UNEMPLOYMENT-RATE DYNAMICS AND
PERSISTENT UNEMPLOYMENT UNDER RATIONAL EXPECTATIONS

by

Michael R. Darby
University of California, Los Angeles
National Bureau of Economic Research

and

John Haltiwanger
University of California, Los Angeles

and

Mark Plant
University of California, Los Angeles

UCLA Dept. of Economics
Working Paper No. 339
August 1984
Unemployment Rate Dynamics And
Persistent Unemployment Under Rational Expectations

ABSTRACT

This paper develops a model of unemployment rate dynamics that provides an explanation of persistent cyclical unemployment that does not involve persistent expectational errors or other nonoptimizing behavior. Our results are based on the interaction of search dynamics and inventory adjustments. An important element in these dynamics appears to be heterogeneity in the labor force which can be characterized as consisting of a relatively small group of high turnover individuals who comprise the bulk of normal unemployment and a larger group of low turnover individuals who dominate movements in cyclical unemployment. Our empirical results provide support for this theory as we demonstrate that the appropriately measured probability of becoming employed during a recovery falls relative to normal because of the unusually high proportion of low turnover individuals who have lost "permanent" jobs. As a result, recovery is much slower than is indicated by normal relationships although each individual is searching optimally.

Michael R. Darby
Department of Economics
University of California, Los Angeles
Los Angeles, CA 90024
(213) 825-3343

John C. Haltiwanger
Department of Economics
University of California, Los Angeles
Los Angeles, CA 90024
(213) 825-6619

Mark W. Plant
Department of Economics
University of California, Los Angeles
Los Angeles, CA 90024
(213) 825-3847
UNEMPLOYMENT-RATE DYNAMICS AND
PERSISTENT UNEMPLOYMENT UNDER RATIONAL EXPECTATIONS

Michael R. Darby
University of California, Los Angeles
National Bureau of Economic Research

John Haltiwanger
University of California, Los Angeles

Mark Plant*
University of California, Los Angeles

Macroeconomists typically build models of the determination of real output with unemployment-rate movements explained, if at all, by an appended Okun's Law relationship. In this paper we show that this approach is misleading because it misses important sources of persistence in cyclical unemployment -- and hence -- real output. In particular, we show that unemployment-rate dynamics imply "humped-shaped" cyclical unemployment characterized by persistence even though unemployed workers have faulty information on wage rates for only a comparatively brief period of time. This result is based on the interaction of two stock or state variables -- inventories and cyclically unemployed workers -- which result in a brief economic shock having a prolonged effect on the economy.

*The authors acknowledge helpful comments from Sean Beckett, Sebastian Edwards, Maxwell Fry, John J. McCall, members of the UCLA Money Workshop, and seminar participants at the UCLA Institute of Industrial Relations. The latter Institute has provided financial support for this research. Any opinions expressed are the opinions of the authors and not of any institutions with which they are affiliated; this is not a report of the National Bureau of Economic Research.
In the first section of the paper we develop the partial-adjustment equation which governs the evolution of the unemployment rate. We show that if individuals differ significantly in their probability of leaving unemployment, unemployment dynamics imply a much slower recovery from recession than would be implied by probability values observed in normal times.

In the second section of the paper we develop new empirical measures of the fraction of the labor force who become unemployed each month and the probability that an unemployed person will become employed or leave the labor force in a month's time. These measures and alternatives — equivalent if all individual probabilities are the same — are used to demonstrate that the unemployed differ substantially in their individual probabilities of leaving unemployment. We can broadly characterize the labor force as consisting of two groups: The first group is characterized by high rates of entry into unemployment and high probability of leaving it; in normal times, the bulk of unemployment comes from this group. The second group consists of individuals with high degrees of specific human capital and "permanent" jobs; so they rarely become unemployed but search for a long time to find a replacement job when they do become unemployed. This second group dominates cyclical unemployment during the recovery from recessions.

Further empirical work reported in the third section of the paper supports the following macroeconomic parable: Money shocks do not directly effect the proximate determinates of unemployment but do so indirectly through inventories. High inventories (relative to sales) lead to abnormally and temporarily high rates of entry of both groups into unemployment. The high inventories also may mildly and temporarily depress the probability of leaving unemployment. The excess unemployment among the high probability group is
quickly eliminated, but those who have lost permanent jobs take many months to find a new job even though each individual's monthly probability of success is no lower that would be the case in normal times. Thus, individual maximizing behavior leads to substantially persistent effects on cyclical unemployment and output.

I. Unemployment Dynamics and Persistence

In this section we demonstrate that the logic of the standard search model implies persistent cyclical unemployment as a result of a single-period forecast error on the part of searching workers. In this theoretical exposition of unemployment dynamics we abstract from the notion that some of the unemployed may not be searchers, but instead may be on temporary layoff, expecting recall. We do this in order to emphasize the separate role of search dynamics in yielding persistent effects from uncorrelated errors. Initially we assume that all workers are identical.

I.A. Identical Workers

The dynamics of the unemployment rate can be described by a simple discrete time model. Let \( u \) represent the unemployment rate, \( s \) be the number of unemployed searchers, and \( n \) be the labor force. By definition:

\[
(1) \quad u = s/n.
\]

It follows directly that

\[
(2) \quad \Delta u = \frac{1}{n} \Delta s - \frac{1}{n} \frac{s-1}{n-1} \Delta n
\]

and therefore:

\[
(3) \quad \Delta u = \frac{1}{n} \Delta s - \frac{\gamma}{1+\gamma} u_{-1}
\]

where \( \gamma = \Delta n/n_{-1} \) is the growth rate of the labor force. Define the search
flow $f$ as the rate (per period) at which people begin search and let $\pi$ be the probability per period that a searcher will become employed (or leave the labor force).\(^1\) The change in searchers $\Delta u$ is simply the inflow during the period less the outflow:

\begin{equation}
\Delta u = \frac{1}{n} (f - s_{n-1}) - \frac{\gamma}{1 + \gamma} u_{n-1}
\end{equation}

Note that $s_{n-1}/n = u_{n-1}/n = u_{n-1}/(1+\gamma)$ and let $\phi = f/n$ denote the search rate (the fraction of the labor force beginning search in the current period); then

\begin{equation}
\Delta u = \phi - \frac{\pi + \gamma}{1 + \gamma} u_{n-1}
\end{equation}

The growth-adjusted probability $\pi^* = (\pi + \gamma)/(1 + \gamma)$ is dominated by $\pi$ empirically, so we can interpret equation (5) as saying that the unemployment rate rises (or falls) as the search rate exceeds (is less than) the adjusted probability of employment times the lagged unemployment rate.

The natural unemployment rate $\bar{u}$ is found by setting $\Delta u = 0$ for the normal or long-run equilibrium values $\bar{\phi}$, $\bar{\pi}$, and $\bar{\gamma}$:

\begin{equation}
\bar{u} = \bar{\phi} \frac{1 + \bar{\gamma}}{\pi + \gamma} = \bar{\phi}/\pi^*
\end{equation}

That is, the natural unemployment rate is the product of the normal search rate $\bar{\phi}$ and adjusted duration of search $1/\pi^*$.\(^2\) For example, suppose that under normal conditions two percent of the labor force begins the search

\(^1\)Clark and Summers (1979) and Ries (1984) indicate that the probability of leaving the labor force (temporarily) is substantial for unemployed people in certain demographic groups — especially married women and teenagers.

\(^2\)Actually $\bar{\phi}$ and $\bar{\pi}$ are proportionally equal underestimates of the true continuous time rates because, in the former case, individuals who both enter and leave unemployment between surveys are omitted and, in the latter case, the effects of continuously compounded attrition. See Footnote 16 below for details of the correspondence between continuous and discrete time measures.
process each month, the probability of finding employment in a month is one third (i.e., the expected duration of search is \(1/\pi\) or 3 months), and the growth rate of the labor force is 0.002/month. Then the natural unemployment rate is

\[
\bar{u} = 0.020 \frac{1.002}{0.3333} = 0.020 \times 2.988 = 0.0598
\]

This is close to the 6 percent rate which we would obtain by ignoring the growth adjustment and simply taking the ratio of \(\phi\) to \(\pi\).

Denote cyclical components with circumflexes so that \(\hat{\phi} = \phi - \bar{\phi}\), \(\hat{\pi} = \pi - \bar{\pi}\), and \(\hat{u} = u - \bar{u}\). Then combining equations (5) and (6) and manipulating the result yields

\[
(7) \quad \Delta u = \hat{\phi} - \hat{\pi}u_{-1} + \hat{\pi}(\bar{u} - u_{-1})
\]

That is, an abnormally high rate of new searchers, or an abnormally low probability of finding an acceptable job tends to increase the unemployment rate, but aside from these shocks the unemployment rate converges to its natural rate with a constant partial adjustment factor \(\bar{\pi}\).

---

3. A form of equation (7) which accounts for \(\pi\) and \(\gamma\) separately is

\[
(7') \quad \Delta u = \hat{\phi} - (\frac{\pi}{1+\gamma} - \frac{\bar{\pi}}{1+\gamma})u_{-1} - (\frac{\gamma}{1+\gamma} - \frac{\bar{\gamma}}{1+\gamma})u_{-1} + (\frac{\gamma+\bar{\gamma}}{1+\gamma}) (\bar{u} - u_{-1})
\]

This form will be used in the empirical work below. The negative sign of the \((\frac{\gamma}{1+\gamma} - \frac{\bar{\gamma}}{1+\gamma})u_{-1}\) term appears puzzling because we naturally think of \(\hat{\phi}\) increasing as \(\gamma\) increases. In a partial sense, however, the more of a given increase in search flow that comes from new entrants, the lower will be the unemployed relative to the labor force. In other words, a given increase in \(\hat{\phi}\) causes a slightly smaller increase in \(\Delta u\) if it comes about from new entrants as compared to the previously employed because in the former case the denominator of the unemployment rate is increased as well as the numerator. This neglects any secondary effect on \(\pi\) should new entrants have an expected search duration which differs from that of the previously employed.

