SIGNALLING IN CREDIT MARKETS*

by

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The credit market is characterized by information asymmetries between lenders and borrowers. In this paper we provide a systematic analysis of the extent to which such asymmetries can be resolved via the signalling process. We then examine the comparative static implications of cyclical shifts in the cost of loanable funds.

In their 1976 paper Jaffee and Russell (J-R) provide key insights into the incentives for loan applicants to "signal" project quality. They did not, however, establish the possibility of equilibrium information transmission through signalling. In addition, the recent discussion between J-R and Vandell in this Journal [November 1984] shows no progress in the application of the signalling idea to explain the borrower-lender relationship in markets with imperfect information.

The J-R model is a two-period Fisherian consumption model. It is assumed that the loan contract is characterized by the loan size and the loan rate. Different borrowers have different default penalty costs or different distributions of second period income. Lenders are unable to observe these differences. The key insight is that the nature of the risk "purchased" by the uninformed lender is dependent on the size of the loan. Therefore, even a perfectly competitive lender will not be indifferent as to loan size.[1] J-R then show that there is no "competitive" (Nash) equilibrium in such a world.

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[1] In his recent comment, Hess [1984] succeeds in muddying the waters by treating the loan rate as constant for a competitive firm, while recognizing that the probability of default does increase with loan size. J-R in their reply challenge this but do not press their case. We address the issue further below and show that it has nothing to do with informational asymmetry but only with informational incompleteness (see also Keeton [1979] for a clear discussion).

A second, closely related strand of the credit market literature begins with the work of Stiglitz and Weiss [1981] (S-W). These authors focus on the potential for adverse selection when banks are unable to distinguish differences between loan applicants. The basic insight is that in contrast to low risk projects the high risk projects, which default unless there is a "big strike", generate a higher expected profit to borrowers. Therefore raising the interest rate to reduce excess demand in a non-cleared loan market tends to force the prefered loan applicants with lower risk projects out of the pool. As a result rationing can occur in equilibrium.[2]

One special feature of the S-W model is that all projects in a given risk pool have the same fixed borrowing requirement. In this paper we relax that assumption and return to the J-R world in which the return to a borrower is a continuous function of loan size. But once loan size is a variable, banks can offer loans of different sizes at different interest rates. This raises the possibility that, by adopting some schedule of loan offers, banks can screen loan applicants. The purpose of this paper is to characterize conditions under which such screening does indeed survive competition among banks. That is, we wish to determine those conditions under which loan applicants who are unobservable **ex ante** are sorted out **ex post** because of their differing incentives to take on risk.

The paper is organized as follows. In Section I a neoclassical model of the loan market is presented and the equilibrium is characterized under the public information assumption. The formal structure is similar to most credit market models. We give a detailed discussion of the public information case in order to have a benchmark to evaluate models with differing

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[2]For a discussion of the macroeconomic implications of such rationing, see also Riley [1986].

informational assumptions. In Section II we assume that loan applicants have private information about the loan quality. That is, we focus on informational asymmetries associated with characteristics of the investment project. This contrasts with J-R's focus on informational asymmetries associated with other sources of future income. It is shown that in equilibrium, applicants with higher quality projects signal this by accepting larger loans.

The positive correlation between loan size and loan quality is not the only possible result. In Section III it is shown that under alternative sets of assumptions about project characteristics, applicants with higher quality projects can signal by accepting smaller loans. In particular, we show that this is likely to be the case under assumptions which parallel closely those made by S-W.

While the formal development is for the case of just two types of loan applicant, the basic insights remain unchanged for an arbitrary number of types. In Section IV we switch to an n type model, where n is large, and consider the market level effects of a change in the lenders' cost of funds. We show that an increase in the cost of loanable funds raises the interest rates offered loan applicants and so reduces the aggregate demand for loans. Whether or not the average quality of funded projects rises with interest rates is shown to hinge critically on the nature of the informational asymmetry facing the lender.

Section V concludes with a brief discussion of the implications of relaxing the crucial assumptions and some conjectures about the possibility of rationing in a more general model. All proofs are relegated to the Appendix.

I. A Neoclassical Model of the Credit Market

Consider a stylized model of the credit market with a neoclassical production technology. Each of a large number of firms seeks to finance a one-period investment project. The gross return, $\tilde{x}$, of a project is stochastic and increasing in the size of the loan L. We assume that the production function can be written as

$$\tilde{x} = Q(\theta, L, \tilde{u}) \tag{1}$$

where $Q(\cdot)$ is a nonnegative increasing function, θ is a quality parameter and u, the realization of the random variable $\tilde{u}$, is unknown, ex ante, to both the loan applicant and the bank. In this section we assume that θ is freely observable by potential lenders. Then, in Sections II and III we introduce informational asymmetry by assuming that only the loan applicant knows the value of θ associated with his project.

To simplify the analysis we shall, at certain points, make the following assumptions about the distribution of $\tilde{u}$.

Assumption A1: The stochastic term $\tilde{u}$ is nonnegative and has a cumulative distribution function $G(u)$ which is differentiable and strictly increasing wherever $0 < G < 1$.

Assumption A2: The hazard rate of G, $G'(u)/(1-G(u))$, increases with u and tends to infinity as $G(u)$ approaches unity.[3]

We shall also appeal to the following restrictions on the form of the

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[3] This assumption is satisfied for a wide range of distribution functions, for example $G(u) = u^{\alpha}$, $\alpha > 0$ and $G(u) = 1 - (1-u)^{\alpha}$, $\alpha > 0$. Actually, all the results in Sections I and II hold under the very weak assumption that

$$d/du[G'(u)/(1-G(u)] \leq -1/u^2.$$

production function.

<u>Assumption B1</u>: The production function, $Q(\theta,L,u)$ is a nonnegative strictly concave function of L.[4] Also

$$\lim_{L\to 0}\left(\frac{Q}{L}\right) = \infty \quad \text{and} \quad \lim_{L\to\infty}\left(\frac{Q}{L}\right) = 0.$$

Given Assumption B1 it follows directly that the scale elasticity

$$\epsilon = \frac{L}{Q}\frac{\partial Q}{\partial L}$$

is less than unity. We now impose further restrictions on this elasticity

<u>Assumption B2</u>: The Scale Elasticity is nondecreasing in θ and nonincreasing in L.

Assumptions B1 and B2 are both relatively weak. They are satisfied, for example, if

$$Q(\theta,L,u) = b(\theta)L^{e}c(u)$$

where the scale elasticity e lies between zero and 1.[5]

We assume that all lenders and loan applicants are risk neutral. As long as the gross return exceeds the interest cost, a successful loan applicant receives the difference $\{Q(\theta,L,\tilde{u}) - RL\}$. Thus a loan applicant's expected return is

$$A(L,R) = E\max\{Q(\theta,L,\tilde{u}) - RL,\ 0\}, \tag{2}$$

where R is the contractual interest rate. Defining u^* to satisfy

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[4] If the loan applicant invests some of his own funds in a project, output may be strictly positive even in the absence of a loan.

[5] Both inequalities in Assumption B2 are satisfied strictly if $Q(\theta,L,u) = (a(\theta)L-b(\theta)L^{2})c(u)$ and $a(\theta)/b(\theta)$ is increasing.

(3) $$Q(\theta,L,u^*) - RL = 0$$

we can rewrite (2) as

(4) $$A(L,R) = \int_{u^*}^{\infty}(Q(\theta,L,u)-RL)\ dG(u).$$

On the supply side of the market there is a large number of lenders (banks). Assuming that the opportunity cost of funds is i, the expected profit to a bank from the project, if it lends L at an interest rate R is

(5) $$\Pi(L,R) = E\ \min\{RL,Q(\theta,L,u)\} - iL.$$

Making use of (3) we can rewrite this expression as

(6) $$\Pi(L,R) = (R-i)L + \int_{0}^{u^*}(Q(\theta,L,u)-RL)\ dG(u).$$

Following Jaffee [1972] and J-R [1976] we proceed by analyzing the preference maps of a typical borrower and lender. For our first model we assume that the production function has the following form.

