and its formal interpretation in terms of tax avoidance will be given. Thus the present paper constructs a bridge between the traditional wisdom for advocating a broad-based tax and the theory of optimal commodity taxation, by establishing a condition under which the traditional wisdom is consistent with tax changes toward the optimal structure.

I. THE MODEL

Household. There is one consumer in this economy.[3] He derives utility v from n private goods and one public good. His utility function is separable in the private goods,[4] so that

$$v = V(U(x),r),$$

where the n-vector x signifies the net consumption of private goods by the consumer and r signifies the amount of the public good. We assume that the function V and U are increasing with respect to their arguments. We also assume that the consumer maximizes v subject to

$$q'x = 0, \tag{1}$$

where the n-vector q signifies the prices faced by the consumer. Moreover, we assume that some goods are factors of production. Hence we have

$$x^i < 0 \qquad i \in F \tag{2}$$

$$x^i > 0 \qquad i \notin F \tag{3}$$

where F is the set of indices for the factors of production. Those goods that are not factors of production will be called commodities.

In view of the monotonicity of the function, the above maximization problem is equivalent to the following

$$\max\ u = U(x) \qquad \text{s.t.} \quad q'x = 0,$$

where u denotes the utility level derived from the private good consumption. In our theorems, r will be kept constant during the tax reform, so that the directions of the changes in v and u are identical. Thus, in the present paper we will study the tax impact on v through that on u.

Let $m(q,u)$ and $x(q,u)$ be the consumer's expenditure and compensated demand functions, respectively. Then we have

(4) $$m(q,u) = q'x(q,u) \quad \text{and}$$

(5) $$m_q(q,u) = x(q,u),$$

where $$m_q \equiv \left(\frac{\partial m}{\partial q^1}, \ldots, \frac{\partial m}{\partial q^n}\right)'.$$

Production. Technology can be represented by a linear production possibility frontier of the form

(6) $$p'x + r = 0,$$

where p is the vector of positive constants. Producers are perfect competitors and in equilibrium p must be proportional to the vectors of producer prices. We choose the unit of currency so that this factor of proportionality is one.

Government. *Ad valorem* excise taxes and income taxes (with possibly different rates for different factor incomes) are imposed. There are no other taxes; in particular, there is no lump-sum tax. Thus

(7) $$q^i = (t^i+1)p^i \qquad i = 1,\ldots,n,$$

where t^i is the <u>ad valorem</u> tax rate of the i^{th} good. We assume

$$t^i + 1 > 0 \qquad i = 1,\ldots,n.$$

This is equivalent to assuming $q > 0$. Since p is fixed in our model, (7) defines the following function:

$$q = q(t), \tag{8}$$

where $t \equiv (t^1,\ldots,t^n)'$. We assume that all the tax revenue is spent on the supply of public goods. Hence

$$(q-p)'x = r. \tag{9}$$

But this is already implied by (1) and (6); the government budget equation (9) is automatically satisfied when (1) and (6) hold.

From (1), (4), and (6), we have

$$m(q,u) = 0 \tag{10}$$

$$p'x(q,u) + r = 0 \tag{11}$$

Substituting (8) for q, we get

$$m(q(t),u) = 0 \tag{12}$$

$$p'x(q(t),u) + r = 0 \tag{13}$$

These two equations contain $n+2$ variables t, u, and r. They describe a complete model when n of them are exogenously given. For example, if t is given, the model determines u and r; if r and $n-1$ elements of t are given, the model determines u and the remaining element of t.

II. EFFECTS OF A CHANGE IN A SINGLE TAX RATE

Suppose that the vector t is exogenously given and u and r are determined by (12) and (13). Then the solution functions for u and r may be written as

(14) $$u = \mu(t) \quad \text{and}$$

(15) $$r = \rho(t).$$

From (12) it is easy to see that

(16) $$\frac{\partial \mu}{\partial t^i} = - \frac{p^i x^i}{m_u} < 0 \quad \text{if} \quad x^i > 0$$

where $m_u = \partial m/\partial u$. In deriving this inequality we have used $\partial q^i/\partial t^i = p^i$, $\partial m/\partial q^i = x^i$, and the fact that $m_u > 0$ from the positivity of the marginal utility of each good. Thus a reduction in a commodity tax rate increases the level of utility derived from private goods. This is only natural since such a tax reduction expands the consumption possibility set of the consumer.

