RATIONAL EXPECTATIONS IN THE AGGREGATE

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I. Introduction

One of the major recent innovations in economic theory is the emergence of the rational expectations hypothesis. By the rational expectations hypothesis we mean the hypothesis that expectations of agents tend to be consistent with the predictions of the relevant economic theory. In this paper we consider the motivation for the use of rational expectations. In particular, we consider the relationship between the way rational expectations is typically employed in practice, and the argument frequently put forth to justify its use.

In practice rational expectations has typically meant what we will refer to as standard rational expectations. By standard rational expectations we mean that the expectation of each agent taken separately is by itself consistent with the predictions of the relevant theory. This, however, is different than the argument frequently put forth by proponents of the rational expectations hypothesis to justify its use. That argument is that on an aggregate level expectations should be consistent with the predictions of the relevant theory. This justification first appeared in Muth's initial treatment of rational expectations, and has appeared more recently in the works of Kantor (1979), Maddock and Carter (1982), and Hoover (1984).

> The hypothesis can be rephrased a little more precisely as follows: that expectations of firms (or, more generally, the subjective probability distribution of outcomes) tend to be distributed, for the same information set, about the prediction of the theory (or the "objective" probability distributions of outcomes). (Muth 1961, p. 316)

What lies behind the above argument is a belief that, if expectations were rational in the aggregate, then expectational deviations across agents would tend to cancel out. The statement of this belief also appeared in Muth, and has appeared as recently as Charles Schultze's 1984 Presidential Address to the American Economic Association.[1]

> ...Allowing for cross-sectional differences in expectations is a simple matter, because their aggregate effect is negligible as long as the deviation from the rational forecast for an individual firm is not strongly correlated with those of the others...
> (Muth 1961, p. 321)

> In a world of auction markets, the fact that forecasts of individual agents are widely distributed around the "true" mean is for most purposes irrelevant.... (Schultze 1985, p. 10)

In this paper we formally investigate the relationship between standard rational expectations and what occurs when expectations are only rational in the aggregate, i.e., what we will refer to as aggregate rational expectations. Our goal is twofold. First, we want to demonstrate that the above view is overly simplistic. That is, it is not the case that an aggregate rational expectations world can in general be accurately modeled using a standard rational expectations assumption. Second, we want to investigate what we consider to be the following even more interesting issue. Given an environment where the standard and aggregate rational expectations equilibria differ, what factors affect the size of the difference.[2]

We investigate the above issues by analyzing a model wherein agents decide which of two activities to participate in, where the return to participating in an activity depends on the total number of agents who choose that activity.[3] This interaction among agents can be either of two types. First, the activities can exhibit congestion effects, i.e., the return to participating in an activity can be negatively related to the total number of agents who participate in that activity. Examples of real world choice situations which exhibit congestion effects are the problem of agents choosing between different roads which lead to the same final destination, and the problem of career choice. Second, the activities can exhibit what we refer to as synergistic effects, i.e., the return to participating in an activity can be <u>positively</u> related to the total number of agents who participate in that activity. A real world situation which exhibits synergistic effects is the problem faced by consumers in choosing a computer hardware system. This choice problem exhibits synergistic effects in that the larger the number of individuals who purchase a particular system, the greater will be the subsequent availability of computer peripherals and software for that system. The reason we do our analysis in the context of the above described model is because of the model's generality. That is, as we demonstrate towards the end of the paper, many more common models are actually special cases of the model we analyze.

Our analysis of the above described model yields the following findings. The first is that only under very special conditions do standard rational expectations and aggregate rational expectations yield equivalent results. That is, as indicated earlier, it is not the case that an aggregate rational expectations world can in general be accurately modeled using a standard rational expectations assumption. The remaining findings concern environments where the standard and aggregate rational expectations equilibria systematically differ. These findings indicate that the difference between the two equilibria is larger when: (i) the divergence in expectations under aggregate rational expectations is large; (ii) in a congestion effects world the severity of the congestion effects is decreased; (iii) in a synergistic effects world the severity of the synergistic effects is increased; and (iv) the activities exhibit synergistic rather than congestion effects.

The outline for the paper is as follows. Section II sets forth our model wherein agents choose between two activities. Section III analyzes the model, where special attention is paid to the factors which affect the size of the difference between standard and aggregate rational expectations equilibria. Section IV presents two special cases of the general model analyzed. In particular, a model where firms choose an output sector is shown to be a special case of the model analyzed, as is a variant of the macroeconomic model contained in Diamond (1982). In addition to showing the general applicability of the model, these examples demonstrate a number of real world implications of our analysis. Section V presents concluding remarks.

II. The Model

In our model each agent must choose between two activities, where this choice is made prior to the realization of the returns to participation. We denote these activities as activity X and activity Y. It is assumed this choice is an irreversible choice, and that it is made simultaneously by all the agents in the population. If agent i chooses activity X, then his utility equals $f_X(N_X) - r_i$. Similarly, if agent i chooses activity Y, then his utility equals $f_Y(N_Y) - (R-r_i)$.[4] $N_X(N_Y)$ denotes the total number of agents who choose activity X(Y), while f'_X and f'_Y can each be either positive (synergistic effects), or negative (congestion effects). Note, r_i and $(R-r_i)$ can be interpreted as representing either agent i's underlying preferences for the two activities, or the costs incurred by agent i in participating. This latter interpretation would be the appropriate one if one were to think of the model as a stylized model of commuting. That is, when thought of as a model of commuting, r_i can be interpreted as the distance between agent i's housing location and the entry ramp to route X (activity X), while $(R-r_i)$ can be interpreted as the distance between the housing location and the entry ramp to route Y (activity Y).

The population consists of a continuum of agents who vary in terms of their values for r_i. In particular, the distribution of r_i's in the population is described by a density function $g(.)$, where $g(.)$ is continuously differentiable and positive in the interval $[0,R]$, and equals zero elsewhere. We also assume $f_X(0) > f_Y(\int_0^R g(r_i)dr_i) - R$ and $f_Y(0) > f_X(\int_0^R g(r_i)dr_i) - R$. This pair of assumptions guarantees that, under both standard and aggregate rational expectations, the equilibrium is necessarily characterized by some agents participating in each activity.

The only aspect of the model remaining to be specified concerns expectations. Expectations are relevant in that, prior to choosing an activity, each agent forms expectations concerning the resulting value for $f_X(N_X) - f_Y(N_Y)$. Recall that our goal is to compare the nature of equilibrium under a standard rational expectations assumption and under an aggregate rational expectations assumption. Standard rational expectations means that each agent taken individually has correct expectations concerning the resulting value for $f_X(N_X) - f_Y(N_Y)$.[5] In contrast, under aggregate rational expectations each agent i's expectation concerning $f_X(N_X) - f_Y(N_Y)$ equals $f_X(N_X) - f_Y(N_Y) + h_i$, where the distribution of h_i's in the population satisfies an aggregate unbiasedness condition. Formally, the distribution of h_i's is described by a density function $k(.)$, where $k(.)$ is continuously differentiable and positive in the interval $[-H,H]$, and equals zero elsewhere. Aggregate unbiasedness translates into the assumption $\int_{-H}^{H} h_i k(h_i)dh_i = 0$. It is also assumed that the values for r_i and h_i are independently distributed in the population, and that $R > H$. The last assumption simply states that individual biases are small relative to the variation of participation cost levels in the population. The purpose of the assumption is to reduce the number of cases which need to be analyzed.