<u>Assumption C</u> (Model I):

The production function has the multiplicative form

(7) $$Q(\theta,L,u) = q(\theta,L)u$$

In the Appendix we derive the following two Propositions.

<u>Proposition 1</u>: Loan Applicant's Indifference Curves (Model I)

If Assumptions A1-C hold then indifference curves for a loan applicant, $A(L,R) = \bar{A}$, drawn with L on the horizontal axis have a unique turning point at $L = L^a(R)$ and slope downwards for larger L. Moreover $L^a(R)$ is decreasing in R.

Proposition 2: Bank's Zero Iso-Profit Curve

If Assumption B1 holds, the iso-profit curve $\Pi(L,R) = 0$ is everywhere upward sloping.

Indifference curves for a loan applicant and iso-profit curves for a bank are depicted in Figure 1. For the applicant, a smaller interest rate is strictly preferable hence higher indifference curves are associated with a smaller expected gain. Let A_0 be the reservation income of a loan applicant. Only contracts below this indifference curve would ever be accepted by the applicant.[6]

For the bank, larger interest rates are preferred, thus lower iso-profit contours are associated with a smaller expected gain. Since we are interested, here, in examining a competitive banking industry, equilibrium must be on the zero iso-profit contour, $\Pi(L,R) = 0$. From (6) and (7) if $\Pi(L,R) = 0$, it follows that

$$R - i = \int_0^{u*} (R-q(\theta,L)u/L)dG(u).$$

From (3) the right hand side approaches zero with L. Thus the iso-profit contour $\Pi(L,R) = \Pi_0 = 0$ goes through the point $<0,i>$, as depicted. Since the bank would never accept any contract below this curve, the set of feasible loan contracts is the shaded region in Figure 1, bounded by $A = A_0$

[6] The borrower's indifference curves continue to have this form in a two-period consumer model with uncertain income endowment $(x_1,\tilde{x}_2)$ and expected utility

$$EU(x_1+L,\max\{\tilde{x}_2-RL,0\}).$$

Indeed the entire analysis of the paper can be extended to cover consumer borrowing if U has the semi-additive form

$$U(c_1,c_2) = V(c_1) + c_2.$$

More generally, however, the convexity of the borrower's return in the second period is offset by the concavity of second period utility.

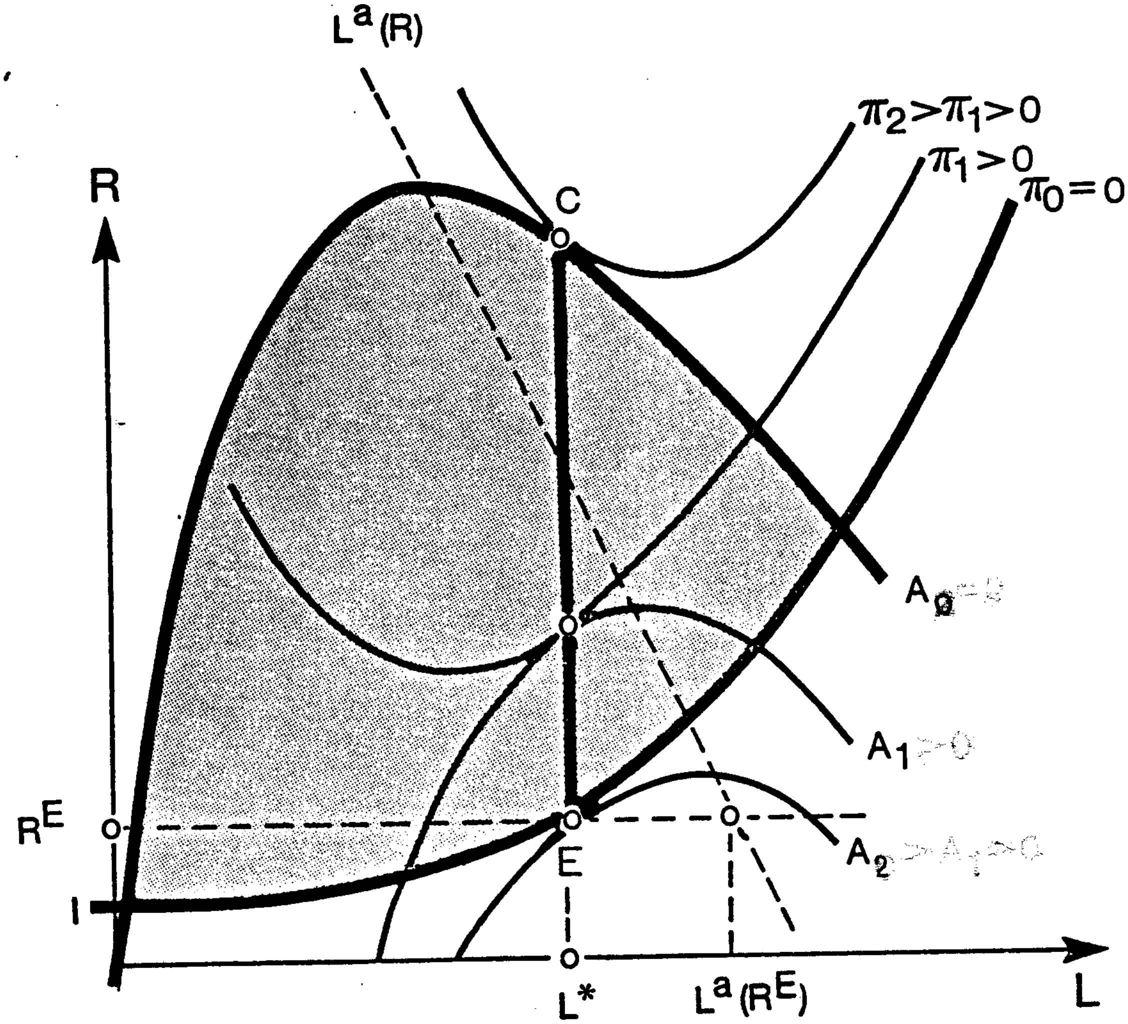

Fig. 1: Preferences of Applicant and Bank

and $\Pi = 0$.

We next ask which of these contracts is Pareto efficient. Graphically, we seek points $\langle L,R \rangle$ at which the indifference curves for applicant and bank are tangential. Note that the total surplus to be distributed is simply a function of the loan size L, that is, from (5), (6) and (7)

$$A(L,R) + \Pi(L,R) = \int_0^\infty Q(\theta,L,u)dG(u) - iL = q(\theta,L)\bar{u} - iL,$$

where $\bar{u} = Eu$. Thus the total gain, $A + \Pi$, is maximized by choosing L^* to satisfy

$$\frac{\partial q}{\partial L}(\theta,L^*)\bar{u} - i = 0 \tag{8}$$

and the contract curve is the vertical line EC in Figure 1.[7]

Perfect competition among banks then results in the equilibrium contract being the point E in Figure 1, where the contract curve intersects $\Pi(L,R) = 0$. From Proposition 2 the equilibrium contract E is a point at which the applicant's indifference curve has positive slope. Thus, at the equilibrium interest rate R^E, the applicant's optimal loan size, $L^a(R^E)$ exceeds L^*. Note that E is the single-contract solution derived in J-R [1976] and in Keeton [1979, p. 52]. While it is perhaps tempting to say that loans are rationed at the equilibrium loan rate R^E, this would be confusing, at best. The curve $L^a(R)$ does not represent the demand for a product of given quality, since the risk characteristics of the project vary as L increases. Recognizing this, a competitive banking industry offers contracts along the "hedonic" offer curve $\Pi(L,R) = 0$. Unconstrained loan applicants then choose the contract E.