We say that the i^{th} tax rate is <u>revenue-increasing</u> if

$$\frac{\partial \rho(t)}{\partial t^i} > 0;$$

<u>viz.</u>, a tax rate is revenue-increasing if raising it increases tax revenue. We say that the i^{th} tax rate is <u>revenue-decreasing</u> if the above inequality is reversed. If a commodity tax rate is revenue-decreasing, that rate should definitely be reduced; in view of (16), such a reduction will increase u as well as the government revenue. When a commodity tax rate is revenue-increasing, however, a change in this rate creates a tradeoff between u and

government revenue.

The following Lemma gives a useful criterion to determine the sign of $\partial\rho/\partial t^i$.

Lemma 1 Let[5] $x_u = (\partial x^1/\partial u, \ldots, \partial x^n/\partial u)'$, $x_i \equiv (\partial x^1/\partial q^i, \ldots, \partial x^n/\partial q^i)'$, and $D^i \equiv (\frac{p'x_u}{m_u} - \frac{p'x_i}{x^i})p^i x^i$. Then the tax rate on good i is revenue-increasing if and only if $D^i > 0$.

Proof. By substituting $\mu(t)$ and $\rho(t)$ for u and r in equation (13), we obtain the following identity:

$$\rho(t) = -p'x\ (q(t), \mu(t)).$$

Differentiating this with respect to t^i and then substituting (16) for $\partial\mu/\partial t^i$, we obtain

$$(17) \qquad \frac{\partial\rho}{\partial t^i} = -p'x_i\ p^i - p'x_u \frac{\partial\mu}{\partial t^i} = D^i. \qquad \text{Q.E.D.}$$

Now suppose that commodities a and b are strong substitutes and that $t^a < t^b$ initially. Then both goods will have high price elasticities of demand. It is readily seen that this creates a tendency for the higher tax rate, say t^b, to be revenue-decreasing.[6] When t^b is increased, the consumer will substitute a for b. This means that the low-taxed good is consumed more and the high-taxed one less, which can easily reduce the net government revenue. When t^b is revenue-decreasing for this reason, then welfare will be improved by a reduction of t^b, i.e., by making the tax rates of the two strong substitutes closer.

Of course, t^b may not be revenue-decreasing, even if a and b are strong substitutes. Our analysis in the next section will show that even in this situation welfare is improved by making tax rates of strong substitutes closer.

III. TAX RATES OF STRONGLY SUBSTITUTABLE GOODS SHOULD BE MADE CLOSER

A. A Criterion for Welfare Improvement

The main aim of this section is to determine how commodity tax rates should be changed when a strongly substitutable commodity pair exists and both rates are revenue-increasing. The following Lemma provides a basis for determining the effects of a tax change on welfare.

Lemma 2. Suppose that goods a and b are commodities and that the tax rates of goods a and b are both revenue-increasing in the model of (12) and (13). Then welfare is increased by a small increase in t^a accompanied by a simultaneous reduction in t^b so as to maintain the initial revenue level, if

$$N^{ab} \equiv \frac{-p'x_a}{x^a} + \frac{p'x_b}{x^b} > 0 \tag{18}$$

Proof: We first evaluate the welfare effect of a change in t^a accompanied by an adjustment in the level of t^b so as to keep the revenue level constant.

Totally differentiating the system of (12) and (13) (letting $dr = 0$), we get

$$\begin{bmatrix} m_u & x^b p^b \\ p'x_u & p'x_b p^b \end{bmatrix} \begin{matrix} du \\ dt^b \end{matrix} = \begin{bmatrix} -x^a p^a \\ -p'x_a p^a \end{bmatrix} dt^a \tag{19}$$

This yields

$$\frac{du}{dt^a} = \frac{(p^a x^a)(p^b x^b)\ N^{ab}}{m_u D^b} \tag{20}$$

Equation (20) implies that an increase in t^a improves utility level if N^{ab} and D^b are both positive. On the other hand, N^{ab} is positive from (18), and D^b is also positive from the revenue-increasingness of b and Lemma 1. Finally, the revenue increasingness of a and b together guarantees that t^b has to be reduced when t^a is increased maintaining the constant government revenue. Q.E.D.

B. Criteria of Strong Substitutability

We now give a criterion of sufficiently strong substitutability that warrants squeezing of their tax rates. Theorem 1 will establish that (18) is indeed satisfied under this criterion. Consider the situation where the initial tax structure satisfies

$$t^a < t^b \tag{21}$$

in the model of (12) and (13).