We can now derive a few preliminary results. Let $N_X^S(N_Y^S)$ denote the number of agents who participate in activity X(Y) under standard rational expectations, and let $D^S = (f_X(N_X^S) - f_Y(N_Y^S) + R)/2$. Given standard rational expectations, agent i will choose to participate in activity X(Y) when[6]

(1) $$r_i <(>) D^S.$$

Equation (1), in turn, yields

(2a) $$N_X^S = \int_0^{D^S} g(r_i)dr_i$$

and

(2b) $$N_Y^S = \int_{D^S}^{R} g(r_i)dr_i.$$

Let $N_X^A(N_Y^A)$ denote the number of agents who participate in activity X(Y) under aggregate rational expectations, and let $D^A = (f_X(N_X^A) - f_Y(N_Y^A) + R)/2$. Given aggregate rational expectations, agent i will choose to participate in activity X(Y) when

(3) $$r_i <(>) D^A + \frac{h_i}{2} .$$

Equation (3) implies that if $h_i > 2(R-D^A)$, then agent i will participate in activity X regardless of his value for r_i. On the other hand, if $h_i < -2D^A$, then agent i will participate in activity Y regardless of his value for r_i. In turn, these two facts combined with (3) yield

(4a) $$N_X^A = \frac{1}{\bar{N}} [\int_{V(D^A)}^{T(D^A)} \int_0^{D^A+\frac{h_i}{2}} g(r_i)dr_i k(h_i)dh_i + \int_{T(D^A)}^{H} \int_0^R g(r_i)dr_i k(h_i)dh_i]$$

and

(4b) $$N_Y^A = \frac{1}{\bar{N}} [\int_{V(D^A)}^{T(D^A)} \int_{D^A+\frac{h_i}{2}}^{R} g(r_i)dr_i k(h_i)dh_i + \int_{-H}^{V(D^A)} \int_0^R g(r_i)dr_i k(h_i)dh_i],$$

where $V(D^A) = \max\{-H,-2D^A\}$, $T(D^A) = \min\{H,2(R-D^A)\}$, and $\bar{N} = \int_0^R g(r_i)dr_i$ $(= \int_{-H}^{H} k(h_i)dh_i)$.[7]

Finally, two restrictions are placed on the model. Specifically, $[f'_X(\int_0^D g(r_i)dr_i) + f'_Y(\int_D^R g(r_i)dr_i)]\, g(D)/2 < 1$ for all $0 < D < R$, and $\frac{1}{2\bar{N}} \int_{V(D)}^{T(D)} [f'_X(\int_0^D g(r_i)dr_i) + f'_Y(\int_D^R g(r_i)dr_i)]\, g(D + \frac{h_i}{2})\ k(h_i)dh_i < 1$ for all $0 < D < R$. The first assumption eliminates the possibility of multiple equilibria under standard rational expectations, while the second eliminates the possibility under aggregate rational expectations. Note, these assumptions are only needed if one or more of the activities exhibit synergistic effects. That is, the congestion assumptions $f'_X < 0$ and $f'_Y < 0$ are in themselves sufficient to ensure uniqueness.

III. Analysis

In this section we analyze the model developed in the previous section. The analysis consists of four parts.[8] First, we compare standard and aggregate rational expectations in terms of the number of agents who participate in each activity. Second, we make the comparison in terms of social welfare. Third, we explore the ramifications of varying the divergence of expectations under aggregate rational expectations. Fourth, we look at the effects of varying the severity of the interaction among agents.

Our first proposition specifies conditions under which the number of agents who participate in each activity is independent of the type of expectations assumed. Note, all proofs are relegated to an Appendix.

Proposition 1: If $\frac{H}{2} < D^S < R - \frac{H}{2}$ and $g(.)$ is a uniform density function, then $N_X^S = N_X^A$ and $N_Y^S = N_Y^A$.

Proposition 1 states that, given two restrictions on the model, standard and aggregate rational expectations result in the same number of agents participating in each activity. The first restriction is that the critical value for r_i under standard rational expectations, i.e., $r_i = D^S$, is further than $\frac{H}{2}$ from the extreme values for r_i. The second restriction is that the distribution of r_i's in the population is described by a uniform density function. The next two propositions demonstrate that, if either of these restrictions is violated, then the number of agents who participate in each activity is no longer independent of the type of expectations assumed.

Proposition 2: If $D^S < \frac{H}{2}$ $(D^S > R - \frac{H}{2})$ and $g(.)$ is a uniform density function, then $N_X^S < N_X^A$ and $N_Y^S > N_Y^A$ $(N_X^S > N_X^A$ and $N_Y^S < N_Y^A)$.

Proposition 3: If $g'(r_i) > 0$ $(g'(r_i) < 0)$ for all $r_i \in [0,R]$ and $D^S < R - \frac{H}{2}$ $(D^S > \frac{H}{2})$, then $N_X^S < N_X^A$ and $N_Y^S > N_Y^A$ $(N_X^S > N_X^A$ and $N_Y^S < N_Y^A)$.

Propositions 2 and 3 demonstrate that standard and aggregate rational expectations do not in general yield equivalent results. That is, unless the model satisfies some fairly strong restrictions, the number of agents who participate in each activity will vary with the type of expectations assumed. The intuition behind these two propositions is as follows. Under aggregate rational expectations there are a set of agents who participate in activity X because, relative to the true returns to participating, they overvalue X and undervalue Y. We refer to these agents as the agents who incorrectly participate in X. Similarly, there are a set of agents who participate in activity Y because, relative to the true returns to participating, they overvalue Y and undervalue X. We refer to these agents as the agents who incorrectly participate in Y. If the number of agents who incorrectly participate in X equals the number of agents who incorrectly participate in Y, then the number of agents who participate in each activity will be independent of the type of expectations assumed. As a general rule, however, there is no guarantee that these two groups will be equal. On the one hand, there could be a truncation problem. This is what underlies Proposition 2. For example, suppose $D^S < \frac{H}{2}$. In this situation the number of agents who incorrectly participate in Y will be relatively small, because the range of participation cost levels from which these agents are drawn is truncated. On the other hand, there could be a weighting problem. This is what underlies Proposition 3. To understand this point suppose $g'(r_i) > 0$ for all $r_i \in [0,R]$. Given this restriction, the number of agents who incorrectly participate in X will tend to be larger than the number who incorrectly participate in Y. This is because the agents who incorrectly participate in X(Y) are drawn from agents with relatively high (low) values for r_i, while, given the restriction, high values for r_i are associated with larger weights than are low values.[9]

We now consider the social welfare aspects of our model. In doing so it is assumed that the externalities in our model are technological, as opposed to pecuniary.[10] Social welfare is defined as the sum of the utilities of all the agents in the population. Additionally, W^S denotes the social welfare which results under a standard rational expectations assumption, while W^A denotes what occurs under aggregate rational expectations.

Not surprisingly, standard and aggregate rational expectations are less likely to yield equivalent results regarding social welfare than they are regarding the number of agents participating in each activity. For example, even when the two assumptions yield equivalent results concerning the number of agents participating in each activity, social welfare is dependent on the type of expectations assumed. Proposition 4 demonstrates this formally.

