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RENEGOTIATION CAN WORSEN WELFARE: AN EXAMPLE

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I. Introduction

The existence of contracting is an integral factor in a wide range of economic settings, and has become a popular topic of analysis among economists. One recent branch of this literature concentrates on the idea that transaction costs preclude the writing of complex contingent claims contracts. Our paper falls into this category, and in particular considers the role played by renegotiation.

The standard role assigned to renegotiation is as a substitute for complex contingent claims contracts (see e.g., Macaulay 1963 and Shavell 1984). This implies that parties to an agreement tend to be better off in an environment where renegotiation is possible, rather than one where it is prohibitively costly. In this paper we will demonstrate that it is actually possible for the opposite to be true. We demonstrate this by employing a variant of the hold-up problem (see Klein, Crawford and Alchian 1978, and Williamson 1975 for discussions of the hold-up problem). Suppose there is a third party whose level of specific investment depends on expectations concerning the future behavior of parties who have a written agreement. If renegotiation is possible, then the potential investor will be concerned with the possibility that the contracting parties will renegotiate to the investor's disadvantage. The result being that the third party will likely have a "low" level of specific investment. In turn, the contracting parties may find that, when the effect on the investor's specific investment is considered, the renegotiation actually makes the contracting parties worse off. This then explains why the parties to an agreement might in fact be better off in an environment where renegotiation is prohibitively costly. That is, if it is impossible to renegotiate then parties to an agreement can bind themselves through the initial contract, and thus avoid behavior which is ex ante non-optimal but ex post attractive.[1]

The outline for the paper is as follows. In Section II we present a simple two period cartel model, wherein prior to period one the members of the cartel meet and agree on a contract which specifies both first and second period cartel prices. In Section III we analyze the model under the assumption that consuming the good produced by the cartel requires the purchase of a nondurable complementary good. Here we find that the model is consistent with the typical role assigned to renegotiation. That is, renegotiation serves as a substitute for complex contingent claims contracts, and thus the ability to renegotiate makes the contracting parties better off. In Section IV we assume that consumption requires the purchase of a durable complementary good, as for example does home heating oil, for which the durable complementary good is an oil burning furnace. Here we find that renegotiation no longer serves simply as a substitute for complex contingent claims contracts. Rather, renegotiation now also serves as a way for the cartel members to hold-up the consumers.[2] The result is that, as is described above, the ability to renegotiate can actually make the contracting parties worse off. Section V presents some concluding remarks.

II. The Model

We consider a two period cartel problem, where the cartel consists of n identical producers of a homogeneous commodity. There are two distinctive aspects of the particular model we analyze. First, prior to period one the members of the cartel meet and agree on a contract which specifies both first and second period cartel prices. Second, consuming the good produced by the cartel requires the purchase of a complementary good.

We begin by characterizing the demand side of the model. Individuals in this model derive utility from the consumption of two goods. The first good, denoted X, is produced by the cartel, while the second is a composite good, denoted B. The i^{th} individual's consumption of goods B and X in period t are denoted as b_i^t and x_i^t respectively. It is assumed that the price of B is the same in each period, and to simplify the exposition we normalize the price to one. It is also assumed that the consumption of B exhibits constant marginal utility. This assumption simplifies the analysis, while leaving the qualitative nature of the results unchanged. As opposed to the consumption of B, the consumption of X is assumed to exhibit decreasing marginal utility. The distinctive characteristic of X, however, is that its consumption requires the purchase of a complementary good. The nature of this complementary good is what varies between Sections III and IV, and in the beginning of each section we discuss more fully the assumptions which pertain therein.

Formally, the i^{th} individual's two period utility function, which is assumed to be additively separable over time, is given by equation (1).

$$(1) \qquad U_i = \sum_{t=1}^{2} b_i^t + v_i F(x_i^t),$$

where $F(0) = 0$, $F'(0) = \infty$, $F'(\infty) = 0$ and $F'' < 0$. Equation (1) tells us that individuals are perfectly identical except for their valuation of good X, where these valuations vary according to the multiplicative factor denoted v_i for individual i. The distribution of v_i's in the population is described by a density $g(v_i)dv_i$ defined on the interval $[0,\overline{v}]$, where $g(.)$ is assumed to be continuously differentiable and nonzero everywhere in the specified interval. Note, as indicated in (1) we are assuming that consumers have a zero rate of discount, and we also assume there is a zero rate of interest. This is done simply for expositional convenience.

Two aspects of the demand side of the model remain to be specified. First, each consumer i's value for v_i is unobservable to anyone but consumer i. Second, consumers have no way of storing X.

We will now describe the supply side of the model. As indicated earlier, there are n identical producers of X who, prior to period one, meet and agree on first and second period cartel prices. It is assumed that in each period t there is a probability p, $0 < p < 1$, that the cartel members produce X at a constant cost $\underline{c}$ per unit, and a probability (1-p) that the members produce X at a constant cost $\overline{c}$ per unit, where $\overline{c} > \underline{c}$. One point to note is that when the cartel members meet prior to period one, they already know whether it will cost $\underline{c}$ or $\overline{c}$ to produce in period one.

To conclude our description of the model we need to specify the renegotiation cost environment. As indicated earlier, our goal is to comapre what occurs when renegotiation is possible with what occurs when it isn't. We therefore analyze the model under the following two polar assumptions concerning the renegotiation cost environment. In the first environment the cartel members incur no costs in getting together prior to period two and renegotiating. This is referred to as a zero renegotiation cost environment. In the second environment the cartel members incur very high costs in renegotiating prior to period two, with the result being that such renegotiation never occurs. This is referred to as a prohibitively high renegotiation cost environment. Finally, we assume consumers have rational expectations concerning the price of X. This last assumption means that if consumers are in a prohibitively high renegotiation cost environment they anticipate that the second period cartel price will be the price specified in the initial contract, while if they are in a zero renegotiation cost environment they realize that the cartel has the ability to reset its price just prior to period two.