Suppose that the tax vector t is a function of a parameter s:

$$t = \alpha(s) = (\alpha^1(s),\dots,\alpha^n(s)),$$

where

$$\frac{d\alpha^i}{ds} = \begin{cases} t^i - t^a & \text{if } t^i < t^a \text{ or if } i = b \\ 0 & \text{otherwise.} \end{cases}$$

Thus the function $\alpha(s)$ represents a tax reform program such that an increase in s causes a simultaneous radial decrease in t^i for all $t^i < t^a$ and a radial increase in t^b, with t^a as the radial center, as is illustrated in Figure 1(a). In other words, $\alpha(s)$ represents a t^a-centered radial expansion of t^b and all the tax rates lower than t^a. If this radial expansion of tax rates increases the compensated demand for a, i.e.,

$$\frac{\partial x^a(q(\alpha(s),u))}{\partial s} > 0, \tag{22}$$

then we say that commodities a and b are <u>strong substitutes relative to the goods with lower tax rates than t^a</u>. When (22) holds, the increase in the compensated demand for x^a caused by an increase in q^b as a result of the radial expansion of the tax rates is so large as to overwhelm the counterveiling effect caused by the reduction in all of the tax rates lower than t^a.

Now suppose that the tax vector t is another function of a parameter s:

$$t = \beta(s) = (\beta^1(s), \ldots, \beta^n(s))',$$

where

$$\frac{d\beta^i}{ds} = \begin{cases} t^i - t^b & \text{if } t^i > t^b \text{ or if } i = a \\ 0 & \text{otherwise.} \end{cases}$$

The function $\beta(s)$ represents a t^b-centered radial expansion of t^a and all the tax rates higher than t^b, as is illustrated in Figure 1(b). If this radial expansion of tax rates reduces the compensated demand for b, i.e.,

$$\frac{\partial x^b(q(\beta(s),u))}{\partial s} < 0, \tag{23}$$

then we say that commodities a and b are strong substitutes relative to the goods with higher tax rates than t^b. When (23) holds, the reduction in the compensated demand for x^b caused by a reduction in q^a as a result of the radial expansion of the tax rates is so large that it overwhelms the counterveiling effect caused by the increase in all of the tax rates higher than t^b.

When $t^a < t^b$, we say that commodities a and b are strong substitutes relative to the outliers if a and b are strong substitutes relative both to the goods with lower tax rates than t^a and to the goods with higher tax rates than t^b, i.e., if (22) and (23) are satisfied.

C. The Theorem

We are now in a position to state and prove the following theorem.

Theorem 1 Suppose that the initial tax structure in the model of (12) and (13) is such that $t^a < t^b$. Suppose also that goods a and b are commodities and that t^a and t^b are both revenue-increasing. Then welfare is increased by a small reduction in t^b accompanied by a simultaneous increase in t^a so as to maintain the initial revenue level, if the following conditions are satisfied:

i) Good a is a substitute for all goods with a higher tax rate than t^a and good b is a substitute for all goods with a lower rate than t^b.

ii) Goods a and b are strong substitutes relative to the outliers.

Proof: The theorem will be proved by establishing that the assumptions made imply inequality (18). Without loss of generality, we can index goods so that

$$(24)\qquad t^1 \leqq t^2 \leqq \dots \leqq t^a \leqq t^{a+1} \leqq \dots \leqq t^{b-1} \leqq t^b \leqq \dots \leqq t^n.$$

From this and (22) we have

$$(25)\qquad \frac{\partial x^a(q(\alpha(s),u)}{\partial s} = \sum_{i<a} \frac{\partial x^a}{\partial q^i}\frac{\partial q^i}{\partial t^i}\frac{d\alpha^i}{ds} + \frac{\partial x^a}{\partial q^b}\frac{\partial q^b}{\partial t^b}\frac{d\alpha^b}{ds}$$

$$= \sum_{i<a} x_i^a p^i(t^i - t^a) + x_b^a p^b(t^b - t^a) > 0 .$$

From (24) and assumption (i) of the theorem, we get

$$\sum_{\substack{i \geqq a \\ i \neq b}} (t^i - t^a)\, p^i x_i^a > 0.$$

This and (25) yield

$$(26)\qquad \sum_{i=1}^{n} (t^i - t^a)\, p^i x_i^a > 0 .$$

Since $q'x_a = 0$ and $x_a^i = x_i^a$, we have

$$(27)\qquad (t^a + 1)\, p'x_a = (t^a + 1)\, p'x_a - q'x_a = \sum_{i=1}^{n} (t^a - t^i)\, p^i x_i^a .$$

Thus, (26) implies $p'x_a < 0$.