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[7]Note that this is a correction of a wrong diagram in Jaffee [1972] where the Pareto optimal locus is not a vertical line. As it is well known from the property rights literature we have to solve a distribution problem (without allocation effects).

As we shall see in the following sections, informational constraints force loan applicants to points like D′ or D″. Again it does not seem helpful to characterize even a point like D″ as a "rationing" solution.

To complete the analysis of the equilibrium with symmetric information, we examine the effect of an increase in the parameter θ. In the Appendix we prove

Proposition 3: If Assumptions B1 and B2 hold, an increase in θ shifts the zero iso-profit contour to the right. Moreover the equilibrium loan, $L^*(\theta)$, is strictly increasing in θ.

The effects of increasing θ are depicted in Figure 2. While the equilibrium for the preferred loan applicants, E_2, lies to the right of E_1, so that $L^*(\theta_2)$ exceeds $L^*(\theta_1)$, the equilibrium contractual interest rate, $R^E(\theta)$, may rise or fall with increasing θ, depending on the precise form of the production function and the c.d.f. $G(u)$. Figure 2 depicts the case of a decreasing rate.

II. Equilibrium With Private Information -- Loan Size as a Signal

We now depart from the assumption that information is imperfect but symmetric, and assume instead that information is imperfect and asymmetric. Each loan applicant has inside information about the quality of his project. Otherwise the bank itself could operate the project. This informational difference between applicant and bank is captured by the parameter θ. Each applicant is assumed to know his own θ, while banks know only the general underlying technology and the probability distribution of θ, $F(\theta)$.

To simplify the exposition we consider the case in which θ takes only two values, θ_1 and θ_2 with $\theta_2 > \theta_1$. From Figure 2 it is clear that

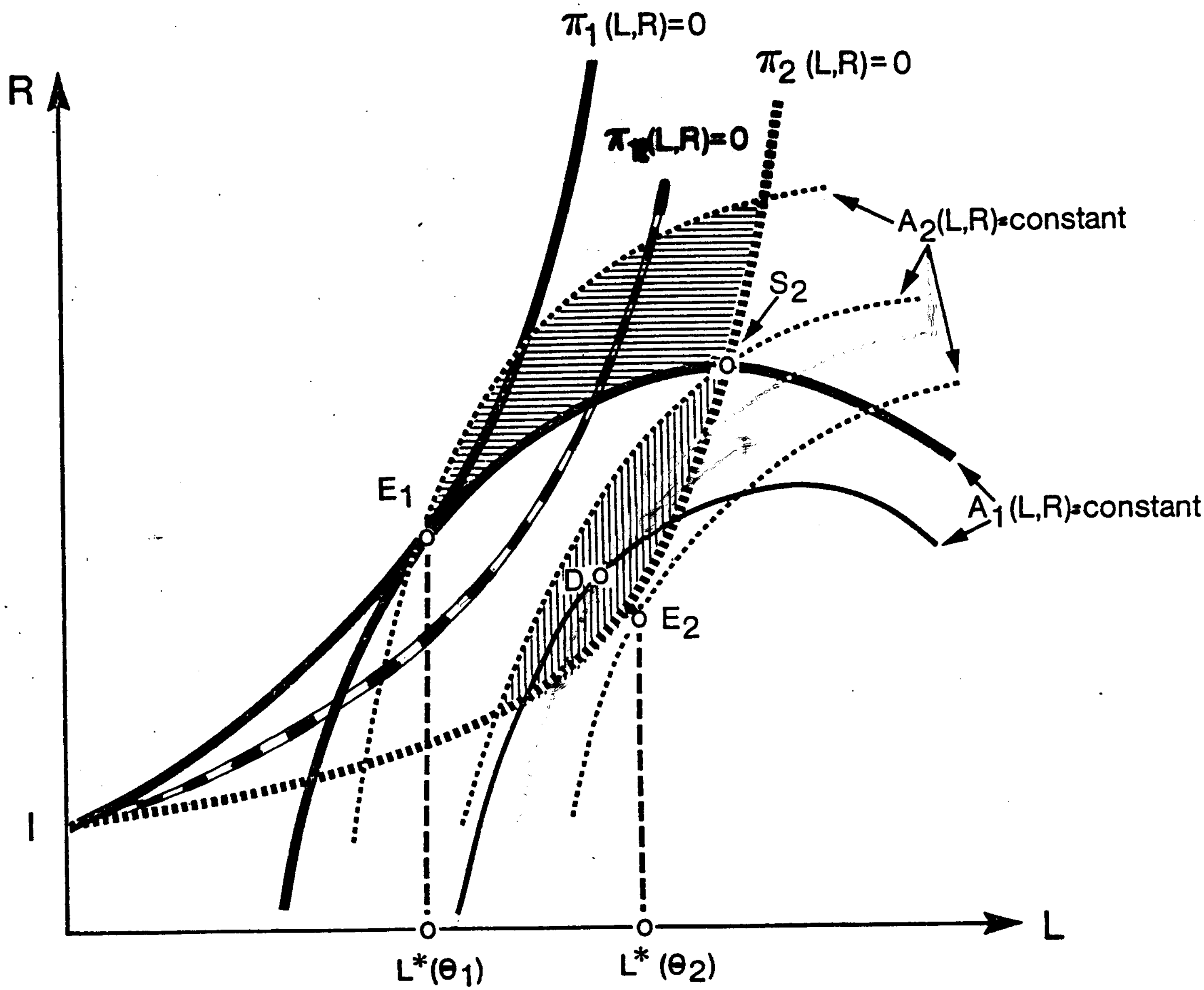

Figure 2: Credit marked equilibrium with Larger Loans signalling higher quality

$\{E_1, E_2\}$, the equilibrium pair of loan contracts, when θ_1 and θ_2 are observable, is not an equilibrium when information is private. Rather than accept the contract E_1, any applicant with a lower quality project is strictly better off accepting the contract E_2. But then E_2 results in losses for the bank.

To separate out the high and low quality applicants, the bank must therefore attempt to exploit differences in the shapes of their preference maps. Throughout this section we assume that the production function $Q(\theta, L, u)$ satisfies Assumption C, that is, the random term is multiplicatively separable. Given this assumption we can prove

Proposition 4: Marginal Willingness to Pay for a Larger Loan (Model I) Given Assumptions A1, A2, B1 and C the marginal increase in interest rate that a loan applicant is willing to accept in order to receive a larger loan is greater for more desirable projects (higher θ). Formally,

$$\frac{\partial}{\partial \theta} \left. \frac{dR}{dL} \right|_A = \frac{\partial}{\partial \theta} \left(- \frac{\partial A/\partial L}{\partial A/\partial R}\right) > 0.$$

This difference in the willingness to pay for a larger loan is illustrated in Figure 2. At each point of intersection of an unbroken indifference curve $(\theta = \theta_1)$ and a dashed indifference curve $(\theta = \theta_2)$ the latter has a greater slope. As a result, there is a set of contracts (the horizontally shaded region in Figure 2), each member of which (i) is strictly preferred over E_1 only by applicants with high quality projects and (ii) yields expected profits to a bank. Of these, the contract S_2 yields the greatest gains to the high quality applicants. Since E_1 is the best contract that low quality applicants can be offered without generating expected losses, and since S_2 is the best separating contract for high quality applicants, the pair $\{E_1, S_2\}$ is Pareto efficient among separating contracts, each of which breaks even.

Since this is not the place for a discussion of the long theoretical debate about equilibrium in signalling models (see, in particular, Wilson [1977], Riley [1979, 1985], Stiglitz and Weiss [1985] and Kreps [1985]) we now make a further simplifying assumption.

Assumption D: The Pareto-efficient **separating** set no-loss contracts is also Pareto efficient among **all** sets of no-loss contracts.