III. Analysis when the Complementary Good is Nondurable

In this section we analyze the model developed in the previous section under the assumption that the complementary good needed to consume X is nondurable. Specifically, for individual i to consume X in period t, that individual must purchase in period t a complementary good which costs an amount q. We also assume there exists an x such that $\bar{v}F(x) > \bar{c}x + q$. This assumption states that, even in high cost states of the world, there are some individuals for whom the consumption of X is socially efficient.

Let P^t be the realized price of X in period t, and for simplicity let all individuals be endowed with the same income, denoted Y. Given the set up of the model, each individual i faces the following two period budget constraint.

$$(2) \qquad \sum_{t=1}^{2} (b_i^t + P^t x_i^t + qK_i^t) \leq Y,$$

where $K_i^t = 1(0)$ if $x_i^t > (=)\ 0$. It is easily demonstrated that (2) must hold as an equality, and therefore each consumer i faces the following maximization problem.[3]

$$(3) \qquad \max_{x_i^1, x_i^2} \; Y + \sum_{t=1}^{2} (v_i F(x_i^t) - P^t x_i^t - qK_i^t)$$

Let $\hat{x}(v_i, P)$ be such that $v_i F'(\hat{x}(v_i, P)) = P$. The following proposition follows immediately from equation (3) . Note, when possible we will simplify expressions by writing $\hat{x}(v_i, P)$ as $\hat{x}_i(P)$.[4]

<u>Proposition 1</u>. $x_i^t = \hat{x}_i(P^t)$ if $v_i F(\hat{x}_i(P^t)) > P^t \hat{x}_i(P^t) + q$, and $x_i^t = 0$ if $v_i F(\hat{x}_i(P^t)) < P^t \hat{x}_i(P^t) + q$.

Proposition 1 tells us two things. First, if an individual consumes X, then he consumes up to the point where the marginal utility of additional consumption equals the price. Second, an individual will consume X if, at this potentially optimal quantity, the individual's valuation of the good exceeds his private cost of consuming it.

We can now proceed to the problem faced by the cartel. This problem is to maximize the expected profits of the cartel, given the constraint that consumer behavior is as described in Proposition 1. Let Π_Z denote the expected profits of the cartel under a zero renegotiation cost environment, Π_H denote expected profits under a prohibitively high renegotiation cost environment, and $v^*(P)$ be such that $v^*(P)F(\hat{x}(v^*(P),P)) = P\hat{x}(v^*(P),P) + q$.[5] We will first consider what occurs under a zero renegotiation cost environment. In this environment the cartel can reset P^2 after observing whether its unit cost of production is $\underline{c}$ or $\bar{c}$. This means that P^1 and P^2 are determined in completely symmetric fashions. That is Π_Z is given by (4).

$$(4)\quad \Pi_Z = 2[\max_P \left(p\int_{v^*(P)}^{\bar{v}} (P-\underline{c})\hat{x}_i(P)g(v_i)dv_i\right) + \max_P \left((1-p)\int_{v^*(P)}^{\bar{v}} (P-\bar{c})\hat{x}_i(P)g(v_i)dv_i\right)]$$

We will now consider what occurs under a prohibitively high renegotiation cost environment. In this environment P^2 equals the second period cartel price specified in the initial contract. Thus Π_H is given by (5).

$$(5)\quad \Pi_H = \max_P \left(p\int_{v^*(P)}^{\bar{v}} (P-\underline{c})\hat{x}_i(P)g(v_i)dv_i\right) + \max_P \left((1-p)\int_{v^*(P)}^{\bar{v}} (P-\bar{c})\hat{x}_i(P)g(v_i)dv_i\right)$$
$$+ \max_P \left(p\int_{v^*(P)}^{\bar{v}} (P-\underline{c})\hat{x}_i(P)g(v_i)dv_i + (1-p)\int_{v^*(P)}^{\bar{v}} (P-\bar{c})\hat{x}_i(P)g(v_i)dv_i\right)$$

A comparison of (4) and (5) leads us to the main proposition of this section.

Proposition 2. If $\bar{c} > \underline{c}$, then $\Pi_Z > \Pi_H$.

Proof: Given (4) and (5), we know the only other possibility is $\Pi_Z = \Pi_H$. This will occur if $\underline{P} = \bar{P}$, where $\underline{P} = \arg\max_P \int_{v^*(P)}^{\bar{v}} (P-\underline{c})\hat{x}_i(P)g(v_i)dv_i$ and $\bar{P} = \arg\max_P \int_{v^*(P)}^{\bar{v}} (P-\bar{c})\hat{x}_i(P)g(v_i)dv$. Taking first order conditions yields

$$\text{(6a)}\qquad \int_{v^*(\underline{P})}^{\bar{v}} \left(\hat{x}_i(\underline{P}) + (\underline{P}-\underline{c}) \frac{d\hat{x}_i(P)}{dP}\right)g(v_i)dv_i - (\underline{P}-\underline{c})\hat{x}_i(\underline{P})g(v^*(\underline{P})) \frac{dv^*(\underline{P})}{dP} = 0,$$

and

$$\text{(6b)}\qquad \int_{v^*(\bar{P})}^{\bar{v}} \left(\hat{x}_i(\bar{P}) + (\bar{P}-\bar{c}) \frac{d\hat{x}_i(\bar{P})}{dP}\right)g(v_i)dv_i - (\bar{P}-\bar{c})\hat{x}_i(\bar{P})g(v^*(\bar{P})) \frac{dv^*(\bar{P})}{dP} = 0.$$

Because $\frac{d\hat{x}_i(P)}{dP} < 0$ and $\frac{dv^*(P)}{dP} > 0$, (6a) and (6b) imply $\underline{P} \neq \bar{P}$. Thus $\Pi_Z > \Pi_H$.

Proposition 2 simply demonstrates the standard view of renegotiation. That is, renegotiation serves as a substitute for complex contingent claims contracts, and thus the ability to renegotiate makes the contracting parties better off.[6] In the following section we demonstrate that, when the complementary good needed to consume X is durable, our model is no longer so supportive of the typical role assigned to renegotiation.