4. The rate \(\bar{\pi}\) is not necessarily constant over time. For example, in the empirical work we show that it varies with the demographic composition of the labor force. For our immediate purposes, however, we may assume that it is constant.
To illustrate the implications of equation (7), suppose for simplicity that an unexpected, restrictive monetary shock increases $\hat{\phi}$ and lowers $\hat{\pi}*$ temporarily until information on the change in policy can be incorporated in the expectations of entrepreneurs, searchers, and the public.\(^5\) Figure 1 illustrates the behavior of the unemployment rate on the assumption that these expectations effects on search flow and the instantaneous probability of employment last for one period only. The figure shows that a one-period expectations error implies a persistent effect on unemployment and hence on real output in the standard search-unemployment model. In the second period, the probability that each searcher finds a job is at the normal level, but the additional unemployment engendered in the first period takes time to work off. If we were to suppose that the initial effect of a restrictive demand shock is in part to build up inventories and that these excessive inventories lead to $\phi > \bar{\phi}$ in successive periods, the interval in which unemployment rates rise would be prolonged.

It may seem strange at first that when workers realize that they made a mistake last period, $\pi^*$ returns only to its normal value rather than going below $\bar{\pi}^*$ to compensate for the error. A bit of overshooting is possible if there are financing constraints, but generally past errors are forever bygones and optimal sequential search will imply a reservation wage which results in the normal probability of successful job search.\(^6\)

\(^5\)A more precise representation of the effect of policy is given below.

\(^6\)See Lippman and McCall (1984). Simple sequential search models imply that the expected remaining duration of search is a constant ($1/\bar{\pi}$) for uncompleted spells of unemployment when workers correctly perceive the potential distribution of offers. Financing constraints could be introduced to make the reservation wage a decreasing function of the length of unemployment experienced.
The stock of search unemployed workers can be increased from its equilibrium value by a single-period expectational error. Once this excess stock comes into existence a fraction is eliminated each period through successful search in the market place. Thus search dynamics alone imply persistent unemployment effect of one-time shocks in the economy. These effects are analogous to those previously discussed by equilibrium theorists for inventories, investment projects in process, and other such state variables. The persistent unemployment and output effects are not due to any persistent errors on the part of workers or firms, but instead due to the dynamic process of search in the labor market. In Sections III and IV below, we present empirical evidence that the interaction of inventory and unemployment dynamics does produce a hump-shaped time path of the unemployment rate in response to brief unexpected monetary shocks.

I.B. Heterogeneous Workers

A second element of employment dynamics which plays an important role in explaining the persistence of unemployment subsequent to a macroeconomic shock is the heterogeneous nature of workers. For simplicity, we can think of workers as being divided into two groups. Those in the first group have little firm-specific human capital and they experience unemployment frequently, but the length of these spells are brief. Thus, $\phi_1$ and $\tau_1^*$ are large since there is little to be gained from extensive search for short-term employment. Members of the second group rarely experience unemployment, but when it occurs, search is extensive and well supported by unemployment compensation, other family income, and assets. So $\phi_2$ and $\tau_2^*$ are both
low. The normal unemployment rate is

\[ \bar{u} = \frac{n_1}{n} \bar{u}_1 + \frac{n_2}{n} \bar{u}_2 = \frac{n_1}{n} \bar{\phi}_1 + \frac{n_2}{n} \bar{\phi}_2. \]

We cannot observe individual values of the \(\pi_i\)'s and \(\phi_i\)'s, but only their appropriately weighted averages. Of particular interest is \(\pi\):

\[ \pi = \sum_{i=1}^{s_i} \pi_i = \pi_2 + (s_{1,-1}/s_{-1}) (\pi_1 - \pi_2) \]

This overall average probability of leaving unemployment can change either because of changes in the individual \(\pi_i\)'s or because of changes in the unemployment shares \((s_i/s)\):

\[ \Delta \pi = \sum_{i=1}^{s_i} \Delta \pi_i = \sum_i \pi_i \Delta (s_{i-1}/s_{-1}) \]

Consider once again a one period shock. To concentrate on issues of persistence, suppose that people always search optimally \((\pi_i = \bar{\pi}_i)\ always\) but that a one-period increase in the \(\phi_i\)'s results in the same proportionate increase in each \(s_i\). In the two group case, \(s_2/s = 1 - (s_1/s)\). Holding \(\pi_1\) and \(\pi_2\) constant, we have from equation (10):

\[ \Delta \pi = (\bar{\pi}_1 - \bar{\pi}_2) \Delta (s_{1,-1}/s_{-1}) \]

For the first two periods, by assumption, lagged \(s_i/s\) is unchanged so the observed \(\pi\) remains constant at \(\bar{\pi}\). For each group unemployment rates will

---

7 We have analyzed multiple groups of workers, but two groups are sufficient to capture the most important empirical features. We here explain the tendency, ceteris paribus, for \(\pi\) to decline as duration of unemployment increases by the sorting hypothesis: The expected duration of unemployment varies inversely with \(\pi\) so that low \(\pi\) individuals comprise a larger share of longer duration relative to short duration unemployment. See Heckman and Borjas (1980) and Carroll and Horrigan (1983). Alternatively, \(\pi\) might itself decrease as duration increased. As discussed below, we believe that the data are best explained by the sorting hypothesis.
follow a pattern like that exhibited in Figure 1, but reflecting the $\bar{w}$ appropriate to the group. In Figure 2 we plot the number of persons unemployed in each group, $s_i$, which is simply the product of the group unemployment rate and the size of the group ($s_i = u_i n_i$). Once the search flows return to normal, the excess unemployment is eliminated in each group at the rate $\bar{w}_i$. At time 2, the lagged values of $s_1$ and $s_2$ have increased by the same proportion so — by equation (11) — $\pi$ is unchanged at $\bar{w}$. But $\pi$ thereafter begins to decline for a number of periods as the share $s_1/s$ drops. This happens because $\pi_1 > \pi_2$ so $s_1$ returns to normal much faster than does $s_2$. However (after about period 6 in the figure), nearly all the adjustment in group 1 is completed so that the decreases in $s_2$ are proportionately larger. Thereafter, $s_1/s$ and hence $\pi$ rise back toward their normal levels.

In conclusion, the slower adjustment of the lower probability group will appear in the aggregate data as a persistent $\hat{\pi}$ even though each individual worker is searching optimally with $\pi_1 = \bar{w}_1$. The problem arises because $\bar{w}$ is based on the normal distribution of unemployment among groups:

$$\bar{w} = \frac{n_1 \bar{u}_1}{nu} \bar{w}_1 + \frac{n_2 \bar{u}_2}{nu} \bar{w}_2$$  (12)

After a recession causes mass disemployment, the low probability group will be overrepresented for a considerable period of time. During the recovery period (with $\phi_1 = \bar{\phi}_1$, $\phi_2 = \bar{\phi}_2$, and $\gamma_1 = \gamma_2 = \bar{\gamma}$), $u_1$ quickly returns to $\bar{u}_1$ and thereafter we observe

8Since we have neglected the growth in the labor force, the transformation from $u$ to $s$ only shifts the relative positions of $u_1$ and $u_2$ so that they can be added together to obtain $u = \frac{n_1}{n} u_1 + \frac{n_2}{n} u_2$. 
FIGURE 2

EFFECT ON S, S1, S2 OF ONE-PERIOD SHOCK
(13) \[ \Delta u = \tau_2 (u - u_{-1}) \]

Were \( \tau_2 \) sufficiently low, the return to the normal unemployment rate could be painfully slow in the absence of a later stimulative monetary or fiscal shock.\(^9\) Note also that during this long recovery period the share of group 2 unemployed will be abnormally large which implies, from equation (9) that \( \tau \) will be abnormally low (\( \hat{\tau} < 0 \)). As a result an analyst looking at only the aggregate equation (7) might incorrectly conclude that convergence to the natural unemployment rate \( \bar{u} \) would be much faster were persistent expectation errors not keeping \( \hat{\tau} \) negative. But we are considering a case in which each individual always correctly perceives the wage distribution so that \( \tau_i = \tau_1 \) always. So persistence in aggregate \( \hat{\tau} \) may reflect expectation error or significant heterogeneity in \( \tau \) across individuals.\(^10\)

This discussion of heterogeneity suggests that considerable care must be taken in defining the natural rate of unemployment. Movements in the unemployment rate will be highly correlated with changes in the composition of the pool of unemployed workers. So any measure of the natural rate that uses weighted averages of normal levels of \( \tau_i \) and \( \phi_i \) where weights are unemployment shares (for example, \( \phi_i \sum_i \phi_i \bar{\tau}_i \)) will result in most movements

\(^9\) This model would appear to provide a new basis for Axel Leijonhufvud's corridor notion (1981) in which the economy converges to long-run equilibrium abnormally slowly after a major perturbation. Note, however, that it is hard to believe that exceedingly low values for \( \tau_2 \) could be socially optimal even if they were privately optimal given our system of transfer payments. It is not clear — of course — that stimulative monetary or fiscal policy has the same power to reemploy group 2 workers as it does to disemploy them through bankruptcies and permanent layoffs.

\(^10\) Recall that in the case just considered the accounting identity (7) will continue to hold although equation (13) is governing the convergence to the natural rate.
in $u$ being explained by movements in the natural rate.\footnote{Some hard-won lessons in this regard are imparted in Section III below.} Adjustment to normal levels of unemployment is reflected in adjustments in the share of unemployment among groups, and thus variations in shares should not be used to capture variation in the natural rate of unemployment.
II. Measurement

In the previous section, we demonstrated that the determinants of search rate $\phi$, the employment probability $\pi$, and the labor-force growth rate $\gamma$ were important in the analysis of the cyclical pattern of unemployment and real output. In this section we first consider the measurement of $\phi$, $\pi$, and $\gamma$, then decompose the measured values into cyclical and normal components, and finally obtain a measure of the heterogeneity of $\pi$ across individuals. At present, we are not able to measure (separately) values of $\phi$ and $\pi$ for those on temporary layoffs and those who are searchers (all others).\textsuperscript{12} We postpone the development of such measures to future research.

II.A. The Measurement of $\phi$, $\pi$, and $\gamma$.

Since the size of the labor force is a regularly reported statistic, we have no difficulty in computing $\gamma$, the growth rate of the labor force. To measure $\phi$ and $\pi$ we would ideally like to have data that reports the gross flows of the number of workers among three states: employed, unemployed, and not in the labor force. Unfortunately complete data of this sort are not regularly and reliably collected.\textsuperscript{13} However, we can obtain very good estimates of $\pi$ and $\phi$ from the available data on the aggregate number, $s$, unemployed each month and the number, $s^{0-4}$, who have been unemployed "0–4 weeks."