Elsewhere Riley [1985] has provided conditions guaranteeing Assumption D. Writing the marginal cost of signalling for type θ_i as MC_i and the frequency of type i in the population as f_i, he shows that the assumption holds if either MC_{i+1}/MC_i or f_{i+1}/f_i is sufficiently small.

Given Assumption D it follows that the pair of contracts $\{E_1, S_2\}$ is a Nash equilibrium, indeed the unique Nash equilibrium.[8] To see this consider Figure 2 once more. Given that E_1 and S_2 are offered there are clearly no profitable alternatives which attract only the low quality types. Moreover, since S_2 is Pareto efficient among separating contracts, there are no profitable alternatives which attracts only the high quality types. The only remaining alternative is an offer like D in the vertically shaded region which is attractive to both types. But, by Assumption D, this set does not intersect with $\Pi_{12}(L,R) = 0$, the locus of contracts which break even when both types accept. Thus D too is unprofitable and so $\{E_1, S_2\}$ is the Nash equilibrium.

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[8] We defer until Section V any discussion of the equilibrium outcome when Assumption D is not satisfied.

Note also that the low quality type is just indifferent between E_1 and S_2. But, from (2) for any loan contract $\langle L,R \rangle$,

$$A(L,R,\theta_2) > A(L,R,\theta_1).$$

Therefore, $A(L^*(\theta_2),R^*(\theta_2),\theta_2) > A(L^*(\theta_1),R^*(\theta_1),\theta_1)$.

While we have examined in detail only the two type case, it should be intuitively clear that adding additional types does not change the qualitative nature of our conclusions. Generalizing to n types $\{\theta_1,\ldots,\theta_n\}$ with $\theta_j < \theta_{j+1}$, we can now state

Proposition 5: Characterization of the Equilibrium (Model I)

Under Assumptions A-D the Nash equilibrium set of contracts $\{(L^*(\theta_j),R^*(\theta_j)\}_{j=1,\ldots,n}$ has the following properties.

(i) the set of funded projects is the same as in a world with full information about θ.

(ii) the lowest quality project which is funded receives the full information efficient loan.

(iii) each contract breaks even

(iv) $(L^*(\theta_j),R^*(\theta_j))$ is strictly increasing in θ_j.

(v) $A(L^*(\theta_j),R^*(\theta_j);\theta_j)$ is strictly increasing in θ_j.

We shall appeal to this theorem in the comparative statics analysis of Section IV.

III. Signalling With Loan Size and Quality Inversely Related

Certainly the strongest of the assumptions made in the previous section is that the production function $Q(\theta,L,\tilde{u})$, should take the multiplicative form

$$\tilde{x} = q(\theta,L)\tilde{u}.$$

While it seems highly plausible that qualitatively similar results will hold under much weaker assumptions, it is not enough to assume that $Q(\theta, L, \tilde{u})$ is a strictly increasing concave function.

In fact, as we shall now show, it is quite possible that applicants with **lower** quality projects are willing to pay a **larger** interest rate premium for a larger loan. In such a world applicants with higher quality projects can signal by accepting a **smaller** rather than a **larger** loan.

We now introduce the following alternative to Assumption C.

Assumption C′ (Model II):

The production function has the additive form

$$Q(\theta, L, \tilde{u}) = \beta(L)L + \tilde{u}L + \theta$$

where $\beta'(L) < 0$ and Q is concave.

Given Assumption C′, the expected social net return is maximized with a loan size L^{**} satisfying

$$\frac{\partial}{\partial L}\{EQ - IL\} = \frac{\partial}{\partial L}\{\beta(L)L + \bar{u}L - iL + \theta\} = 0$$

Note that L^{**} is independent of the loan quality parameters.

Arguing almost exactly as in the previous section, we can establish that the zero iso-profit contour shifts rightwards as θ increases. Thus the equilibrium, when both banks and applicants know θ, is as depicted in Figure 3. The exact counterpart to Proposition 4 above is

Proposition 6: Marginal Willingness to Accept a Smaller Loan (Model II)
Given Assumptions A1, A2 and C′ the decrease in interest rate that a loan applicant requires, in order to accept a smaller loan, is smaller for more desirable projects (greater θ). Formally,

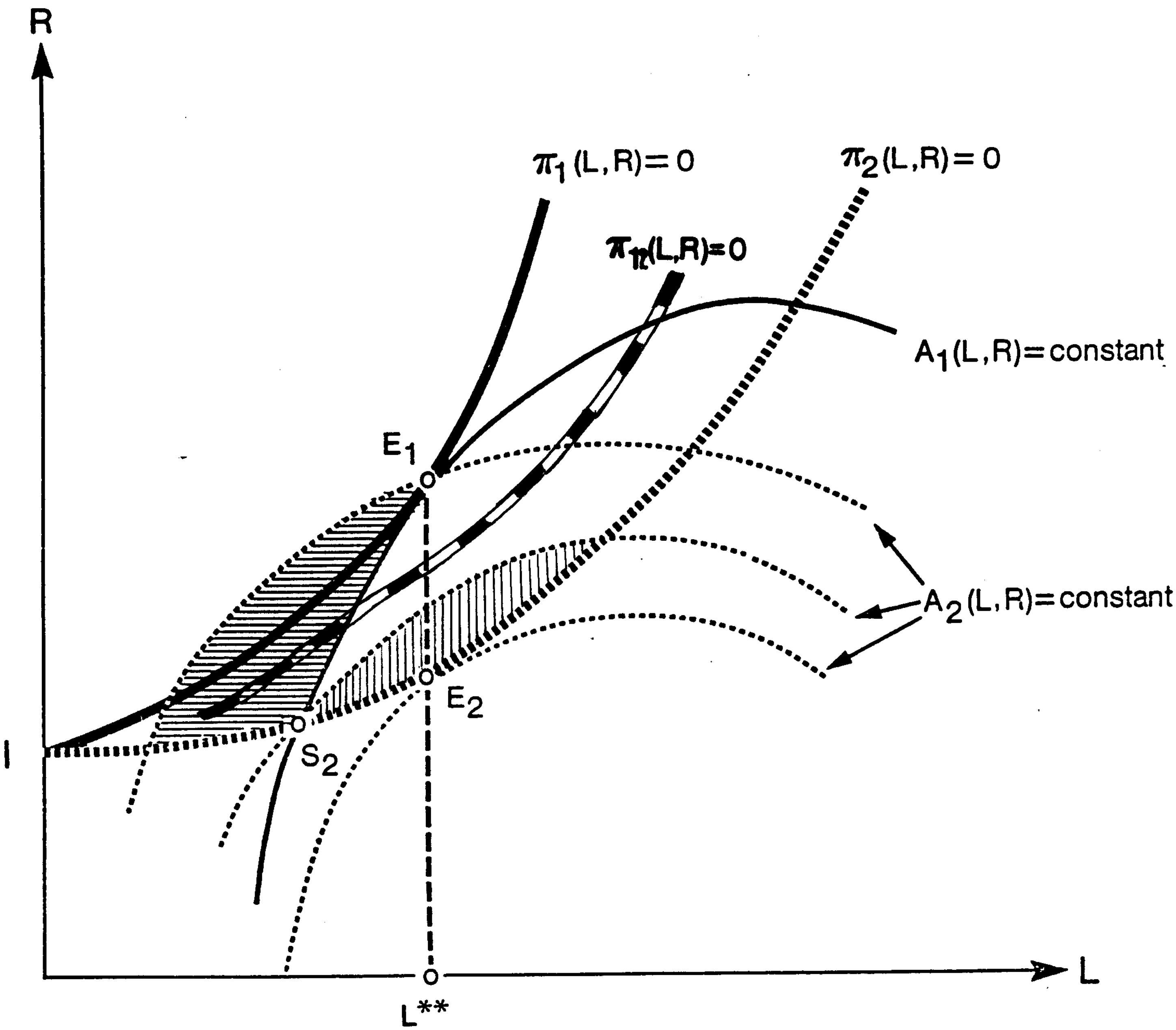

Figure 3: Credit market equilibrium with smaller loans signalling higher quality

$$\frac{\partial}{\partial\theta}\left.\frac{dR}{dL}\right|_A = \frac{\partial}{\partial\theta}\left(-\frac{\partial A/\partial L}{\partial A/\partial R}\right) < 0.$$

