

A COMPARISON OF MARKETS AND TOURNAMENTS

by

Bruce H. Kobayashi and Ben T. Yu

University of California, Los Angeles

UCLA Working Paper No. 330

June 1984

A COMPARISON OF MARKETS AND TOURNAMENTS

Bruce H. Kobayashi and Ben T. Yu

University of California, Los Angeles

Recently, literature on the incentives provided by competition in the market have emphasized that markets exhibit features similar to those of tournament contracts.¹ The results of these papers parallel those of the tournament literature. Specifically, with risk neutral agents facing perfectly correlated shocks, markets will yield the first-best allocation of resources.²

This paper examines the implicit rules of a market in the presence of perfectly correlated and uncorrelated uncertainties. Although competitive markets are similar to tournament contracts in the sense that competing agents are rewarded based upon relative performance, tournaments and markets differ in terms of the type of rewards offered. In tournaments, winning prizes are preset at levels higher than expected productivity; efforts influence the probability of winning, but do not affect the size of the prize. In contrast, competing agents in the market often anticipate higher winning prizes via expanding efforts. Thus, depending on how a competing agent formulates his objective function, his efforts may or may not correspond to those in a tournament.

This paper examines a particular market interaction that emphasizes reward by relative performance. Our analysis suggests that a large winning prize is not necessary to induce the first-best allocation of resources. However, markets and tournaments differ in terms of their reaction to large common uncertainties. While tournament contracts perform best in situations involving large common uncertainties, market interactions may break down in

exactly these situations.³

We concentrate on an environment where information is expected to be revealed sequentially. Competing agents exert their efforts based upon prior information that may or may not be perfectly correlated. After efforts are expended, additional information is obtained, and on the basis of the latter information, the winner will be selected. We study how prior information and the anticipation of the final evaluation criteria affect the decisions of competing agents.

The environment described above is particularly suitable for applying the concept of a tournament in analyzing the relative efficiency of private versus public provision of research and development. It is well known that one of the main reasons for public research and development is to alleviate the free-riding problem caused by costly enforcement of property rights.⁴ However, public research and development, like many public enterprises, is often marred by the problems of malincentives — the willingness of the government to subsidize research does not necessarily imply fruitful research. The tournament literature is suggesting an incentive mechanism to resolve this principal-agent problem — create a prize higher than expected productivity and let researchers compete. To the extent that incentives in such contracts are created by the force of competition, and since competition always exists in the market, one may ask: what is the difference between private and public provision of research and development? Wouldn't a more stringent enforcement of patent rights (thus relying on market competition) be equally effective? A study comparing markets and tournaments in this informational environment may shed some light on this issue.⁵

The organization of the paper will be as follows: Section II sets up a model of cost reducing innovation incorporating the feature of sequential

revelation of information. We identify the condition that would lead to evaluation based on final information. A market evaluation is assumed to take exactly this form. We argue that the condition most likely to result in evaluation by the market is one where competing agents share identical general information, but divergent ex-post information. Section III characterizes the first-best allocation of resources under the informational assumptions of Section II. Section VI establishes a tournament contract and examines its response to general information. It is shown that the tournament contract provides perfect flexibility in revealing general information. Section V models the decision of an individual firm in a competitive market setting. The main result of this section is that, although firms competing in a market interaction may have the correct marginal incentives, this equilibrium will break down in cases where perfectly correlated prior information is important relative to technique specific ex-post uncertainty.

II. The Model

We consider a situation where x identical individuals place a value of K in a unit of good q . For simplicity, we assume each individual consumes exactly one unit of the good. The demand conditions associated with this market are

$$TUV(q) = \begin{cases} Kq & q < x \\ Kx & q > x \end{cases} \quad (1)$$

$$MUV(q) = \begin{cases} K & q < x \\ 0 & q > x \end{cases}$$

This demand curve allows us to set aside losses due to output restriction and concentrate on the supply side of the market.