\textsuperscript{12}If we could, we would like to measure $\phi_L$ as the fraction of the labor force beginning temporary layoff within the month and $\pi_L$ as the probability of being recalled from temporary layoff within the month.

\textsuperscript{13}For a discussion of the availability of and problems with the gross flow labor force data, see Smith and Vanski (1979).
Recall that the Current Population Survey is conducted each month during the week which contains the 12th day of the month. Thus the typical year has twelve surveys of which 8 are conducted 4 weeks subsequent to the previous survey and 4 are 5 weeks subsequent to the previous survey. We find it convenient to use standard months of 4.35 weeks (30.4 days = 365/12). Since \( s^{0-4} \) reports the number of people unemployed 31 days or less, it is a good measure of the people who have become unemployed over the last month.\(^{14}\) The number unemployed at the last survey is used to approximate \( s_{-1} \), the number unemployed exactly one month earlier. The equivalence is exact whenever \( u_{-1} \) equals the value \( \phi \frac{1+\gamma}{\pi+\gamma} \) toward which the unemployment rate currently converges as it would, for example, in steady-state equilibrium; otherwise the error introduced in estimating \( \pi \) is bounded by 0.02 for plausible values of the relevant parameters.\(^{15}\) These data are sufficient to calculate \( \pi \) as

\[
(14) \quad \pi = 1 - \frac{s - s^{0-4}}{s_{-1}}
\]

since \( 1-\pi \) is the fraction of individuals unemployed last month who are still unemployed.

Our measured \( \pi \) tells us how much longer the average currently unemployed person would be unemployed under current conditions: \( 1/\pi \) months. This would be the expected total duration of the average newly unemployed person only if the ratio of search flows, \( f_1/f_2 \), equals the ratio of the unemployment shares, \( s_1/s_2 \). In steady-state equilibrium, for example,

---

\(^{14}\)Census enumerators, who have a calendar before them during an interview, round unemployment duration to the nearest whole number of weeks. Correction for the difference between 30.4 and 31 days is not attempted.

\(^{15}\)Since \( \pi \) normally lies between 0.4 and 0.6, this is an acceptable margin of error. See Appendix A for details.
\( f_1/f_2 \) exceeds \( s_1/s_2 \) so that the average expected duration of newly begun unemployment is less than the average expected remaining duration of the currently unemployed.

The corresponding measure of \( \phi \) is

\[
(15) \quad \phi = s^{0.4}/n
\]

where \( n \) is the civilian labor force. Note that this discrete time measure of \( \phi \) measures the flow of people who become unemployed between monthly observations and are still unemployed at this monthly observation. The continuous-time search rate would be higher.\(^1\text{6}\)

Monthly values of our estimates of \( \phi \) and \( \pi \) are reported in the Data Appendix. Figure 3 plots quarterly averages of the monthly values of \( \pi \). The mean value of 0.46 is interpreted as saying that on average 46 percent of the people who are unemployed at the beginning of a month will find a job, leave the labor force, or be recalled from layoff by the end of the month. The vertical lines mark cyclical peaks (P) and troughs (T) on the NBER

---

\(^1\text{6}\) Let \( \psi \) be the continuous time search rate, \( \theta \) be the Poisson parameter such that the probability of finding a job between \( t \) and \( t + dt \) is \( \theta dt \), and \( \gamma \) be continuously compounded monthly growth rate of \( n \). Then the observed value of \( \phi \) is given by

\[
\phi = (\int_0^1 n_1 e^{\gamma(1-t)} \psi e^{-\theta t} dt)/n = \psi(1-e^{-\theta-\gamma})/(\theta+\gamma).
\]

Note that the observed value of \( \pi \) is simply \( 1-e^{-\theta} \). Taking \( \gamma = 0 \), the following correspondence are observed:

\[
\begin{array}{ccc}
\pi & \theta & \phi/\psi = \pi/\theta \\
0.2 & .223 & .896 \\
0.3 & .357 & .841 \\
0.4 & .511 & .783 \\
0.5 & .693 & .721 \\
0.6 & .916 & .655 \\
\end{array}
\]

Thus the observed monthly probability of finding a job \( \pi \) is less than its continuous time equivalent \( \theta \), and the observed monthly flow into unemployment \( \phi \) is less than the continuous time equivalent \( \psi \).
reference cycle chronology. We note first that the values of \( \pi \) tended to be rather higher (around 0.6) during the Korean and Vietnamese eras and also that the monthly probability of getting a job drops sharply during a recession and rises in a boom. Our estimate of \( \bar{\pi} \) is derived and discussed in Section II.B below.

Figure 4 similarly plots quarterly averages of monthly values of \( \phi \). We see that \( \phi \) generally follows an upward trend although it is sharply below trend in both the Korean and Vietnamese eras. The cyclical behavior is sharply contracyclical — rising in recessions and falling in booms.

II.B. Cyclical and Normal Components

In equations (6) and (7) we analyze unemployment rate dynamics in terms of normal values \( \bar{\phi}, \bar{\pi}, \bar{\gamma} \), and \( \bar{u} \) and the corresponding cyclical components \( \hat{\phi}, \hat{\pi}, \hat{\gamma}, \) and \( \hat{u} \). In this section we develop estimates of these quantities. Following Barro (1977, 1978) and Wachter (1976) we develop measures of \( \bar{\pi}, \bar{\phi}, \) and \( \bar{\gamma} \) which reflect the effects of the military draft and the age-sex composition of the labor force on the natural rate of unemployment and derive a measure of \( \bar{u} \).

We define \( \bar{\pi} \) by

\[
\bar{\pi} = \sum_{i} \frac{\bar{n}_{i,-1} \bar{u}_{i,-1}}{\sum_{j} \bar{n}_{j,-1} \bar{u}_{j,-1}} \bar{\pi}_{i}
\]

(16)

where the summations are over age-sex cells.\(^{17}\) Labor force shares and

\(^{17}\)We must be careful to use normal unemployment weights in formula (16) since we have seen that after a recession causes mass disemployment low probability groups — predominant among prime-age males — will be overrepresented for some time. Therefore the use of actual unemployment weights will lead to spuriously procyclical movements in measured \( \bar{\pi} \) and hence contracyclical movements in \( \bar{u} \). This type of problem seems to be at the root of other estimates of \( \bar{u} \) which fluctuate sharply with the business cycle.
FIGURE 4

NET SEARCH RATE

QUARTERLY AVERAGE OF MONTHLY VALUES
unemployment rates by age and sex are available monthly but the number who have become unemployed during the last month by age and sex is only available as an annual monthly average beginning in 1967.

For all age and sex groups other than males 16-19 and males 20-24 we hypothesize that \( \bar{\pi}_i \) is constant over time. This allows us to use the available data to measure \( \bar{\pi}_i \) with the following steady-state version of equation (14) for group \( i \) where \( A(\cdot) \) denotes annual averages:\(^{18}\)

\[
\bar{\pi}_i = 1 - \text{mean} \left( \frac{A(s_i^1) - A(s_{i-1}^0)}{A(s_{i-1}^1)} \right)
\]

(17)

For males 16-19 and 20-24 the normal \( \bar{\pi}_i \) is allowed to vary over time with the nature of the military draft. To measure \( \bar{\pi}_i \) for these two young male groups we estimate the following equation:\(^{19}\)

\[
A(\pi_i^1) = \alpha_0 + \alpha_1 A(MI) + \alpha_2 A(\pi_j^1 - \bar{\pi}_j^1) + \alpha_3 A(\pi_k^1 - \bar{\pi}_k^1)
\]

(18)

where \( \pi_i \) refers to either males 16-19 or 20-24, MIL is Robert Barro's military draft variable, and \( \pi_j^1 - \bar{\pi}_j \) and \( \pi_k^1 - \bar{\pi}_k \) are the deviations from the mean of the annual average of monthly \( \pi_j \) and \( \pi_k \) for males 35-44 and 45-54 respectively.\(^ {20}\) The latter two variables are included to account for cyclical variation in \( \pi_i^1 \). Using the estimated coefficients from this equation, \( \bar{\pi}_i \)

---

\(^{18}\)All measures of normal \( \bar{\pi}, \bar{\phi}, \bar{\gamma} \) and \( \bar{u} \) are based on data from 1967-1983. \( A(s_{i-1}^1) \) denotes the annual average of data from the prior December through November of the current year, and the mean is taken over the period 1967-83.

\(^{19}\)This equation is estimated using annual data from 1967-1983.

\(^{20}\)The military variable is zero in the years in which there was no draft (January 1970 to December 1983), and equal to the ratio of the number of military personnel to the male population age 16 to 34 in years in which a draft was present. The draft after 1970 was a lottery draft. Following Barro (1977, 1978), we include this in the "non-draft" period since the incentives to search were considerably decreased by this process of conscription. Monthly data for this variable is available beginning 1953-58.
for males 16-19 and 20-24 is calculated as:

\[ \overline{\pi}_i = \hat{\alpha}_0 + \hat{\alpha}_1 \text{MIL} \]

We define \( \overline{\phi} \) as:

\[ \overline{\phi} = \sum_I \frac{n_I}{n} \overline{\phi}_I \]

where the summations are over age-sex cells.

Similar to our measurement of \( \overline{\pi}_i \), the normal \( \overline{\phi}_i \) for group \( i \) for age-sex groups other than males 16-19 and 20-24 is measured by the 1967-83 mean of equation (14) for group \( i \):

\[ \overline{\phi}_i = \text{mean} \left( \frac{A(s)^{0-4}}{A(n_i)} \right) \]

For males 16-19 and 20-24 \( \overline{\phi}_i \) is measured using an adjustment for the effect of the military draft similar to that used to adjust \( \overline{\pi}_i \) for the military draft.

We define \( \overline{\gamma} \) as:

\[ \overline{\gamma} = \sum_I \frac{n_I}{n} \overline{\gamma}_I \]

where the summations are over age-sex cells and \( \overline{\gamma}_I \) is measured as the mean of the observed monthly growth rates for group \( i \).