In graphical terms, the indifference curve for a low quality loan applicant, at any point $<L,R>$ is steeper than the indifference curve for a high quality applicant. As in the previous section, $\{E_1,E_2\}$, the equilibrium with θ observable, is no longer feasible since all loan applicants prefer E_2 to E_1. However, given the difference in the preference maps there is again a set of loan contracts which, if offered in conjunction with E_1, separate out the two types of applicant. This is the horizontally shaded region in Figure 3. Then the best pair of separating contracts, which also at least break even, is the pair $\{E_1,S_2\}$. As in the previous section, this pair is the Nash equilibrium if Assumption D holds. The argument is exactly as before. The only possible way of making a profit, given that other firms offer the contracts $\{E_1,S_2\}$ is by attracting both types. In Figure 3 such contracts lie in the vertically shaded region. As long as the zero profit curve for both types, $\Pi_{12}(L,R) = 0$, does not intersect this region, none of the pooling contracts are profitable. From the figure it can be seen that the smaller the proportion of high quality types (so that $\Pi_{12}(L,R) = 0$ lies nearer $\Pi_1(L,R) = 0$) and the flatter the indifference curves for high quality types, the more likely it is that Assumption D will be satisfied.

Also, since $A(L,R,\theta)$ is strictly increasing in θ higher quality loan applicants are again better off, in equilibrium. Generalizing to the n-type case we can now state

<u>Proposition 7</u>: Characterization of the Equilibrium (Model II)

Under Assumptions A, C′ and D the Nash equilibrium set of contracts

$\{L^*(\theta_j), R^*(\theta_j)\}_{j=1,\ldots,n}$ has the following properties

(i)-(iii) as in Proposition 5.

(iv) $(L^*(\theta_j), R^*(\theta_j))$ is strictly <u>decreasing</u> in θ_j

(v) $A(L^*(\theta_j), R^*(\theta_j), \theta_j)$ is strictly increasing in θ_j.

For our final example we present a model very close in spirit to that analyzed by Stiglitz and Weiss. While for our first two examples a higher quality parameter implies a higher mean return, S-W consider a case in which all the projects in the risk pool have the same mean but some are more risky in the sense of second order stochastic dominance (implying higher variance). While S-W consider projects with fixed loan requirements, we modify this model assuming that the gross return increases with the size of the loan.

As a preliminary we strengthen the notion of "more risky" in the following definition.

<u>Definition</u>: Fat Tail Property

The random variable $\tilde{X}$ has a fatter right tail than $\tilde{Y}$ if, for all a,b $a < b$

$$\mathrm{Prob}\{\tilde{X}>a\} > \mathrm{Prob}\{\tilde{Y}>a\} \Rightarrow \mathrm{Prob}\{\tilde{X}>b|\tilde{X}>a\} \geq \mathrm{Prob}\{\tilde{Y}>b|\tilde{Y}>a\}$$

In the following Lemma we summarize two useful implications of the fat tail property.

<u>Lemma 2</u>: If $\tilde{X}$ and $\tilde{Y}$ have the same mean and the former has a fatter right tail then

(i) $\tilde{X}$ is a mean preserving spread of $\tilde{Y}$, and

(ii) for all a $E\{\tilde{X}|\tilde{X}\geq a\} \geq E\{\tilde{Y}|\tilde{Y}\geq a\}$.

We now impose the following further restriction on the family of production functions

Assumption C" (Model II):
The production function has the form

$$Q(\theta, L, \tilde{u}) = q(L)\tilde{V}_{\theta}$$

where $E\tilde{V}_{\theta} = \bar{v}$, independent of θ and $\theta_1 < \theta_2$ implies that $\tilde{V}_{\theta_1}$ has a fatter right tail than $\tilde{V}_{\theta_2}$.

Given part (i) of Lemma 2, the fact that the loan applicant's profit function

$$A(L,R,\theta_j) = \max\{q(L)V_{\theta_j} - RL, 0\},$$

is convex in V, implies that, for any loan offer $\langle L,R \rangle$,

$$A(L,R,\theta_1) \geq A(L,R,\theta_2).$$

This is the key point made by S-W. Making use of part (ii) of Lemma 2 we can also prove that Proposition 6 continues to hold when Assumption C' is replaced by Assumption C". (See Corollary 3 in the Appendix.) Generalizing once more to the n-type case we can now state

Proposition 8: Characterization of the Equilibrium (Model III)
Under Assumptions A, C" and D the Nash equilibrium set of contracts $\{L^*(\theta_j), R^*(\theta_j)\}_{j=1,\ldots,n}$ has the following properties.
(i)-(iii) as in Proposition 5
(iv) $(L^*(\theta_j), R^*(\theta_j))$ is strictly **decreasing** in θ_j
(iv) $A(L^*(\theta_j), R^*(\theta_j), \theta_j)$ is strictly **decreasing** in θ_j.

We next examine the comparative static-effects of a change in the banking industry's cost of loanable funds. As we shall see, despite complete separation of loan applicants **ex post**, adverse selection continues to play a distinctive role in the S-W type model III.

IV. Changes in the Cost of Loanable Funds

The above analysis is partial in nature in that it examines the aggregate demand for loanable funds assuming that these are available in unlimited amounts at a fixed interest rate i. In this section we consider the demand for loanable funds as a function of this interest rate. We shall argue that aggregate demand is downward sloping so that the demand and supply of loanable funds will be equated at a unique equilibrium rate i^*. Holding the demand curve constant, cyclical shifts in supply then generate procyclical shifts in the equilibrium interest rate.

To avoid discussing all possible permutations, we shall assume that the bank borrowing rate i, and hence the implied interest rate schedule for loan applicants,

$$R = r(L;i),$$

is sufficiently high that, under complete information about θ, a strict subset of the set of potential loan applicants would be funded. For the remainder there is no profitable contract which yields an expected return as high as the applicants' reservation income A_o.

From the three characterization theorems of the previous sections, we know that for each model the equilibrium interest rate schedule $r(L;i)$ is strictly increasing in L as depicted in Figure 4. In each case the point X represents the lowest quality loan contract and Y the highest quality loan contract. For model I, depicted in Figure 4.1, we also know that the equilibrium loan to type θ, $L^*(\theta)$, and expected payoff, $A^*(\theta)$, are strictly increasing functions of θ. Then it is contract X which represents the marginal loan applicant who is just indifferent between