IV. Analysis when the Complementary Good is Durable

In this section we analyze the model developed in the previous section under the assumption that the complementary good needed to consume X is durable. The specifics concerning the complementary good are as follows. For individual i to consume X in period 1, that individual must purchase in period 1 a complementary good which costs an amount $2q$. If individual i makes this purchase in period 1, then to consume X in period 2 he need make no further purchase of the complementary good. If, however, he does not purchase the complementary good in period 1, then to consume X in period 2 he must purchase in period 2 the complementary good which again costs an amount $2q$. Finally, we retain the assumption that there exists an x such that $\bar{v}F(x) > \bar{c}x + q$, and we also assume that the salvage value of the complementary good is always zero.

Let $L_i^t = 1$ if period t is the first period consumer i purchases a positive quantity of X, and let $L_i^t = 0$ otherwise. Each individual i now faces the following two period budget constraint.

$$\sum_{t=1}^{2} (b_i^t + P^t x_i^t + 2qL_i^t) \leq Y \tag{7}$$

It is easily demonstrated that (7) must hold as an equality, and therefore the budget constraint can be substituted directly into the utility function, i.e.,

$$U_i = Y + \sum_{t=1}^{2} (v_i F(x_i^t) - P^t x_i^t - 2qL_i^t). \tag{8}$$

Equation (8) immediately yields the following proposition.

<u>Proposition 3</u>. $x_i^2 = \begin{cases} \hat{x}_i(P^2) & \text{if } x_i^1 > 0 \text{ or } v_i F(\hat{x}_i(P^2)) > P^2\hat{x}_i(P^2) + 2q \\ 0 & \text{if } x_i^1 = 0 \text{ and } v_i F(\hat{x}_i(P^2)) < P^2\hat{x}_i(P^2) + 2q \end{cases}$

Proposition 3 is similar to Proposition 1 of Section II. It first tells us that if an individual consumes X in period 2, then he consumes up to the point where the marginal utility of additional consumption equals the price. Second, it states that an individual won't consume X in period 2 if he did not consume X in period 1, and if the cost of the complementary good is prohibitive relative to the returns from consuming X.

The period 1 consumption decision is somewhat more complex. The reason is that the period 1 consumption decision depends on expectations concerning the period 2 price. As mentioned earlier, to analyze this aspect of the problem we assume that consumers have rational expectations. This means that if consumers are in a prohibitively high renegotiation cost environment they anticipate that the second period cartel price will be the price specified in the initial contract, while if they are in a zero renegotiation cost environment they realize that the cartel has the ability to reset P^2 just prior to period 2.

Our next step is to separately analyze what happens under the two different types of environments. We first consider the prohibitively high renegotiation cost environment. In this environment there is no uncertainty concerning what the second period cartel price will be. This, in turn, yields the following proposition.

<u>Proposition 4</u>. Under a prohibitively high renegotiation cost environment $x_i^1 = \hat{x}_i(P^1)$ if $v_i F(\hat{x}_i(P^1)) + v_i F(\hat{x}_i(P^2)) > P^1\hat{x}_i(P^1) + P^2\hat{x}_i(P^2) + 2q$, while $x_i^1 = 0$ if $v_i F(\hat{x}_i(P^1)) + v_i F(\hat{x}_i(P^2)) < P^1\hat{x}_i(P^1) + P^2\hat{x}_i(P^2) + 2q$.

<u>Proof</u>: Suppose $v_i F(\hat{x}_i(P^2)) > P^2\hat{x}_i(P^2) + 2q$. This implies $v_i F(\hat{x}_i(P^1)) + v_i F(\hat{x}_i(P^2)) > P^1\hat{x}_i(P^1) + P^2\hat{x}_i(P^2) + 2q$. We also know from Proposition 3 that $x_i^2 = \hat{x}_i(P^2)$. This tells us that there is no cost in terms of the durable good of buying X in period 1, which given (8) yields $x_i^1 = \hat{x}_i(P^1)$. Suppose $v_i F(\hat{x}_i(P^2)) < P^2\hat{x}_i(P^2) + 2q$. Proposition 3 tells us that $x_i^2 = \hat{x}_i(P^2)$ if $x_i^1 > 0$, and $x_i^2 = 0$ if $x_i^1 = 0$. Given this, (8) in turn implies

$x_i^1 = \hat{x}_i(P^1)$ if $v_i F(\hat{x}_i(P^1)) + v_i F(\hat{x}_i(P^2)) > P^1\hat{x}_i(P^1) + P^2\hat{x}_i(P^2) + 2q$, while $x_i^1 = 0$ if $v_i F(\hat{x}_i(P^1)) + v_i F(\hat{x}_i(P^2)) < P^1\hat{x}_i(P^1) + P^2\hat{x}_i(P^2) + 2q$.

We can now derive an expression for the expected profits of the cartel under a prohibitively high renegotiation cost environment. Let $v'(P^1,P^2)$ be such that $v'(P^1,P^2)F(\hat{x}(v'(P^1,P^2),P^1)) + v'(P^1,P^2)F(\hat{x}(v'(P^1,P^2),P^2)) = P^1\hat{x}(v'(P^1,P^2),P^1) + P^2\hat{x}(v'(P^1,P^2),P^2) + 2q$.[7] Propositions 3 and 4 yield (9). Note, in the following we delete the arguments of v'.

$$\Pi_H = p\max_{P^1,P^2} \left[\int_{v'}^{\overline{v}} (P^1-\underline{c})\hat{x}_i(P^1)g(v_i)dv_i + J(P^1,P^2)\right] \tag{9}$$

$$+ (1-p) \max_{P^1,P^2} \left[\int_{v'}^{\overline{v}} (P^1-\overline{c})\hat{x}_i(P^1)g(v_i)dv_i + J(P^1,P^2)\right],$$

where $J(P^1,P^2) = p\int_{v'}^{\overline{v}}(P^2-\underline{c})\hat{x}_i(P^2)g(v_i)dv_i + (1-p)\int_{v'}^{\overline{v}} (P^2-\overline{c})\hat{x}_i(P^2)g(v_i)dv_i$.