Having measured \( \overline{\pi}_i \), \( \overline{\phi}_i \), and \( \overline{\gamma}_i \) for each group \( i \), we can measure \( \overline{u}_i \) by an age-sex specific version of equation (6). The aggregate values \( \overline{\pi} \) and \( \overline{\phi} \) are obtained from equations (16) and (20) and used with \( \overline{\gamma} \) and equation (6) to obtain the aggregate normal unemployment rate \( \overline{u} \) which rises (see Figure 5) until the late 1970s and then begins to drift downward.

Examining Figures 3 and 4, we see that \( \overline{\pi} \) is relatively constant and the key factor moving \( \overline{u} \) is \( \overline{\phi} \) which indeed increases during the 1970s and then
falls off somewhat in the early 1980s. This can be explained by examining the estimates of the $\bar{w}_1$ and $\bar{\phi}_1$ values.

Table 1 reports the estimated normal values for $\bar{w}_1$, $\bar{\phi}_1$, $\bar{\gamma}_1$, and $\bar{u}_1$ by age and sex. During the 1970s the labor force share of the young increased and then decreased slightly in the early 1980s as the baby boom generation grew older. Since, as reported in Table 1, the young have relatively high probabilities of leaving unemployment, this accounts for the small movements in $\bar{w}$ over this period.

Figure 4 includes a plot of the quarterly averages of monthly values of $\bar{\phi}$. We see that $\bar{\phi}$ follows a general upward trend that is dramatic in the early to mid-1970s but falls off in the early 1980s. Since the $\bar{\phi}_1$'s are relatively constant over time, equation (20) indicates that this variation in $\bar{\phi}$ must be the result of variation in labor force shares. Our examination of the labor force share data reveals that the upswing in $\bar{\phi}$ in the early to middle 1970s is accounted for by the influx of the young and women into the labor force; both are relatively high turnover groups as evidenced by Table 1. Accordingly, the falloff in $\bar{\phi}$ in the early 1980s is accounted for by the decreased share of the young in the labor force as the baby boom generation grew older. The estimated $\bar{\phi}$ values move more dramatically than $\bar{w}$ because of the much greater variation in $\phi_1$ than in $\pi_1$ values.

II.C Measurement of Heterogeneity in $\pi$

In Section I,B we discussed some implications of having two groups with different values of $\pi$ and $\phi$. In this section we develop a measure of the empirical importance of this heterogeneity. We do so by reference to $d_v$, which is defined as the average duration of unemployment this month for those who were unemployed last month and are still unemployed.
### TABLE 1

**Estimated Normal Values of $\pi$ and $\phi$ by Age and Sex**

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma_1$</td>
<td>$\pi_1$</td>
</tr>
<tr>
<td>16-19 years(^a)</td>
<td>0.0023</td>
<td>0.5366</td>
</tr>
<tr>
<td>20-24 years(^a)</td>
<td>0.0036</td>
<td>0.4373</td>
</tr>
<tr>
<td>25-34 years</td>
<td>0.0033</td>
<td>0.3847</td>
</tr>
<tr>
<td>35-44 years</td>
<td>0.0005</td>
<td>0.3571</td>
</tr>
<tr>
<td>45-54 years</td>
<td>-0.0002</td>
<td>0.3379</td>
</tr>
<tr>
<td>55-64 years</td>
<td>0.0003</td>
<td>0.3164</td>
</tr>
<tr>
<td>65 years and over</td>
<td>-0.0004</td>
<td>0.3449</td>
</tr>
</tbody>
</table>

Calculated as described in text.

\(^a\)Reported values of $\pi_1$ and $\phi_1$ for young males (16-24) are the values since 1970:1 with MIL = 0. Prior to 1970:1, these values varied monthly with the draft variable MIL.
Suppose that all individuals have the same value of \( \pi \) at any time. Then \( d_v \) will be equal to the lagged average unemployment duration for all unemployed persons plus 1 month or \( d_{-1} + 1 \). If individuals have different values of \( \pi \), then \( d_v \) will generally exceed \( d_{-1} + 1 \) by a positive amount \( h \) because a disproportionate share of high duration (low \( \pi \)) people remain unemployed at the end of a month's time.

We can infer the average value of \( h \) from the available data by noting that average duration is the weighted average of \( d_v \) and \( d_n \) (the average duration of people who became unemployed in the last month):

\[
d = \frac{s - s^{0.4}}{s} (d_{-1} + 1 + h) + \frac{s^{0.4}}{s} d_n
\]

Solving for \( h \) yields:

\[
h = \frac{d - d_n (s^{0.4}/s)}{1 - (s^{0.4}/s)} - d_{-1} - 1
\]

All of the right-hand variables are available except \( d_n \) which lies between 0.4267 and 0.4691.\(^{21}\) To compute the mean value of \( h \) over 1953:8 through

\(^{21}\) Those who became unemployed within the month will have an average duration of less than one half month because those who became unemployed earlier in the month are more likely to have found a job than the more recently unemployed. Assuming the Poisson distribution of note 16 above and \( \gamma = 0 \), the average duration of those who became unemployed within the month is

\[
d_n = \int_0^1 te^{-\theta t} dt = \frac{1 - e^{-\theta}}{\theta} - \frac{e^{-\theta}}{\theta(1-e^{-\theta})} = \frac{1}{\theta} - \frac{e^{-\theta}}{1-e^{-\theta}}
\]

Recall that \( \theta = -\log (1-\pi) \). The minimum and maximum values of \( \pi \) observed over 1953:8 - 1983:12 are 0.31 and 0.59 for which the corresponding \( d_n \) values are 0.4691 and 0.4267, respectively.
1983:12, we took \( d_n \) to be constant at 0.4495 which corresponds to the mean \( \bar{w} \) value of 0.45674. The resulting average value for \( \bar{h} \) is 0.973.

For the two group case of Section I.B, it is straightforward to determine the steady-state (normal) value \( \bar{h} \) for alternative values of \( \bar{w}_1 \) and \( \bar{w}_2 \) given that the steady-state ratio of inflows \( \bar{f}_{1}/\bar{f}_{2} \) adjusts so that the aggregate \( \bar{w} \) is constrained to equal its sample mean of about 0.46.\(^{22}\) The relevant pairs are those for which the calculated \( \bar{h} \) equals the value 0.973 which we calculated directly from the data. Table 2 reports these pairs together with the implied values of \( \bar{f}_{1}/\bar{f}_{2} \) and \( \bar{h} \).

We can get some idea of potential values for \( \bar{w}_1 \) and \( \bar{w}_2 \) from reviewing the range of values for broad age-sex cells. These values (see Table 1) range from 0.31 to 0.57. Suppose that the average \( \bar{w}_1 \) for 16-19 year olds of both sexes — 0.55 — reflects \( \bar{w}_1 \), because these youngsters have not yet formed any permanent job attachments nor acquired specific human capital.\(^{23}\) Table 2 tells us that \( \bar{w}_2 \) would be about 0.112. If the normal unemployment rate among group 2 were about half the overall rate — say 2.75 percent — then \( \bar{w}_2 \) would be 0.00308. Put differently the average duration of these "permanent" jobs would be about 325 months or 27 years. This number is consistent with the stylized facts about the labor market.\(^{24}\) A \( \bar{w}_2 \) of 0.112 would imply that during the recovery from a recession when \( \bar{w}_2 \) controls the adjustment rate, cyclical unemployment would fall by about a ninth in one month, a third in a quarter, a half in a half year, and three quarters in a

\(^{22}\)See Appendix B for details.

\(^{23}\)The correct \( \bar{w}_1 \) could be even a bit lower than this if the average high-probability individual has a lower probability than these youngsters of getting a job or leaving the labor force.

\(^{24}\)See Hall (1982).
### TABLE 2

<table>
<thead>
<tr>
<th>$\bar{\pi}_1$</th>
<th>$\bar{\pi}_2$</th>
<th>$\bar{t}_1/\bar{t}_2$</th>
<th>$\bar{h}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90</td>
<td>0.225</td>
<td>4.72</td>
<td>0.977</td>
</tr>
<tr>
<td>0.85</td>
<td>0.217</td>
<td>4.72</td>
<td>0.976</td>
</tr>
<tr>
<td>0.80</td>
<td>0.208</td>
<td>5.01</td>
<td>0.973</td>
</tr>
<tr>
<td>0.75</td>
<td>0.197</td>
<td>5.60</td>
<td>0.973</td>
</tr>
<tr>
<td>0.70</td>
<td>0.183</td>
<td>6.70</td>
<td>0.976</td>
</tr>
<tr>
<td>0.65</td>
<td>0.166</td>
<td>8.70</td>
<td>0.974</td>
</tr>
<tr>
<td>0.60</td>
<td>0.143</td>
<td>13.00</td>
<td>0.976</td>
</tr>
<tr>
<td>0.55</td>
<td>0.112</td>
<td>24.85</td>
<td>0.968</td>
</tr>
<tr>
<td>0.54</td>
<td>0.104</td>
<td>29.96</td>
<td>0.969</td>
</tr>
<tr>
<td>0.53</td>
<td>0.095</td>
<td>37.35</td>
<td>0.975</td>
</tr>
<tr>
<td>0.52</td>
<td>0.086</td>
<td>47.83</td>
<td>0.971</td>
</tr>
<tr>
<td>0.51</td>
<td>0.076</td>
<td>64.52</td>
<td>0.968</td>
</tr>
<tr>
<td>0.50</td>
<td>0.064</td>
<td>95.14</td>
<td>0.981</td>
</tr>
<tr>
<td>0.49</td>
<td>0.052</td>
<td>153.44</td>
<td>0.975</td>
</tr>
<tr>
<td>0.48</td>
<td>0.038</td>
<td>303.88</td>
<td>0.985</td>
</tr>
<tr>
<td>0.47</td>
<td>0.023</td>
<td>892.5</td>
<td>0.981</td>
</tr>
<tr>
<td>0.46</td>
<td>0.006</td>
<td>14,156.7</td>
<td>0.980</td>
</tr>
</tbody>
</table>

Notes: $\bar{\pi}_2$ is the value to three digits which yields $\bar{h}$ nearest to 0.973 given that $\bar{t}_1/\bar{t}_2$ adjusts so that $\bar{\pi} = 0.45674$. See Appendix B for details.
year's time if all $\phi$'s and $\pi$'s were at their normal values. Such persistence may be sufficient to explain the puzzle of why unemployment is persistent without need for recourse to any substantially persistent expectational errors which cause $\pi_1$ and $\pi_2$ to differ from their normal values for long periods of time. This hypothesis is considered further in Section III.