Fig 4.I

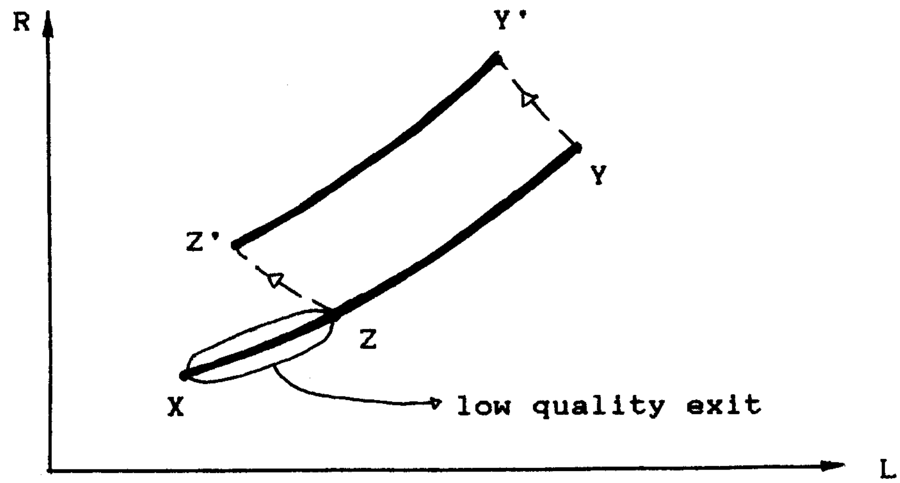

Fig 4.II

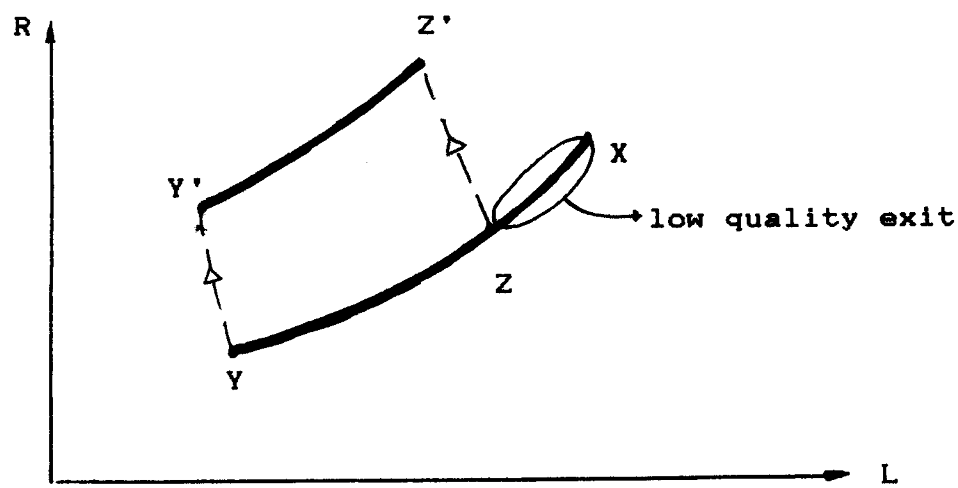

Fig 4.III

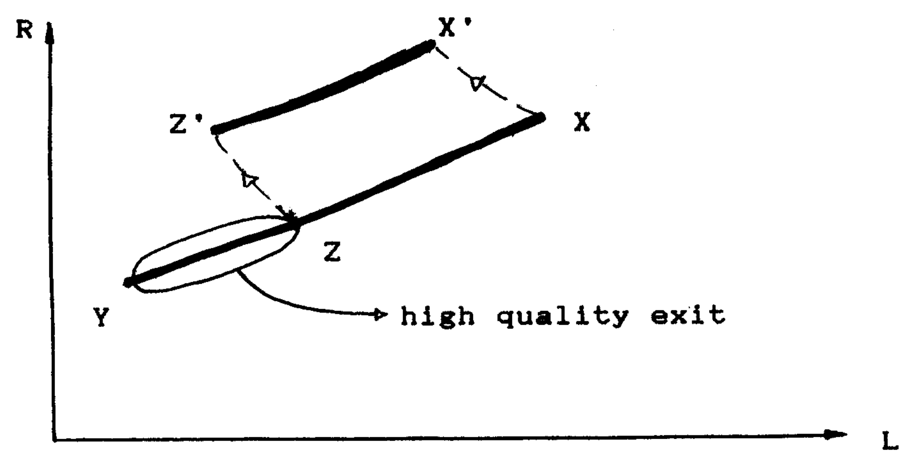

Figure 4: Effects of an increase in the banks' cost of funds

funding his project at the terms offered and accepting his reservation income A_o.

Now suppose the banking industry's cost of funds, i, increases. Since full information efficient loan size and expected return for each type declines and since the marginal loan applicant receives the full information efficient loan, it follows that there is exit from the industry. Moreover some initially inframarginal type moves from Z to Z' and becomes marginal.

Without further analysis it is not possible to conclude that loan size declines for all types. However, the intuition as to why this should be the case is strong. With exit, each quality type has less inferior types from which it must be distinguished. Therefore the difference between the equilibrium loan size and the full information efficient loan size declines. Intuitively then, the equilibrium loan rate schedule must shift as depicted.

A similar analysis holds for model II. However now $L^*(\theta_i)$ is a <u>decreasing</u> function of quality level θ and so exit from the industry takes place at the right hand end of the loan rate schedule. We can therefore again conclude that an increase in the cost of loanable funds results in a northwesterly shift in the equilibrium loan rate schedule. This is depicted in Fig. 4.2.

Finally, we turn to model III. From Proposition 8 it is now the highest quality loan applicants who are marginal. Therefore an increase in the interest rate forces those borrowing the least from the market. Again the shift in the equilibrium interest rate schedule is to the Northwest. However, in contrast with the previous cases, the average quality of the funded loan applications declines. Therefore, while models I and II generate **<u>procyclical</u>** shifts in the interest rate and loan quality, model III generates a procyclical shift in the interest rate but an <u>countercyclical</u> shift in loan quality.

This last point is the counterpart of the S-W rationing conclusion. Their analysis led to rationing because of adverse selection and an inability to signal. We have shown here how loan size might be used to separate out loan types. As we have seen, if separation is complete, rationing can no longer occur. However, adverse selection continues to play a potentially important role. For even with separation, a change in the cost of bank funds changes the marginal loan applicant. In the S-W world this loan applicant is, from the bank's viewpoint, the best loan applicant, hence the countercyclical effect on the average quality of funded loans.

V. <u>Concluding Remarks</u>

In this paper we have analyzed credit market equilibrium under private information. Asymmetric default information is transmitted by signalling. Instead of applicants with preferred projects withdrawing from the market, banks are able to exploit differences in applicants' marginal rates of substitution and so sort out different risk classes. In contrast with the S-W pooling and rationing equilibrium we derive a market equilibrium which is characterized by separation and no rationing. And in contrast with J-R's conclusions we find that the multiple-contract solution is a stable equilibrium.

While we have emphasized the potentially important role that signalling can play in eliminating, or at least reducing incentives to ration credit, we certainly do not wish to suggest that there is anything wrong with the formal logic of the J-R and S-W papers. J-R correctly point out that with a particular form of informational asymmetry there is no "competitive" equilibrium. In our model, suppose the borrower has an additional source of income z which can be used as partial collateral. Then, from Section I, a loan applicant's expected return becomes

$$(2') \qquad A(L,R) = E\max\{z+Q(\theta,L,\tilde{u}) - RL, 0\}.$$

If each borrower has private unverifiable information about z we have a model which is formally equivalent to that of J-R. Thus the practical implications of the two models hinge critically on the relative importance of different types of informational asymmetry.

The S-W model is special in that the technology is of the simple point-input, point-output type so that there is no opportunity for lenders to use loan size in the screening process. As we have seen, under more neoclassical assumptions a competitive banking industry may well be able to use loan size as a way of screening for differences in project quality.

However each of the three models presented here are also based on strong simplifying assumptions. In particular each is constructed so that preferences satisfy the "single crossing" property. A more general analysis would attempt to encompass aspects of all three models by introducing multi-dimensional informational asymmetry. Clearly with two or more unobservable characteristics it would be necessary to have two signals for separation of loan applicants to be possible.[9]

Engers [1987] has established conditions under which, with multi-dimensional asymmetry, complete separation is achieved in equilibrium. As is intuitively clear, such results require strong assumptions about

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[9] Recent papers by Besanko and Thakor [1984], Bester [1985], Chan and Kanatas [1985] and Clemenz [1985] have all pointed to the potential role of collateral as an additional screening mechanism. Intuitively it will be applicants with higher quality loans who are more willing to put up their personal assets as collateral.

preferences which generalize the "single crossing" property. In the absence of such assumptions the most that one would expect would be partial screening and hence partial pooling. What remains to be analyzed is the extent of such pooling.