The supply side of the market is characterized by n ex-ante identical production techniques, each producing q according to

$$MC_1(q) = K - R(u_1, \theta_1, \varepsilon_1) \quad (2)$$

$$AC_1(q) = K - R(u_1, \theta_1, \varepsilon_1) + \frac{v(u_1)}{q} \quad (3)$$

Marginal cost is constant with respect to q , but varies with respect to alternative improvement techniques in the production processes. Each entrepreneur (or inventor) of a production technique chooses a level of effort u_1 at a cost $v(u_1)$, to reduce the cost of production according to $R(u_1, \theta_1, \varepsilon_1)$, $v'(u) > 0$, $v''(u) > 0$. The realization of cost reduction is characterized by an information production function $R(\cdot)$, which is affected by two random variables, $\theta_1 \sim h(\bar{\theta}, \Omega)$, observed before the level of effort u_1 is chosen, and $\varepsilon_1 \sim g(0, \Sigma)$, observable after u_1 is chosen. Neither random variable is observable to persons other than the entrepreneurs. We also assume a linear form, i.e.,

$$R(\cdot) = \theta_1 u_1 + \varepsilon_1 \quad (4)$$

The stochastic terms can be broken up into two parts — a perfectly correlated term, and another specific term, distributed i.i.d. across production techniques. The covariance matrices can be written as

$$\Omega = \sigma_1 [1_{1j}] + \sigma_2 I \quad (5)$$

$$\Sigma = \sigma_3 [1_{1j}] + \sigma_4 I \quad (6)$$

where I is the identity matrix and $[1_{1j}]$ is a matrix of 1's. By varying the magnitude of σ_1 to σ_4 we may generate any partial correlation structure.

Alternative techniques can be sorted out by a large variety of competitive processes. To concentrate on situations where evaluation will be based upon markets or tournaments, it is not necessary to consider all possible correlation structures. Under a large variety of covariance assumptions, the sorting of alternative techniques does not have to wait until ϵ_i is observed. For example, if the variance in ϵ_i is zero and the variation in θ_i is uncorrelated, individuals observing their own θ_i can infer their relative ranking based upon the knowledge of the distribution θ_i . Individuals with a high draw of θ_i can signal his relative position to his competitors, and agents with low draws may be discouraged. Thus, sorting of the best technique will take place immediately after θ is drawn, but before the agents exert effort. This corresponds to the "dynamic strategies" described in Dasgupta and Stiglitz (1980, p. 8). There are other possibilities; for example, if the variance of ϵ_i is zero and the variation of θ is correlated, alternative techniques are both ex-ante and ex-post identical. The competitive outcome in this case will be either a mixed strategy (see Dasgupta and Stiglitz (1980), and Gilbert and Stiglitz (1979)), or some prior contracting arrangement would be developed, (see Yu, (1981)).

The case where individual production techniques are most likely to subject themselves to a market evaluation is when the ex-ante stochastic term is perfectly correlated, but the ex-post stochastic elements are distributed i.i.d. across production techniques. Intuitively, this corresponds to a situation where individuals cannot agree between themselves upon which technique is better before effort is expended, and thus it is not possible to sort out the best technique until after ϵ_i is observed.

III. The First-Best Solution

Suppose there is a social dictator interested in maximizing the expected welfare of his constituency, (the consumers). The problem faced by the dictator is

$$\max_{q,u,n} E(TUV(q)) - MC(q, n)q - nv(u) \quad (7)$$

where

$$MC(q, n) = \min_i (MC(q_i)).$$

The first order conditions are

$$MUV(x) > E(MC(x, n)) \quad (8a)$$

$$\theta x = n v'(u) \quad (8b)$$

$$\frac{\partial}{\partial n} E[MC(q, n)q] + v(u) = 0 \quad (8c)$$

We denote the solution to (8) as (x, u^s, n^s) .

The second order conditions (setting aside 8a) are given by

$$H = \begin{vmatrix} -n v''(u) & -v'(u) \\ -v'(u) & \frac{\partial^2}{\partial n^2} [MC(q, n)q] \end{vmatrix}$$

is positive definite, which yields the following conditions:

$$\begin{aligned} v''(u) &> 0 \\ \frac{\partial^2}{\partial n^2} \hat{\epsilon}_{(1)} &< 0 \\ -n v''(u) \frac{\partial^2}{\partial n^2} \hat{\epsilon}_{(1)} - (v'(u))^2 &> 0 \end{aligned} \quad (9)$$

where $\hat{\epsilon}_{(1)}$ is the i^{th} order statistic from n , i.i.d. draws of ϵ_1 . We note that the second order conditions do not depend upon the parameters θ

and x. Equations (8) and (9) characterize the welfare maximizing allocation of resources.