A final implication of using $\bar{\pi}_1 = .55$ and $\bar{\pi}_2 = .112$ is that normally 79% of the unemployed would be from group 1 and only 21% from group 2.25 However, if we assume that the $\pi_1$ values were at their normal levels when $\pi$ fell to 0.313 in November 1982, then we conclude that $s_1/s$ fell to only about 0.459. That is, normal unemployment among group 1 would account for 4.77 percentage points of their estimated total contribution of 4.91 percentage points.26 So group 2 would have accounted for 4.49 percentage points of the total 4.63 percentage points of cyclical unemployment.

Thus we can show that numbers consistent with the data are also consistent with our theoretical analysis of Sections I.A and I.B: Normal unemployment consists primarily of high turnover individuals, but cyclical unemployment is dominated during the recovery from a recession by individuals who have lost "permanent" jobs and are searching for a replacement. This search process — possibly involving relocation or shifting industries — is a lengthy one so that cyclical unemployment falls much more slowly than would be suggested by a normal $\bar{\pi}$ of almost one half.

25This is computed as $\bar{s}_1/s = (0.45674-0.112)/(0.55-0.112) = 0.787$.

26We calculate $s_1/s = (0.313-0.112)/(0.55-0.112) = 0.459; \bar{u}_1 = (s_1/s)(\bar{u}) = (0.787)(6.066\%) = 4.77\%; u_1 = (s_1/s)(u) = (0.455)(10.7\%) = 4.91\%$. 
III. An Empirical Model of Cyclical Unemployment

We can identify four separate potential sources of persistence in cyclical unemployment \( \hat{u} \): (a) persistence due to the partial adjustment of \( u \) toward \( \bar{u} \) with \( \phi, \pi, \) and \( \gamma \) at their normal levels, (b) reduced adjustment speed of \( u \) toward \( \bar{u} \) due to heterogeneity with individual \( \pi_i, \phi_i, \) and \( \gamma_i \) at their normal levels, (c) autocorrelation in \( \hat{\phi}, \hat{\pi}, \) and \( \hat{\gamma} \) due to equilibrium behavior, and (d) autocorrelation in \( \hat{\phi}, \hat{\pi}, \) and \( \hat{\gamma} \) due to autocorrelated expectational errors. The first three sources of persistence are consistent with the hypothesis that economic agents form their expectations rationally while the fourth source contradicts that hypothesis.

In this section we assess the relative importance of these alternative sources of persistence by first complementing our explanations of \( \bar{\phi}, \bar{\pi}, \) and \( \bar{\gamma} \) with variables which determine the movement of \( \phi, \pi, \) and \( \gamma \) around their normal levels. These equations — together with supplementary equations which explain the movements of inventories and money — are then estimated.

Although this initial analysis of the data raises many interesting questions for future research, the data appear to be consistent with equilibrium models of persistence — sources (a), (b) and (c) — with little if any indication of a significant role for autocorrelated expectational errors.

III.A. Behavior of the Employment Probability \( \pi \)

The standard search model of unemployment states that \( \hat{\pi} \) is a function of unexpected changes in aggregate demand and hence the derived demand for labor. The theory states that an unexpected increase (decrease) in aggregate demand shifts the actual distribution of wage offers right (left) relative to the expected distribution so that \( \pi \) is increased (decreased) relative to normal. Another, possibly complementary, story says that the probability of receiving any offer increases (decreases) so that the probability \( \pi \) of an
acceptable offer increases (decreases). Following Barro (1977, 1978), we use unexpected money \( \hat{M} \) as an indicator of shifts in aggregate demand. The strictest interpretation of rational expectations, would posit that only the current value of \( \hat{M} \) should affect \( \hat{\pi} \).\(^{27}\) We investigate this hypothesis by including in our \( \hat{\pi} \) regression specification a distributed lag on the current and first 11 lags of \( \hat{M} \).\(^{28}\)

The cyclical component \( \hat{I} \) of the inventory-sales ratio may affect \( \hat{\pi} \) for three reasons:\(^{29}\) First, our measure of \( \hat{\pi} \) is actually a weighted average

\(^{27}\)For such an interpretation, see McCallum (1979). The basic idea is that all past information will be incorporated in current expectations and so will have no effect on real variables except through past effects on current state variables such as inventories. If lagged values of \( \hat{M} \) were to enter we would conclude (a) expectations are not formed rationally, (b) the relevant horizon for forming expectations is longer than 1 month, or (c) some significant state variables have been omitted from the regression. In the latter case, the lagged \( \hat{M} \) coefficients would reflect past effects on the omitted state variable(s) and current effects of the state variable(s).

\(^{28}\)We measure \( \hat{M} \) as the residual from an ARIMA(0,2,4) process fit to log M. Our money series is the current Federal Reserve M\(_1\) series for 1959-1 through 1983-12 which we have extended back to 1953-6 by a ratio splice at 1959-1 to the old M\(_1\) series in Board of Governors of the Federal Reserve System (1976). (This splice preserves the growth rates in the two series which are practically identical in 1959.) After allowing for taking second differences, \( \hat{M} \) estimates are available from 1953-8. This start date for estimating \( \hat{M} \) was chosen for two reasons: (1) This avoids essentially all of the period during which the Fed was pegging government bond prices. The process determining money growth is potentially different during that period; see Friedman and Schwartz (1963, pp. 613, 625). (2) This is the latest start date for \( \hat{M} \) for which availability of \( \hat{M} \) observations does not reduce the period over which we can estimate the regressions reported in this section.

\(^{29}\)We measure \( \hat{I} \) as the deviation from a linear trend fit to Citibase data on the inventory-sales ratio for manufacturing and trade. This ratio -- the one-month lagged value of Business Conditions Digest series number 77 -- has the beginning-of-month total book value (in 1972 dollars) of manufacturing and trade inventories as the numerator and manufacturing and trade sales for the prior month (in 1972 dollars) as the denominator. This dating is appropriate both with respect to the decision-making of the firm and for the intertemporal decision-making of individuals.
of the probability that a searcher finds an acceptable job, the probability that a searcher leaves the labor force, and the probability that a laid-off worker is recalled. The probability of recall from layoff depends positively on the extent to which excess inventories have been eliminated. Second, when inventories are abnormally high, the discounted marginal value product of labor is abnormally low. Optimal intertemporal leisure substitution implies that some searchers should then drop out of the labor force, but we expect that in fact many of these individuals would be counted as unemployed due to unemployment benefit rules. Finally, cyclical fluctuations in inventories could shift the actual wage offer distribution relative to the expected distribution. To capture these inventory effects, the \( \hat{\mathbf{w}} \) equation also includes a 12 month distributed lag on \( \hat{I} \). Intertemporal substitution considerations would suggest that \( \hat{I}_{-1} \) should enter because it is the most recent information available to workers during the month. From the point of view of firms, lagged as well as current values of \( \hat{I} \) could enter for the following reason: Given that there are adjustment costs associated with varying employment, it will be optimal for firms to use inventories to buffer short-run monthly variations in sales.\(^{30}\) However, given that there are also costs of varying inventories, successive monthly variations in the inventory-

---

**Fn. 29 (cont.)**...

The linear trend was not estimated directly, but instead is that implied by our regression of \( I \) on both trend and cyclical variables (see Table 5 below). This approach assures consistency in the simulations reported in Section IV below, but the difference is not a substantive one: The correlation coefficient between our \( \hat{I} \) and the residuals from a linear trend regression is 0.95. Details of how to impute \( \bar{I} \) from the Table 5 regression are given in footnote 39 below.

\(^{30}\) See Haltiwanger and Maccini (1984) for theoretical results which support this statement and Blanchard (1983) for empirical support.
sales ratio in the same direction will make it optimal for a firm eventually to change production and hence employment. It is to capture this optimal lagged response of employment to inventories that a polynomial distributed lag on \( \hat{I} \) is used.

To capture the effects of heterogeneity on \( \hat{I} \), we use the variables that indicate how the composition of the unemployed has changed over time. In particular, we are interested in the changes in the share of the unemployed who exhibit high turnover and low duration relative to the share of the unemployed who exhibit low turnover and high duration. Our initial approach is to include the one period lagged share of unemployed who have been unemployed less than 5 weeks, the one period lagged share of the unemployed who have been unemployed 15-26 weeks and the one period lagged share of the unemployed who have been unemployed more than 27 weeks as explanatory variables in the \( \hat{I} \) equation. However, while subdividing the unemployed into four separate groups provides a means for distinguishing between multiple categories of duration groups, it can reasonably be argued that these variables are collinear. Hence, in addition to estimating the \( \hat{I} \) equation using four duration groups, we estimate the \( \hat{I} \) equation using only two groups; in particular, we estimate \( \hat{I} \) including only the one period lagged share of the unemployed who have been unemployed less than 5 weeks to capture heterogeneity.

The first column of Table 3 reports the results of regressing \( \hat{I} \) on the lagged unemployment shares by duration using four groups, and the 12 month distributed lags on \( \hat{I} \) and \( \hat{W} \) using OLS.\(^{31}\) The coefficients on each of the

\(^{31}\) The D-W statistics from the \( \hat{I} \) regressions indicate some residual positive autocorrelation. This can be eliminated by using the current unemployment shares by duration rather than the lagged but we eschew this approach because of the simultaneity problems it creates.
included unemployment shares by duration is positive indicating that as each
share increases relative to the excluded group (the share of the unemployed
with 5-14 weeks of unemployment) \( \hat{\pi} \) increases. That the coefficients on the
unemployment shares of those with 15-26 and 27+ weeks of unemployment are both
positive is somewhat surprising. It may indicate that at the upper tail of
the duration distribution there is a degree of negative duration dependence.
However, this may also be due to the multicollinearity between the unemploy-
ment shares by duration. As confirmation of this, the second column in Table
3 reports the results of regressing \( \hat{\pi} \) on the lagged unemployment shares by
duration using only two groups as well as the 12 month distributed lags on \( \hat{\bar{\pi}} \)
and \( \bar{\pi} \). Under either specification, as a group the unemployment shares by
duration have explanatory power which is significant at the 99% confidence
level.\(^{32}\) The negligible drop in \( R^2 \) associated with using two duration
groups rather than four indicates that multicollinearity may underly the
surprising pattern of coefficients.