Proposition 4: If $\frac{H}{2} < D^S < R - \frac{H}{2}$ and $g(.)$ is a uniform density function, then $W^S > W^A$.

The intuition for Proposition 4 is as follows. Under aggregate rational expectations individual mistakes are being made. Thus, even when the number of agents participating in each activity is independent of the type of expectations assumed, social welfare is lower under aggregate rational expectations because agents do not efficiently sort themselves among activities.

There is a second issue which is important when social welfare is considered. Specifically, social welfare can be affected by changes in the number of agents participating in each activity. The following propositions explore this issue. So that the reader can more easily understand the forces at work, in Propositions 5 and 6 we only allow one of the activities to exhibit an externality.

Proposition 5: If $f'_Y = 0$, $f'_X < 0$ and either i) or ii) holds, then $W^S > W^A$.

i) $D^S < \frac{H}{2}$ and $g(.)$ is a uniform density function;

ii) $g'(r_i) > 0$ for all $r_i \in [0,R]$ and $D^S < R - \frac{H}{2}$.

Proposition 6: If $f'_Y = 0$, $f'_X > 0$ and either i) or ii) holds, then $W^S > W^A$.

i) $D^S > R - \frac{H}{2}$ and $g(.)$ is a uniform density function;

ii) $g'(r_i) < 0$ for all $r_i \in [0,R]$ and $D^S > \frac{H}{2}$.

Consider first Proposition 5. There are two forces at work. As before, W^A tends to be less than W^S because under aggregate rational expectations agents do not sort themselves in an efficient manner. The second force works through the number of agents participating in each activity. When activity X displays congestion effects and activity Y displays no externality, the standard rational expectations equilibrium has more agents choosing X than would be optimal from a social welfare point of view. Further, in Proposition 5 we restrict ourselves to situations where the aggregate equilibrium has an even higher number of agents participating in activity X. Thus, in these situations both forces work in the same direction, with the final result being $W^S > W^A$. In situations where the aggregate equilibrium has a lower number of agents participating in activity X, the two forces work in opposite directions and a comparison of social welfare measures would yield ambiguous results.

As for Proposition 6, the intuition is the same as for Proposition 5 except that with synergistic effects the standard rational expectations equilibrium has less agents participating in activity X than would be optimal from a social welfare point of view.

For the next step of our analysis we investigate the following issue. Consider a situation in which standard and aggregate rational expectations do not yield equivalent results. The question is, how is the size of the difference between the standard equilibrium and the aggregate equilibrium affected by an increase in the divergence in expectations under aggregate rational expectations? Proposition 7 addresses this issue.

Proposition 7: Let $j(h_i)$ be a density function defined on the interval $[-H,H]$ which is the result of a mean preserving spread of $k(h_i)$, and let $K(a) = \int_{-H}^{a} k(h_i)dh_i$ and $J(a) = \int_{-H}^{a} j(h_i)dh_i$. Also, suppose $K(0) = J(0)$ and $K(a) \neq J(a)$ for all $a \neq 0$ and $-H < a < H$. If i), ii) or iii) holds, then this mean preserving spread causes $|N_X^S - N_X^A|$ and $|N_Y^S - N_Y^A|$ to increase.

i) $g(.)$ is a uniform density function and either $D^S < \frac{H}{2}$ or $D^S > R - \frac{H}{2}$;

ii) $g'(r_i) > 0$ for all $r_i \in [0,R]$ and $D^S < R - \frac{H}{2}$;

iii) $g'(r_i) < 0$ for all $r_i \in [0,R]$ and $D^S > \frac{H}{2}$.

Proposition 7 demonstrates that, given an environment where the standard and aggregate equilibria systematically differ, the size of the difference is positively related to the divergence in expectations under aggregate rational expectations. That is, in terms of numbers participating, a mean preserving spread of the expectations distribution will drive the aggregate equilibrium away from the standard equilibrium as long as the mean preserving spread satisfies a particular single crossing property.

We next investigate the following issue. Again consider a situation in which standard and aggregate rational expectations do not yield equivalent results. The question is, how is the size of the difference between the standard equilibrium and the aggregate equilibrium affected by an increase in the severity of the interaction among agents? Consider a world in which both activities exhibit congestion (synergistic) effects. To answer the proposed question we below define what we refer to as a normalized increasing congestion (synergistic) transformation of $f_X(\cdot)$ and $f_Y(\cdot)$. Specifically, $\hat{f}_X(\cdot)$ and $\hat{f}_Y(\cdot)$ are a normalized increasing congestion (synergistic) transformation of $f_X(\cdot)$ and $f_Y(\cdot)$ if the following two conditions are satisfied. First, $\hat{f}'_X(z) < (>) f'_X(z)$ and $\hat{f}'_Y(\bar{N}-z) < (>) f'_Y(\bar{N}-z)$ for all $0 < z < \bar{N}$, where in addition for each z in the specified range at least one must hold as a strict inequality. Second, N_X^S and N_Y^S must be independent of whether the expected returns to participating are given by $f_X(\cdot)$ and $f_Y(\cdot)$, or are given by $\hat{f}_X(\cdot)$ and $\hat{f}_Y(\cdot)$. In other words, a normalized increasing congestion (synergistic) transformation is one which increases the severity of the congestion (synergistic) effects, but, in terms of numbers participating in each activity, leaves the standard rational expectations equilibrium undisturbed.[11] We now proceed to the propositions.

<u>Proposition 8</u>: If $f'_X < 0$, $f'_Y < 0$, and i), ii) or iii) of Proposition 7 holds, then a normalized increasing congestion transformation of $f_X(\cdot)$ and $f_Y(\cdot)$ causes $|N_X^S - N_X^A|$ and $|N_Y^S - N_Y^A|$ to decrease.

Proposition 9: If $f'_X > 0$, $f'_Y > 0$, and i), ii) or iii) of Proposition 7 holds, then a normalized increasing synergistic transformation of $f_X(.)$ and $f_Y(.)$ causes $|N_X^S - N_X^A|$ and $|N_Y^S - N_Y^A|$ to increase.

Proposition 8 considers an environment where both activities exhibit congestion effects. It shows that if standard and aggregate rational expectations equilibria systematically differ, then the size of the difference is negatively related to the severity of the congestion effects. That is, if in such an environment one were to increase the severity of the congestion effects, but at the same time leave the standard rational expectations equilibrium undisturbed, then in terms of numbers participating the aggregate equilibrium would be driven towards the standard equilibrium.

On the other hand, Proposition 9 considers an environment which exhibits synergistic effects. It shows that if standard and aggregate rational expectations equilibria systematically differ, then the size of the difference is positively related to the severity of the synergistic effects. That is, if in such an environment one were to increase the severity of the synergistic effects, but at the same time leave the standard rational expectations equilibrium undisturbed, then in terms of numbers participating the aggregate equilibrium would be driven away from the standard equilibrium.

One might at first think that Propositions 8 and 9 are incompatible. This is because Proposition 8 states that by increasing the severity of the interaction among agents the aggregate equilibrium is driven towards the standard equilibrium, while Proposition 9 states that increasing the severity of the interaction drives the aggregate equilibrium away from the standard equilibrium. In actuality, however, the two propositions are quite compatible. They both state that by transforming the interaction such that it is more positive, i.e., increasing both f'_X and f'_Y, then the aggregate equilibrium is driven away from the standard equilibrium.

In the following we end this section by presenting a proposition which follows directly from Propositions 8 and 9.