IV. Tournaments and General Information

The essential features of a tournament contract are the following: first, the winning and losing agents are sorted out on the basis of their relative performance. Second, agents are rewarded with ex-ante fixed prizes, with the winning prize W_1 set above expected productivity, and the losing prize W_2 set below expected productivity. To induce the appropriate effort in cost reducing activity, the social dictator would first select n ex-ante identically capable researchers according to equation (8c); then he would set up a tournament with the above properties to achieve efforts consistent with equation (8b). If the social dictator can evaluate the reduced marginal cost of the competing agents, he awards the winning prize to the agent who has the lowest cost. In the two agent case, agent i wins if

$$\begin{aligned} MC_i &< MC_j \\ K - \theta u_i - \varepsilon_i &< K - \theta u_j - \varepsilon_j \\ \varepsilon_j &< \varepsilon_i + \theta(u_i - u_j) \end{aligned} \tag{10}$$

The probability of winning conditional on ε_i is

$$\begin{aligned} P|\varepsilon_i &= \text{prob} (\varepsilon_j < (\varepsilon_i + \theta(u_i - u_j))) \\ &= G(\varepsilon_i + \theta(u_i - u_j)) \end{aligned}$$

Thus, the probability of winning is given by

$$P = \int_{-\infty}^{\infty} G(\varepsilon_i + \theta(u_i - u_j)) g(\varepsilon_i) d\varepsilon_i \tag{11}$$

Agent i maximizes his expected profit with respect to his efforts

$$u_1^* = \arg \max_{u_1} (P(W_1 - W_2) + W_2 - v(u_1)) \quad (12)$$

The first order condition for a maximum is given by

$$(W_1 - W_2) \frac{dP}{du_1} - v'(u_1) = 0 \quad (13)$$

To induce the first-best level of effort, the optimal tournament contract sets

$$(W_1 - W_2) \frac{dP}{du_1} = \frac{\theta x}{2} \quad (14)$$

in symmetric equilibrium

$$(W_1 - W_2) = x / (2 \int_{-\infty}^{\infty} g^2(\varepsilon_1) d\varepsilon_1) \quad (15)$$

Figure 1 illustrates the mechanics of the tournament. The marginal gain facing a competing agent depends upon the magnitude of the gap between the winning and the losing prize $(W_1 - W_2)$, the number of individuals that would be utilizing the cost reducing activity, θx , and the distribution of the specific stochastic term, $g(\varepsilon_1)$. By manipulating the size of the gap, the marginal gain (and therefore effort) can be shifted to any level. If the gap is set according to (15), the efficient level of effort would result.

The interesting thing about the optimal gap in (15) is that it is not a function of the variable representing general information. Once the gap is set, competing agents will react appropriately to all values of privately observed θ . Thus, from the social dictator's point of view, a tournament scheme processes general information efficiently. Note: this result is not dependent upon any of the agents being risk averse, an assumption commonly used in the tournament literature.)

Furthermore to check the second order conditions:

$$(W_1 - W_2) \theta^2 \int_{-\infty}^{\infty} g'(\varepsilon_1 + (u_1 - u_j)) g(\varepsilon_1) d\varepsilon_1 - v''(u_1) < 0$$