Under both specifications neither the current money shock nor all 12
coefficients as a group are significantly different from zero.\(^{33}\) In contrast,
using four duration groups, the coefficient on \( \hat{\bar{\pi}} \) is negative and marginally
significant and the distributed lag on \( \hat{\bar{\pi}} \) as a group is marginally
significant as well.\(^{34}\) Moreover, using two duration groups, the coefficient

\(^{32}\) With four duration groups, the \( F(3,324) \) statistic for testing the
null hypothesis that all the coefficients on the unemployment share by
duration are zero is 344.32.

\(^{33}\) With four duration groups, the \( F(12,324) \) statistic for testing the
null hypothesis that all the coefficients on \( \bar{\bar{\pi}} \) are zero is 0.524. Using two
duration groups, the analogous \( F(12,326) \) statistic is 0.501.

\(^{34}\) Using four duration groups the \( F(12,324) \) statistic for testing the
null hypothesis that all of the coefficients on \( \hat{\bar{\pi}} \) are zero is 1.532 which
implies rejecting the null hypothesis at the 11% level. Note that if the
### TABLE 3

Determinants of $\hat{\pi}$

<table>
<thead>
<tr>
<th></th>
<th>Four Duration Groups</th>
<th>Two Duration Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-1.2799 (0.1971)</td>
<td>-0.4827 (0.0155)</td>
</tr>
<tr>
<td>Lagged Share of Unemployed with 0-4 Weeks Duration</td>
<td>2.1322 (0.2589)</td>
<td>1.0628 (0.0342)</td>
</tr>
<tr>
<td>Lagged Share of Unemployed with 15-26 Weeks Duration</td>
<td>1.6834 (0.5368)</td>
<td>-</td>
</tr>
<tr>
<td>Lagged Share of Unemployed with 27+ Weeks Duration</td>
<td>0.8365 (0.2138)</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{i}$</td>
<td>-0.2659 (0.1567)</td>
<td>-0.3079 (0.1604)</td>
</tr>
<tr>
<td>$\hat{i}(-1)$</td>
<td>0.0913 (0.2089)</td>
<td>-0.1453 (0.2048)</td>
</tr>
<tr>
<td>$\hat{i}(-2)$</td>
<td>0.0032 (0.2022)</td>
<td>-0.0771 (0.2060)</td>
</tr>
<tr>
<td>$\hat{i}(-3)$</td>
<td>0.0056 (0.2074)</td>
<td>0.1133 (0.2057)</td>
</tr>
<tr>
<td>$\hat{i}(-4)$</td>
<td>-0.0253 (0.2106)</td>
<td>0.1972 (0.2060)</td>
</tr>
<tr>
<td>$\hat{i}(-5)$</td>
<td>-0.1988 (0.2016)</td>
<td>-0.1478 (0.2063)</td>
</tr>
<tr>
<td>$\hat{i}(-6)$</td>
<td>0.3462 (0.2051)</td>
<td>0.2365 (0.2075)</td>
</tr>
<tr>
<td>$\hat{i}(-7)$</td>
<td>-0.1423 (0.2038)</td>
<td>-0.0638 (0.2075)</td>
</tr>
<tr>
<td>$\hat{i}(-8)$</td>
<td>-0.2042 (0.2060)</td>
<td>-0.0708 (0.2081)</td>
</tr>
<tr>
<td>$\hat{i}(-9)$</td>
<td>0.1775 (0.2044)</td>
<td>0.1885 (0.2095)</td>
</tr>
<tr>
<td>$\hat{i}(-10)$</td>
<td>-0.0242 (0.2053)</td>
<td>-0.0776 (0.2101)</td>
</tr>
<tr>
<td>$\hat{i}(-11)$</td>
<td>0.2247 (0.1591)</td>
<td>0.1638 (0.1564)</td>
</tr>
</tbody>
</table>

(continued)
<table>
<thead>
<tr>
<th></th>
<th>Four Duration Groups</th>
<th>Two Duration Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{H}$</td>
<td>-0.6606 (0.4534)</td>
<td>-0.6711 (0.4644)</td>
</tr>
<tr>
<td>$\hat{H}(-1)$</td>
<td>0.1702 (0.4582)</td>
<td>0.4234 (0.4658)</td>
</tr>
<tr>
<td>$\hat{H}(-2)$</td>
<td>-0.1600 (0.4565)</td>
<td>-0.2526 (0.4670)</td>
</tr>
<tr>
<td>$\hat{H}(-3)$</td>
<td>-0.0737 (0.4587)</td>
<td>-0.0153 (0.4699)</td>
</tr>
<tr>
<td>$\hat{H}(-4)$</td>
<td>0.3268 (0.4630)</td>
<td>0.3293 (0.4746)</td>
</tr>
<tr>
<td>$\hat{H}(-5)$</td>
<td>0.2172 (0.4730)</td>
<td>0.2577 (0.4841)</td>
</tr>
<tr>
<td>$\hat{H}(-6)$</td>
<td>0.4644 (0.4831)</td>
<td>0.3098 (0.4897)</td>
</tr>
<tr>
<td>$\hat{H}(-7)$</td>
<td>-0.1367 (0.4831)</td>
<td>0.0056 (0.4937)</td>
</tr>
<tr>
<td>$\hat{H}(-8)$</td>
<td>-0.3641 (0.4790)</td>
<td>-0.2787 (0.4904)</td>
</tr>
<tr>
<td>$\hat{H}(-9)$</td>
<td>0.2806 (0.4847)</td>
<td>0.4130 (0.4950)</td>
</tr>
<tr>
<td>$\hat{H}(-10)$</td>
<td>-0.2682 (0.4821)</td>
<td>-0.2192 (0.4941)</td>
</tr>
<tr>
<td>$\hat{H}(-11)$</td>
<td>0.2030 (0.4774)</td>
<td>0.2458 (0.4892)</td>
</tr>
</tbody>
</table>

$R^2$  
0.778  
0.767

S.E.E.  
0.031  
0.032

D-W  
1.326  
1.264

Note: Standard errors in parentheses. Period of estimation is 1954-8 through 1983-12.
on \( \hat{I} \) is negative and significant and the distributed lag on \( \hat{I} \) is is significant as well.\(^{35}\)

We interpret these results as indicating that the major factor determining cyclical variations in the probability of leaving unemployment is heterogeneity. Inventory innovations appear to play a minor role and surprisingly money shocks play no significant role here. That money shocks are insignificant may be because they operate only through the \( \hat{I} \) or because of measurement error problems, but there is certainly no evidence here of persistent expectational errors.

III.B. Behavior of \( \hat{\phi} \)

Since firings and layoffs are the complement of firms' decisions with respect to new hires and recalls, we would expect \( \hat{\phi} \) to be increased by high cyclical inventories and decreased by positive money shocks. High inventories or low sales due to a negative \( \hat{M} \) present the best time for firms to cull their labor force of marginal workers; when these conditions are persistent, temporary layoffs will result. In the opposite direction, low \( \hat{I} \) and high \( \hat{M} \) would tend to induce firms to retain otherwise unsatisfactory workers temporarily and would reduce the aggregate incidence of new temporary layoffs below its normal level. So as with \( \hat{\pi} \), we include 12 month distributed lags on \( \hat{I} \) and \( \hat{M} \) in our regressions explaining \( \hat{\phi} \).

\(^{34}\) Insignificant distributed lag is deleted from this \( \hat{\pi} \) equation then the \( \hat{I} \) distributed lag is significant at the 1% level.

\(^{35}\) In this case, the \( F(12,326) \) statistic for testing the null hypothesis that all of the coefficients on \( \hat{I} \) are zero is 2.974. Note that if the insignificant distributed lag on \( \hat{M} \) is dropped from the \( \hat{\pi} \) equation with two duration groups then the \( \hat{I} \) distributed lag is significant at the 1% level.
Heterogeneity in the labor force in terms of turnover propensities is a third factor which may influence \( \hat{\phi} \). Our measure of \( \hat{\phi} \) already controls for such heterogeneity that is associated with the age-sex composition of the labor force. However, other characteristics of individuals may be related to heterogeneity in turnover propensities. In an attempt to capture some of this residual heterogeneity we include measures of the industrial composition of the labor force in the \( \hat{\phi} \) regressions.

The first column of Table 4 reports the results of regressing \( \hat{\phi} \) on the 12 month distributed lags on \( \hat{I} \) and \( \hat{M} \) and on the industrial composition of the labor force. The shares of the labor force by industry proved to be significant indicating heterogeneity in turnover propensities across industries.\(^{36}\) Relatively high turnover industries include construction, mining and manufacturing (non-durables) whereas relatively low turnover industries include transportation and manufacturing (durables). As a group, the distributed lag on \( \hat{I} \) proved quite significant in explaining \( \hat{\phi} \).\(^{37}\) While the large, significant, positive coefficients on \( \hat{I} \) and \( \hat{I}(-1) \) probably reflect fluctuations in temporary layoffs, the significant negative coefficient on \( \hat{I}(-11) \) suggests that variation in the rate of separation for marginal workers is also involved: If a marginal worker is fired earlier due to high cyclical inventories, \( \hat{\phi} \) will decrease below normal levels during the latter period when the worker would have otherwise been fired. Nonetheless, the sum of the coefficients is positive confirming the notion that high cyclical inventories cause temporary layoffs of employees who otherwise would never

\(^{36}\) The \( F(7,321) \) statistic for testing the null hypothesis that all labor force share by industry coefficients are zero is 161.39.