Similarly, (23) and assumption (i) yield $p'x_b > 0$. Hence we have $N^{ab} > 0$. In view of Lemma 2, the theorem is proved. Q.E.D.

The underlying intuition of the theorem is that since different tax rates on two strongly substitutable goods causes a substantial distortion in resource allocation, making their tax rates closer will have a tendency towards improving welfare. The problem, of course, is that reducing the distortion between the two goods in question will, in general, result in some increase in the distortions between the two goods and other goods. In the case of Theorem 1, the increase in t^a (the decrease in t^b) increases the distortion between good a (b) and goods with tax rates lower than t^a (higher than t^b). Conditions (22) and (23) define the circumstances under which the positive welfare effect of reducing the distortion between a and b dominates the negative effects due to the accompanied increase in distortions.[7] Finally, notice that (22) and (23) are empirically testable conditions, since the LHS of (22), for example, can be equivalently restated in terms of demand parameters as in (25).

IV. AN INTERPRETATION IN TERMS OF REFLECTION[8]

In the Introduction, we stated that intuitive justification usually given for the broad-based tax is that if a lower tax rate is imposed on one of the substitutable goods, tax avoidance is possible by substituting it for the commodity with the high tax rate. A formal interpretation of this intuition was already given for the case where the higher tax rate is revenue-decreasing due to the substitutability. Let us now give an interpretation to this intuition for the case where both rates are revenue-increasing.

We first examine the revenue effect of a change in a single tax rate, allowing the utility level to vary freely. From (8) and (9), we have

$$r = (q(t) - p)'\ x(q(t),u).$$

Since the solution of r and u in the model of (12) and (13) for given t necessarily satisfy this equality, we can substitute $\rho(t)$ for r and

$\mu(t)$ for u to obtain:

$$\rho(t) \equiv (q(t) - p)' \, x(q(t), \mu(t)) \quad \text{for all} \quad t.$$

Differentiating this identity yields

$$\frac{\partial \rho}{\partial t^j} = \underset{\substack{\text{[primary} \\ \text{effect]}}}{p^j x^j} + \underset{\substack{\text{[real income} \\ \text{effect]}}}{\tau' x_u \frac{\partial \mu}{\partial t^j}} + \underset{\substack{\text{[substitution} \\ \text{effect]}}}{\tau' x_j p^j} \tag{28}$$

where $\tau \equiv q-p$. (This is of course equivalent to (17)). The first term on the RHS gives the revenue increase from the tax on the j^{th} good that would accrue if the quantity demanded for good j were kept constant. The second term represents the revenue change due to the (negative) income effect of the price increase of the j^{th} good. The last term captures the revenue change due to the substitution effects of the price effect in the j^{th} good.

Now consider an increase in t^j that reduces the utility level by one unit. The resulting revenue change is given by $\frac{\partial \rho}{\partial t^j} \Big/ \left(-\frac{\partial \mu}{\partial t^j}\right)$. From (16) and (28) we have

$$-\frac{\partial \rho}{\partial t^j} \Big/ \frac{\partial \mu}{\partial t^j} = \underset{\substack{\text{normalized} \\ \text{primary} \\ \text{effect}}}{m_u} - \underset{\substack{\text{normalized} \\ \text{income} \\ \text{effect}}}{\tau' x_u} + \underset{\substack{\text{normalized} \\ \text{substitution} \\ \text{effect}}}{\frac{\tau' x_j}{x^j} m_u}$$

Once tax increases are thus normalized, the primary and income effects are common for all j; only the normalized substitution effect differs from one tax rate to another. To put it differently, the substitution effect is the only factor that makes the effects of normalized tax increases differ among the tax rates on various goods. If we increase t^a as much as to lower the utility level by one dollar's worth, and reduce t^b as much as to exactly offset the welfare effect of t^a, the net revenue changes, $\Delta\rho$, is given by