Suppose, for example, that the unobservable θ is a two-dimensional vector distributed continuously over the unit square. As in the analysis of Sections II and III, it seems reasonable to anticipate that there would be an equilibrium loan rate schedule $R = r(L)$. It would be very useful to know whether the fraction of loan applicants choosing loan sizes in an interval $(L', L'+\delta)$ typically approaches zero with δ or whether finite fractions tend to be pooled at a particular loan size. To the extent the latter is true, the rationing conclusion emphasized by Stiglitz and Weiss again reenters the picture.

We end with a few remarks about equilibrium when the simplifying Assumption D is dropped. From the early work of Rothschild and Stiglitz [1976] it is well known that in general there is no Nash equilibrium (in pure strategies) when each uninformed agent must make the first move, offering contracts to the informed agents.

The several attempts to resolve the nonexistence paradox (Wilson [1977], Miyazaki [1977] and Riley [1979]) all appeal to the idea that a potential "defector" from a Nash equilibrium is likely to be deterred if predictable reactions by others will make his initial defection unprofitable. While the details of the resulting equilibria differ, the conclusions are qualitatively similar. Most important for our purposes here, the "anticipatory" or "reactive" equilibrium yields results similar to those of Sections II and III.[10]

Other authors, in particular Stiglitz and Weiss [1985] and Kreps [1985] have considered an alternative game in which it is the informed agents who must make the first move. In the context of our analysis, each loan applicant first selects his loan requirements and then presents the case for funding to a bank. As Stiglitz and Weiss point out, there is a multitude of Nash equilibria in such a world if out-of-equilibrium beliefs are never tested. However, when the set of equilibria are refined to eliminate "unreasonable" beliefs, the number of equilibria declines dramatically. Indeed Kreps has argued that the unique equilibrium is precisely the allocation analyzed here. That is, the refined Nash equilibrium is the Pareto efficient separating set of no-loss contracts.

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[10] Wilson's conclusions differ from those of Riley in that there is less than complete separation of types in his equilibrium.

APPENDIX

Lemma 1: If Assumption A2 holds, that is the hazard rate of G, $G'/(1-G)$ is increasing, then for any x such that $G(x) < 1$,

$$H(x) = \frac{\int_x^{\infty} (1-G(u))du}{1 - G(x)}, \tag{9}$$

is a decreasing function of x.

Proof: Let $\hat{x}$ be the upper support of $G(u)$. By l'Hôpital's rule

$$H(\hat{x}) = \lim_{x\to\hat{x}} \frac{-(1-G(\hat{x}))}{-G'(\hat{x})} = 0, \tag{9}$$

since, by Assumption A2, $G'/(1-G)$ tends to infinity as $x \to \hat{x}$.

Differentiating (9) by x we obtain

$$H'(x) = -1 + \frac{G'(x)}{1-G(x)} H(x). \tag{10}$$

Differentiating again

$$H''(x) = \frac{d}{dx} \left(\frac{G'}{1-G}\right) H(x) + \left(\frac{G'}{1-G}\right) H'(x) \tag{11}$$

By hypothesis, $G'/(1-G)$ is increasing. Therefore, whenever $H(x)$ is positive the first term on the right hand side of equation (11) is positive. Then

$$H'(x) = 0 \Rightarrow H''(x) > 0.$$

Thus any turning point of $H(x)$ is a minimum. But $H(x) \geq 0$ and $H(\hat{x}) = 0$. Thus there can be no turning points and hence $H(x)$ is decreasing everywhere. Q.E.D.

<u>Proposition 1</u>: Loan Applicant's Indifference Curves

If Assumptions A1-C hold then indifference curves for a loan applicant, $A(L,R) = \bar{A}$, drawn with L on the horizontal axis, have a unique turning point at $L = L^a(R)$ and slope downwards for larger L. Moreover $L^a(R)$ is decreasing in R.

<u>Proof</u>: From equation (4) and (7)

$$A(L,R) = q(\theta,L) \int_{u^*}^{\infty} u\,dG(u) - (1-G(u^*))RL \tag{12}$$

where, from (3)

$$u^* = \frac{RL}{q(\theta,L)} \tag{13}$$

Since q is concave q/L declines with L and hence u^* is an increasing function of L. Also since q is increasing in θ, u^* is a decreasing function of θ. Summarizing, we have

$$\frac{\partial u^*}{\partial L} > 0, \quad \frac{\partial u^*}{\partial \theta} < 0. \tag{14}$$

From (12)

$$\left.\frac{dR}{dL}\right|_A = \frac{-\dfrac{\partial A}{\partial L}}{\dfrac{\partial A}{\partial R}} = \frac{\dfrac{\partial q}{\partial L} \displaystyle\int_{u^*}^{\infty} u\,dG - (1-G(u^*))R}{(1-G(u^*))L} \tag{15}$$

$$= \frac{R}{L}\left[\frac{L}{q}\frac{\partial q}{\partial L}\frac{\displaystyle\int_{u^*}^{\infty} u\,dG}{u^*(1-G(u^*)} - 1\right].$$

Also, by Assumption B1

$$\lim_{L\to\infty} q(\theta,L)/L = 0.$$

Hence $\lim_{L\to\infty} u^* = \infty$.

Integrating by parts

$$\int_{u^*}^{\infty} udG = u^*(1-G(u^*)) + \int_{u^*}^{\infty} (1-G(u)du.$$

Substituting this expression into (15) we therefore obtain

$$\left.\frac{dR}{dL}\right|_A = \frac{R}{L}\left[\frac{L}{q}\frac{\partial q}{\partial L}\left(1 + \frac{H(u^*)}{u^*}\right) - 1\right] \tag{16}$$

where $H(\cdot)$, defined in Lemma 1 is a decreasing function.

By hypothesis the scale elasticity $\epsilon = L/q\ \partial q/\partial L$ is nonincreasing in L. Moreover, from (14) u^* is increasing in L and hence $H(u^*)/u^*$ is decreasing in L. Therefore, from (16)

$$\left.\frac{dR}{dL}\right|_A = 0 \Rightarrow \frac{\partial}{\partial L}\left(\left.\frac{dR}{dL}\right|_A\right) < 0.$$

Thus, drawn with L on the horizontal axis, a loan applicant's indifference curves are strictly quasi-concave.

From Lemma 1 $H(u^*) = 0$ for u^* sufficiently large. Therefore, for L sufficiently large it follows from (16) that

$$\left.\frac{dR}{dL}\right|_A = \frac{R}{L}\left[\frac{L}{q}\frac{\partial q}{\partial L} - 1\right].$$

Thus by Assumption B1, the slope is negative for sufficiently large L.

Finally, from (13), u^* is increasing in R and hence $H(u^*)/u^*$ is decreasing in R. Therefore, from (16)

$$\left.\frac{dR}{dL}\right|_A = 0 \Longrightarrow \frac{\partial}{\partial R}\left(\left.\frac{dR}{dL}\right|_A\right) < 0$$

Thus $L^a(R)$ is decreasing in R. Q.E.D.

Proposition 2: Bank's Zero Iso-Profit Curve

If Assumption B1 holds, the iso-profit curve $\Pi(L,R) = 0$ is everywhere upward sloping.