Proposition 10: Consider a congestion (synergistic) effects world. If i), ii) or iii) of Proposition 7 holds, then a transformation of $f_X(\cdot)$ and $f_Y(\cdot)$ which changes the world into a synergistic (congestion) effects world, but leaves N_X^S and N_Y^S undisturbed, causes $|N_X^S - N_X^A|$ and $|N_Y^S - N_Y^A|$ to increase (decrease).

Proposition 10 tells us that, given an environment in which standard and aggregate rational expectations equilibria systematically differ, the size of the difference is larger when there are synergistic effects than when there are congestion effects.

IV. Applications

The above analysis is conducted in the context of a general model for which many more common models are special cases. In this section we consider two such special cases. This serves the purpose of both demonstrating the general applicability of the above model, and yielding additional insights concerning the economic implications of the analysis.

Application 1: Choice of an Output Sector

The first application we consider is the choice of output sector by firms. While this has many possible interpretations itself, we will focus on the choice by agricultural enterprises of what to produce.

Consistent with the above analysis, we assume there are a continuum of risk neutral farms. Each farm must decide how much of its acreage to put into corn production and how much to put into wheat production. For the time period under consideration each farm makes an irreversible choice concerning this decision, and all farms make this choice simultaneously and prior to the realization of the prices of corn and wheat. Further, units of measurement are normalized such that if farm i puts a proportion v of its land into corn (wheat) production, then exactly v units of corn (wheat) are produced.

Each farm is a price taker due to the fact that there are a continuum of them. If farm i were to put a proportion v of its land into corn production and $(1-v)$ into wheat production, then profits would be given by $(P_c-m_c-r_i)v + (P_w-m_w-R+r_i)(1-v)$. $P_c(P_w)$ denotes the price of corn (wheat), while $m_c-r_i(m_w-R+r_i)$ is the constant marginal cost of producing corn (wheat). The term r_i reflects heterogeneity across farms in terms of comparative advantage in wheat versus corn production. That is, farms with a relatively low marginal cost of producing corn have by construction a relatively high marginal cost of producing wheat. The distribution of r_i's across farms is described by a density function $g(.)$ which has the same properties as the analogous density function in the general analysis above.

We denote the total production of corn and wheat as Q_c and Q_w. The prices of corn and wheat are given by the demand equations, i.e.,

(5) $$P_c = f_c(Q_c), \quad f_c' < 0,$$

and

(6) $$P_w = f_w(Q_w), \quad f_w' < 0.$$

Demand and cost conditions are assumed to be such that, first, all farms produce, and second, some of both corn and wheat is produced. In turn, the assumption of risk neutrality ensures that each farm will specialize in production, i.e., each farm will either put all of its land into corn production or all of its land into wheat production.

As in the general analysis, there are two assumptions concerning expectations. The standard rational expectations assumption is that each farm taken individually has correct expectations concerning the resulting value for $P_c - P_w$. In contrast, under aggregate rational expectations each agent i's expectation concerning $P_c - P_w$ equals $P_c - P_w + h_i$, where the distribution of h_i's in the population has the same properties as the analogous distribution in the general analysis above.

Let $P_c^S(P_w^S)$ denote the price for corn (wheat) which holds under standard rational expectations, and $P_c^A(P_w^A)$ denote the price for corn (wheat) which holds under aggregate rational expectations. Given standard rational expectations, farm i will choose to produce corn (wheat) when

(7) $$r_i <(>) D^S,$$

where $D^S = (P_c^S - P_w^S - m_c + R + m_w)/2$. Similarly, given aggregate rational expectations, farm i will choose to produce corn (wheat) when

(8) $$r_i <(>) D^A + \frac{h_i}{2},$$

where $D^A = (P_c^A - P_w^A - m_c + R + m_w)/2$. Given this specification, it should be obvious that Q_c^S, Q_w^S, Q_c^A and Q_w^A are determined by equations similar to (2) and (4) above.

There is a one-to-one correspondence between the current model and the one found in the general analysis above. To indicate the implications for the current analysis, we provide a series of corollaries that follow immediately from the propositions in the general analysis.

Corollary 1: Standard and aggregate rational expectations equilibria are only guaranteed to have the same prices and quantities if $g(.)$ is a uniform density function, and $\frac{H}{2} < D^S < R - \frac{H}{2}$.

The first corollary indicates that standard and aggregate rational expectations equilibria will not in general be the same. As earlier, differences can be attributed to either a truncation problem or a weighting problem. Corollaries analogous to Propositions 2 and 3 are immediately derivable, but for the sake of brevity are omitted. Of more interest are the following corollaries which follow from Propositions 7 and 8.[12]

<u>Corollary 2</u>: Given a situation where the two equilibria are systematically different, an increase in the dispersion of expectations will increase $|Q_c^S - Q_c^A|$, $|Q_w^S - Q_w^A|$, $|P_c^S - P_c^A|$, and $|P_w^S - P_w^A|$. That is, in terms of both prices and quantities, an increase in the dispersion of expectations causes the aggregate rational expectations equilibrium to diverge from the standard rational expectations equilibrium.

<u>Corollary 3</u>: Given a situation where the two equilibria are systematically different, the more inelastic the demand for corn and/or wheat, the smaller are $|Q_c^S - Q_c^A|$ and $|Q_w^S - Q_w^A|$, and the greater are $|P_c^S - P_c^A|$ and $|P_w^S - P_w^A|$. That is, the more inelastic the demand for the products, the greater is the difference between the equilibria in terms of prices, and the smaller is the difference in terms of quantities.

These two corollaries yield a number of insights concerning the implications of aggregate rational expectations within a typical market context. First, dispersion of expectations is seen to be a potentially important determinant of the difference between prices and quantities under aggregate relative to standard rational expectations. Second, price elasticities of demand are seen to be important for whether the impact of changes in the dispersion of expectations will be greater on prices or quantities. That is, Corollaries 2 and 3 suggest that, the more inelastic the product demands under consideration, the more will an increase in the dispersion of expectations be reflected in prices rather than quantities.

There is one insight from this application that is not readily apparent from the general analysis. This concerns the "trade-off" between prices and quantities with respect to the impact of aggregate rational expectations. That is, suppose a standard rational expectations assumption is employed to model a market situation similar to that considered here, when in fact the situation under consideration is better characterized by aggregate rational expectations. This analysis suggests that if product demands are inelastic, then the standard rational expectations assumption will yield relatively good predictions concerning quantities. However, inelastic product demands means that if a price differential exists between standard and aggregate rational expectations, then it will be relatively large. Hence, while the model may yield reasonably accurate predictions for quantity behavior, the predictions for price behavior are likely to be much less accurate.

Application 2: Trading Externalities and Aggregate Output

The previous application involved a typical market situation which is characterized by what we have referred to as congestion effects. In this application we consider a situation characterized by synergistic effects. Recent theoretical research in macroeconomics has investigated the consequences of trading externalities for the determination of aggregate output and employment. In particular, Diamond (1982) and Howitt (1985) consider the idea that in a many agent economy there are trade frictions making coordination of trade difficult. They suggest the existence of a trading externality due to the notion that an increase in the number of trading partners makes trade easier. In our terminology this type of trading externality is a synergistic effect. In this section we develop a simple model with this type of trading externality, and investigate the ramifications of aggregate rational expectations in this context.