We now consider the zero renegotiation cost environment. In this environment the cartel can reset P^2 after observing whether the second period cost of production is $\underline{c}$ or $\overline{c}$. Let $\underline{P}^2$ denote the second period price which results when the cost of production is $\underline{c}$, and $\overline{P}^2$ denote the second period price which results when the cost of production is $\overline{c}$. Our rational expectations assumption yields the following proposition. Note, because of the complexity of the problem, we restrict Proposition 5 to the case $2q + \underline{c}\hat{x}(\overline{v},\underline{c}) > \overline{v}F(\hat{x}(\overline{v},\underline{c}))$. This restriction guarantees that consumers never purchase X in the second period without purchasing X in the first period. In the Appendix we present Proposition 5a, which is simply Proposition 5 in the absence of this restriction.

Proposition 5. Consider a zero renegotiation cost environment wherein $2q + \underline{c}\hat{x}(\bar{v},\underline{c}) > \bar{v}F(\hat{x}(\bar{v},\underline{c}))$. In this situation $x_i^1 = \hat{x}_i(P^1)$ if $v_iF(\hat{x}_i(P^1)) + pv_iF(\hat{x}_i(\underline{P}^2)) + (1-p)v_iF(\hat{x}_i(\bar{P}^2)) > P^1\hat{x}_i(P^1) + p\underline{P}^2\hat{x}_i(\underline{P}^2) + (1-p)\bar{P}^2\hat{x}_i(\bar{P}^2) + 2q$, and $x_i^1 = 0$ if $v_iF(\hat{x}_i(P^1)) + pv_iF(\hat{x}_i(\underline{P}^2)) + (1-p)v_iF(\hat{x}_i(\bar{P}^2)) < P^1\hat{x}_i(P^1) + p\underline{P}^2\hat{x}_i(\underline{P}^2) + (1-p)\bar{P}^2\hat{x}_i(\bar{P}^2) + 2q$.

Proof: Because the cartel can reset P^2 after the first period consumption decisions have already been made, we know $\underline{P}^2, \bar{P}^2 > \underline{c}$. Given the restriction $2q + \underline{c}\hat{x}(\bar{v},\underline{c}) > \bar{v}F(\hat{x}(\bar{v},\underline{c}))$, this, in turn, implies $v_iF(\hat{x}_i(\underline{P}^2)) < \underline{P}^2\hat{x}_i(\underline{P}^2) + 2q$ for all i, and $v_iF(\hat{x}_i(\bar{P}^2)) < \bar{P}^2\hat{x}_i(\bar{P}^2) + 2q$ for all i. Given this, Proposition 3 yields $x_i^2 = 0$ unless $x_i^1 > 0$. Using (8) this implies $x_i^1 = \hat{x}_i(P^1)$ if $v_iF(\hat{x}_i(P^1)) + pv_iF(\hat{x}_i(\underline{P}^2)) + (1-p)v_iF(\hat{x}_i(\bar{P}^2)) > P^1\hat{x}_i(P^1) + p\underline{P}^2\hat{x}_i(\underline{P}^2) + (1-p)\bar{P}^2\hat{x}_i(\bar{P}^2) + 2q$, and $x_i^1 = 0$ if $v_iF(\hat{x}_i(P^1)) + pv_iF(\hat{x}_i(\underline{P}^2)) + (1-p)v_iF(\hat{x}_i(\bar{P}^2)) < P^1\hat{x}_i(P^1) + p\underline{P}^2\hat{x}_i(\underline{P}^2) + (1-p)\bar{P}^2\hat{x}_i(\bar{P}^2) + 2q$.

We can now use Proposition 5 to derive an expression for the expected profits of the cartel under a zero renegotiation cost environment. Let $v''(P^1,\underline{P}^2,\bar{P}^2)F(\hat{x}_i(\underline{P}^2))$ be such that $v''(P^1,\underline{P}^2,\bar{P}^2)F(\hat{x}_i(P^1)) + pv''(P^1,\underline{P}^2,\bar{P}^2) + (1-p)v''(P^1,\underline{P}^2,\bar{P}^2)F(\hat{x}_i(\bar{P}^2)) = P^1\hat{x}_i(P^1) + p\underline{P}^2\hat{x}_i(\underline{P}^2) + (1-p)\bar{P}^2\hat{x}_i(\bar{P}^2) + 2q.$[8] Propositions 3 and 5 yield that if $2q + \underline{c}\hat{x}(\bar{v},\underline{c}) > \bar{v}F(\hat{x}(\bar{v},\underline{c}))$, then Π_Z is defined by (10). Note, in the Appendix we present equation (10a), which is simply (10) in the absence of this restriction. Also, in the following we delete the arguments of v''.[9]

$$(10) \quad \Pi_Z = p \max_{P^1,\underline{P}^2,\bar{P}^2} \left[\int_{v''}^{\bar{v}} (P^1-\underline{c})\hat{x}_i(P^1)g(v_i)dv_i + pM(P^1,\underline{P}^2,\bar{P}^2) + (1-p)N(P^1,\underline{P}^2,\bar{P}^2)\right]$$

$$\text{s.t.} \quad \underline{P}^2 = \arg\max_P \hat{M}(P;P^1,\underline{P}^2,\bar{P}^2)$$

$$\bar{P}^2 = \arg\max_P \hat{N}(P;P^1,\underline{P}^2,\bar{P}^2)$$

$$+ (1-p) \max_{P^1,\underline{P}^2,\bar{P}^2} \left[\int_{v''}^{\bar{v}} (P^1-\bar{c})\hat{x}_i(P^1)g(v_i)dv_i + pM(P^1,\underline{P}^2,\bar{P}^2) + (1-p)N(P^1,\underline{P}^2,\bar{P}^2)\right]$$

$$\text{s.t.} \quad \underline{P}^2 = \arg\max_P \hat{M}(P;P^1,\underline{P}^2,\bar{P}^2)$$

$$\bar{P}^2 = \arg\max_P \hat{N}(P;P^1,\underline{P}^2,\bar{P}^2),$$

where $M(P^1,\underline{P}^2,\bar{P}^2) = \int_{v''}^{\bar{v}} (\underline{P}^2-\underline{c})\hat{x}_i(\underline{P}^2)g(v_i)dv_i$, $\hat{M}(P;P^1,\underline{P}^2,\bar{P}^2) = \int_{v''}^{\bar{v}} (P-\underline{c})\hat{x}_i(P)g(v_i)dv_i$, $N(P^1,\underline{P}^2,\bar{P}^2) = \int_{v''}^{\bar{v}} (\bar{P}^2-\bar{c})\hat{x}_i(\bar{P}^2)g(v_i)dv_i$, and $\hat{N}(P;P^1,\underline{P}^2,\bar{P}^2) = \int_{v''}^{\bar{v}} (P-\bar{c})\hat{x}_i(P)g(v_i)dv_i$.