(29)
$$\Delta\rho \equiv \left(-\frac{\partial\rho}{\partial t^a} \Big/ \frac{\partial\mu}{\partial t^a} + \frac{\partial\rho}{\partial t^b} \Big/ \frac{\partial\mu}{\partial t^b}\right)/m_u$$
$$= \frac{\tau' x_a}{x^a} - \frac{\tau' x_b}{x^b}.$$

Since $\tau' x_b = - p' x_b$; (18), and this yield

(30)
$$\Delta\rho = N^{ab}.$$

A critical step in the proof of Theorem 1 was the signing of N^{ab}. Equation (30) gives a direct interpretation to N^{ab} in terms of the revenue effect of a welfare-maintaining tax squeeze.[9] Since N^{ab} is positive when a and b are sufficiently strong substitutes as was shown in the proof of Theorem 1, so is $\Delta\rho$; i.e., the government revenue is increased by a utility-maintaining squeeze of t^a and t^b if goods a and b are sufficiently strong substitutes.[10]

Thus, we can conclude that if the tax rates of two goods are revenue increasing and if the goods are so strongly substitutable as to make $\Delta\rho$ positive, then welfare is improved by making their tax rates closer in such a way as to maintain the revenue constant. When the tax rates of the pair of substitutes are both revenue increasing this may be interpreted as a precise restatement of the notion that the tax rates of two strong substitutes should be made closer because the tax revenue will erode otherwise.[11]

V. A DIAGRAMATIC ILLUSTRATION OF THE RESULTS

We now give a diagramatic illustration of the results obtained thus far. Suppose that all tax rates other than the two commodity tax rates t^a and t^b are fixed in the model of (12) and (13). We call the locus of the points in the t^a-t^b plane that gives a prescribed utility level a <u>tax indifference curve</u>. The three downward sloping curves in Figure 2 depict the tax indifference curves for the utility levels u_1, u_2, and u_3. By definition, these curves are the level curves for the function $\mu(t)$ when all tax rate other than t^a and t^b are fixed at given levels. The tax indifference curves must be downward sloping, because $\partial\mu/\partial t^a < 0$ and $\partial\mu/\partial t^b < 0$ from (16). Also note that a curve for a higher combination of t^a and t^b corresponds to a lower utility level.

When all the tax rates other than t^a and t^b are fixed at the same levels as above, we can define an <u>iso-revenue curve</u> as the locus of the points in the t^a-t^b plane for a prescribed revenue level. Two concentric curves in Figure 2 depict the iso-revenue curves for the revenue levels r_1 and r_2. By definition, these curves are the level curves for the function $\rho(t)$ when all tax rates other than t^a and t^b are fixed at the given levels. When a tax rate is close to zero, it must be revenue-increasing, but as the rate is increased, sooner or later it will become revenue-decreasing. Thus there must be a combination of t^a and t^b that maximizes tax revenue given other rates. Point M represents this combination. Notice that an iso-revenue curve is downward-sloping if both t^a or t^b are revenue-increasing or if both are revenue-decreasing. If one of t^a and t^b is revenue-increasing and the other is revenue-decreasing, the iso-revenue curve must be upward-sloping. Thus the zone AMBO represents the area where both tax rates are revenue-increasing. This area will be called the <u>normal rate zone</u>.

At a point in the $t^a - t^b$ plane where an iso-revenue curve intersects a tax indifference curve, it is possible to improve utility by changing tax rates along the iso-revenue curve, as in the case of moving from F to G in Figure 2. We will call the tangency locus between the system of iso-revenue curves and the system of the tax indifference curves the tax efficiency locus. At a point on this locus, say at point L in Figure 2, it is impossible to improve the utility level without reducing the tax revenue.

Notice that this diagram illustrates that if t^a is revenue-decreasing, as at E, a reduction in t^a, keeping all other tax rates intact, will increase both utility and revenue. In this case, Theorem 1 is not necessary to find out a desirable direction of the tax reform. When t^b is revenue-decreasing, as at D, a reduction in t^b can be similarly justified.

If (t^a, t^b) is in the normal rate zone, however, one tax rate must be increased if the other is decreased to avoid reducing tax revenue; there is no obvious way to identify desirable directions of tax change. It is this case where Theorem 1 must be invoked to identify desirable tax changes.