The model we use is an adaptation of the static version of Diamond's model (see Diamond (1982), pp. 886-87). Workers and firms are not distinguished. Rather, each agent i must decide whether or not to undertake a production project which produces y units of output, and which costs an amount r_i. This heterogeneity in costs can be thought of as either heterogeneity in "reservation wages," or just variance in costs across projects. The latter interpretation simply means that, prior to deciding whether or not to produce, each agent i draws a production project from the distribution of projects. The distribution of r_i's in the population is described by a density function $g(.)$ which has the same properties as the analogous density function in the general analysis above. Note, further, the output y and the costs r_i are denominated in the same units.

The key restriction on behavior is that each individual cannot consume what he himself produces, but must rather trade his own output for that which is produced by others. This assumption reflects the advantage that specialized production and trade have over self-sufficiency. Let Y be the aggregate output level. The probability of making a trade is given by $p(Y)$, where the assumption $p' > 0$ captures the trading externality. Untraded output is assumed to be wasted. Further, agents must decide whether or not to produce prior to the realization of $p(Y)$, and it is over this probability that agents form expectations. Finally, agents are assumed to have standard rational expectations or aggregate rational expectations, where each type is specified in exactly the same manner as in the general analysis above.

Let Y^S denote aggregate output under standard rational expectations, and Y^A denote it under aggregate rational expectations. Agent i will undertake his production project if it has positive expected value. Given standard rational expectations, this implies agent i will (will not) undertake his production project when

(9) $$p(Y^S)y >(<) r_i.$$

Similarly, given aggregate rational expectations, agent i will (will not) undertake his production project when

(10) $$p(Y^A)y + h_i >(<) r_i.$$

In turn, the above implies that Y^S and Y^A are determined by equations similar to equations (2) and (4) found in the general analysis. Further, conditions similar to those specified in the general analysis ensure that a unique interior solution for aggregate output exists.[13]

The correspondence between the current model and the one found in the general analysis allows us to establish a series of corollaries that follow immediately from the propositions in the general analysis. The following corollary follows immediately from Proposition 1.

Corollary 4: Standard and aggregate rational expectations equilibria are only guaranteed to have the same aggregate output if $g(.)$ is a uniform density function, and $H \leq p(Y^S)y \leq R - H$.

Corollary 4 indicates that standard and aggregate rational expectations equilibria will not in general be the same. As earlier, differences can be attributed to either a truncation problem or a weighting problem. The following two corollaries follow immediately from Propositions 7 and 9.

Corollary 5: Given a situation where the two equilibria are systematically different, an increase in the dispersion of expectations will result in an increase in $|Y^S - Y^A|$.

<u>Corollary 6</u>: Given a situation where the two equilibria are systematically different, an increase in the severity of the trading externality will result in an increase in $|Y^S - Y^A|$.

Corollaries 5 and 6 indicate some of the factors which can affect the size of the aggregate output difference between standard and aggregate rational expectations equilibria. Of particular interest is the result concerning dispersion of expectations. A number of empirical studies have found that dispersion of expectations concerning inflation is an important factor in the determination of aggregate output.[14] In particular, increases in the dispersion of expectations have in general been found to lead to decreases in aggregate output. Although the expectations here do not concern inflation, Corollary 5 states that in our model an increase in the dispersion of expectations can yield a similar result. That is, even constraining expectations to be correct in the aggregate, our analysis also states that an increase in the dispersion of expectations can depress aggregate output.[15] This suggests that theoretical work employing an aggregate rational expectations assumption may prove fruitful in the explanation of this empirical observation.

One final comment concerns the relationship between the results derived here and those of Diamond and Howitt. Diamond and Howitt deal solely with a standard rational expectations assumption. Further, in order for their model to be consistent with fluctuations in aggregate output, they assume the trading externality is sufficiently severe that multiple equilibria exist. Our analysis suggests that the existence of multiple equilibria may not be necessary for trading externalities to be important for explaining fluctuations in aggregate output. Rather, under an aggregate rational expectations assumption, changes in the dispersion of expectations yield interesting fluctuations in aggregate output even when multiple equilibria are ruled out.

V. Conclusion

In practice rational expectations has typically meant that the expectation of each agent taken separately is by itself consistent with the predictions of the relevant economic theory, i.e., what we refer to as standard rational expectations. This differs, however, from the argument frequently put forth by proponents of the rational expectations hypothesis to justify its use. That argument is that on an aggregate level it would be surprising if expectations were inconsistent with the predictions of the relevant theory. The employment of the stronger assumption of standard rational expectations is then justified by the belief that, if expectations were rational in the aggregate, then expectational deviations across agents would tend to cancel out. In this paper we have conducted a formal investigation of the relationship between standard rational expectations and what we refer to as aggregate rational expectations. Our goal was twofold. First, we wanted to show that the above argument frequently used to justify a standard rational expectations assumption is incorrect. That is, expectational deviations across agents do not always cancel out, and thus standard and aggregate rational expectations equilibria may be quite different. Second, we wanted to investigate the question which our first conclusion immediately brings to mind. That is, given an environment where the standard and aggregate rational expectations equilibria differ, what factors tend to affect the size of this difference?

This paper explored the above issue by analyzing a model wherein agents decide which of two activities to participate in. Complicating the model is the assumption that, for each agent i, the return to participating in an activity depends on the total number of agents who choose to participate in that activity. The reason we conduct our analysis in the context of this model is the model's generality. That is, as we demonstrated in Section IV, many more common models are actually special cases of the model analyzed. Our analysis yielded five major findings. First, only under very special conditions do standard rational expectations and aggregate rational expectations yield equivalent results. Second, given an environment where the standard and aggregate rational expectations equilibria systematically differ, the size of the difference tends to be positively related to the divergence in expectations under aggregate rational expectations. Third, if in a congestion effects world conditions are such that the standard and aggregate equilibria systematically differ, then an increase in the severity of the congestion effects tends to decrease the magnitude of the difference. Fourth, if in a synergistic effects world conditions are such that the standard and aggregate equilibria systematically differ, then an increase in the severity of the synergistic effects tends to increase the magnitude of the difference. Fifth, again given an environment where the standard and aggregate equilibria systematically differ, the size of the difference tends to be larger under synergistic effects than under congestion effects. Finally, as indicated earlier a number of special cases were analyzed. These examples served to demonstrate both the general applicability of the model, as well as a number of real world implications of our analysis.

One might ask what conclusions can be drawn from the above results regarding the common practice of employing a standard rational expectations assumption. To gain perspective on this issue it is helpful to review briefly the related research reported in Haltiwanger and Waldman (1985). In that paper we used a model similar to that analyzed in this paper but considered what happens when the population consists in part of a set of agents who satisfy a standard rational expectations assumption, and in part of a set of agents all of whom have the same incorrect expectations. The analysis yielded two major results. First, in a world characterized by congestion effects, the agents who satisfy standard rational expectations tend to have a disproportionately large effect on equilibrium. Second, in a world characterized by synergistic effects, the agents with incorrect expectations tend to have a disproportionately large effect on equilibrium. Overall, then, both papers suggest that for situations characterized by congestion effects there are relatively strong justifications for making the standard rational expectations assumption. However, for situations characterized by synergistic effects the employment of a standard rational expectations assumption would seem to be less defensible.