By considering equations (9) and (10) we can see that the basic distinction between the prohibitively high renegotiation cost environment and the zero renegotiation cost environment is the same as in Section III. That is, under the zero renegotiation cost environment the cartel can make P^2 contingent on the period 2 cost realization, while under the prohibitively high renegotiation cost environment this is not feasible. Notice, however, there is one difference. Under the zero renegotiation cost environment in Section III there was no constraint on the manner with which P^2 could be made contingent on the period 2 cost realization. Here there are constraints, as evidenced by the four incentive compatibility constraints contained in equation (10). The logic is that the cartel is now constrained to a second period price which is optimal given the consumer purchases in period 1, and such a price will not in general be the optimal ex ante price. In the following we demonstrate that this latter factor can be the dominant one. That is, because under the zero renegotiation cost environment the cartel is constrained to ex post optimal prices, it is possible that the cartel will actually be worse off when it has the ability to renegotiate.

Proposition 6. If $\bar{c} = \underline{c}$, then $\Pi_Z < \Pi_H$.

Proof: Given $\bar{c} = \underline{c}$, (9) reduces to

$$\Pi_H = \max_{P^1,P^2} \left[\int_{v'}^{\bar{v}} (P^1-\underline{c})\hat{x}_i(P^1)g(v_i)dv_i + \int_{v'}^{\bar{v}} (P^2-\underline{c})\hat{x}_i(P^2)g(v_i)dv_i\right], \tag{11}$$

while (10a) reduces to

$$\Pi_Z = \max_{P^1,P^2} \left[\int_{v'}^{\bar{v}} (P^1-\underline{c})\hat{x}_i(P^1)g(v_i)dv_i + \int_{v'}^{\bar{v}} (P^2-\underline{c})\hat{x}_i(P^2)g(v_i)dv_i\right] \tag{12}$$

$$\text{s.t. } P^2 = \arg\max_{P} \int_{v'}^{\bar{v}} (P-\underline{c})\hat{x}_i(P)g(v_i)dv_i.$$

Denote the price vector which solves (11) as (P_H^1, P_H^2), and the price vector which solves (12) as (P_Z^1, P_Z^2). (11) and (12) yield $\Pi_Z < \Pi_H$ unless $P_H^1 = P_Z^1$ and $P_H^2 = P_Z^2$.[10] The first order conditions for (11) are

$$\int_{v'}^{\bar{v}} \left(\hat{x}_i(P^1) + (P^1-\underline{c}) \frac{d\hat{x}_i(P^1)}{dP^1}\right)g(v_i)dv_i - \frac{dv'}{dP^1}(P^1-\underline{c})\hat{x}_i(P^1)g(v') = 0 \tag{13}$$

and

$$\int_{v'}^{\bar{v}} \left(\hat{x}_i(P^2) + (P^2-\underline{c}) \frac{d\hat{x}_i(P^2)}{dP^2}\right)g(v_i)dv_i - \frac{dv'}{dP^2}(P^2-\underline{c})\hat{x}_i(P^2)g(v') = 0, \tag{14}$$

while the first order condition for the maximization problem in the constraint in (12) is

$$\int_{v'}^{\bar{v}} \left(\hat{x}_i(P^2) + (P^2-\underline{c}) \frac{d\hat{x}_i(P^2)}{dP^2}\right)g(v_i)dv_i = 0. \tag{15}$$

A comparison of (14) and (15) yields it is not the case that both $P_H^1 = P_Z^1$

and $P_H^2 = P_Z^2$. Thus, $\Pi_Z < \Pi_H$.

The intuition behind Proposition 6 is straightforward. As indicated earlier, there are two factors which determine whether the cartel is more profitable with or without the ability to renegotiate. On the one hand, the cartel's capacity under the zero renegotiation cost environment to make the second period price contingent on the second period cost realization tends to make the cartel more profitable when it has the ability to renegotiate. On the other hand, the fact that under the zero renegotiation cost environment the cartel is restricted to second period prices which are ex post optimal tends to make the cartel more profitable when it does not have the ability to renegotiate. In Proposition 6 we have restricted the analysis to situations where there is no cost uncertainty. This means that the first factor is absent, and thus the cartel is better off when it does not have the ability to renegotiate. In ending this section we demonstrate that even when cost uncertainty is present, it is possible for the cartel to be worse off when it has the ability to renegotiate. Note, because of the complexity of the problem when cost uncertainty is present, in the following we consider a somewhat simplified version of this section's model.

Let $g(.)$ be a uniform density function, i.e., $g(v_1) = \hat{g}$ for all $0 < v_1 < \bar{v}$, and let X be an indivisible commodity such that each individual consumes either zero units or one unit of X, where $F(0) = 0$ and $F(1) = 1$. Also, let there be no cost uncertainty in the first period such that the unit cost of production in that period equals $\underline{c}$ with probability one, while for the second period the unit cost of production remains $\underline{c}$ with probability p and $\bar{c}$ with probability $(1-p)$. Finally, we assume $\bar{v} > \bar{c} + q$. This assumption is analogous to our previous assumption that there exists an x such that $\bar{v}F(x) > \bar{c}x + q$. Analysis of this specification yields the following.