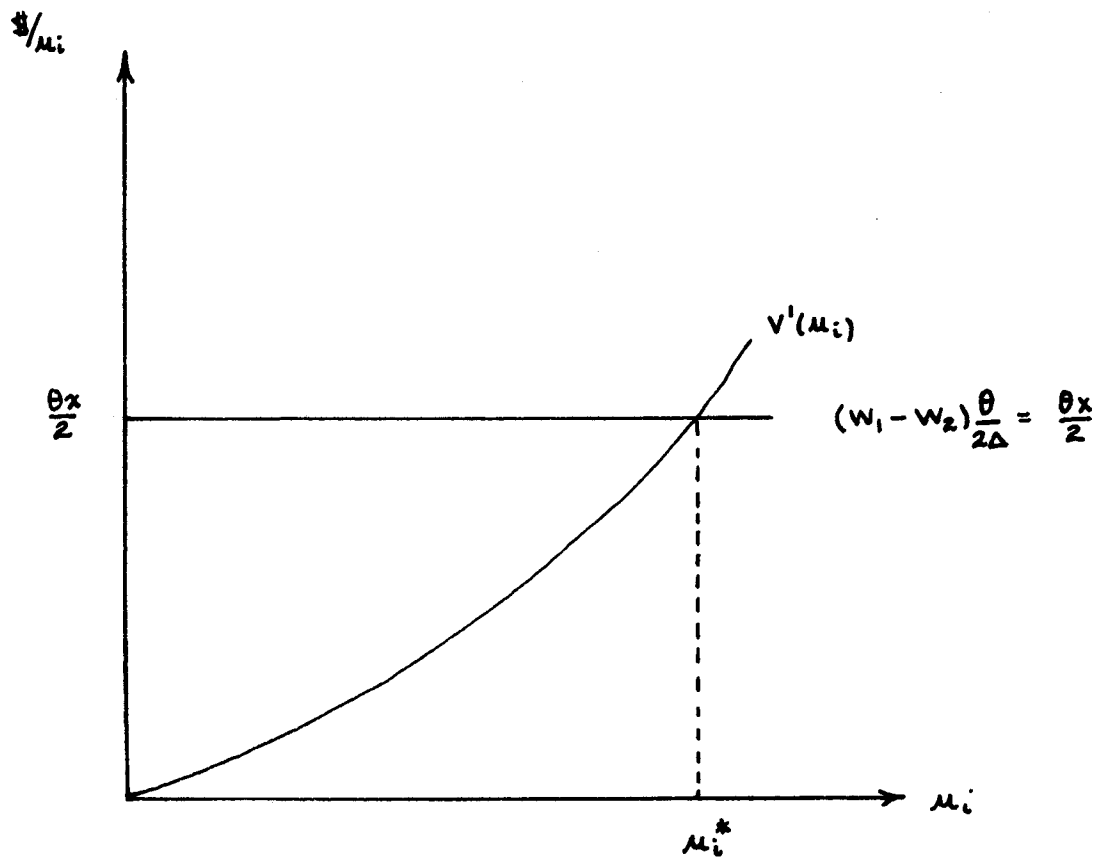


Figure 1.

In symmetric equilibrium, $u_i = u_j$

$$(W_1 - W_2) \theta^2 E(g'(\epsilon_1)) - v''(u_1) < 0 \quad (17)$$

In the uniform case, and in general for all symmetric distributions

$$E(g'(\epsilon)) = \int_{-\infty}^{\infty} g'(\epsilon)g(\epsilon)d\epsilon = 0 \quad (18)$$

Therefore the second order conditions for a maximum hold at the symmetric equilibrium for all values of θ .

V. The Market Interaction

In the sense that price cutting in the market will sort out the lower cost production technique, competing agents in the market are in effect joining a contest where rewards are based upon relative performance. The agent's payoff in this contest is contingent on whether his realized marginal cost is above or below his competitors. With constant returns to scale in production, and with two competing agents, the gain facing agent i is given by

$$G = \begin{cases} (MC_j - MC_i)x & \text{if } MC_i < MC_j \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

or

$$G = \begin{cases} \theta(u_i - u_j) + \epsilon_i - \epsilon_j & \text{if } \epsilon_j < \theta(u_i - u_j) + \epsilon_i \\ 0 & \text{otherwise} \end{cases}$$

Assuming the agents entertain Cournot conjectures about his competitors level of effort u , the expected gain is given by

$$E(G) = \int_{-\infty}^{\infty} \int_{-\infty}^{\theta(u_i - u_j) + \epsilon_i} [\theta(u_i - u_j) + \epsilon_i - \epsilon_j] g(\epsilon_j) g(\epsilon_i) d\epsilon_j d\epsilon_i \quad (20)$$

The first and second order conditions become

$$\frac{dE(G)}{du_1} = x\theta \int_{-\infty}^{\infty} [G(\theta(u_1 - u_j) + \varepsilon_1) g(\varepsilon_1) d\varepsilon_1 = v'(u_1) \quad (21)$$

$$\frac{d^2E(G)}{du_1^2} = x\theta^2 \int_{-\infty}^{\infty} g(\theta(u_1 - u_j) + \varepsilon_1) g(\varepsilon_1) d\varepsilon_1 < v''(u_1) \quad (22)$$

Assuming $\varepsilon_1, \varepsilon_j$ are distributed i.i.d. uniform $(-\Delta, \Delta)$, (21) and (22) become

$$\frac{\theta x}{2} + \frac{\theta^2 x}{2\Delta} (u_1 - u_j) = v'(u_1) \quad (23)$$

$$\frac{\theta^2 x}{2\Delta} < v''(u) \quad (24)$$

In symmetric equilibrium, $u_1 = u_j$. The first order condition (21) reduces to

$$\frac{\theta x}{2} = v'(u) \quad (25)$$

which is identical to the social first order condition (8b) with $n = 2$. Thus the implication seems to be that a competing agent facing price cutting in the output market will exert the first best level of effort.