Theorem 1 states that if t^a and t^b are revenue-increasing and $t^a \neq t^b$ initially, if both commodities a and b are substitutes for all other goods, and if a and b are sufficiently strong substitutes themselves, welfare will be improved by making t^a and t^b closer. Therefore, welfare is improved by moving along a given iso-revenue curve in Figure 2 toward the direction of the 45 degree line through the origin (not drawn). Thus Theorem 1 implies that under its assumptions the tax efficiency locus must lie close to the 45 degree line. In the extreme case, where a and b are substitutes but are "unrelated" to all other goods (i.e., $x_j^a = x_j^b = 0$ for all $j \neq a,b$), then the 45 degree line is the tax efficiency locus.

VI. CONCLUDING REMARKS

It is commonly held that one should impose the lowest tax rates on the commodities with the highest price elasticities of demand. The underlying intuition of imposing the lowest tax rates on the commodities with the highest price elasticities of demand is of course to minimize the deviations from the nondistortive, pre-tax allocation. However, in general, the reason why a commodity has a high price elasticity is that it has close substitutes. Our analysis suggests that even if the price elasticity of a commodity is extremely high, its optimal tax rate may be quite high as long as a uniform tax rate is imposed on this good and its close substitutes.

Recent developments in the theory of optimal commodity taxation have led to an emphasis on the non-uniformity of the optimal commodity tax structure in an economy without a lump-sum tax. This paper has shown that the traditional wisdom of advocating a broad-based tax for close substitutes is consistent with this theory. We have shown that piecemeal movements towards uniform taxation amongst groups of commodities that are close substitutes improves welfare. Our formal analysis has provided precise measures of the degree of substitutability which warrants such tax rate "squeezing".

Reconciliation between our results which advocate movements towards uniform taxation and the non-uniformity generally found in the optimal taxation literature may lie in recognizing the contrasting requirements of tax reform and tax optimality. In the region of the optimum, it may well be the case that the criterion for strong substitutibility given in Theorem 1 will not be satisfied. Yet this will be true because close to the optimum there will not be a disparity between the tax differentials across goods and their relative substitutibility. Our results provide a simple, piecemeal approach to eliminate any such disparities from a given tax structure.

FOOTNOTES

[1]For example, Musgrave and Musgrave (1980, p. 286) state,

> ...the price elasticity for a group of products (such as cars in general) will tend to be lower than for a particular item in that group (such as blue Pintos with air conditioning). The reason, of course, is that substitution is easier in the latter case. Selective taxes thus leave the consumer in a better position to avoid payment than do broad-based taxes.
>
> A tax on Pintos can be avoided by buying some other car; a tax on cars in general can be avoided, if less conveniently, by taking buses or by flying; but a general sales tax can be avoided only by consuming less and saving.

[2]See Ramsey (1927), Diamond and Mirrlees (1971), Sandmo (1976),and Sadka (1977), for an example. In an economy where a lump-sum tax is available, however, uniform commodity taxation is optimal. Even then squeezing of tax rates of any pair of substitutes can not be generally justified when the initial tax structure is arbitrary and nonuniform. This is because of the "second best" problem, as pointed out by Lipsey and Lancaster (1956).

[3]Here we follow the tradition of Ramsey (1927), Corlett and Hague (1953-54), Sandmo (1976), Sadka (1977), and others. As all of these authors were aware, a lump-sum tax is feasible in the single consumer economy without any distributional side effects. Nevertheless, they find it useful to study the single-consumer economy without a lump-sum tax, because they can focus on the efficiency aspects of their problems. For example, this assumption enables them to derive the nonuniformity of optimal commodity taxation from efficiency grounds alone, without bringing in distributional considerations. In the present paper, we show desirability of changing the tax rates of close substitutes toward uniformity under the same assumption as the aforementioned articles make. A very similar model as the present paper was also used in Hatta (1981).

[4]The theorem in the present paper holds without assuming the separability of the utility function as long as an innocuous assumption to preclude an abnormally strong complementarity between the public good and private goods is made. But the separability greatly simplifies the exposition. This is particularly so in Sections IV and V.