\(^{37}\) The \( F(12,321) \) statistic for testing the null hypothesis that all \( \hat{I} \) coefficients are zero is 16.01.
Table 4
Determinants of \( \hat{\phi} \)

<table>
<thead>
<tr>
<th></th>
<th>With ( \hat{I} ) and ( \hat{M} )</th>
<th>With ( \hat{I} ) only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.0325 (0.04363)</td>
<td>-0.0277 (0.0425)</td>
</tr>
<tr>
<td>LF share of Transportation</td>
<td>-0.5475 (0.0966)</td>
<td>-0.5295 (0.0946)</td>
</tr>
<tr>
<td>LF share of Finance and Services</td>
<td>0.1107 (0.0484)</td>
<td>0.1056 (0.0468)</td>
</tr>
<tr>
<td>LF Share of Manufacturing-Durables</td>
<td>-0.3020 (0.0669)</td>
<td>-0.3052 (0.0644)</td>
</tr>
<tr>
<td>LF Share of Construction</td>
<td>0.3513 (0.0762)</td>
<td>0.3474 (0.0746)</td>
</tr>
<tr>
<td>LF Shares of Mining</td>
<td>1.1473 (0.1324)</td>
<td>1.1457 (0.1298)</td>
</tr>
<tr>
<td>LF Share of Wholesale and Retail Trade</td>
<td>0.0171 (0.0850)</td>
<td>0.0062 (0.0842)</td>
</tr>
<tr>
<td>LF Shares of Manufacturing-Non-Durables</td>
<td>0.8229 (0.0590)</td>
<td>0.8054 (0.0579)</td>
</tr>
<tr>
<td>( \hat{I} )</td>
<td>0.0355 (0.0072)</td>
<td>0.0376 (0.0067)</td>
</tr>
<tr>
<td>( \hat{I}(-1) )</td>
<td>0.0187 (0.0092)</td>
<td>0.0196 (0.0090)</td>
</tr>
<tr>
<td>( \hat{I}(-2) )</td>
<td>-0.0062 (0.0091)</td>
<td>-0.0077 (0.0089)</td>
</tr>
<tr>
<td>( \hat{I}(-3) )</td>
<td>-0.0039 (0.0091)</td>
<td>-0.0038 (0.0089)</td>
</tr>
<tr>
<td>( \hat{I}(-4) )</td>
<td>0.0044 (0.0092)</td>
<td>-0.0014 (0.0090)</td>
</tr>
<tr>
<td>( \hat{I}(-5) )</td>
<td>-0.0078 (0.0092)</td>
<td>-0.0074 (0.0090)</td>
</tr>
<tr>
<td>( \hat{I}(-6) )</td>
<td>0.0032 (0.0093)</td>
<td>-0.0024 (0.0090)</td>
</tr>
<tr>
<td>( \hat{I}(-7) )</td>
<td>-0.0055 (0.0092)</td>
<td>-0.0071 (0.0090)</td>
</tr>
<tr>
<td>( \hat{I}(-8) )</td>
<td>(0.0065 (0.0093)</td>
<td>0.0079 (0.0090)</td>
</tr>
</tbody>
</table>

continued
(Table 4 continued)

<table>
<thead>
<tr>
<th></th>
<th>With ( \hat{i} ) and ( \hat{N} )</th>
<th>With ( \hat{i} ) only</th>
</tr>
</thead>
</table>
| \( \hat{i} \)  | 0.0013  
(0.0093)         | 0.0023  
(0.0090)         |
| \( \hat{i} \)  | -0.0081  
(0.0094)         | -0.0061  
(0.0090)         |
| \( \hat{i} \)  | -0.0233  
(0.0072)         | -0.0210  
(0.0068)         |
| \( \hat{N} \)  | -0.0012  
(0.0209)         | -            |
| \( \hat{N} \)  | -0.0155  
(0.0210)         | -            |
| \( \hat{N} \)  | -0.0139  
(0.0210)         | -            |
| \( \hat{N} \)  | -0.0420  
(0.0213)         | -            |
| \( \hat{N} \)  | -0.0109  
(0.0215)         | -            |
| \( \hat{N} \)  | -0.0275  
(0.0221)         | -            |
| \( \hat{N} \)  | -0.0102  
(0.0225)         | -            |
| \( \hat{N} \)  | 0.0147   
(0.0227)         | -            |
| \( \hat{N} \)  | 0.0165   
(0.0223)         | -            |
| \( \hat{N} \)  | -0.0025  
(0.0226)         | -            |
| \( \hat{N} \)  | 0.0047   
(0.0226)         | -            |
| \( \hat{N} \)  | 0.0217   
(0.0223)         | -            |

Sum of DL coefficients on \( \hat{i} \): 0.0310
Sum of DL coefficients on \( \hat{N} \): -0.0442

<table>
<thead>
<tr>
<th></th>
<th>With ( \hat{i} ) and ( \hat{N} )</th>
<th>With ( \hat{i} ) only</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R^2 )</td>
<td>0.790</td>
</tr>
<tr>
<td></td>
<td>S.E.E.</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>D-W</td>
<td>1.217</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. Period of estimation is 1954-8 through 1983-12.
have been unemployed. A similar pattern of coefficients appears in the distributed lag on $\hat{M}$. A high value of $\hat{M}$ reduces the likelihood that workers will be terminated in the first few months, but there is essentially no effect over a year's time as indicated by the insignificance of money shocks as a group.\textsuperscript{38}

The right-hand column of Table 4 reports the results of a regression which drops the insignificant $\hat{M}$ distributed lag. The "S" shaped pattern of $\hat{I}$ coefficients is more pronounced than in the previous regression. The marginal increase in the $R^2$ indicates that no explanatory power is lost by dropping the $\hat{M}$. Again we conclude that lagged expectational errors from money shocks account for no detectible persistence in the unemployment rate except as it is incorporated in $\hat{I}$.

III.C. Cyclical Behavior of $\hat{\gamma}$

Given potential discouraged and added worker effects over the cycle, both $\hat{I}$ and $\hat{M}$ might influence $\hat{\gamma}$. In regressions not reported here, we tried regressing $\hat{\gamma}$ on 12 month distributed lags on these variables. The $F$ statistic on each group of parameters was insignificant and the $R^2$ negligible. We conclude that, while various subgroups of the labor force may have cyclically sensitive participation notes, the overall growth rate of the labor force is not sensitive to cyclical factors.

III.D. Cyclical Behavior of the Inventory-Sales Ratio $\hat{I}$

Our results so far indicate that to understand the cyclical behavior of the unemployment rate the determinants of the inventory-sales ratio must be

\textsuperscript{38}The $F(12,321)$ statistic for testing the null hypothesis that all $\hat{M}$ coefficients are zero is 0.716.
understood as well. Much research on the cyclical behavior of I is already underway; so, for our present purposes, it suffices to estimate a simple partial-adjustment regression of I on time, the lagged dependent variable, and a 12 month distributed lag on $\bar{M}_{-1}$ as our indicator of aggregate demand shocks. Since I is the ratio of beginning inventories to last month's sales, the current money shock $\bar{M}$ occurs too late to affect its value. This regression is reported in Table 5.

Although the equation is a simple one, it in fact explains the behavior of the inventory-sales ratio very well. The coefficient on $I_{-1}$ indicates that 9 percent of cyclical inventories are eliminated per month; this corresponds to 25 percent in three months, 44 percent in six months, and 68 percent in a year's time. The long-run effect of the time trend term is only 0.0004 per annum. The distributed lag on $\bar{M}$ indicates that positive money shocks significantly decrease inventories from the first through seventh month. The gradual build up of production relative to final sales is consistent with shock-absorber money demand and costs of changing production levels.

---

39If we define $\bar{T}_t$ as the value to which $I_t$ converges in the absence of any money shocks or random disturbances; we can write $\bar{T}_t = \alpha + \beta t$ in this case. If in the specified regression equation, $a$ is the constant, $b$ the coefficient of time, and $c$ the coefficient of $I_{-1}$, it can be shown that the long run values are found as $\beta = b/(1-c)$ and $\alpha = \frac{a}{1-c} - \frac{bc}{(1-c)^2}$. The $bc/(1-c)^2$ adjustment in computing $\alpha$ arises because normal growth in $\bar{T}$ is conventionally included in the constant term instead of appearing explicitly in the partial adjustment mechanism. [Were it included there, we would have a partial adjustment term like $(1-c)(\bar{T}_t - \beta - I_{t-1})$.] Our estimates imply $\alpha = 0.3682$ and $\beta = 0.0004$.

40On the shock-absorber approach to money demand, see Darby (1972) and Carr and Darby (1981).
### Table 5

**Determinants of I**

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.0340</td>
<td>(0.0072)</td>
</tr>
<tr>
<td>Time</td>
<td>0.00004</td>
<td>(0.00009)</td>
</tr>
<tr>
<td>I_{-1}</td>
<td>0.9086</td>
<td>(0.0190)</td>
</tr>
<tr>
<td>( \hat{H} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{H}(-1) )</td>
<td>-0.4129</td>
<td>(0.1583)</td>
</tr>
<tr>
<td>( \hat{H}(-2) )</td>
<td>-0.3672</td>
<td>(0.1577)</td>
</tr>
<tr>
<td>( \hat{H}(-3) )</td>
<td>-0.3467</td>
<td>(0.1563)</td>
</tr>
<tr>
<td>( \hat{H}(-4) )</td>
<td>-0.4125</td>
<td>(0.1559)</td>
</tr>
<tr>
<td>( \hat{H}(-5) )</td>
<td>-0.5682</td>
<td>(0.1591)</td>
</tr>
<tr>
<td>( \hat{H}(-6) )</td>
<td>-0.4715</td>
<td>(0.1605)</td>
</tr>
<tr>
<td>( \hat{H}(-7) )</td>
<td>-0.3466</td>
<td>(0.1622)</td>
</tr>
<tr>
<td>( \hat{H}(-8) )</td>
<td>-0.2325</td>
<td>(0.1657)</td>
</tr>
<tr>
<td>( \hat{H}(-9) )</td>
<td>-0.3219</td>
<td>(0.1655)</td>
</tr>
<tr>
<td>( \hat{H}(-10) )</td>
<td>-0.0832</td>
<td>(0.1664)</td>
</tr>
<tr>
<td>( \hat{H}(-11) )</td>
<td>-0.1791</td>
<td>(0.1686)</td>
</tr>
<tr>
<td>( \hat{H}(-12) )</td>
<td>-0.2422</td>
<td>(0.1695)</td>
</tr>
</tbody>
</table>

Sum of DL Coefficients on \( \hat{H} \): \(-3.985\)

\( R^2 \): \(0.950\)

S.E.E.: \(0.012\)

Durbin's h: \(-1.569\)

**Note:** Standard errors in parentheses. Period of estimation is 1954-8 through 1983-12.
IV. Conclusions

This paper has demonstrated that the fraction $\phi$ of the labor force becoming unemployed in a month and the probability $\pi$ of leaving unemployment over a month are useful constructs for understanding how the unemployment rate fluctuates around its normal level. Furthermore, these variables have empirical counterparts which we can readily calculate from the aggregate data on numbers and duration of unemployment.