Appendix

The proofs in this Appendix are abbreviated versions of the more detailed proofs found in our working paper (Haltiwanger and Waldman, 1984). Before proceeding to the proofs we define a term denoted $F(Z)$, i.e.,

$$\text{(1A)} \qquad F(Z) = Z - \frac{1}{\bar{N}}\left[\int_{V(D)}^{T(D)} \int_{0}^{D+\frac{h_i}{2}} g(r_i)dr_i k(h_i)dh_i + \int_{T(D)}^{H} \int_{0}^{R} g(r_i)dr_i k(h_i)dh_i\right],$$

where $D = (f_X(Z) - f_Y(\bar{N}-Z) + R)/2$. The assumptions at the end of Section II used to eliminate the possibility of multiple equilibria yield $F' > 0$, and in turn

$$\text{(2A)} \qquad F(Z) \gtreqless 0 \quad \text{if} \quad Z \gtreqless N_X^A.$$

We can now proceed to the proofs.

<u>Proof of Proposition 1</u>: Suppose $N_X^S < N_X^A$. Given equation (2A), this yields $F(N_X^S) < 0$. Now, the assumption that $g(.)$ is uniform means that $g(r_i) = \bar{N}/R$ for $r_i \in [0,R]$. Given (1A), $\frac{H}{2} < D^S < R - \frac{H}{2}$, $\int_{-H}^{H} k(h_i)dh_i = \bar{N}$ and $\int_{-H}^{H} h_i k(h_i)dh_i = 0$, this in turn implies

$$\text{(3A)} \qquad F(N_X^S) = N_X^S - \frac{\bar{N}}{R} D^S.$$

Given (2A), (3A) implies $F(N_X^S) = 0$, i.e., a contradiction. Thus, $N_X^S \not< N_X^A$. It can similarly be shown that $N_X^S \not> N_X^A$; therefore it must be that $N_X^S = N_X^A$ and $N_Y^S = N_Y^A$.

<u>Proof of Proposition 2</u>: Consider the case $D^S < \frac{H}{2}$. Now suppose $N_X^S > N_X^A$. This implies $F(N_X^S) > 0$. Further, the assumption that $g(.)$ is uniform

combined with $D^S < \frac{H}{2}$, $\int_{-H}^{H} k(h_i)dh_i = \bar{N}$ and $\int_{-H}^{H} h_i k(h_i)dh_i = 0$ yields:

$$\text{(4A)} \qquad F(N_X^S) = \frac{1}{R} \int_{-H}^{-2D^S} (D^S + \frac{h_i}{2})\, k(h_i)dh_i.$$

Equation (4A) implies $F(N_X^S) < 0$, i.e., a contradiction. This yields that if $g(.)$ is a uniform density function and $D^S < \frac{H}{2}$, then $N_X^S < N_X^A$ and $N_Y^S > N_Y^A$.

The remainder of the proposition follows from the symmetry of the model.

<u>Proof of Proposition 3</u>: Consider the case $g'(r_i) > 0$ for all $r_i \in [0,R]$ and $D^S < R - \frac{H}{2}$. Now suppose $N_X^S > N_X^A$. This implies $F(N_X^S) > 0$. Further, $D^S < R - \frac{H}{2}$ implies

$$\text{(5A)} \qquad F(N_X^S) = N_X^S - [\frac{1}{\bar{N}} \int_{V(D^S)}^{H} \int_{0}^{D^S + \frac{h_i}{2}} g(r_i)dr_i\, k(h_i)dh_i].$$

Given that $\int_{-H}^{H} k(h_i)dh_i = \bar{N}$, we know $N_X^S = \frac{1}{\bar{N}} [\int_{-H}^{H} \int_{0}^{D^S} g(r_i)dr_i k(h_i)dh_i]$.
Given (5A) and $g'(r_i) > 0$, this implies

$$\text{(6A)} \qquad F(N_X^S) < -\frac{1}{\bar{N}} [g(D^S) \int_{V(D^S)}^{H} \frac{h_i}{2} k(h_i)dh_i].$$

Because $\int_{-H}^{H} h_i k(h_i)dh_i = 0$, (6A) implies $F(N_X^S) < 0$, i.e., a contradiction. Thus, if $g'(r_i) > 0$ for all $r_i \in [0,R]$ and $D^S < R - \frac{H}{2}$, then $N_X^S < N_X^A$ and $N_Y^S > N_Y^A$.

The remainder of the proposition follows from the symmetry of the model.

<u>Proof of Proposition 4</u>: Our restrictions immediately yield

$$\text{(7A)} \quad W^S = N_X^S f_X(N_X^S) + N_Y^S f_Y(N_Y^S) - \frac{\bar{N}}{R} [\int_{0}^{D^S} r_i dr_i + \int_{D^S}^{R} (R-r_i)dr_i],$$

and

(8A) $$W^A = N_X^A f_X(N_X^A) + N_Y^A f_Y(N_Y^A) - \frac{1}{R}\Big[\int_{V(D^A)}^{T(D^A)} \int_0^{D^A + \frac{h_i}{2}} r_i dr_i k(h_i)dh_i$$

$$+ \int_{T(D^A)}^{H} \int_0^R r_i dr_i k(h_i)dh_i + \int_{V(D^A)}^{T(D^A)} \int_{D^A + \frac{h_i}{2}}^{R} (R-r_i)dr_i k(h_i)dh_i$$

$$+ \int_{-H}^{V(D^A)} \int_0^R (R-r_i)dr_i k(h_i)dh_i\Big].$$

Given Proposition 1, (8A) reduces to:

(9A) $$W^A = W^S - \frac{1}{R}\Big[\int_{-H}^{H} (h_i(D^S - \frac{R}{2}) + \frac{h_i^2}{4})k(h_i)dh_i\Big].$$

Because $\int_{-H}^{H} h_i k(h_i)dh_i = 0$, we know $\int_{-H}^{H} (h_i(D^S - \frac{R}{2}) + \frac{h_i^2}{4})\, k(h_i)dh_i > 0$. Thus, (9A) yields $W^S > W^A$.

<u>Proof of Proposition 5</u>: Consider i). Given our restrictions, (7A) and (8A) are still correct. Let W^* be given by

(10A) $$W^* = N_X^A f_X(N_X^A) + N_Y^A f_Y(N_Y^A) - \frac{\bar{N}}{R}\Big[\int_0^{D^*} r_i dr_i + \int_{D^*}^{R} (R-r_i)dr_i\Big],$$

where D^* is such that $N_X^A = \frac{\bar{N}}{R}\int_0^{D^*} r_i dr_i$. Using arguments similar to those given in the proof of Proposition 4, it can be demonstrated that $W^* > W^A$. We therefore need only show that $W^S > W^*$.