Equation (25) as implied by the Cournot interaction in this market is not at all intuitive. Recall that θx is general information available to all competing agents in the market. In terms of the shifting of marginal cost, all competing agents' expected marginal cost would shift down by the same amount, θu , in symmetric equilibrium. If price cutting is anticipated, the gain from reacting to the general information will not be capturable. Yet the model is suggesting competing agents will exert the appropriate effort, acting as if the general information is fully capturable. When a competing agent spends a dollar of effort in reducing marginal cost, he anticipates a $1/n$ probability of getting the full return. This would be a rational calculation had agents competed in a winner take all patent race, where the winner is

given a monopoly rent based solely upon the timing of innovation.⁷ But this is not the criteria of competition examined here. The market evaluation mechanism used here sorts out the winner on the basis of realized marginal costs through price cutting. If agents are perfectly rational, they should anticipate the noncapturability of θ .

The dilemma of the market interaction is partially the result of the Cournot assumptions -- a competing agent reacts to the general information assuming the other agents effort is held constant. This conceptual assumption has often been criticized because agents expectations concerning his competitors reaction is constantly being nullified. However, the alternative expectation that the competitor will react identically to his action is equally implausible because that implies both competing agents will expend zero effort. Thus equation (25) can be interpreted as a statement similar to the "Prisoners' Dilemma" which is typical of situations involving conflicts.

The interesting thing about equation (25) is the sustainability of the interaction in situations involving large θ . An increase in general information in the market interaction described above may lead the agents to call off the type of interaction altogether. To see this point, the marginal gain and cost curves of a competing agent are plotted in Figure 2. An increase in θ not only shifts the marginal gain curve upward, it also increases its slope. At a certain critical value of $\theta = \hat{\theta}$, the second order condition facing a competing agent no longer applies, and the Cournot market interaction breaks down. Similar breakdowns occur if Δ is small. However, this latter type of breakdown also occurs in tournament contracts and therefore is not something unique to the Cournot market interaction under examination.