Proof: From equation (6)

$$\left.\frac{dR}{dL}\right|_{\Pi} = - \frac{\frac{\partial \Pi}{\partial L}}{\frac{\partial \Pi}{\partial R}} = - \frac{R-i + \int_0^{u*} \left[\frac{\partial Q}{\partial L} - R\right] dG(u)}{L(1-G(u*))}$$

$$= - \left[\frac{(R-i)L + \int_0^{u*} (\epsilon(\theta,L,u)Q(\theta,L,u)-RL)dG(u)}{L^2(1-G(u*))}\right]$$

$$= \frac{-\Pi(L,R) + \int_0^{u*} (1-\epsilon)QdG(u)}{L^2(1-G(u*)}.$$

By Assumption B1 the scale elasticity ϵ is less than unity. Therefor e

$$\Pi(L,R) = 0 \Rightarrow \left.\frac{dR}{dL}\right|_{\Pi} > 0. \qquad \text{Q.E.D.}$$

Proposition 3: If Assumptions B1 and B2 hold, an increase in θ shifts the zero iso-profit contour to the right. Moreover the equilibrium loan $L*(\theta)$ is strictly increasing in θ.

Proof: We first show that, for any R, the zero iso-profit curve shifts to the right. Totally differentiating

$$\Pi(L,\tilde{R},\theta) = 0$$

by θ we obtain

$$(17) \qquad \frac{\partial \Pi}{\partial L}\frac{dL}{d\theta} + \frac{\partial \Pi}{\partial \theta} = 0.$$

But, from Proposition 2

$$\left.\frac{dR}{dL}\right|_{\Pi=0} = -\frac{\frac{\partial \Pi}{\partial L}}{\frac{\partial \Pi}{\partial R}} > 0$$

Then, since $\partial\Pi/\partial R$ is positive, $\partial\Pi/\partial L$ is negative. From (5) $\partial\Pi/\partial\theta$ is positive and so, from (17) $dL/d\theta$ is positive.

To complete the proof we must show that the efficient loan size $L^*(\theta)$ is increasing. But $L^*(\theta)$ satisfies the first order condition.

$$E\left\{\frac{\partial}{\partial L} Q(\theta,L,u)\right\} = i.$$

From Assumption B2, $\partial Q/\partial L$ is an increasing function of θ and from Assumption B1 $\partial^2 Q/\partial L^2$ is negative. Differentiating the first order condition totally by θ then yields the desired result. Q.E.D.

<u>Proposition 4</u>: Marginal Willingness to Pay for a Larger Loan (Model I)
Given Assumptions A1, A2, B1 and C the increase in interest rate that a loan applicant is willing to accept in order to receive a loan is greater for more desirable projects (higher θ). Formally,

$$\frac{\partial}{\partial\theta}\left.\frac{dR}{dL}\right|_{A} = \frac{\partial}{\partial\theta}\left(-\frac{\partial A/\partial L}{\partial A/\partial R}\right) > 0.$$

<u>Proof</u>: From (16)

$$\left.\frac{dR}{dL}\right|_{A} = \frac{R}{L}\left[\epsilon\left(\frac{H(u^*)}{u^*}\right) - 1\right].$$

From (14) u^* declines as θ increases. Therefore, since $H(\cdot)$ is a decreasing function, $H(u^*)/u^*$ increases as θ increases. Finally, given

Assumption B2, the scale elasticity, ϵ, is nondecreasing in θ. Therefore

$$\frac{\partial}{\partial\theta}\left(\left.\frac{dR}{dL}\right|_{A}\right) > 0. \qquad \text{Q.E.D.}$$

<u>Proposition 6</u>: Marginal Willingness to Accept a Smaller Loan (Model II) Given Assumptions A1, A2 and C′, the decrease in interest rate that a loan applicant requires, in order to accept a smaller loan, is smaller for more desirable projects (greater θ).

<u>Proof</u>: From (4), the expected gain to a loan applicant is

$$(18) \qquad \bar{A}(L,R) = \int_{u*}^{\infty} (\beta(L)L + uL + \theta - RL)dG(u)$$

where

$$(19) \qquad \beta(L)L + u*L + \theta - RL = 0.$$

Then

$$\left.\frac{dR}{dL}\right|_{\bar{A}} = -\frac{\frac{\partial\bar{A}}{\partial L}}{\frac{\partial\bar{A}}{\partial R}} = \int_{u*}^{\infty} \frac{[\beta+L\beta' + u - R]\ dG(u)}{(1-G(u*))L}$$

$$= \left[\beta + L\beta' - R + \frac{\int_{u*}^{\infty} udG(u)}{(1-G(u*))}\right]/L$$

$$= [\beta + L\beta' - R + u* + H(u*)]/L.$$

Hence

$$\frac{\partial}{\partial\hat{u}}\left(\left.\frac{dR}{dL}\right|_{\bar{A}}\right) = \frac{1 + H'(u*)}{L}.$$

From (19) an increase in θ lowers $u*$. From (10), $1 + H'(u*)$ is positive. Therefore

$$\frac{\partial}{\partial \theta}\left(\frac{dR}{dL}\bigg|_{\tilde{A}}\right) < 0. \qquad \text{Q.E.D.}$$

Lemma 2: If $\tilde{X}$ and $\tilde{Y}$ have the same mean and the former has a fatter right tail then

(i) $\tilde{X}$ is a mean preserving spread of $\tilde{Y}$, and

(ii) for all a $E\{\tilde{X}|\tilde{X} \geq a\} \geq E\{\tilde{Y}|\tilde{Y} \geq a\}$.

Proof: Without loss of generality we may assume that realizations of both random variables lie in the interval $[0,1]$. Let the c.d.fs be $G_X(\cdot)$ and $G_Y(\cdot)$. Since $\tilde{X}$ and $\tilde{Y}$ have the same mean, there must be some α such that

$$G_X(\alpha) = G_Y(\alpha) < 1.$$

Suppose that, for some $a < \alpha$,

$$\text{Prob}\{\tilde{X} \geq a\} = 1 - G_X(a) > 1 - G_Y(a) = \text{Prob}\{\tilde{Y} \geq a\}.$$

By the fat tail property it follows that, for all $b > a$

$$\frac{1-G_X(b)}{1-G_X(a)} \geq \frac{1-G_Y(b)}{1-G_Y(a)} .$$

But this cannot be true at $b = \alpha$. Therefore

$$G_X(a) > G_Y(a), \quad \text{for all} \quad a < \alpha.$$

Moreover, since this inequality holds for each intersection point α, and since

$$\int_0^1 G_X(a)da = \int_0^1 G_Y(a)da,$$

it follows that there can be only one such intersection point. Q.E.D.(i)

To complete the proof we next note that

$$E\{\tilde{X}|\tilde{X}\geq a\} = \int_a^1 \frac{vdG_X(v)}{1-G_X(a)} = \int_a^1 \left(\frac{1-G_X(v)}{1-G_X(a)}\right)dv + a.$$

If $a > \alpha$, $G_X(a) < G_Y(a)$ and (ii) follows immediately from the fat tail assumption. If $a < \alpha$,

$$E\{\tilde{X}|\tilde{X}\geq a\} > \int_a^1 \left(\frac{1-G_X(v)}{1-G_Y(a)}\right)dv, \quad \text{since} \quad G_Y(a) < G_X(a)$$

$$\geq \int_a^1 \left(\frac{1-G_Y(v)}{1-G_Y(a)}\right)dv, \quad \text{since } \tilde{X} \text{ is a mean preserving spread of } \tilde{Y}$$

$$= E\{\tilde{Y}|\tilde{Y} \geq a\}$$ Q.E.D.(ii)

Corollary 3: Proposition 6 continues to hold when Assumption C′ is replaced by Assumption C".

Proof: $$A(L,R,\theta) = \int_{v*}^1 (q(L)v-RL)dG_\theta(v)$$

where $q(L)v* = RL$. Hence

$$\left.\frac{dR}{dL}\right|_A = \frac{\partial A}{\partial L} \Big/ \frac{\partial A}{\partial R} = \frac{\partial q}{\partial L} \int_{v*}^1 \frac{vdG_\theta(v)}{1-G(v*)} = \frac{\partial q}{\partial L}E\{v|v\geq v*\} - R.$$

The result then follows directly from Lemma 2. Q.E.D.

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