Consider equation (11A)

(11A) $$\tilde{W} = \tilde{N}_X f_X(\tilde{N}_X) + \tilde{N}_Y f_Y(\tilde{N}_Y) - \frac{\bar{N}}{R}\Big[\int_0^{\tilde{D}} r_i dr_i + \int_{\tilde{D}}^{R} (R-r_i)dr_i\Big],$$

where $\tilde{N}_X = \frac{\bar{N}}{R}\int_0^{\tilde{D}} dr_i$ and $\tilde{N}_Y = \frac{\bar{N}}{R}\int_{\tilde{D}}^{R} dr_i$. From Proposition 2 we know $N_X^S < N_X^A$, which yields $D^S < D^*$. Hence, if $\frac{d\tilde{W}}{d\tilde{D}} < 0$ for all $\tilde{D} > D^S$, then $W^S > W^*$. $\frac{d\tilde{W}}{d\tilde{D}}$ is given by

$$\text{(12A)} \qquad \frac{d\tilde{W}}{d\tilde{D}} = \frac{\bar{N}}{R}\,[(f_X(\tilde{N}_X)-\tilde{D}) - (f_Y(\tilde{N}_Y) - (R-\tilde{D})) + \tilde{N}_X f_X'(\tilde{N}_X) - \tilde{N}_Y f_Y'(\tilde{N}_Y)].$$

By definition, $f_X(N_X^S) - D^S = f_Y(N_Y^S) - (R-D^S)$. This combined with the fact that $f_X' < 0$ and $f_Y' = 0$ yields that at $\tilde{D} = D^S$, $\frac{d\tilde{W}}{d\tilde{D}} < 0$. This, in turn, yields that for any $\tilde{D} > D^S$, $\frac{d\tilde{W}}{d\tilde{D}} < 0$. Thus, $W^S > \tilde{W}$, which implies $W^S > W^A$.

The proof for ii) follows along similar lines.

<u>Proof of Proposition 6</u>: Given the restriction which eliminates the possibility of multiple equilibria under standard rational expectations, the proof of Proposition 6 follows along the same arguments as given for Proposition 5.

<u>Proof of Proposition 7</u>: Following Rothschild and Stiglitz (1970), $j(h_i)$ being the result of a mean preserving spread of $k(h_i)$ means $\int_{-H}^{y} (J(x) - K(x))dx > 0$ for all $-H < y < H$ and $\int_{-H}^{H} (J(x) - K(x))dx = 0$. Combining this with the assumption $K(0) = J(0)$ and $K(a) \neq J(a)$ for all $a \neq 0$ and $-H < a < H$ yields

$$\text{(13A)} \quad \int_0^a (k(h_i) - j(h_i))dh_i > 0 \quad \text{for all } 0 < a < H,$$

and

$$\text{(14A)} \quad \int_a^0 (k(h_i) - j(h_i))dh_i > 0 \quad \text{for all } -H < a < 0.$$

In (15A) we define $F_j(Z)$, i.e.,

$$\text{(15A)} \quad F_j(Z) = Z - \frac{1}{\bar{N}}\,[\int_{V(D)}^{T(D)}\int_0^{D+\frac{h_i}{2}} g(r_i)dr_i j(h_i)dh_i + \int_{T(D)}^{H}\int_0^{R} g(r_i)dr_i j(h_i)dh_i].$$

Further, let $N^A_{X,j}$ and $N^A_{Y,j}$ be such that $F_j(N^A_{X,j}) = 0$ and $N^A_{Y,j} = \bar{N} - N^A_{X,j}$. We can demonstrate the proposition by showing that, given i), ii) or iii), then $|N^S_X - N^A_X| < |N^S_X - N^A_{X,j}|$ and $|N^S_Y - N^A_Y| < |N^S_Y - N^A_{Y,j}|$.

First consider i) under the additional assumption $D^S < \frac{H}{2}$. Proposition 2 yields $N^S_X < N^A_X$ and $N^S_X < N^A_{X,j}$. Thus, we need only prove $N^A_X < N^A_{X,j}$. Using the restrictions on $K(.)$ and $J(.)$, $F_j(N^A_X) - F(N^A_X)$ can be written as

$$(16A) \quad F_j(N^A_X) - F(N^A_X) = \frac{1}{R}\left[\int_{2D^A}^{D^A + \frac{H}{2}} \int_{-H}^{2(D^A - r_i)} (k(h_i) - j(h_i))dh_i dr_i\right].$$

Given (14A) and $K(0) = J(0)$, (16A) implies $F_j(N^A_X) < F(N^A_X)$. Since $F(N^A_X) = 0$, this implies $F_j(N^A_X) < 0$. Since $F_j(N^A_{X,j}) = 0$ and $F'_j > 0$, this in turn yields $N^A_X < N^A_{X,j}$. Thus, given i) under the additional assumption $D^S < \frac{H}{2}$, we know that $|N^S_X - N^A_X| < |N^S_X - N^A_{X,j}|$ and $|N^S_Y - N^A_Y| < |N^S_Y - N^A_{Y,j}|$.

Now consider ii). Proposition 3 yields $N^S_X < N^A_X$ and $N^S_X < N^A_{X,j}$. Thus, we need only prove $N^A_X < N^A_{X,j}$. Using the restrictions on $K(.)$ and $J(.)$ and $g'(r_i) > 0$ yields in this case:

$$(17A) \qquad F_j(N^A_X) - F(N^A_X) = \frac{g(D^A)}{\bar{N}}\left[\int_{2D^A}^{D^A + \frac{H}{2}} \int_{-H}^{2(D^A - r_i)} (k(h_i) - j(h_i))dh_i dr_i\right]$$

(17A) is sufficiently similar to (15A) that the proof for ii) now follows from the same logic used to prove i) above.

The remainder of the proposition follows from the symmetry of the model.

<u>Proof of Proposition 8</u>: Let $\hat{f}_X(.)$ and $\hat{f}_Y(.)$ be an arbitrary normalized increasing congestion transformation of $f_X(.)$ and $f_Y(.)$, and let $\hat{N}^S_X$, $\hat{N}^S_Y$, $\hat{D}^C$, $\hat{N}^A_X$, $\hat{N}^A_Y$, $\hat{D}^A$ be the transformation values for N^S_X, N^S_Y, D^S, N^A_X, N^A_Y, D^A. We can prove the proposition by demonstrating that, given i), ii) or iii), then

$|\hat{N}_X^S - \hat{N}_X^A| < |N_X^S - N_X^A|$ and $|\hat{N}_Y^S - \hat{N}_Y^A| < |N_Y^S - N_Y^A|$.

Consider either i) under the additional assumption $D^S < \frac{H}{2}$ or ii). Either Proposition 2 or Proposition 3 yields $N_X^S < N_X^A$. By definition we also know $\hat{N}_X^S = N_X^S$, which in turn yields $\hat{D}^S = D^S$ and by Proposition 2 or 3 that $\hat{N}_X^S < \hat{N}_X^A$. Thus, we need only prove $\hat{N}_X^A < N_X^A$. Suppose $\hat{N}_X^A > N_X^A$. Because of the definition of D^A and that $\hat{D}^S = D^S$, $\hat{N}_X^S = N_X^S$, and $\hat{N}_Y^S = N_Y^S$, the first property of a normalized increasing congestion transformation now yields $\hat{D}^A < D^A$. However, given equation (4a), this in turn yields $\hat{N}_X^A < N_X^A$, i.e., a contradiction. Thus, given either i) under the additional assumption $D^S < \frac{H}{2}$ or ii), we know that $|\hat{N}_X^S - \hat{N}_X^A| < |N_X^S - N_X^A|$ and $|\hat{N}_Y^S - \hat{N}_Y^A| < |N_Y^S - N_Y^A|$.

The remainder of the proposition follows from the symmetry of the model.