Proposition 7. There exists a value q^*, $0 < q^* < \frac{\bar{v}+\bar{c}}{4}$, such that if $q >(<) q^*$, then $\Pi_H >(<) \Pi_Z$.

Proof: See Appendix.

Proposition 7 tells us that, as regards whether the cartel is more profitable with or without the ability to renegotiate, it is possible for either to be true. When the cost of the complementary good is "large," then the hold-up problem is the dominant factor and the end result is that Π_H exceeds Π_Z. On the other hand, when the cost of the complementary good is "small," then the ability to make the second period price contingent on the second period cost realization is the dominant factor, and the end result is that Π_Z exceeds Π_H.

V. Conclusion

In terms of the welfare of the contracting parties, the standard interpretation is that having the ability to renegotiate is a positive factor. The logic is that renegotiation serves as a substitute for complex contingent claims contracts. In this paper we have demonstrated that having the ability to renegotiate can actually lower the welfare of the contracting parties. We demonstrated this by employing a variant of the hold-up problem. Suppose there is a third party whose level of specific investment depends on expectations concerning the future behavior of parties who have a written agreement. In such an environment the contracting parties may find it to their advantage to guarantee that in the future they will not hold-up the third party, and thus the parties to the agreement might in fact be better off in an environment where renegotiation is not possible. That is, if it is impossible to renegotiate then parties to an agreement can bind themselves through the initial contract, and in this way avoid behavior which is ex ante non-optimal but ex post attractive.

In the present paper we formalized the above argument in a two period cartel model, wherein prior to period one the members of the cartel meet and agree on a contract which specifies both first and second period cartel prices. We showed that when consumption of the cartel's output requires the purchase of a durable complementary good, then having the ability to renegotiate could actually make the cartel worse off. That is, consistent with the above discussion, not having the ability to renegotiate in the presence of a durable complementary good could be beneficial, because the cartel members could then guarantee that they would not hold-up the consumers. Note, finally, although the formal analysis of the paper is confined to a specific cartel environment, our feeling is that the basic result concerning renegotiation is applicable to a wide variety of contracting situations. In fact, as suggested in footnote 1, the result should be applicable even more broadly than the third party hold-up problem considered here.

Appendix

Proposition 5a: Under a zero renegotiation cost environment $x_i^1 = \hat{x}_i(P^1)$ if i), iii), iv), or vi) holds, while $x_i^1 = 0$ if ii), v), or vii) holds.

i) $v_i F(\hat{x}_i(\underline{P}^2)) < \underline{P}^2 \hat{x}_i(\underline{P}^2) + 2q$, $v_i F(\hat{x}_i(\overline{P}^2)) < \overline{P}^2 \hat{x}_i(\overline{P}^2) + 2q$, and

$v_i F(\hat{x}_i(P^1)) + p v_i F(\hat{x}_i(\underline{P}^2)) + (1-p) v_i F(\hat{x}_i(\overline{P}^2)) > P^1 \hat{x}_i(P^1)$

$+ p\underline{P}^2 \hat{x}_i(\underline{P}^2) + (1-p)\overline{P}^2 \hat{x}_i(\overline{P}^2) + 2q$;

ii) $v_i F(\hat{x}_i(\underline{P}^2)) < \underline{P}^2 \hat{x}_i(\underline{P}^2) + 2q$, $v_i F(\hat{x}_i(\overline{P}^2)) < \overline{P}^2 \hat{x}_i(\overline{P}^2) + 2q$, and

$v_i F(\hat{x}_i(P^1)) + p v_i F(\hat{x}_i(\underline{P}^2)) + (1-p) v_i F(\hat{x}_i(\overline{P}^2)) < P^1 \hat{x}_i(P^1)$

$+ p\underline{P}^2 \hat{x}_i(\underline{P}^2) + (1-p)\overline{P}^2 \hat{x}_i(\overline{P}^2) + 2q$;

iii) $v_i F(\hat{x}_i(\underline{P}^2)) > \underline{P}^2 \hat{x}_i(\underline{P}^2) + 2q$ and $v_i F(\hat{x}_i(\overline{P}^2)) > \overline{P}^2 \hat{x}_i(\overline{P}^2) + 2q$;

iv) $v_i F(\hat{x}_i(\underline{P}^2)) > \underline{P}^2 \hat{x}_i(\underline{P}^2) + 2q$, $v_i F(\hat{x}_i(\overline{P}^2)) < \overline{P}^2 \hat{x}_i(\overline{P}^2) + 2q$, and

$v_i F(\hat{x}_i(P^1)) + (1-p) v_i F(\hat{x}_i(\overline{P}^2)) > P^1 \hat{x}_i(P^1) + (1-p)\overline{P}^2 \hat{x}_i(\overline{P}^2) + (1-p)2q$;

v) $v_i F(\hat{x}_i(\underline{P}^2)) > \underline{P}^2 \hat{x}_i(\underline{P}^2) + 2q$, $v_i F(\hat{x}_i(\overline{P}^2)) < \overline{P}^2 \hat{x}_i(\overline{P}^2) + 2q$, and

$v_i F(\hat{x}_i(P^1)) + (1-p) v_i F(\hat{x}_i(\overline{P}^2)) < P^1 \hat{x}_i(P^1) + (1-p)\overline{P}^2 \hat{x}_i(\overline{P}^2) + (1-p)2q$;

vi) $v_i F(\hat{x}_i(\underline{P}^2)) < \underline{P}^2 \hat{x}_i(\underline{P}^2) + 2q$, $v_i F(\hat{x}_i(\overline{P}^2)) > \overline{P}^2 \hat{x}_i(\overline{P}^2) + 2q$, and

$v_i F(\hat{x}_i(P^1)) + p v_i F(\hat{x}_i(\underline{P}^2)) > P^1 \hat{x}_i(P^1) + p\underline{P}^2 \hat{x}_i(\underline{P}^2) + p2q$;

vii) $v_i F(\hat{x}_i(\underline{P}^2)) < \underline{P}^2 \hat{x}_i(\underline{P}^2) + 2q$, $v_i F(\hat{x}_i(\overline{P}^2)) > \overline{P}^2 \hat{x}_i(\overline{P}^2) + 2q$, and

$v_i F(\hat{x}_i(P^1)) + p v_i F(\hat{x}_i(\underline{P}^2)) < P^1 \hat{x}_i(P^1) + p\underline{P}^2 \hat{x}_i(\underline{P}^2) + p2q$.