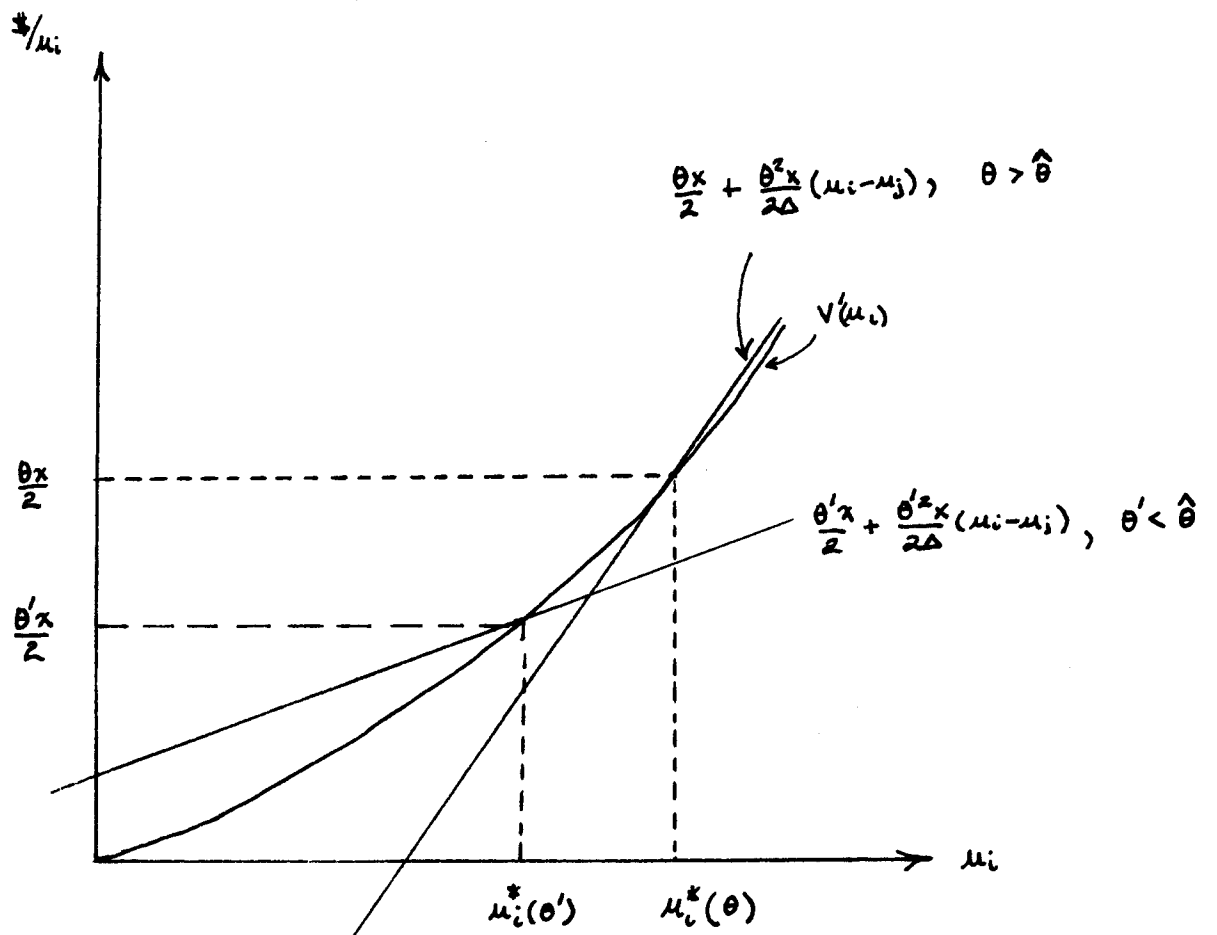


Figure 2.

In general, the existence of the Cournot symmetric equilibrium in the market depends upon the relationship between the distribution of the common stochastic term θ , and the variance of the specified stochastic term σ_ϵ^2 . Again using the uniform case, the symmetric equilibrium will not exist if the realization of

$$\theta > \hat{\theta} \equiv (2\Delta v''(u)/x)^{1/2}$$

The problem may be phrased in terms of the benevolent dictator (or the competing agents) trying to decide ex-ante on a contract to compete. If θ is distributed normally with mean $\bar{\theta}$ and variance σ^2 , the probability that the market competition will fail is given by $P(\theta > \hat{\theta}) = 1 - H(\hat{\theta})$. It is obvious that the probability of failure is increasing in $\bar{\theta}$. If $\hat{\theta} > \bar{\theta}$, the probability of failure is also increasing for mean preserving spreads. Thus, when the distribution of θ has a high mean or a large variance, the increased probability of failure will lead the benevolent dictator to search for alternative contractual arrangements to facilitate efficient utilization of general information. It is possible that a tournament contract is such an arrangement.

VI. Conclusion

Although markets and tournaments are similar in that they both base rewards on relative performance, they are not identical. The major difference between markets and tournaments is the prize awarded to the winner. In a market interaction, agents will anticipate the ex-post rent being dissipated away via price cutting. The tournament, however preserves the ex-post rent by fixing the winning prize ex-ante above expected productivity. In a related paper, (Kobayashi and Yu (1984)), we show how independent competing agents may

design alternative competitive interactions that have the properties of tournaments. This paper suggests a reason for doing this. In situations where general information is important relative to specific uncertainties, the market's sorting of a winner via price cutting will not be able to induce the level of effort in symmetric equilibrium. Tournaments, by preserving the rent, are able to induce appropriate levels of effort when general information is important.

Footnotes

*We are indebted to Michael Waldman who made valuable suggestions on the market interaction model in the paper. All errors are ours.

¹The idea of tournament as incentive devices is discussed in Lazear and Rosen (1981), Nalebuff and Stiglitz (1983 a, b), Carmichael (1983), Green and Stokey (1983), and O'Keefe, et. al. (1984). This concept was extended to a market setting by Hart (1983) and Nalebuff and Stiglitz (1983b).

²See Nalebuff and Stiglitz (1983b, p. 281). Hart (1983, p. 373-375), analyzed a similar problem but in the context of risk averse agents.