Proof of Proposition 9: Let $\hat{f}_X(.)$ and $\hat{f}_Y(.)$ be an arbitrary normalized increasing synergistic transformation of $f_X(.)$ and $f_Y(.)$, and let $\hat{N}_X^S$, $\hat{N}_Y^S$, $\hat{D}^S$, $\hat{N}_X^A$, $\hat{N}_Y^A$, $\hat{D}^A$ be the transformation values for N_X^S, N_Y^S, D^S, N_X^A, N_Y^A, D^A. We can prove the proposition by demonstrating that, given i), ii) or iii), then $|\hat{N}_X^S - \hat{N}_X^A| > |N_X^S - N_X^A|$ and $|\hat{N}_Y^S - \hat{N}_Y^A| > |N_Y^S - N_Y^A|$.

In equation (18A) we define $\hat{F}(Z)$.

$$\text{(18A)}\quad \hat{F}(Z) = Z - \frac{1}{\bar{N}}\left[\int_{V(\hat{D})}^{T(\hat{D})} \int_0^{\hat{D}+\frac{h_i}{2}} g(r_i)dr_i k(h_i)dh_i + \int_{T(\hat{D})}^{H} \int_0^{R} g(r_i)dr_i k(h_i)dh_i\right],$$

where $\hat{D} = (\hat{f}_X(Z) - \hat{f}_Y(\bar{N}-Z) + R)/2$. The restriction which eliminates the possibility of multiple equilibria under aggregate rational expectations (see footnote 10) yields $\hat{F}' > 0$, and, in turn,

$$\text{(19A)}\qquad \hat{F}(Z) \gtrless 0 \quad \text{for} \quad Z \gtrless \hat{N}_X^A.$$

By definition $\hat{N}_X^S = N_X^S$, which implies $\hat{D}^S = D^S$ and $\hat{F}(\hat{N}_X^S) = F(N_X^S)$. Combining these facts with the first property of a normalized increasing synergistic transformation yields

(20A) $$\hat{F}(Z) \lesseqgtr F(Z) \quad \text{for} \quad Z \gtreqless N_X^S.$$

Now consider either i) under the additional assumption $D^S < \frac{H}{2}$ or ii). Either Proposition 2 or Proposition 3 yields $N_X^S < N_X^A$. As noted previously, we also know $\hat{N}_X^S = N_X^S$ and $\hat{D}^S = D^S$, which by Proposition 2 or 3 yields $\hat{N}_X^S < \hat{N}_X^A$. Thus, we need only prove $\hat{N}_X^A > N_X^A$. Suppose $\hat{N}_X^A < N_X^A$. Given (20A) and $\hat{F}' > 0$, this implies $\hat{F}(\hat{N}_X^A) < F(N_X^A)$. This, however, implies a contradiction since by definition $\hat{F}(\hat{N}_X^A) = F(N_X^A) = 0$. Thus, given either i) under the additional assumption $D^S < \frac{H}{2}$ or ii), we know $|\hat{N}_X^S - \hat{N}_X^A| > |N_X^S - N_X^A|$ and $|\hat{N}_Y^S - \hat{N}_Y^A| > |N_Y^S - N_Y^A|$.

The remainder of the proposition follows from the symmetry of the model.

Footnotes

[1]Schultze questions the validity of Muth's claim for the analysis of environments with implicit contracts. From this perspective our analysis can partially be interpreted as saying that, even in the absence of implicit contracts, Muth's claim is incorrect.

[2]Some authors have addressed related issues. For example, Sharpe (1964) and Lintner (1969) contain formal analyses of asset markets which suggest that expectational deviations do tend to cancel out. More recently Jarrow (1980) and Mayshar (1983) have shown that, if there are restrictions on short sales, then expectational deviations may not cancel out. Note, however, none of the above authors specifically consider the question in a world where expectations are constrained to be rational in the aggregate.

[3]The basic model analyzed in this paper initially appeared in Haltiwanger and Waldman (1985). In that paper we considered what happens when the population consists in part of a set of agents who have rational expectations, and in part of a set of agents all of whom have the same incorrect expectations. Other papers which consider that type of heterogeneity include Conlisk (1980), Akerlof and Yellen (1985a,b), and Russell and Thaler (1985).

[4]It is assumed that there is no futures market on the returns to participating in each activity. For most of the real world examples that have been mentioned, the lack of an organized futures market is in accordance with empirical observation. That is, for commuting, career choice, purchase of a computer, and the macroeconomic application involving a trading externality (see Section IV, application 2), our assumption matches empirical observation. However, for the market application in Section IV involving agricultural enterprises, our assumption is less consistent with what is

observed. For a discussion of what factors determine the existence of futures markets and how the presence or absence of futures markets influence expectations formation, see Russell and Thaler (1985).

[5]In Haltiwanger and Waldman (1985) we show how the specification presented here can be derived from a world where stochastic elements affect the utility of participating in each activity. Thus, for the above specification standard rational expectations does not necessarily imply perfect foresight.

[6]In equation (1) we do not specify what happens when $r_i = D^C$, while in equation (3) we do not specify what happens when $r_i = D^A + \frac{h_i}{2}$. We can ignore each of these situations because each concerns a set of agents whose weight is zero.

[7]It is easily demonstrated that $V(D^A) < T(D^A)$.

[8]Our analysis derives properties which standard and aggregate rational expectations equilibria must display. In a mathematical supplement available from the authors upon request we demonstrate that an equilibrium exists and is unique under both standard and aggregate rational expectations.

[9]Proposition 3 demonstrates that standard and aggregate rational expectations yield different results when $g(.)$ is not uniform, and g' is always the same sign. In the mathematical supplement mentioned in footnote 8 we show that a similar conclusion follows when $g(.)$ is not uniform, but g' is not always of the same sign. The advantage of having g' always be of the same sign in Proposition 3 is that there is then a systematic difference between the two equilibria.

[10]See Scitovsky (1954) for a discussion of this distinction. Note, the first application in Section IV deals with a pecuniary externality, and thus the social welfare results derived here do no necessarily hold for that

application.

[11]For $\hat{f}_X(\cdot)$ and $\hat{f}_Y(\cdot)$ to be a normalized increasing synergistic transformation of $f_X(\cdot)$ and $f_Y(\cdot)$, we also require the following to be true: $[\hat{f}_X'(\int_0^D g(r_i)dr_i) + \hat{f}_Y'(\int_D^R g(r_i)dr_i)]\, g(D)/2 < 1$ for all $0 < D < R$, and $\frac{1}{2\bar{N}} \int_{V(D)}^{T(D)} [\hat{f}_X'(\int_0^D g(r_i)dr_i) + \hat{f}_Y'(\int_D^R g(r_i)dr_i)]\, g(D + \frac{h_i}{2})\, k(h_i)dh_i < 1$ for all $0 < D < R$.

[12]When we state that the two equilibria are systematically different, we mean that either i), ii), or iii) of Proposition 7 holds.

[13]That we restrict the severity of the trading externality so that a unique equilibrium exists represents a significant departure from Diamond and Howitt. Further discussion of this difference is provided below.

[14]See, for example, Mullineaux (1980) and Levi and Makin (1980). These empirical studies are an outgrowth of a discussion in Friedman (1977) concerning the influence of an increase in the dispersion of expectations on aggregate economic activity.

[15]This will be true if conditions are such that $Y^S > Y^A$. By Proposition 3, a reasonable condition that will yield this result is $g'(r_i) > 0$ for all r_i. This condition essentially requires that low cost projects be less likely than high cost projects.

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