[5]A functional symbol with a subscript will indicate the derivative, the gradient, or the Jacobian matrix of the original function with respect to the variable or the vector denoted by the subscript. For example,

$$m_u \equiv \frac{\partial m}{\partial u},\ x_i^j \equiv \frac{\partial x^j}{\partial q^i},\ x_i \equiv (x_i^1,\ldots,x_i^n)',\ x_q \equiv \frac{\partial x}{\partial q},\ x_u^i \equiv \frac{\partial x^i}{\partial u},\ \text{and}\ \ x_u = (x_u^1,\ldots,x_u^n)'.$$

[6]This can be illustrated formally using Lemma 1. Similarly as (27) we can prove that $(t^b + 1)\, p'x_b = \sum_{i=1}^{n} (t^b - t^i)\, p^i x_i^b$. Thus we can rewrite D^b as:

$$D^b = \left[\frac{p'x_u}{m_u} - \frac{\sum_{i=1}^{n} (t^b - t^i)\, p^i x_i^b}{(t^b + 1)\, x^b} \right] (p^b x^b)$$

If $t^b > t^a$ and commodities a and b are strong substitutes, then $(t^b - t^a)\, p^a x_a^b$ is positive with a large absolute value and from this expression we can see that there is a tendency for $D^b < 0$. From Lemma 1, if $D^b < 0$, then t^b is revenue decreasing.

[7]In a model similar to the present one Hatta (1981) has shown that in many situations welfare may be improved by simultaneously reducing the highest tax rates among all goods and increasing the lowest tax rates among all goods, i.e., by changing the tax structure towards uniformity. Our Theorem 1 contains as a special case the theorem in Hatta (1981) for the situation where a commodity, not an input, has the lowest tax rate among all goods. Our Theorem

1, however, neither contains nor is contained by any of the theorems in Hatta (1981) concerning situations in which leisure has the lowest tax rate.

[8]In this section, the utility change under the revenue constraint is studied through its reflection i.e., the revenue change under the utility constraint. Often a reflection has been called by economists a problem dual to the original problem. But the distinction is useful because in the economics literature the term "dual" has been also used in the sense of mathematical duality. See Peter Newman (1982, esp. p. 118) on this. The term "reflection" employed here is his.

[9]In terms of the revenue impact of the tax change, Lemma 2 can be restated as follows:

"Suppose that both t^a and t^b are revenue-increasing and that $\Delta\rho > 0$. Then welfare is improved by making t^a and t^b closer in such a way as to maintain revenue constant."

[10]Note that just because a and b are substitutes does not imply that making t^a and t^b closer keeping utility constant increases revenue. Again this is because a reduction in distortion between a and b generally increases distortions between the two goods and other goods, contributing to a reduction in the revenue level. This is the reason why goods a and b must be sufficiently strong substitutes in order to make $\Delta\rho$ positive.

[11]It may be useful at this point to note that by definition $\Delta\rho$ is the maximum amount of the additional tax payment the consumer is willing to pay in a lump sum fashion if tax rates are changed as in the definition of $\Delta\rho$. In other words, $\Delta\rho$, and hence N^{ab}, is a measure of excess burden associated with this particular tax change.

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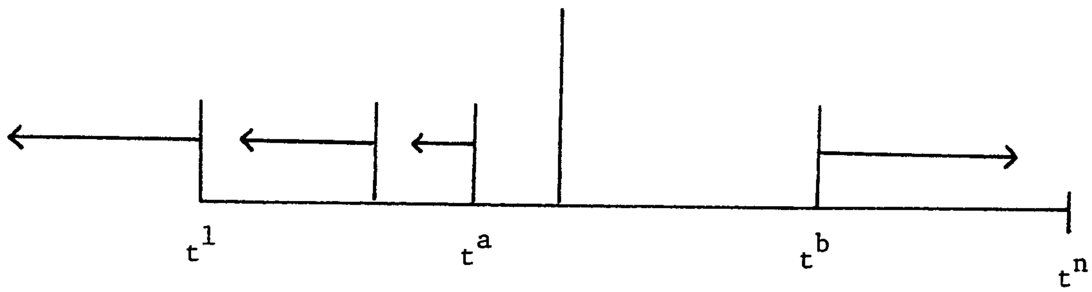
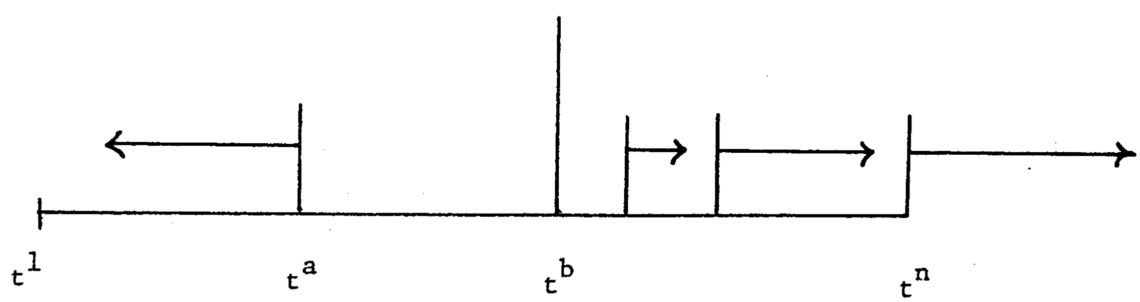