³The existence of common uncertainties is crucial to the viability of any relative performance scheme. Holmstrom (1982, 1983) shows that relative performance schemes only have value if one agents output provides information about another agents state uncertainty, or equivalently, if the agents face common uncertainties. Similarly, rank order tournaments dominate piece rates when the variance of a common error term is large relative to that of a specific error term (see Green and Stokey (1983), Lazear and Rosen (1981)).

⁴See Arrow (1962).

⁵See Wright (1983) for a study on the relative merits of alternative systems for innovation. There, patents and prizes are treated as equivalent. The comparison between markets and tournaments in this paper examines whether patent and prizes are in fact identical. On this issue, see also Kitch (1978).

⁶This illustrates the idea of parallel research. In the case where $\sigma_{\epsilon}^2 = 0$, parallel research is purely duplicative. Thus, it is socially efficient to have more than one mechanism in the market only if $\sigma_{\epsilon}^2 \neq 0$. (See R. Nelson (1961), Arditti and Levy (1980)).

⁷See Barzel (1968), Kamien and Schwartz (1975), Loury (1979), and Dasgupta and Stiglitz (1980).

REFERENCES

- Arditti, F.P. and H. Levy, "A Model of Parallel Team Strategy in Product Development," American Economic Review, 70 (1980), pp. 1089-1097.
- Arrow, K., "Economic Welfare and the Collection of Resources for Innovation," in R. Nelson (ed.), The Rate and Direction of Economic Activity, N.B.E.R. (Princeton: 1962).
- Barzel, Y., "Optimal Timing of Innovation," Review of Economics and Statistics, 50 (1968), pp. 348-355.
- Carmichael, H.L., "The Agents-Agents Problem: Payment by Relative Output," Journal of Labor Economics, 1 (1983), pp. 50-65.
- Dasgupta, P. and J. Stiglitz, "Uncertainty, Industrial Structure, and the Speed of R & D," Bell Journal of Economics, 11 (1980), pp. 1-28.
- Gilbert, R., and J. Stiglitz, "Entry, Equilibrium and Welfare," Paper presented to Conference on Theoretical Explorations in Industrial Organization, Montreal (October 1979).
- Green, J. and N. Stokey, "A Comparison of Tournaments and Contracts," Journal of Political Economy, 91 (1983), pp. 349-364.
- Hart, O.D., "The Market Mechanism as an Incentive Scheme," Bell Journal of Economics, 14 (1983), pp. 366-382.
- Holmstrom, B., "Moral Hazard in Teams," Bell Journal of Economics, 13 (1982), 324-340.
- _____, "Moral Hazard and Observability," Bell Journal of Economics, 10 (1979), pp. 74-91.
- Kamien, N. and N. Schwartz, "Market Structure and Innovation: A Survey," Journal of Economic Literature, 13 (1975), pp. 1-37.

- Kitch, E., "The Nature and Function of the Patent System," Journal of Law and Economics, 20 (1977), p. 265.
- Kobayashi, B.H. and B.T. Yu, "Market and Non-Market Tournaments," Mimeo UCLA (1984).
- Lazear, E. and S. Rosen, "Rank Order Tournaments as Optimum Labor Contracts," Journal of Political Economy, 89 (1981), pp. 841-864.
- Loury, G., "Market Structure and Innovation," Quarterly Journal of Economics, 93 (1979), pp. 395-410.
- Nalebuff, B. and J. Stiglitz, "Prizes and Incentives: Towards a General Theory of Compensation and Competition," Bell Journal of Economics, 14 (1983a), pp. 21-43.
- _____ and _____, "Information, Competition, and Markets," American Economic Review, 73 (1983b) pp. 278-283.
- Nelson, R., "Uncertainty, Learning and the Economics of Parallel Research and Development Efforts," Review of Economics and Statistics, 43 (1961), pp. 351-364.
- O'Keefe, M., W.K. Viscusi and R.J. Zeckhauser, "Economic Contests: Comparative Reward Structures," Journal of Labor Economics, 2 (1984), pp. 29-55.
- Wright, B., "The Economics of Innovation Incentives," American Economic Review, 73 (1983).
- Yu, B.T. "Potential Competition and Contracting in Innovation," Journal of Law and Economics, 24 (1981), 215-238.