Proof: i) and ii) follow from the proof of Proposition 5. Consider iii). Proposition 3 yields $x_i^2 = \hat{x}_i(P^2)$, which using (8) yields $x_i^1 = \hat{x}_i(P^1)$. Consider iv) and v). Proposition 3 yields $x_i^2 = \hat{x}_i(\underline{P}^2)$ if the cost of production is $\underline{c}$, while, unless $x_i^1 > 0$, $x_i^2 = 0$ if the cost of production is $\overline{c}$. (8), in turn, yields $x_i^1 = \hat{x}_i(P^1)$ if $v_i F(\hat{x}_i(P^1)) + (1-p)v_i F(\hat{x}_i(\overline{P}^2)) >$ $P^1\hat{x}_i(P^1) + (1-p)\overline{P}^2\hat{x}_i(\overline{P}^2) + (1-p)2q$, while $x_i^1 = 0$ if $v_i F(\hat{x}_i(P^1)) +$ $(1-p)v_i F(\hat{x}_i(\overline{P}^2)) < P^1\hat{x}_i(P^1) + (1-p)\overline{P}^2\hat{x}_i(\overline{P}^2) + (1-p)2q$. The other two cases follow similarly.

We can now consider Π_Z in the absence of the restriction $2q+\underline{c}\hat{x}(\overline{v},\underline{c}) >$ $\overline{v}F(\hat{x}(\overline{v},\underline{c}))$. Let $\hat{v}''(P^1,\underline{P}^2,\overline{P}^2)$ be such that $x_i^1 = \hat{x}_i(P^1)$ if $v_i >$ $\hat{v}''(P^1,\underline{P}^2,\overline{P}^2)$, while $x_i^1 = 0$ if $v_i < \hat{v}''(P^1,\underline{P}^2,\overline{P}^2)$. Note, we are not able to provide a more precise definition because of the complexity of Proposition 5a. Also, let $\hat{v}^*(P)$ be such that $\hat{v}^*(P)F(\hat{x}(\hat{v}^*(P),P)) = P\hat{x}(\hat{v}^*(P),P) + 2q$. Π_Z is now given by (10a). As previously, we delete the arguments of $\hat{v}''$.

$$\text{(10a)}\quad \Pi_Z = p\max_{P^1,\underline{P}^2,\overline{P}^2,\hat{v}''} \left[\int^{\overline{v}}(P^1-\underline{c})\hat{x}_i(P^1)g(v_i)dv_i + pM^*(P^1,\underline{P}^2,\overline{P}^2) + (1-p)N^*(P^1,\underline{P}^2,\overline{P}^2)\right]$$

$$\text{s.t. } \underline{P}^2 = \arg\max_P \hat{M}^*(P;P^1,\underline{P}^2,\overline{P}^2)$$

$$\overline{P}^2 = \arg\max_P \hat{N}^*(P;P^1,\underline{P}^2,\overline{P}^2)$$

$$+ (1-p)\max_{P^1,\underline{P}^2,\overline{P}^2,\hat{v}''} \left[\int^{\overline{v}}(P^1-\overline{c})\hat{x}_i(P^1)g(v_i)dv_i + pM^*(P^1,\underline{P}^2,\overline{P}^2) + (1-p)N^*(P^1,\underline{P}^2,\overline{P}^2)\right]$$

$$\text{s.t. } \underline{P}^2 = \arg\max_P \hat{M}^*(P;P^1,\underline{P}^2,\overline{P}^2)$$

$$\overline{P}^2 = \arg\max_P \hat{N}^*(P;P^1,\underline{P}^2,\overline{P}^2),$$

where $M^*(P^1,\underline{P}^2,\overline{P}^2) = \int_{v^+}^{\overline{v}}(\underline{P}^2-\underline{c})\hat{x}_i(\underline{P}^2)g(v_i)dv_i$, $\hat{M}^*(P;P^1,\underline{P}^2,\overline{P}^2) =$ $\int_{v^+}^{\overline{v}}(P-\underline{c})\hat{x}_i(P)g(v_i)dv_i$, $N^*(P^1,\underline{P}^2,\overline{P}^2) = \int_{v^+}^{\overline{v}}(\overline{P}^2-\overline{c})\hat{x}_i(\overline{P}^2)g(v_i)dv_i$, $\hat{N}^*(P;P^1,\underline{P}^2,\overline{P}^2) = \int_{v^+}^{\overline{v}}(P-\overline{c})\hat{x}_i(P)g(v_i)dv_i$, and $v^+ = \min\{\hat{v}''(P^1,\underline{P}^2,\overline{P}^2), \hat{v}^*(P)\}$.

<u>Proof of Proposition 7</u>. It is easy to demonstrate that $P_H^1 < P_H^2 + 2q$. Thus,

$$(16) \qquad \Pi_H = \max_{P^1,P^2} \hat{g}[\int_{\tilde{v}_H}^{\bar{v}} (P^1-\underline{c})dv_i + p \int_{\tilde{\tilde{v}}_H}^{\bar{v}} (P^2-\underline{c})dv_i + (1-p) \int_{\tilde{\tilde{v}}_H}^{\bar{v}} (P^2-\bar{c})dv_i],$$

where $\tilde{v}_H = P^1 + 2q - \max\{\tilde{v}_H - P^2, 0\}$ and $\tilde{\tilde{v}}_H = \max\{\tilde{v}_H, P^2\}$. This means Π_H is given by (17a) or (17b), where $\tilde{P}_H = P_H^1 + P_H^2$.