In addition, we have shown that significant heterogeneity across individuals in $\pi$ implies a much slower convergence of $u$ toward $\bar{u}$ than would be hypothesized from current or normal values of $\pi$. We demonstrate that this heterogeneity is quite substantial in the aggregate data.

The hypotheses supported by our initial exploration of these new data can be summarized by means of Figure 6. In this figure we distinguish between two groups: the first is characterized by high values of $\hat{\phi}_1$ and $\bar{\pi}_1$ compared to the second group. Money shocks do not directly affect the proximate contemporaneous determinants ($\pi_1$ and $\phi_1$) of each group's cyclical unemployment rate $\hat{u}_1$, but do so indirectly through cyclical inventories $\hat{i}$. A restrictive monetary policy causes $\hat{i}$ to gradually build up. There is scant evidence that high $\hat{i}$ values may depress the $\pi_1$ directly, but their main effect seems to be on the cyclical search rates $\phi_1$ and $\phi_2$. High values of the $\hat{\phi}_1$ (and perhaps negative $\bar{\pi}_1$) build up $\hat{u}_1$ and especially $\hat{u}_2$ for a period of some months. Then as the effect of the money shock on inventories is attenuated, $u_1$ quickly returns to normal ($\hat{u}_1$ goes to 0). But the low turnover group is characterized by a very low normal probability $\bar{\pi}_2$, and this value governs the speed of adjustment of $\hat{u}_2$ (and eventually $\hat{u}$) toward 0.
FIGURE 6
Determination of Group and Aggregate Values of $\phi$, $\pi$, $u$
Indeed, it appears that in normal times the bulk of unemployment is comprised of group 1 individuals. Major recessions have the effect of disemploying very large numbers of group 2 individuals who otherwise have nearly permanent jobs. Their lengthy process of search for a new permanent job appears to dominate the recovery period and explain substantial persistence in unemployment. It is not that any individual is taking unusually long to find a job; it is simply unusual to have so many slow searchers unemployed at once.

To return to Figure 6, we note that given exogenous labor force shares and growth rates, we can infer the aggregate values of $\hat{\phi}$, $\hat{\pi}$, and $\hat{u}$ from the corresponding values for each group. At present, we are constrained to work with these aggregate data and infer compositional effects thorough proxies and other indirect evidence. Our first task for future research will be to develop new measures and evidence to permit us to observe more directly the nature of labor-force heterogeneity and its influence on unemployment rate dynamics.

For now we conclude that the observed persistence in unemployment appears to be consistent with equilibrium models and rational expectations since we are unable to detect any effect of lagged — or even current — money shocks on $\pi$. Inventories appear to be the key channel transmitting the effects of money shocks to the proximate determinants of unemployment.
APPENDIX A

CALENDAR BIAS IN MEASURED $\pi$ DUE TO SURVEY TIMING

The potential calendar bias in $\pi$ is associated with our estimate of $s_{-1}$ which is supposed to be the number unemployed exactly one standard month ago (4.35 weeks). However, in months in which there are 4(5) weeks between surveys our estimates of $s_{-1}$ is actually the number unemployed 4(5) weeks ago. Denote $u_{-1}$, $u_{-1}^4$, and $u_{-1}^5$ as the unemployment rate 4.35, 4, and 5 weeks ago, respectively, and $\pi^4$ and $\pi^5$ as our estimates of $\pi$ for 4 week and 5 week intervals between surveys, respectively. Then, neglecting any growth in the labor force within a week the calendar bias is given by

$$\pi - \pi^4 = (1-\pi) \left( \frac{u_{-1}}{u_{-1}^4} - 1 \right)$$

or

$$\pi - \pi^5 = (1-\pi) \left( \frac{u_{-1}}{u_{-1}^5} - 1 \right)$$

Note first that there is no calendar bias in the stationary state. Moreover, if as we believe is generally the case that $(1-\pi)$ is approximately 0.5 and either $((u_{-1}/u_{-1}^4) - 1)$ or $((u_{-1}/u_{-1}^5) - 1)$ is less than 0.04 in absolute value then we can conclude that the calendar bias is negligible in magnitude.\(^1\)

\(^1\)An increase of 1 percent (.01) per day translates into a 30 percent increase in the unemployment rate over a month. This would be 1.5 percentage points on a base of 5 percentage points.
APPENDIX B

CALCULATIONS OF HETEROGENEITY STATISTICS

We measure heterogeneity across individual \( \pi_1 \) values by \( h \), the amount by which the unemployment duration \( d_v \) of those currently unemployed since last month exceeds 1 month plus the average unemployment duration of all persons last month \( (d_{-1}) \). This \( h \), which is 0 if all \( \pi_1 \) values are equal, has a statistical interpretation in terms of the covariance between duration and changes in shares by duration which can be useful for certain problems.\(^1\) However, we need not be concerned with that interpretation to understand the derivation of Table 2 in the text.

The table is derived by considering the steady-state solution to the two-group heterogeneity model of Section I.B. Define \( \tilde{\beta}_1 \) and \( \tilde{\beta}_2 \) to be the continuous time equivalents of \( \pi_1 \) and \( \pi_2 \), respectively. Hence \( \tilde{\beta}_1 = -\log(1 - \pi_1) \) and \( \tilde{\beta}_2 = -\log(1 - \pi_2) \). Given \( \tilde{\beta}_1 \) and \( \tilde{\beta}_2 \) we can write \( \tilde{\pi} \) as:

\[
\tilde{\pi} = \frac{(\tilde{\pi}_1/\tilde{\beta}_1)(\pi_1/\tilde{\beta}_1) + (\pi_2/\tilde{\beta}_2)}{(\pi_1/\tilde{\beta}_1)/\tilde{\beta}_1 + 1/\tilde{\beta}_2}
\]  

(B1)

which in turn allows us to write \( (\tilde{\pi}_1/\tilde{\pi}_2) \) as:

\[\text{\( \text{\( h = d_v - d_{-1} - 1 = m \text{ cov}(d_{i,-1}, \Delta q_{i}) \)}\)\]

where \( d_{i,-1} \) refers to one of the \( m \) durations observed last month and \( \Delta q_{i} \) refers to (a) the ratio of people with duration \( d_{i,-1} + 1 \) this month to total people with duration of one month or over minus (b) the ratio of number unemployed last month with duration \( d_{i,-1} \) to the total unemployed last month.
\[(\bar{\bar{E}}_1/\bar{\bar{E}}_2) = \frac{\bar{\theta}_1 \bar{\bar{\pi}}_2 - \bar{\pi}}{\bar{\theta}_2 \bar{\bar{\pi}} - \bar{\pi}_1}\]

Hence, (B2) defines the steady state ratio of inflows \(\bar{\bar{E}}_1/\bar{\bar{E}}_2\) that is consistent with given values of \(\bar{\bar{\pi}}_1, \bar{\bar{\pi}}_2,\) and \(\bar{\pi}\).

Define \(\bar{d}^1\) and \(\bar{d}^2\) to be the steady state average duration of group 1 and 2 respectively. Then \(\bar{d}^1 = 1/\bar{\theta}_1\) and \(\bar{d}^2 = 1/\bar{\theta}_2\). Also, since all individuals within each group are assumed to be homogeneous then \(\bar{d}^1_\nu = \bar{d}^1 + 1\) and \(\bar{d}^2_\nu = \bar{d}^2 + 1\). Define \(\bar{\sigma}^1\) as the steady state share of unemployed from group 1. Then:

\[(B3) \quad \bar{\sigma}^1 = \frac{(\bar{\bar{E}}_1/\bar{\bar{E}}_2)/\bar{\theta}_1}{(\bar{\bar{E}}_1/\bar{\bar{E}}_2)/\bar{\theta}_1 + 1/\bar{\theta}_2}\]

and \(\bar{\sigma}^2 = 1 - \bar{\sigma}^1\). This allows us to write \(d\) as:

\[(B4) \quad d = \bar{\sigma}^1 \bar{d}^1 + \bar{\sigma}^2 \bar{d}^2 = \frac{(\bar{\bar{E}}_1/\bar{\bar{E}}_2)/(\bar{\theta}_1)^2 + (1/\bar{\theta}_2)^2}{(\bar{\bar{E}}_1/\bar{\bar{E}}_2)/\bar{\theta}_1 + 1/\bar{\theta}_2}\]

Given (B2) and the definitions of \(\bar{\bar{\theta}}_1\) and \(\bar{\bar{\theta}}_2\), (B4) allows us to compute the steady state value of \(d\) associated with any combination of \(\bar{\bar{\pi}}_1, \bar{\bar{\pi}}_2,\) and \(\bar{\pi}\).

In a similar fashion we can define \(\bar{\sigma}^1_\nu\) and \(\bar{\sigma}^2_\nu\) as the share of last period's unemployed from groups 1 and 2 respectively. This, in turn, allows us to write \(d_\nu\) as:

\[(B5) \quad d_\nu = \bar{\sigma}^1_\nu \bar{d}^1 + \bar{\sigma}^2_\nu \bar{d}^2 = \frac{[(1 + \bar{\theta}_1)(1 - \bar{\pi}_1)(\bar{\bar{E}}_1/\bar{\bar{E}}_2)/(\bar{\theta}_1)^2] + [(1 + \bar{\theta}_2)(1 - \bar{\pi}_2)/(\bar{\theta}_2)^2]}{[(1 - \bar{\pi}_1)(\bar{\bar{E}}_1/\bar{\bar{E}}_2)/\bar{\theta}_1] + [(1 - \bar{\pi}_2)/\bar{\theta}_2]}\]
Since $\bar{h} = d_v - d - 1$, using (B2), (B4) and (B5) we can calculate $(\bar{f}_1/\bar{f}_2)$ and $\bar{h}$ for any given combination of $\bar{\pi}_1$, $\bar{\pi}_2$, and $\bar{\pi}$. In Table 2 we use the estimated value of $\bar{\pi} = 0.45674$ and $(\bar{f}_1/\bar{f}_2)$ and $\bar{h}$ are calculated in this manner for the values of $\bar{\pi}_1$ and $\bar{\pi}_2$ as given.
BIBLIOGRAPHY