(a)



(b)

Figure 1

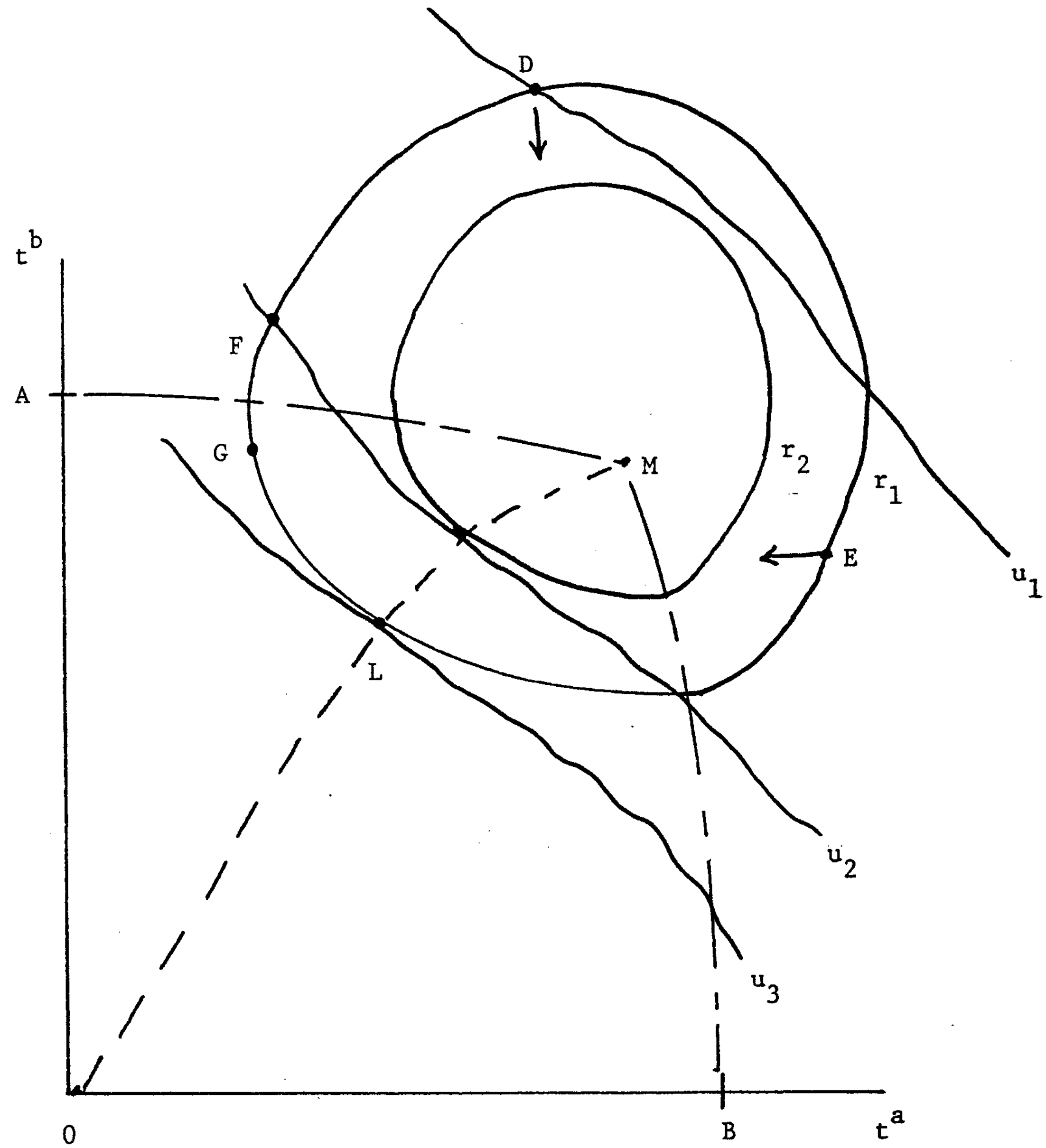


Figure 2