$$(17a) \qquad \Pi_H = \hat{g}[(\bar{v} - P_H^1 - 2q)(P_H^1 - \underline{c}) + (\bar{v} - P_H^2)(P_H^2 - p\underline{c} - (1-p)\bar{c})]$$

$$(17b) \qquad \Pi_H = \hat{g}[(\bar{v} - (\frac{\tilde{P}_H+2q}{2}))\ (\tilde{P}_H - (1+p)\underline{c} - (1-p)\bar{c})]$$

It can also be shown that $P_Z^1 < P_Z^2 + 2q$ and $P_Z^1 + 2q > \frac{\bar{v}+\underline{c}}{2}$. Thus,

$$(18) \qquad \Pi_Z = \max_{P^1} \hat{g}[\int_{P^1+2q}^{\bar{v}} (P^1-\underline{c})dv_i + p \int_{P^1+2q}^{\bar{v}} (P^1+2q-\underline{c})dv_i + (1-p)\int_{\tilde{v}_Z}^{\bar{v}} (\tilde{v}_Z-\bar{c})dv_i],$$

where $\tilde{v}_Z = \max\{P^1+2q, \frac{\bar{v}+\bar{c}}{2}\}$. This means Π_Z is given by either

$$(19a) \qquad \Pi_Z = \hat{g}[(\bar{v} - P_Z^1 - 2q)(2P_Z^1 + 2q - (1+p)\underline{c} - (1-p)\bar{c})],$$

or

$$(19b) \qquad \Pi_Z = \hat{g}[(\bar{v} - P_Z^1 - 2q)((1+p)(P_Z^1-\underline{c}) + 2pq) + (1-p)\ (\frac{\bar{v}-\bar{c}}{2})^2].$$

If $q=0$, then the logic used in the proof of Proposition 2 yields $\Pi_Z > \Pi_H$. Now suppose $q > \frac{\bar{v}+\bar{c}}{4}$. (18) yields that the second period price will not be contingent on the second period cost realization.[11] Thus, by definition, $\Pi_H > \Pi_Z$. This, in turn, implies that to prove the proposition we need only demonstrate that $\frac{d\Pi_Z}{dq} < \frac{d\Pi_H}{dq}$ over the relevant range.

There are two regimes for Π_Z. Consider first (19b). In this case P_Z^1 is given by

(20) $$P_Z^1 = \frac{\bar{v}+\underline{c}}{2} - \frac{q(1+2p)}{(1+p)},$$

which in turn yields

(21) $$\frac{d\Pi_Z}{dq} = -\hat{g}[\bar{v} + \underline{c} + \frac{2q}{1+p}].$$

Now consider (19a). In this case P_Z^1 is given by

(22) $$P_Z^1 = \frac{2\bar{v} - 6q + (1+p)\underline{c} + (1-p)\bar{c}}{4},$$

which in turn yields

(23) $$\frac{d\Pi_Z}{dq} = -\hat{g}\ [\bar{v} - q - \frac{(1+p)\underline{c}}{2} + \frac{(1-p)\bar{c}}{2}].$$

These are also two regimes for Π_H. Consider first (17a). In this case P_H^1 is given by

(24) $$P_H^1 = \frac{\bar{v}+\underline{c}}{2} - q,$$

which in turn yields

(25) $$\frac{d\Pi_H}{dq} = -\hat{g}[\bar{v} - \underline{c} - 2q].$$

Comparing (25) with both (21) and (23) yields $\frac{d\Pi_Z}{dq} < \frac{d\Pi_H}{dq}$.

Now consider (17b). In this case $\tilde{P}_H$ is given by

(26) $$\tilde{P}_H = \bar{v} - q + \frac{(1+p)\underline{c} + (1-p)\bar{c}}{2},$$

which in turn yields

(27) $$\frac{d\Pi_H}{dq} = -\hat{g}\ [\bar{v} - q - \frac{(1+p)\underline{c}}{2} - \frac{(1-p)\bar{c}}{2}].$$

Comparing (27) with both (21) and (23) yields $\frac{d\Pi_Z}{dq} < \frac{d\Pi_H}{dq}$.

Footnotes

[1]As is suggested by this last statement, the welfare of contracting parties can be lowered by the ability to renegotiate in a wider range of circumstances than just the hold-up problem. Specifically, this issue will arise whenever parties to an agreement face a time inconsistency problem (see Strotz 1955-56, and Kydland and Prescott 1977 for discussions of the time inconsistency problem).

[2]Thompson (1983) considers a similar hold-up problem in the context of a price discriminating monopolist.

[3]We are assuming Y is large enough such that the constraint, $b_i^1 + b_i^2 > 0$, is never binding.

[4]Because it concerns a set of agents whose weight is zero, we need not specify what happens when $v_i F(\hat{x}_i(P^t)) = P^t \hat{x}_i(P^t) + q$.

[5]It is easy to demonstrate that $v^*(.)$ is uniquely defined for any value for P.

[6]Note, as indicated in the statement of the proposition, this is only true if $\bar{c} > \underline{c}$. If $\bar{c} = \underline{c}$, there is no advantage to having a contingent contract and thus no benefit to having the ability to renegotiate. This case will play a more prominent role in the next section.

[7]It is easy to demonstrate that $v'(.,.)$ is uniquely defined for any (P^1,P^2) pair.

[8]It is easy to demonstrate that $v''(.,.,.)$ is uniquely defined for any $(P^1,\underline{P}^2,\bar{P}^2)$ triplet.

[9]In (10) we are implicitly imposing the restrictions $\underline{P}^2 > \underline{c}$ and $\bar{P}^2 > \bar{c}$. We can restrict the analysis in this way because the cartel has the ability to reset P^2 after observing its second period cost of production.

[10]The argument goes through even if there is either not a unique price vector which solves (11), or a unique price vector which solves (12). The proof then follows from the fact that of the price vectors which solve (11) and the price vectors which solve (12), there cannot be a pair such that $P_H^1 = P_Z^1$ and $P_H^2 = P_Z^2$.

[11]Throughout the analysis we are restricting the cartel to positive prices. With negative prices consumers with no intention of purchasing the complementary good would have an incentive to purchase X. This, in turn, would obviously lead to large negative profits for the cartel.

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